



UIT

THE ARCTIC
UNIVERSITY
OF NORWAY

School of Business and Economics

Bleeding for profit

Researching the efficiency of positively skewed portfolios in the Norwegian financial markets

—

Espen Forså

Master's thesis in Business Administration - June 2017

FOREWORD

This thesis has been written as the final part of my Master's degree in Business Administration at School of Business and Economics, University of Tromsø.

I have since I first became an undergraduate student looked forward to writing my Master's thesis. It has been an interesting semester, filled with challenges and a lot of learning.

I would like to thank my family for their support and patience, my fellow students for fruitful discussions and camaraderie, and my supervisor, Espen Sirnes, for excellent advice and input.

Tromsø, June 2017

Espen Forså

ABSTRACT

Portfolios created by hyper defensive and hyper aggressive derivatives aims to limit the size of potential downside returns, whilst at the same time benefit from potentially large returns. However, the portfolio will experience long periods of small losses, bleeding. This thesis has empirically researched bleeding portfolios in the Norwegian financial markets. The research question that has been examined is:

Could a barbell portfolio with extremely positively skewed derivatives create risk-adjusted excess returns in the Norwegian financial market between 2005-2015 compared to alternative investments?

By creating portfolios of OBX-total return index put options and Norwegian treasury bills, there has been created six portfolios. The portfolios have varied in time horizon, 3 or 6 months, and risk balance; 90%, 80% or 70% in treasury bills. Furthermore, they have invested with both varying and constant monthly investments. To evaluate return, risk, risk-adjusted performance and other characteristics, several measurements have been calculated and compared to a benchmark portfolio. This benchmark portfolio was created by investments in OBX-total return index.

The empirical analysis found that the bleed portfolios performed worse than the OBX-portfolio when evaluating risk-adjusted performance. However, it was found some characteristics with the bleed portfolios that investors are known to appreciate: skewness, “floor” on negative returns and potential high upside. Furthermore, it was found that, due to the illiquid Norwegian out-of-the-money put option market and few observations, the evaluation of these bleed portfolios cannot be generalized. There is large uncertainty regarding the evaluation of skewed portfolios, in accordance with the law of large numbers.

Keywords: Skewness, barbell strategy, downside risk, put options, bleed portfolio

Table of Contents

1 INTRODUCTION.....	1
1.1 Background	1
1.2 Research question.....	2
2 THEORETICAL FRAMEWORK	4
2.1 Black swan events	4
2.2 Barbell strategy	5
2.3 The Black-Scholes option pricing model	6
2.3.1 Assumptions	6
2.3.2 The model.....	6
2.3.3 Volatility smiles	7
2.4 Statistical moments and the capital asset pricing model	8
2.4.1 Mean return	8
2.4.2 Variance and standard deviation	9
2.4.3 Skewness	10
2.4.4 Kurtosis	11
2.4.5 Capital asset pricing model (CAPM)	12
2.5 Simple or logarithmic returns.....	13
2.6 The law of large numbers (LLN)	14
2.7 Portfolio performance measurements.....	16
2.7.1 Simple benchmarking.....	16
2.7.2 The Sharpe ratio	17
2.7.3 Measuring performance with regards to downside risk	17
3 RESEARCH METHODOLOGY	20
3.1 Developing the research question	20
3.2 Choice of research design	22

3.3 Choice of research method	23
3.4 Collection of data	23
3.5 Choice of research units	24
3.6 Analysis of data, analysis of findings and interpretation of results	24
4 PRESENTATION OF DATA	25
4.1 The time frame	25
4.2 The risk free	25
4.3 OBX put options.....	27
4.4 OBX-Total Return Index.....	29
5 EMPIRICAL RESEARCH	32
5.1 Trading rules	32
5.2 Portfolios with adjusted investments.....	32
5.2.1 3-month horizon	34
5.2.2 6-month horizon	36
5.3 Portfolios with constant investments.....	39
5.3.1 3-month horizon	39
5.3.2 6-month horizon	42
5.4 Analysis of chosen options.....	43
5.4.1 3-month options.....	44
5.4.2 6-month options.....	44
6 DISCUSSION/CONCLUSION	46
6.1 Discussion	46
6.2 Weaknesses of the thesis	47
6.2.1 Few observations.....	47
6.2.2 Illiquid out-of-the-money put option market	47
6.2.3 Logarithmic or simple returns	47

6.3 Future research	48
6.3.1 Relaxing some assumptions	48
6.3.2 Choosing options differently	48
6.3.3 Derivatives with more available observations	48
6.3.4 Logarithmic returns	49
7 CONCLUSION	50
8 REFERENCE LIST.....	51
APPENDICES.....	A
Appendix 1: R-code for Treasury bills and OBX	A
Appendix 2: R-code for 6-month portfolios with variable investments.....	F

Table of figures

Figure 1: Returns of two lotteries.....	2
Figure 2: Barbell strategy	5
Figure 3 The volatility smile	8
Figure 4: Skewness.....	11
Figure 5: The law of large numbers	15
Figure 6: Treasury bills	26
Figure 7: Returns of 3-month out-of-the-money put options	28
Figure 8: Returns of 6-month out-of-the-money put options	29
Figure 9: OBX – Total return index	30
Figure 10: Daily returns of OBX – Total return index.....	31
Figure 11: 3-month horizon portfolios with variable investments	34
Figure 12: 6-month horizon portfolios with variable investments	36
Figure 13: 3-month horizon portfolios with constant investments	39
Figure 14: 6-month horizon portfolios with constant investments	42

Table of tables

Table 1: Research variables, unites and values	21
Table 2: Annualized returns of treasury bills	26
Table 3: Adjusting investments for portfolios.....	33
Table 4: 3-month horizon portfolios with variable investments. Monetary returns.....	34
Table 5:3-month horizon portfolios with variable investments. Descriptive statistics	35
Table 6: 6-month horizon portfolios with variable investments. Monetary returns.....	37
Table 7: 6-month horizon portfolios with variable investments. Descriptive statistics.	38
Table 8: 3-month horizon portfolios with constant investments. Monetary returns.	39
Table 9: 3-month horizon portfolios with constant investments. Descriptive statistics.....	41
Table 10: 6-month horizon portfolios with constant investments. Monetary returns.	42
Table 11: 6-month horizon portfolios with constant investments. Descriptive statistics.....	43
Table 12: No spread rule. 3-month horizon.	44
Table 13: No spread rule. 6-month horizon.	45

1 INTRODUCTION

1.1 Background

“Most traders are «picking pennies in front of a steamroller» exposing themselves to the high-impact rare event yet sleeping like babies, unaware of it.”

Nassim Taleb (2007, p. 19)

Financial investors have in modern times experienced several brutal downfalls in the financial markets. Famous examples are the great depression in 1929, black Monday 1987 and most recently the global financial crisis in 2008. These had a huge impact on the world and they came as a shock to everyone. In the early 2000s professor and trader Nassim Taleb wrote the book series *Incerto*. Attracting considerable attention to his views on extreme and rare events, and randomness. He named these extreme and rare events black swans. Events that are highly unexpected, carries large consequences and is subject to ex-post rationalization (Taleb, 2007).

Taleb argues that people tend to underestimate the randomness they face and are prone to hindsight bias. More specifically, financial professionals are, per Taleb, taking huge unknown risks that eventually might blow up and they are not in position to survive it. The issue is not to forecast these events, that is impossible, but rather to be robust to them. Or even be in a position to benefit when they happen (Taleb, 2007).

The Norwegian financial market has not received much attention with regards to black swan exploration in academia. However, the Norwegian markets has experienced huge downfalls as well. Prior to this, the Norwegian OBX index ("Titlon,") experienced several good years, reaching a high of 462,5 on the 22.05.2008. In the autumn, during the global financial crisis, the OBX index experienced 10 days with descents of more than 8%. The largest downfall was seen 06.11.2008 where the index fell 10,66% and Friday 21.11.2008 the index had fallen all the way down to 162,92. It took nearly 5 years for the OBX to recover, in August 2013. For investors that were not robust to these changes, the ramifications were presumably gigantic.

As a trader and writer Taleb has practiced and advocated a strategy to be robust towards, and benefit from, black swans. He suggests investing in a portfolio consisting of hyper defensive

and hyper aggressive derivatives. Combining treasury bills and buying far-out-of-the-money put options. The latter will be referred to as a bleed derivative. A derivative that has a large chance of losing small, bleeding, and a small chance of winning big.

To illustrate the bleed derivative. Let us imagine two lotteries. Lottery A is a coin toss where one can win 1\$ or lose 1\$ at a 50/50 probability. Lottery B one can win 999 with a 0,01% chance or lose 1\$ with 99,99% chance. Both lotteries have expected values of 0.

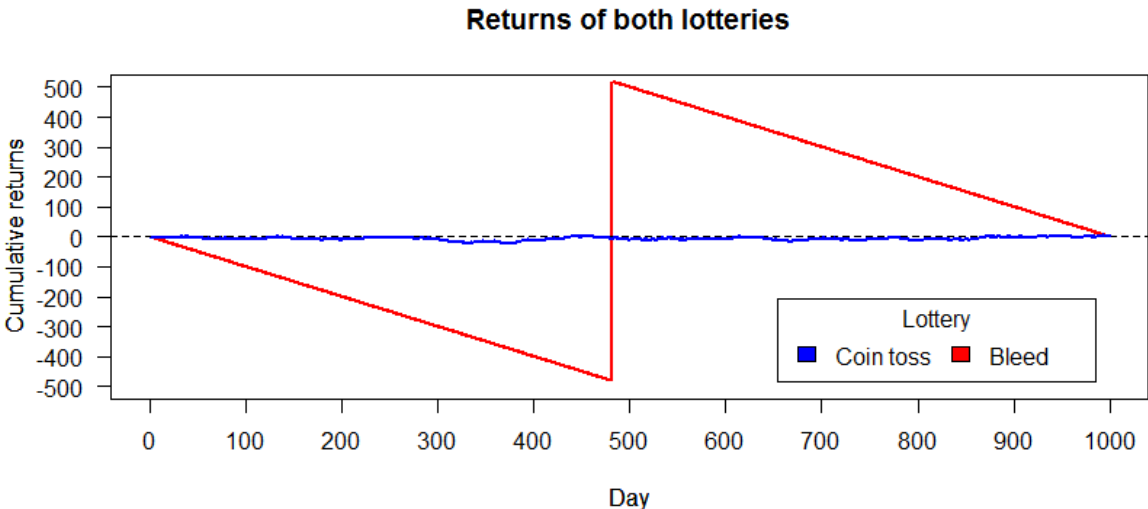


Figure 1: Returns of two lotteries

As we can see if one participates in lottery B every day one will experience long periods of small losses. However, after some time a bet won and the profit was huge. The coin toss distribution has zero skewness. The bleed derivative has high positive skewness as most of the returns are lower than the mean. This distribution characteristic is of interest to this topic.

1.2 Research question

The aim of this thesis is to explore the potential success of a highly positively skewed portfolio in the Norwegian financial market. The main research question is formulated to be:

Could a barbell portfolio with extremely positively skewed derivatives create risk-adjusted excess returns in the Norwegian financial market between 2005-2015 compared to alternative investments?

Furthermore, the thesis aims to explore relevant statistical properties of such a portfolio.

Therefore, the research sub-question is:

What statistical properties does bleed derivatives carry and what implications might they have in the context of pricing theory?

The sub-question may help understand bleed derivatives and how to evaluate their performance compared to other strategies.

2 THEORETICAL FRAMEWORK

The aim of the theoretical framework is to present and investigate the theories and terms that are relevant when researching barbell strategies, options and positively skewed distributions. Perhaps most importantly the available tools to evaluate the results of the portfolios will be outlined for use in the empirical research.

2.1 Black swan events

Prior to the discovery of Australia, the West believed to have empirical evidence of all swans being white. However, a single observation of a black swan falsified this and has later become a well-known anecdote to introduce the main idea of famous scientific philosopher Karl Popper. Popper believed that true science could only exist of testable hypothesis and theories, anything else he would classify as pseudo-science. Thus, a black swan became a synonym to the extremely rare event.

Nassim Taleb has popularized the term in finance and introduced it as a topic of discussion with his book series in the 2000s. In his book he defines a Black swan as an event that carries three attributes(Taleb, 2007, p. xxii Prologue):

1. It is an outlier.
2. It carries extreme impact.
3. Human tend to retrospectively explain and predict the event.

Black Swans can happen in all aspects of life. Politics, natural disasters and terror are some examples that can influence the financial markets. A modern example is 19th of October 1987, also known as “Black Monday”, were the global markets experienced the largest single-day drop in modern history(Taleb, 2007, p. 18).The event was not predicted by professionals and carried extreme impact all over the world and many countries took years to recover.

The essence of Nassim Taleb’s writing and trading is that we know black swans occur. But since they are impossible to predict we must be robust to them and possibly be in a position to benefit from them.

2.2 Barbell strategy

“If you know that you are vulnerable to prediction error, and if you accept that most “risk measures” are flawed, because of the Black Swan, then your strategy is to be as hyperconservative and hyperaggressive as you can be instead of being mildly aggressive or conservative”

Nassim Taleb (2007, p. 205)

A barbell strategy has generally been referred to as a strategy where the portfolio is split, typically in half, between short- and long-term bonds(Fooladi & Roberts, 1992, p. 5). Its name originated from the fact that the portfolios invested in the both ends of the duration spectrum but stayed away from the middle. Therefore, the portfolio could look like a barbell. In recent years it has also been a term for portfolios split between high-risk derivatives and low-risk derivatives like Taleb described it as(Weinberg). It is Taleb’s definition of a barbell strategy that will be used in this thesis.

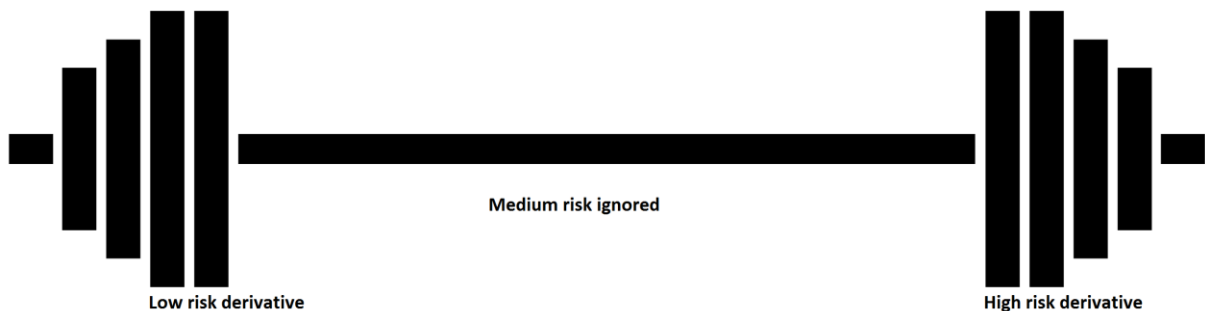


Figure 2: Barbell strategy("Finkin", Text added.)

The idea is to limit the potential downside from black swans by creating a “floor”, while at the same time keeping the possibility of large returns. These characteristics must be kept in mind when comparing the barbell strategy with other portfolio strategies.

Taleb(2007, p. 205) exemplified a portfolio that would fit to such a strategy as having 85-90% in treasury bills and the remaining portfolio in options. This type of portfolio will be the basis for this thesis’ research on skewed portfolios.

2.3 The Black-Scholes option pricing model

The Black-Scholes option pricing model is the most widely known model for pricing options theoretically. To understand to what extent option pricing takes potential black swans into account this subsection will outline the theory behind the model. The model calculates the price for European options. In other words, options that can only be exercised at the date of maturity.

2.3.1 Assumptions

The Black-Scholes formula has several assumptions and some of them can and have been relaxed or criticized by academics or professionals. This thesis will assume the assumptions presented by John Hull(2015, p. 331):

1. The stock price follows a process given by $\frac{dS}{S} = \mu dt + \sigma dz$. Known as a Wiener process or a Brownian motion.

Where

$\frac{dS}{S}$ is the relative change in the stock price.

μ is the expected return of the stock.

dt is the change in time t .

σ is the stocks volatility.

dz is a variable z that follows a Wiener process and $dz = \varepsilon\sqrt{\Delta t}$. Where ε has a standard normal distribution $N(0,1)$.(Hull, 2015, p. 304 and 309)

2. No limitations in short selling.
3. No transaction costs or taxes.
4. No dividends.
5. No riskless arbitrage opportunities.
6. Security trading is continuous.
7. The risk-free rate of interest, r , is constant and the same for all maturities.

2.3.2 The model

With the assumptions in mind the theoretical BS option price can be calculated as follows(Hull, 2015, pp. 335-335):

$$\text{Call option price} = c = S_0 N(d_1) - Ke^{-rT} N(d_2)$$

$$\text{Put option price} = p = Ke^{-rT} N(-d_2) - S_0 N(-d_1)$$

Where

S_0 is the underlying stocks price.

T is time to maturity.

σ is the stock price volatility.

$N(d_1)$ is the probability of d_1 or less than d_1 , where $d_1 = \frac{\ln\left(\frac{S_0}{X}\right) + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}$, in a standard normal distribution.

$N(d_2)$ is the probability of d_2 or less than d_2 , where $d_2 = d_1 - \sigma\sqrt{T}$, in a standard normal distribution.

K is the strike price.

X is the strike price of the option.

r is the risk-free interest rate.

With regards to the topic at hand it is worth noting that the Black-Scholes-Merton formula assumes that the underlying stocks logarithmic returns are normally distributed. If this assumption does not hold, the consequences will be largest when operating in the tails. I.e. with far-out-of-the-money or far-in-the-money options where it is essential that the model can accurately say something about the probability of a large ascent or descent in the stock value. This is very hard, and according to Taleb, not possible. It is better to be robust or even being in a position to benefit from them.

2.3.3 Volatility smiles

When comparing the theoretical BS option prices to actual market prices the difference appears to follow a pattern. In fact, the further away from the spot the strike is, the bigger is the difference between the BS and the market price. The reason for this is that the BS model assumes constant volatility. In reality this is not the case. When calculating the implied volatility from a market price, the volatility that the model would have to assume to achieve the correct market price, a pattern can be seen that can be reminiscent of a smile.

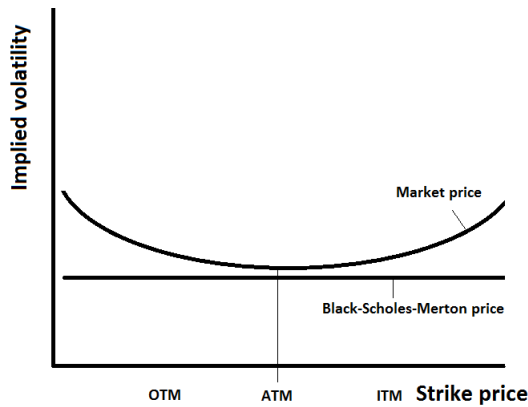


Figure 3 The volatility smile. Similar to Naylor, Wongchoti and Chen (2011, p. 23)

As the figure shows the theoretical price assumes constant volatility while the market price implies a higher volatility the further away from the spot the strike is. Since the aggressive derivative of the portfolio in question are far-out-of-the-money. The options will most likely be overpriced according to the theoretical price.

2.4 Statistical moments and the capital asset pricing model

Statistical moments are calculated to evaluate and interpret the behaviors of distributions, for example portfolio distributions. The understanding of the rational investors preference to relevant statistical moments is of essence to compare performance. This subsection aims to present an overview of their properties.

2.4.1 Mean return

The first moment is the mean return. Mean return is the most intuitive of the moments and simply represents the average return for each investment period. There are two forms of mean return, arithmetic average and geometric average:

$$\text{Arithmetic average} = \mu = \frac{1}{n} \sum_{i=1}^n x_i$$

Where

n is the number of returns.

x_i is the i th return.

$$\text{Geometric average} = \mu = \sqrt[n]{x_1 \times x_2 \times \dots \times x_n} - 1$$

Where

n is the number of returns.

x_1, x_2 and x_n are the first, second and n th number of return respectively.

A further discussion of arithmetic geometric mean will be conducted in chapter 2.5.

2.4.2 Variance and standard deviation

The second moment is the variance of the returns. It is a measurement of how spread the data are from the mean. The higher variance the more spread the observations are. It is calculated as the expected value of the squared deviation from the mean:

$$\text{Var}(X) = \sigma^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \mu)^2$$

Where

n is the number of observations.

x_i is the i th observation.

μ is the mean of the data.

In finance, the standard deviation is usually used to represent volatility. Standard deviation is the square root of the variance.

$$\text{SD}(X) = \sigma = \text{volatility} = \sqrt{\text{Var}(X)}$$

In financial context, a rational investor is assumed to prefer lower volatility. A high volatility results in higher risk of going broke which leads to loss of further liquidity and potential income.

In financial context, a rational investor can be assumed to favor high positive returns. However, when calculating volatility large positive returns can punish it. This is especially the case for positively skewed distributions which often experience gains that are far above the mean, but rarely losses that are far below the mean. In an attempt to give a more correct view of the risk one can calculate the semivariance and by extension semideviation, known as downside deviation in financial literature. The downside deviation looks specifically at the

values below a chosen threshold, e.g. 0 or the mean of the data, and calculate the deviation of the disadvantageous returns. Its formula can be written as(Nawrocki, 1999):

$$\text{Semivariance} = \frac{1}{k} \sum_{j=1}^k (x_j - T)^2$$

Where

- k is the number of observations below a chosen threshold.
- x_j is the jth observation.
- T is the chosen threshold.

Furthermore, downside deviation can be calculated from the semivariance:

$$\text{Semideviation} = \text{Downside deviation} = \sqrt{\text{Semivariance}}$$

It will be illustrated in chapter 2.7 that the volatility and downside deviation can tell a different story about the risk.

2.4.3 Skewness

The third of the statistical moments is the skewness of the distribution. It describes the inclination of the distribution, or in other words the symmetry on both sides of the mean. It is defined as(DeCarlo, 1997):

$$\sqrt{\beta_1} = \frac{\sum(X_i - \bar{X})^3/n}{(\sum(X_i - \bar{X})^2/n)^{3/2}}$$

Where

\bar{X} is the mean of X.

n is the number of observations.

A distribution is said to be positively skewed if the long tail is above the mean.

The capital asset pricing model, CAPM, is a model to theoretically price assets based on mean and variance. In other words, skewness was not a part of the original CAPM. However, later work did introduce it into the model. Works by amongst others Kraus and Litzenberger (Kraus Litzenberger 1976). This and several later works has confirmed that ex-ante positive skewness correlates with lower expected returns (Boyer 2010, Conrad 2013, Barberis and

Huang 2007) and implying a skewness price on assets. This means that investors have a preference for positive skewness.

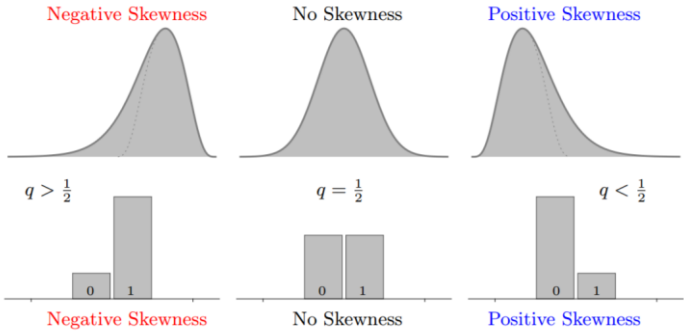


Figure 4: Skewness(Goulding, 2015, p. 66)

The standard normal distribution has a skewness of 0(Weisstein), thus the tails are of equal size on each side of the mean. Positively skewed distributions experience more observations below its mean, however in a financial setting the positive observations hopefully give a larger payoff.

2.4.4 Kurtosis

The fourth statistical moment is the kurtosis of the distribution. It describes the fatness of the tails of the distribution and is formally defined as(DeCarlo, 1997):

$$\beta_2 = \frac{E(X - \mu)^4}{(E(X - \mu)^2)^2} = \frac{\mu_4}{\sigma^4}$$

Where

E is the expectation operator.

μ is the mean.

μ_4 is the fourth moment about the mean.

σ is the standard deviation.

Kurtosis has received some interest by investors and academics, albeit not as much as the three first moments. Scott and Horvath(1980)proved that a positive preference for skewness implies a negative preference for kurtosis.

2.4.5 Capital asset pricing model (CAPM)

In the classical modern portfolio theory introduced by Harry Markowitz in the 1950s it is assumed that a rational investor wishes to maximize expected return and minimize variance. More formally it can be formulated as by Constantinides and Malliari's(1995, p. 4):

$$\text{Minimize } \sigma_p^2 = \mathbf{x}^T \mathbf{V} \mathbf{x}$$

$$\text{Subject to } \mathbf{x}^T \mathbf{1} = 1$$

$$\mathbf{x}^T \mathbf{R} = R_p$$

Where

σ_p^2 is the portfolio variance.

\mathbf{x} is an n-column vector representing the investors proportion of investment in the x_1, \dots, x_n assets.

\mathbf{x}^T is the transposed \mathbf{x} vector.

\mathbf{V} is the n*n covariance matrix with σ_{ij} where $i,j=1,2,\dots,n$.

\mathbf{R} is an n-column vector of mean returns R_1, \dots, R_n .

R_p is the portfolio mean.

This means that a rational investor wants to minimize his portfolios risk when earning an expected return R_p . A decade after the introduction of modern portfolio theory. Its ideas developed into the capital asset pricing model (CAPM). This is to this day a popular asset pricing model and is formulated as:

$$E(R_i) - r_f = \beta_i(E(R_M) - r_f)$$

Where

$E(R_i)$ is the expected return of asset i.

r_f is the risk-free asset return.

β_i is the sensitivity asset i has to movements in the market m. $\beta_i = \frac{Cov(R_i, R_M)}{Var(R_M)}$

$E(R_M)$

2.5 Simple or logarithmic returns

When calculating the periodic returns of a portfolio and conducting an analysis of them, there are two main ways to do it. The choice of calculation will carry some implications and the most relevant will be presented here. The two types of returns are calculated by:

$$\text{Simple return}_n = r_{n,S} = \frac{\text{Portfolio value}_n - \text{Portfolio value}_{n-1}}{\text{Portfolio value}_{n-1}}$$

$$\text{Logarithmic return}_n = r_{n,L} = \log\left(\frac{\text{Portfolio value}_n}{\text{Portfolio value}_{n-1}}\right) - 1$$

Where

n is the n th period of the portfolio.

The benefits of using logarithmic returns are according to Hudson and Gregoriou(2010, p. 5):

1. They act as continuously compounded returns. Meaning that the frequency of compounding does not matter.
2. Multi-period return is easily calculated as the sum of the logarithmic returns.
3. Security prices cannot become negative.
4. For security prices following a Wiener process, the logarithmic returns are normally distributed. A characteristic that can be of use when analyzing them.
5. Logarithmic returns will give a better forecasting than simple returns.
6. Logarithmic returns are approximately equal to simple returns.

The disadvantages of using logarithmic returns are according to the same authors(Hudson & Gregoriou, 2010, pp. 5-6):

1. The logarithmic returns do not represent a correct measure of the monetary change.
2. The variance of the returns will affect the mean logarithmic return and the difference between it and mean from simple returns. The approximate relationship is given by:

$$r_{n,L} = r_{n,S} - 0,5\sigma_S^2$$

3. The simple returns mean cannot be deducted from the logarithmic returns mean.

Because variance might be an inaccurate measurement for highly skewed portfolios. And since geometric returns are affected by variance. This thesis will assume that arithmetic mean

is more accurate than geometric mean. Furthermore, the empirical research will be conducted with simple returns. This must be viewed as a simplification, but might be an interesting topic of future works.

2.6 The law of large numbers (LLN)

The law of large numbers, hereafter LLN, is relevant when evaluating the uncertainty of the statistical moments from a sample set. LLN states that a sample set obtained from a distribution will have a sample mean that converges to the distributions mean as the size n of the sample set increases. However, the size n needed to say something about what values the distribution converges to, can vary greatly upon the distribution. This is called the rate of convergence and says something about how fast a distribution closes in on its true mean. There is a weak and a strong law of large numbers and they have been defined by Klenke(2013, p. 109):

Let $(X_n)_{n \in \mathbb{N}}$ be a sequence of real random variable in $\mathcal{L}^1(\mathbf{P})$ and let $\tilde{s}_n = \sum_{i=1}^n (X_i - E[X_i])$.

(i) We say that $(X_n)_{n \in \mathbb{N}}$ fulfills the weak law of large numbers if

$$\lim_{n \rightarrow \infty} \mathbf{P} \left[\left| \frac{1}{n} \tilde{s}_n \right| > \varepsilon \right] = 0 \text{ for any } \varepsilon > 0.$$

(ii) We say that $(X_n)_{n \in \mathbb{N}}$ fulfills the strong law of large numbers if

$$\mathbf{P} \left[\lim_{n \rightarrow \infty} \sup \left| \frac{1}{n} \tilde{s}_n \right| = 0 \right] = 1.$$

distributions true mean and sup is

Where

$\mathcal{L}^1(\mathbf{P})$ is the distributions probability function.

ε is a chosen boundary from the distributions true mean.

sup refers to “the largest of”.

Extremely skewed distributions will converge very slowly towards the mean and needs a significantly larger sample size for us to be certain about its validity. To illustrate this, two distributions with the same mean, but different skewness, will be introduced:

$$g(x) = \begin{cases} 1000, & x = 1 \\ 0, & \text{otherwise} \end{cases} \text{ for } x \in \{0, 1, \dots, 1000\}.$$

$$f(x) = \begin{cases} 2, & x = 1 \\ 0, & x = 0 \end{cases} \text{ for } x \in \{0,1\}$$

Both the distributions have a mean of 1. However, $g(x)$ is heavily positively skewed with potential large payouts, but many instances of 0 return. Simulating for a sample size of $n = 50000$ trials and $\varepsilon = 0.05$ the difference in speed of convergence between the two distributions is clearly illustrated.

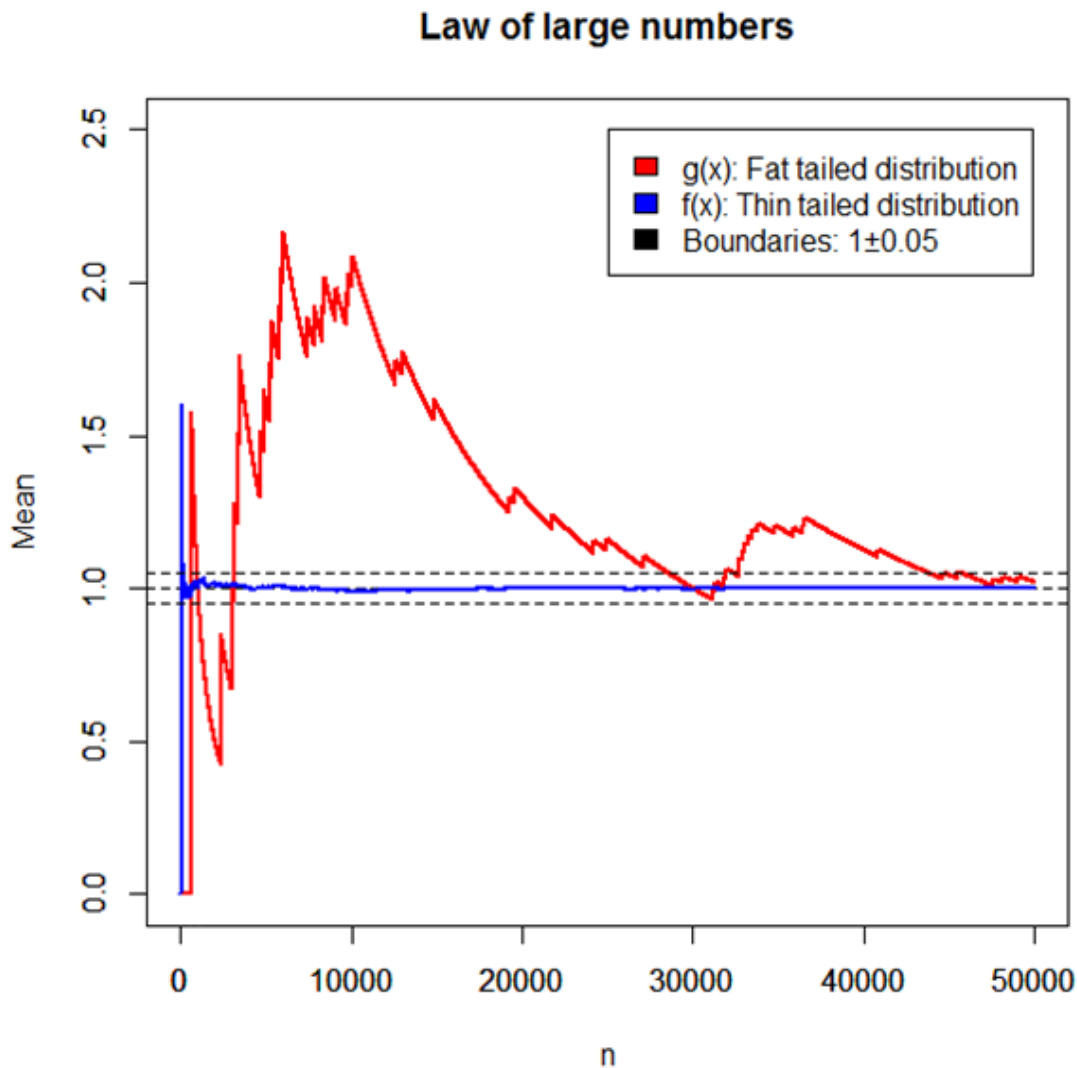


Figure 5: The law of large numbers

In the above figure the thin-tailed distribution quickly approaches the distributions true mean and after $n = 141$ the sample average is inside the average 1 ± 0.05 and has fulfilled the strong law of large numbers. The skewed distribution on the other hand takes a long time to reach the mean and is not steadily within the boundaries until $n = 45\,715$. This clearly illustrates

that to say something about a distributions statistical properties. Large sample sizes might be needed to be certain, this depends on the characteristics of the distribution.

The understanding of this “law” does not come naturally to most people which may lead to the “Belief in the law of small numbers”(Tversky & Kahneman, 1971). That is inferring the statistical properties from a viewable selection that is not a large enough sample size. The misguided or excessive confidence in early trends, and perhaps especially what can be regarded as “early”, is a common human error when interpreting data generated by skewed distributions. The robustness of the statistical moments from the bleed-portfolios will be in question and will be considered when analyzing the empirical results.

2.7 Portfolio performance measurements

To evaluate and compare the performances of the bleed portfolios and alternative benchmarks there are several measurements that can be used. In this sub-section, some of the most relevant will be introduced and discussed.

2.7.1 Simple benchmarking

The simplest form of performance measurement is to look at the difference in return between the portfolio in question and some benchmark. The benchmark is usually chosen as some alternative investment like treasury bills or index portfolios.

$$\text{Difference in terminal wealth} = \sum_{i=1}^n (1 + r_{P,i}) - \sum_{i=1}^n (1 + r_{B,i})$$

$$\text{Average excess return} = \frac{1}{N} \left\{ \sum_{i=1}^n (1 + r_{P,i}) - (1 + r_{B,i}) \right\}$$

Where

n is the number of returns.

$r_{P,i}$ is the i th number of return for the portfolio.

$r_{B,i}$ is the i th number of return for the benchmark.

These sort of simple benchmarking measurements however has the drawback that they do not take risk into account. In a mean-variance universe where higher mean and lower variance is preferred by the rational investor it is problematic to evaluate based on only one of these. These values can be interesting when portfolios of the same risk profile are compared, but they are flawed when comparing distributions with high abnormal returns like bleed portfolios and more normally distributed portfolios like an OBX-index portfolio.

2.7.2 The Sharpe ratio

To take into account the fact that a rational investor requires higher mean to accept higher variance William Sharpe developed the Sharpe ratio. The aim of the Sharpe ratio is to evaluate the premium return over the “risk-free” alternative, in relation to the risk one has bear to achieve it. The Sharpe ratio is both easy to calculate and understand and is widely used in the financial industry:

$$\text{Realized Sharpe ratio} = \frac{\mu_P - \mu_{rf}}{\sigma_P}$$

Where

μ_P is the mean return of the portfolio.

μ_{rf} is the mean risk free return.

σ_P is the portfolios standard deviation.

The Sharpe ratio is intuitive and easy to calculate but its biggest flaw is that it punishes both positive and negative variance equally. This is especially problematic when evaluating positively skewed distributions, as the potentially huge winnings will be punished by the Sharpe ratio, even though these types of fluctuations are more than welcome by the investors.

2.7.3 Measuring performance with regards to downside risk

As the Sharpe ratio can punish “upside risk”, several attempts have been made to combat this problem when measuring risk-adjusted performance. To do this a measurement must only take “downside risk” into account. Downside risk can be defined as the risk of delivering returns below a threshold return T. It can be formulated as(Rollinger & Hoffman, 2013):

$$\text{Downside risk deviation} = DD = \sqrt{\frac{1}{N} \sum_{i=1}^N (\text{Min}(0, r_i - T))^2}$$

Where

N is the number of returns.

r_i is the return of the i^{th} return

T is the benchmark threshold

The Sortino ratio is an adjustment of the Sharpe ratio that evaluates excess return with regards to the downside risk deviation instead of the standard deviation. The result is that “upside risk” is not punished. It is formulated as (Chaudhry & Johnson, 2008):

$$\text{Sortino ratio} = \frac{\mu_P - T}{DD}$$

Where

μ_P is the mean return of the portfolio.

T is the benchmark threshold.

DD is the downside deviation.

The Sortino ratio was found to have little difference in ranking power compared to the Sharpe ratio under normally or symmetric return distributions. However, the Sortino ratio showed more accurate results when the distributions were positively skewed (Chaudhry & Johnson, 2008).

A similar attempt to only punish “downside risk” was made by Keating and Shadwick (2002) when they introduced Omega:

$$\text{Omega} = \Omega = \frac{\int_T^b (1 - F(r)) dx}{\int_a^T F(x) dx}$$

Where

(a,b) is the interval of the returns.

T is the benchmark threshold.

F is the cumulative distributions of returns.

The Omega is the probability weighted ratio between the returns above and below the target threshold. One of its strengths is that it is as statistically significant as the returns itself and is not bothered by potential sampling uncertainty as it is derived from the returns themselves(Keating & Shadwick, 2002).It further carries the interesting property that if the mean return is equal to the target return, Ω is equal to 1.

The Sortino ratio and the Omega are closely related, as they can both be derived from the generalized Kappa. It is defined as(Kaplan & Knowles, 2004):

$$\text{Kappa of the } n^{\text{th}} \text{ moment} = K_n = \frac{\mu_p - T}{\sqrt{\frac{1}{N} \sum_{i=1}^N (\text{Min}(0, r_i - T))^n}}$$

Where

μ_p is the mean return of the portfolio.

r_i is the return of the i^{th} observation.

T is the benchmark threshold.

Furthermore, $\Omega = K_1 + 1$ and Sortino ratio = K_2 as shown by Kaplan and Knowles(2004). They further show that the ranking of portfolios can vary according to the choice of Kappa variant. For the purpose of better robustness, it might be useful to use several Kappa variants when evaluating performance.

3 RESEARCH METHODOLOGY

The aim of research is to answer questions through scientific procedures and methodology(Kothari, 2004). This chapter aims to review the scientific methodology and the choices that have been made when conducting this research.

The research methodology is the structuring and description of how to answer the research question or questions. A clear and thorough research plan is essential in order to achieve valid and reliable answers to the research question. This means that the research methodology has succeeded in measuring the intended measurements, research validity, and in a way that ensures that the results are trustworthy and replicable, research reliability. This can be obtained by having a systematic plan for the collection and processing of the data, and the interpretation of the results.

The research process can be divided into eight phases of research according to Jacobsen(Jacobsen, 2005). Jacobsen is mainly focused on research that uses qualitative interviews or quantitative questionnaires. Despite the different approach from this thesis, the same research process has been conducted. The eight phases of the research process are:

1. Developing the research question
2. Choice of research design
3. Choice of research method
4. Choice of research units
5. Analysis of data
6. Analysis of findings
7. Interpretation of results

3.1 Developing the research question

The research question is the concretized formulation of the question(s) the research initially aims to answer and is formulated in such a way that it can be answered empirically(Jacobsen, 2005). Developing a good research question involves narrowing the field of research according to the available time and researches (Jacobsen). But in such a way that the research does not lose its academic interest by being too narrow. When developing the research

question the context has to be specified, according to Jacobsen the context can be defined as the framework of units, variables and values the research operates under.

The aim of this thesis is to investigate the effectiveness of positively skewed portfolios in financial markets. The research question is formulated as:

Could a barbell portfolio with extremely positively skewed derivatives create risk-adjusted excess returns in the Norwegian financial market between 2005-2015 compared to alternative investments?

The initial question is narrowed by specifying the context and its units, variables and values. The research question implies that the variables that will be investigated are risk and (excess) return, the units are the barbell portfolio(s) and the alternative portfolio(s) and values these variables can take are well known to be mean and volatility, but also other measurements will be investigated

Table 1: Research variables, unites and values

Variables	Unites	Values
Return	Barbell portfolio	Mean, monetary value etc.
Return	Alternative investment 1	Mean, monetary value etc.
Return	Alternative investment 2	Mean, monetary value etc.
Risk	Barbell portfolio	Volatility etc.
Risk	Alternative investment 1	Volatility etc.
Risk	Alternative investment 2	Volatility etc.

According to Jacobsen(2005, p. 72), a research question can be analyzed along three main dimensions.

- Clarity
- Explanatory or descriptive
- Generalization

The chosen research question can be evaluated as clear. The variables return and risk are well known in financial academia. The units are not entirely clear yet and must be investigated

during the research. For example, what derivatives does a barbell portfolio with an extremely positively skewed derivative consist of? And what exactly are the alternative investments? An experienced financial academic would probably assume that it entails the market return and risk, but this must be specified and its answer can change during the research. Furthermore, the values the variables can take are normally mean and volatility in financial academia, but further measurements will be investigated during the research.

The difference between an explanatory and a descriptive research question is that an explanatory question aims to explain relationships in the phenomenon through causal analysis. A descriptive research question on the other hand mainly aims to describe the situation without saying why it is like it is (Jacobsen, 2005, p. 75). This research question can be said to be of a descriptive character as its aim is to describe how a specified portfolio would do, without saying much about why.

The research question does not aim to generalize as it investigates the entire population and does not want to say anything about other populations than the Norwegian financial market during 2005-2015.

Jacobsen (2005, pp. 81-82) states that a good research question meets three requirements: It has to be exciting, it has to be simple and it has to be able to provide empirically interesting results. It is of the author's opinion that these requirements have been met.

What statistical properties does bleed derivatives carry and what implications might they have in the context of pricing theory?

Keeping the research question's characteristics in mind we can now make further choices in our research methodology.

3.2 Choice of research design

When choosing our research design, we want to choose the design that can give us the most reliable results to our research question, given our time and resource constraints. Research design can be classified through two dimensions according to Jacobsen (2005, p. 87):

- Extensive or intensive
- Descriptive or explanatory

The extensiveness of the research design tells something about how many research units the design aims to explore and the intensity tells about how many variables we research. Due to time and budget constraints, it is rare for research to examine large amounts of both units and variables, so it is often a choice between the two. This research can be said to lean towards an extensive design. We aim to research two variables, risk and return, via different measurements. And we aim to explore 8 portfolios, but these units are built up by many observations from the put option market, the OBX index and the treasury bills market. MER

3.3 Choice of research method

The main distinction when choosing research method is between a qualitative and a quantitative approach. A qualitative approach involves analysis involving subjective assessments of phenomena that are hard or impossible to quantify in objective numbers. Typically, this approach involves interviews or questionnaires. Quantitative research on the other hand is research involving quantifiable measurements. The field of finance has historically focused on the quantifiable sizes like profit or risk through mean, variance and similar units that aim to describe financial phenomena. However, the field of behavioral finance has received more attention the previous decades and the field has rapidly developed.

This thesis aims to answer the research question through quantitative analysis. The main advantages of quantitative analysis are that the results generally gives good external validity, the data is easy to process and there is often smaller cost attached to the collection of quantitative data(Jacobsen, 2005, p. 132). On the other hand, a quantitative analysis generally gives a less in-depth analysis of a phenomenon and it gives less flexibility for the researcher compared to a design involving for example interviews.

3.4 Collection of data

When collecting data for research purposes we can generally divide between primary data and secondary data. Primary data is data gathered by the researcher, whilst secondary data is gathered by a secondary source. For this thesis' purpose, secondary data has to be used. First and foremost because the data is historical and cannot be observed directly by us. They are gathered from Norges Bank (risk-free derivative) and Titlon via Norges Bank (OBX-index and put-option prices). Often the use of secondary data can carry problems regarding reliability and are often initially gathered for different purposes. These problems are small or

non-existing in our case as the data are from reliable sources and presented in a standardized financial way.

3.5 Choice of research units

The research units of this research are the different portfolios built up according to the research problem and the comparable portfolios used to compare performance. The empirical research will create portfolios based on duration and risk balance. There will be two durations, 3 and 6 months. The risk balance will vary between 90%, 80% and 70% in the risk-free derivative. The idea behind examining several portfolios is to get the most robust results, and by comparing similar and different properties of the portfolios, new knowledge or ideas may arise.

3.6 Analysis of data, analysis of findings and interpretation of results

Analysis of data can be found in chapter 4.

Analysis of findings and interpretation of results can be found in chapters 5,6 and 7.

4 PRESENTATION OF DATA

In this chapter a presentation of the data that has been used to conduct the research will be made. The aim of the chapter is to describe the treatment of the raw data in such a way that the research and its results can be easily understood and replicated.

4.1 The time frame

Before 2005, the Norwegian financial market out-of-the-money put options occasionally. Therefore, the time frame has been chosen to be from January 2005. The end of the active portfolio investment will be said to be June 2015. This means that the final cash flows will be found in the subsequent months depending on the portfolio horizon.

4.2 The risk free

The risk-free derivative of the portfolio are Norwegian treasury bills. The duration of them will be equal to the length of the portfolio, this means 3- and 6-month duration treasury bills. Norwegian treasury bills are close to risk free. The rates has been obtained from Norges Bank("Norges Bank,") and they are presented as yearly rates based on the monthly averages of daily quotes collected at 16.00 each day. As the portfolio operates on a 3- and 6-month duration horizon, the yearly rates has been recalculated as follows to get the 3- and 6-month rates:

$$3 \text{ month rate} = (1 + \text{Yearly rate}_{3 \text{ month}})^{\frac{3}{12}} - 1$$

$$6 \text{ month rate} = (1 + \text{Yearly rate}_{6 \text{ month}})^{\frac{6}{12}} - 1$$

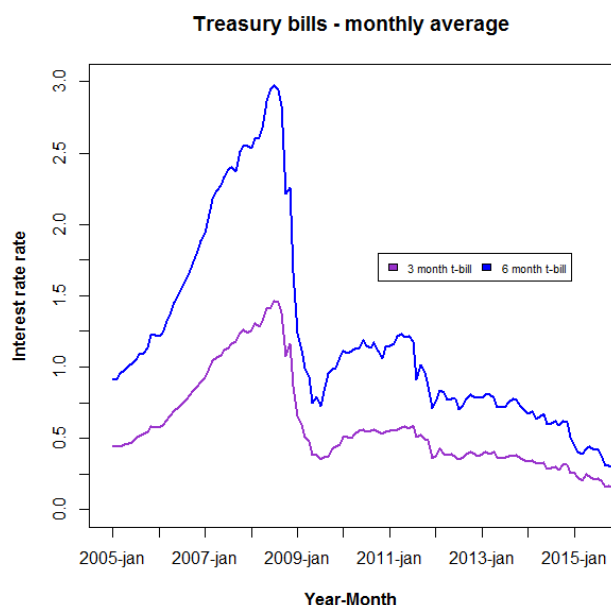


Figure 6: Treasury bills

The Norwegian treasury bills were at a high during the global financial crisis. However, after 2009 it has been fairly stable around the 0.3-0.5% range for the 3-month treasury bills and 0.5-1.0% range for the 6-month treasury bills. As the chosen portfolios commits the money in treasury bills for short durations, it is interesting to see if they are punished for this during the period January 2005-June 2015.

Table 2: Annualized returns of treasury bills

Duration	Average annualized rate	126-month return
3 month	2.44 %	28.80 %
6 month	2.50 %	29.60 %
9 month	2.53 %	30.00 %
3 years	2.70 %	32.28 %
5 years	2.95 %	35.70 %
10 years	3.40 %	42.06 %

The treasury bills on average pays more for longer durations. For a 126-month period, like the one in question, there is not a huge difference between the 3-year treasury bills and the sub

12-months. However, when looking at especially the 10-year treasury bills the difference becomes significant. The choice of 3- and 6-month treasury bills seems to be satisfactory.

4.3 OBX put options

The bleed derivative of the portfolio has been chosen to be far out-of-the money put options, with the OBX total return index as the underlying. The choice is the equivalent of Naylor, Chen and Wongchoti's research(2011) when they chose S&P 500-puts when analyzing the American market. This was to avoid unsystematic risk from options with individual companies as underlying and since the S&P 500-options were the most liquid and offered most alternative strikes.

The historical prices for the OBX-put options has been acquired from Titlon("Titlon,"). When processing the data, some values have been calculated in Excel before importing them into R for empirical analysis.

$$\text{Moneyness} = \frac{\text{Spot}}{\text{Strike}}$$

Moneyness is the measurement of how in- or out-of-the-money an option is. If the moneyness is 1 the option is said to be at-the-money, whilst if it is out-of-the-money (in-the-money) the moneyness is below (above) 1.

$$\text{Spread} = \frac{\text{Best bid price}}{\text{Best ask price}}$$

The spread is the difference between the best ask and the best bid price. A large spread indicates low liquidity and might be a problem in the data in question. For this research the spread has been defined as above to get the relative difference in the spread. If the relative spread is 1 the spread is 0, whilst a very small relative spread indicates large spread.

$$\text{Duration} = \text{Date}_{\text{Strike}} - \text{Date}_{\text{Spot}}$$

The duration is simply the number of days between the date of the strike and the issue date.

$$\text{MonthIndex} = \text{Month} + 12 * (\text{Year} - 2005)$$

Where

Month is the number of the month. January = 1, February = 2 etc.

To easily treat the data, an index has been created for the portfolios lifetime. January 2005 has been defined as month number 1 in the data set and it goes up until 126 which is June 2015 for the last investments. Also, note that month number 129, September 2015, and 132, December 2015 are the last payouts for the 3-month and 6-month portfolios respectively.

$$\text{Payoff} = \begin{cases} \text{Strike} - \text{Spot}_{\text{Expiration}} - \text{Price}_{\text{Put}}, & \text{when Strike} > \text{Spot}_{\text{Expiration}} \\ -\text{Price}_{\text{Put}}, & \text{when Strike} < \text{Spot}_{\text{Expiration}} \end{cases}$$

The payoff for the put options are calculated as the payments from the put option at expiration minus the put price. The percentage payoff is further calculated as:

$$\text{Percentage payoff} = \frac{\text{Payoff}}{\text{Best ask price}}$$

The percentage payoff from individual options are presented on normal form, in other words not as logarithmic returns.

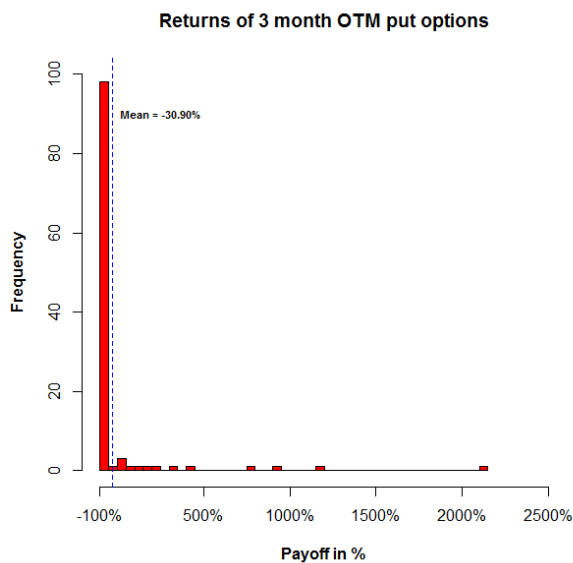


Figure 7: Returns of 3-month out-of-the-money put options

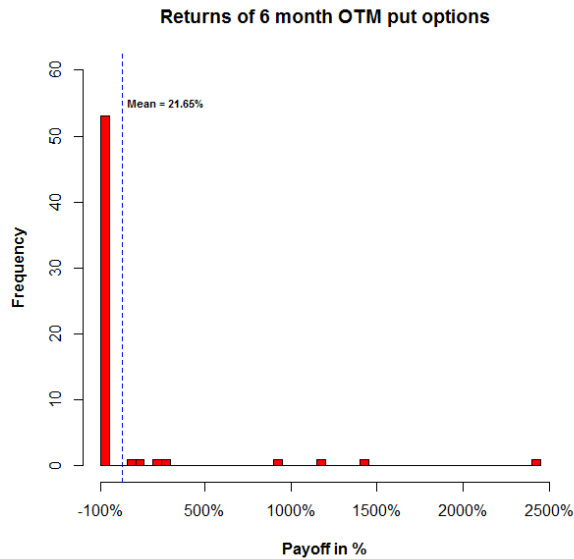


Figure 8: Returns of 6-month out-of-the-money put options

The out-of-the-money put options for both the 3- and 6-month durations are clearly skewed in their payoffs and we can see similarities to Goulding's figure from page 11. The 3-month options have a skewness of 5.57 while the skewness of the 6-month horizon is 4.20. This is satisfactory for our research purposes where we want the risky derivative of the portfolio to be extremely positively skewed.

4.4 OBX-Total Return Index

A portfolio investing in the OBX-total return index is chosen as the benchmarking portfolio. It is chosen to avoid most unsystematic risk and because it shows the general performance of the Norwegian financial markets. It is interesting to compare the barbell strategy to this. In addition, it has very little skewness which makes it a good benchmark for our positively skewed derivatives.

The historical OBX total return index prices have been acquired from Titlon("Titlon,").

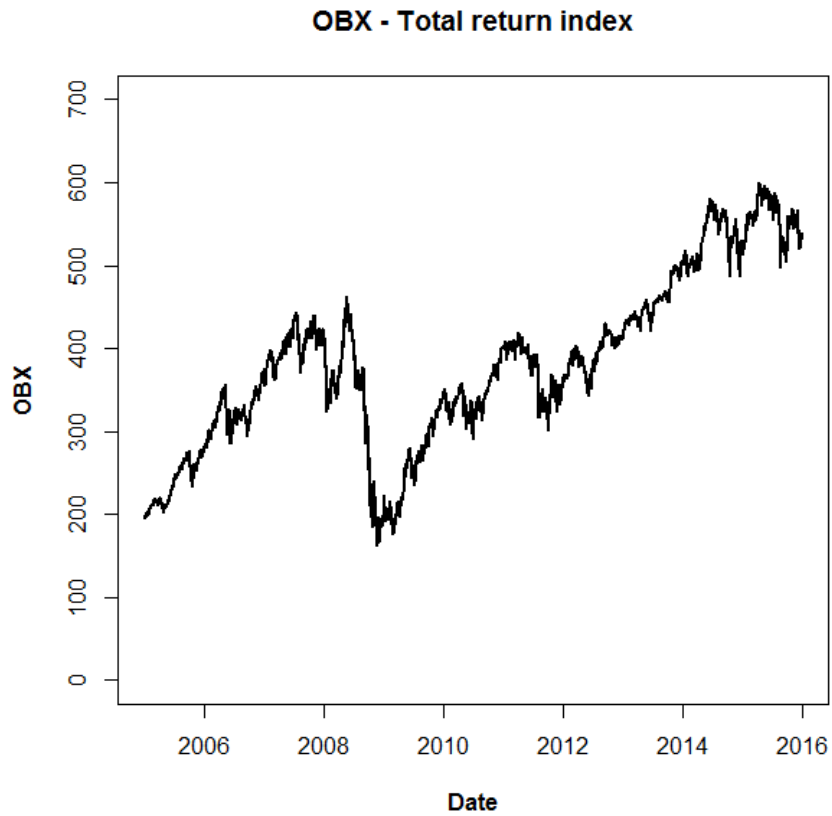


Figure 9: OBX – Total return index

The normal returns for the OBX Total return index are calculated:

$$\text{Daily return}_n = \frac{\text{OBX}_n - \text{OBX}_{n-1}}{\text{OBX}_{n-1}}$$

$$\text{Monthly return}_k = \frac{\text{OBX}_k - \text{OBX}_{k-1}}{\text{OBX}_{k-1}}$$

However, when calculating the portfolio returns in the empirical research. The change in portfolio value is calculated relative to the invested capital that lead to the change. Meaning the investment 3- or 6-months in advance.

$$\text{Monthly return}_k = \frac{\text{Portfolio}_k - \text{Portfolio}_{k-1}}{\text{Invested capital}_{k-n}}$$

Where

k is the kth month.

n is the time horizon. 3 or 6 months.

When choosing a portfolio of OBX-index investments as the benchmark. A part of the reason was that it presumably would carry little skewness. This is confirmed by analyzing the data.

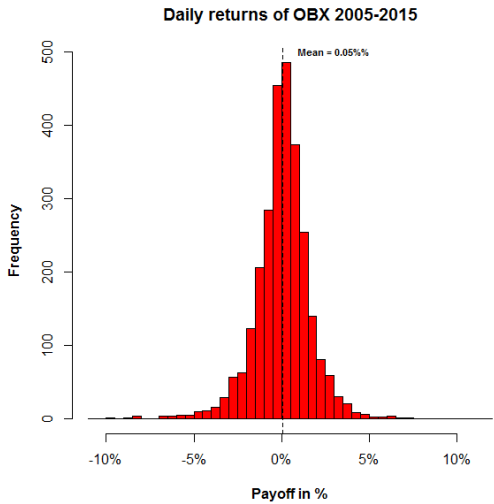


Figure 10: Daily returns of OBX – Total return index

The daily returns from the OBX total return index for the period 01.01.2005 to 30.06.2015 appears to have little skewness based on its histogram. This is confirmed by calculations that shows a skewness of -0.33, meaning that there are slightly more observations above the mean than below. In the data there are 1437 observations above the mean and 1325 observations below it.

5 EMPIRICAL RESEARCH

To research the profitability of the bleed strategy on the Norwegian market, several portfolios will be constructed and evaluated. Portfolios are constructed with respect to duration, liquidity and balance between risky and “risk free” instrument. Furthermore, portfolio management with both constant and adjusted investments will be conducted.

For simplicity, some assumptions have been made:

- Every put option within a month can be bought at the beginning of the month.
- An option with duration from 80 to 100 days is defined as a 3-month option.
- An option with duration from 160 to 200 days is defined as a 6-month option.

5.1 Trading rules

The portfolio will start with 1000 NOK. For each month, a third of the portfolios value will be invested and balanced between 3-month treasury bills and one OBX-put option by either a 90/10, 80/20 or a 70/30 distribution. When choosing the put option to invest in each month, the available put option with the lowest moneyness will be chosen. This means that it is the option that is most out-of-the-money, and should have the most skewness. To account for low liquidity and especially unfavorable prices the options must have a spread of more than 0.80 to be eligible for selection. If there is no eligible put option for the month, 100% of the invested amount will be invested in treasury bills. The reasoning behind this rule is to avoid buying clearly overpriced options since they are not liquid, creating an unrealistic ask price.

5.2 Portfolios with adjusted investments

To emulate a portfolio that has budget or liquidity constraints, portfolios that adjust their investments according to the current portfolio value will be constructed. The main point of the “risk-free” part of the barbell strategy is to fund the bleeding part of the portfolio. It is useful to see to what extent the funding can be maintained and possible implications.

These portfolios start with a value of 1000. Where n is the horizon in months, $\frac{1}{n}$ of the portfolios value will be invested each month according to the barbell strategy. With long bleeding streaks the portfolio runs the risk of not being able to profit enough from the

successful periods due to the potentially low funds. The calculations for month n can be generalized as following.

Table 3: Adjusting investments for portfolios

Month	n
Portfolio value	$PV_{n-1} - [IRf_{n-h} * (1 + r_{n-h}) + IP_{n-h} * (1 + p_{n-h})]$
Invested in "risk free"	$\frac{1}{h} * (1 - k) * PV_n$
Invested in puts	$\frac{1}{h} * k * PV_n$
Payout from "risk free"	$IRf_{n-h} * (1 + r_{n-h})$
Payout from puts	$IP_{n-h} * (1 + p_{n-h})$

Where

PV is the portfolio value.

h is the horizon.

IR_f is the amount invested in "risk-free".

IP is the amount invested in puts.

r_n is the payoff on "risk free".

p_n is the payoff on put.

k is the percentage of the portfolio that goes into the risky instrument.

5.2.1 3-month horizon

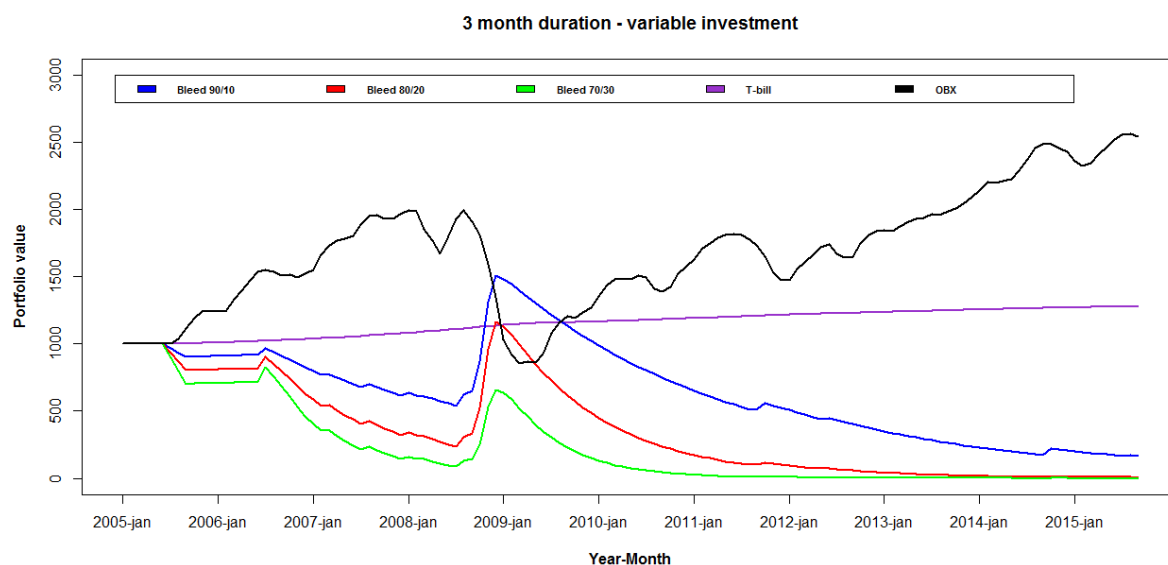


Figure 11: 3-month horizon portfolios with variable investments

Table 4: 3-month horizon portfolios with variable investments. Monetary returns

	90/10	80/20	70/30	OBX	T-bills
Lifetime returns	-836.11	-992.78	-999.90	1547.74	280.92
Lifetime % returns	-83.61 %	-99.28 %	-99.99 %	154.77 %	28.09 %
Obs. w/ return > 0	27	27	27	87	126
Obs. w/ option	112	112	112	0	0
Total obs.	129	129	129	129	129

The bleed portfolios show weak monetary return for the period in question. The 80/20- and 70/30-portfolios basically go broke whilst the 90/10 also show great losses. The OBX appears to have done well compared to the treasury bills.

The bleed portfolios initially experience a 43-month period of bleeding and months without option investments. Then the global financial crisis hit the markets in the autumn of 2008 and for the next 5 months the bleed portfolios increased by 279%, 498% and 775% respectively. It is worth noting that the pure monetary increase in the same period were 973 NOK, 933 NOK and 579 NOK. Meaning that the 90/10 portfolio gained more as a result of preserving the

capital to harvest in good times. However, after this successful period the bleed portfolios bled for most of its remaining life time and only a few options had positive payoff and only month 118 had relatively large payoffs of 25%, 50% and 75% respectively. In the end the 3-month bleed portfolios.

Table 5:3-month horizon portfolios with variable investments. Descriptive statistics

	90/10	80/20	70/30	OBX	T-bills
Arithmetic Mean	-0.0078	-0.0176	-0.0274	0.0112	0.0020
Mean: Standard error	0.0076	0.0151	0.0226	0.0037	0.0001
Geometric Mean	-0.0406	-0.1073	-0.1921	0.0235	0.0059
Std. Dev.	0.0860	0.1715	0.2571	0.0424	0.0011
Downside deviation*	0.0293	0.0585	0.0877	0.0281	0.0006
Skewness	6.0334	6.0373	6.0385	-0.6730	1.0872
Kurtosis	41.1979	41.2545	41.2721	4.0572	0.3562
Sharpe ratio	-0.1140	-0.1143	-0.1144	0.2170	0.0000
Omega*	0.6188	0.6179	0.6175	1.8720	1.0000
Sortino ratio*	-0.3338	-0.3343	-0.3345	0.3292	0.0000
Kappa 3rd moment*	-0.3188	-0.3194	-0.3196	0.2079	0.0000
Min.	-0.0327	-0.0661	-0.0995	-0.1744	0.0000
Max.	0.7084	1.4119	2.1155	0.1720	0.0049

The arithmetic and geometric mean returns were negative for all the bleed portfolios. It is worth noting that the geometric mean present worse results than the arithmetic means due to the high variances, see equation page 13. It is assumed that arithmetic mean might give the most accurate results for skewed portfolios. However, this is not obvious and can be regarded as a simplification. Furthermore, the standard errors for the bleed portfolios are clearly larger than for the OBX arithmetic mean. Meaning that it is greater uncertainty about its “true” value. This is per the law of large numbers, discussed in chapter 2.6.

The volatilities are very high for all the bleed portfolios. The 90/10 is twice as volatile as the OBX-portfolio. But as discussed this might not be an accurate representation of “unwanted volatility”. The downside deviations for the 90/10- and OBX-portfolios are very close, and the gap to the 80/20- and 70/30-portfolios has narrowed greatly compared to the standard

deviation. The “worst case”-scenarios, represented by the portfolios worst months, are significantly better for the bleed-portfolios than the OBX-portfolio. This is due to the “floor” created by the barbell strategy. On the other side the “best case”-scenarios, represented by the portfolios best returns, are much larger for all the bleed portfolios than the OBX-portfolio.

The risk-adjusted performance measurements Sharpe ratio, Sortino ratio, Omega and third moment Kappa all show poor results for the bleed portfolios compared to the OBX-portfolio. This is natural as the mean returns for the bleed portfolios are all lower than the threshold return, the treasury bill mean return. Whilst the OBX-portfolio mean return is higher.

The bleed portfolios are all highly positively skewed whilst the OBX-portfolio is slightly negatively skewed. This is a portfolio characteristic investors appreciate, and are willing to pay a premium for. Furthermore, the bleed portfolios have high positive kurtosis compared to the OBX-portfolio. This is a trait investors are averse to.

5.2.2 6-month horizon

For the initial 30 months of the bleed-portfolios there was no 6 month out-of-the-money put options. This is also the case when we disregard the spread rule. Therefore, the bleed portfolios follow the treasury bill-portfolio for a long time, underlining the lack of liquidity in the Norwegian out-of-the-money put option market.

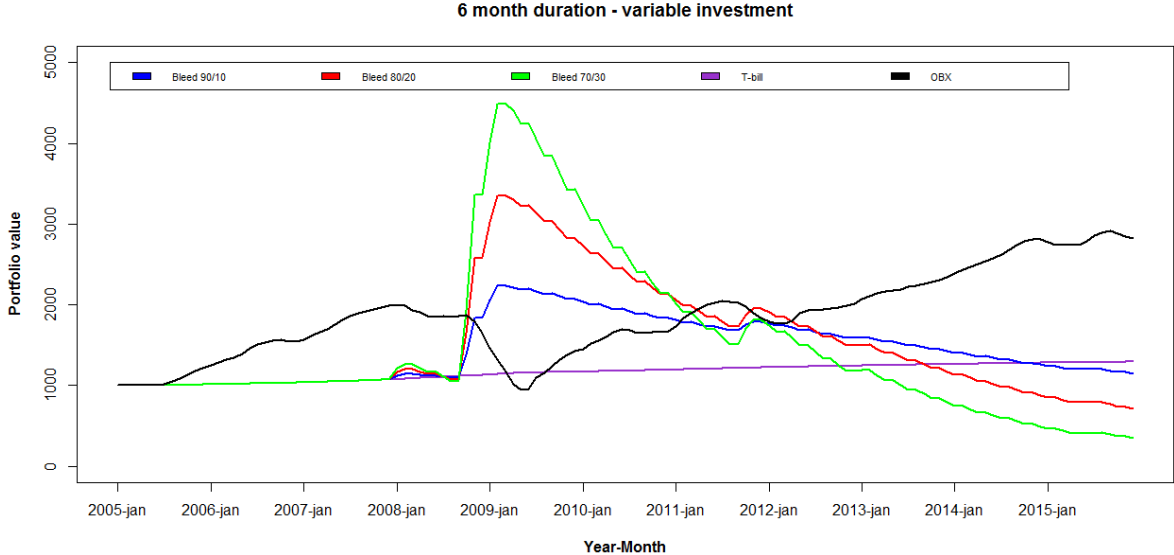


Figure 12: 6-month horizon portfolios with variable investments

Table 6: 6-month horizon portfolios with variable investments. Monetary returns.

	90/10	80/20	70/30	OBX	T-bills
Lifetime returns	143.97	-287.19	-650.88	1821.66	292.42
Lifetime % returns	14.40 %	-28.72 %	-65.09 %	182.17 %	29.24 %
Obs. w/ return > 0	73	73	73	91	126
Obs. w/ option	61	61	61	0	0
Total obs.	132	132	132	132	132

After the 11-year portfolio life time the bleed portfolios end up with disappointing monetary results. The 90/10-portfolio show a slight monetary gain, whilst both the 80/20- and 70/30- portfolios lost money over the period. On the other hand, the OBX-portfolio appears to have made a decent gain. It is worth noting that the bleed portfolios only invested in put options in 61 of the 126 active months according to the trading rule.

Similar to the 3-month duration portfolio the 6-month duration portfolio show great returns in a period during the global financial crisis. From September 2008, month 45 of the portfolio, the bleed portfolios increased the portfolio values 103%, 210% and 325% respectively. The monetary gains were 1130, 2266 and 3430 NOK. Due to the lack of available options, and thus a lack of bleeding, in the months before the success the 90/10-portfolio were not in a better position to benefit from gains as it was in the 3 month-horizon portfolios.

During the same 5-month period in 2008-2009 the OBX-portfolio value fell by 29%, whilst the treasury bill-portfolio increased by 2.4%.

Table 7: 6-month horizon portfolios with variable investments. Descriptive statistics.

	90/10	80/20	70/30	OBX	T-bills
Arithmetic mean	0.0035	0.0051	0.0067	0.0106	0.0020
Mean: Standard error	0.0042	0.0084	0.0126	0.0026	0.0001
Geometric mean	0.0074	-0.0127	-0.0428	0.0490	0.0116
Std. Dev.	0.0484	0.0964	0.1443	0.0297	0.0012
Downside deviation*	0.0109	0.0215	0.0322	0.0194	0.0007
Skewness	6.3896	6.4089	6.4149	-0.8661	0.9610
Kurtosis	45.1237	45.3909	45.4759	2.9585	0.2903
Sharpe ratio	0.0310	0.0322	0.0326	0.2896	0.0000
Omega*	1.2217	1.2272	1.2291	2.2401	1.0000
Sortino ratio*	0.1451	0.1465	0.1469	0.4482	0.0000
Kappa 3rd moment*	0.1246	0.1258	0.1262	0.2871	0.0000
Min.	-0.0161	-0.0328	-0.0495	-0.9970	0.0000
Max.	0.4126	0.8204	1.2282	0.0968	0.0050

Arithmetic means for the bleed portfolios are positive and higher than the treasury bill return mean. However, the OBX-portfolio return mean is higher than all the bleed portfolios. As for the 3 month-portfolios the geometric mean show a worse situation for the bleed portfolios due to the high variance. However as discussed in chapter 2.5 they do represent the compound return, which indeed is negative in this instance as the life time returns are negative for the 80/20- and 70/30-portfolios. Again, there is high uncertainty about the true means for the bleed portfolios means, this is clearly shown by the high standard errors.

As there are many months of investments in only treasury bills the downside deviations are relatively lower for the bleed portfolios compared to the 3 month-horizon. In this instance the downside deviation is nearly twice as large for the OBX-portfolio compared to the 90/10-portfolio. The “worst case”-scenarios show a similar story for this horizon, the bleed portfolios have clearly higher “floors” than the OBX-portfolio. At the same time the best periods are clearly higher than the best OBX-portfolio period.

All the risk-adjusted measurements show a preference for the OBX-portfolio over the bleed portfolios. Whilst again the bleed portfolios have significantly higher skewness and kurtosis.

5.3 Portfolios with constant investments

These portfolios will assume that there are no budget constraints. They will invest a constant amount for each month, The portfolios for both the 3- and 6-month horizon will invest the same amount, $1000/3=333.33$ NOK, each month. As for the variable investment portfolios, the initial value of the portfolios will be 1000 NOK.

5.3.1 3-month horizon

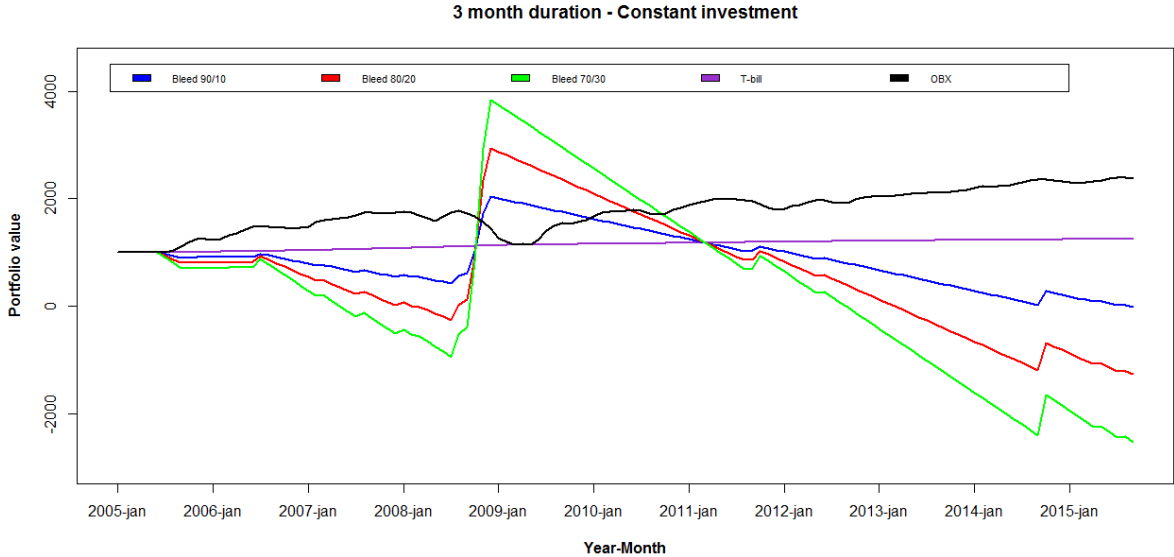


Figure 13: 3-month horizon portfolios with constant investments

Table 8: 3-month horizon portfolios with constant investments. Monetary returns.

	90/10	80/20	70/30	OBX	T-bills
Lifetime returns	-1007.65	-2268.90	-3530.15	1444.78	253.60
Lifetime % returns	-100.77 %	-226.89 %	-353.02 %	144.48 %	25.36 %
Obs. w/ return > 0	27	27	27	87	126
Obs. w/ option	112	112	112	0	0
Total obs.	129	129	129	129	129

Similar to the variable investment portfolios the bleed portfolios perform poorly over the period. When the successful 5-month period came in the autumn of 2008, there is more capital to benefit from the gains due to the constant investment sizes. At the same time, the bleeding is smaller for the portfolio as long as its value is below 1000. However, when the portfolio value is below 1000 the bleeding will be steeper. In the end the bleeding made the bleed portfolios end up with significant losses. On the other side, the OBX-portfolio performed well, albeit slightly worse than for the varying investment sizes strategy.

Table 9: 3-month horizon portfolios with constant investments. Descriptive statistics.

	90/10	80/20	70/30	OBX	T-bills
Arithmetic mean	-0.0234	-0.0528	-0.0821	0.0336	0.0059
Mean: Standard error	0.0227	0.0453	0.0679	0.0112	0.0003
Geometric mean	NA	NA	NA	NA	NA
Std. Dev.	0.2579	0.5146	0.7713	0.1273	0.0034
Downside deviation*	0.0879	0.1755	0.2631	0.0842	0.0018
Skewness	6.1763	6.1803	6.1815	-0.6889	1.1130
Kurtosis	42.8885	42.9473	42.9656	4.2669	0.4183
Sharpe ratio	-0.1136	-0.1141	-0.1141	0.2176	0.0000
Omega*	0.6188	0.6179	0.6175	1.8720	1.0000
Sortino ratio*	-0.3338	-0.3343	-0.3345	0.3292	-0.0022
Kappa 3rd moment*	-0.3188	-0.3194	-0.3196	0.2079	0.0000
Min.	-0.0981	-0.1983	-0.2985	-0.5233	0.0000
Max.	2.1252	4.2358	6.3464	0.5159	0.0146

The arithmetic means are negative for the bleed portfolios. Although the standard errors are high. The geometric means are not calculable as the logarithmic value cannot be calculated for negative numbers. This is the case when the portfolio value goes negative, which happens with these portfolios.

Regarding risk the standard deviations are clearly higher for the bleed portfolios compared to the OBX-portfolio, whilst the downside deviation is comparable for the 90/10- and OBX-portfolio. Furthermore, the 80/20- and 70/30-portfolios has clearly lower downside deviation than standard deviation. However, the “worst case” and “best case” scenarios are again in the bleed portfolios favor.

Investors will according to current theory be happy with the high positive skewness of the bleed portfolios, but dissatisfied with the high kurtosis.

5.3.2 6-month horizon

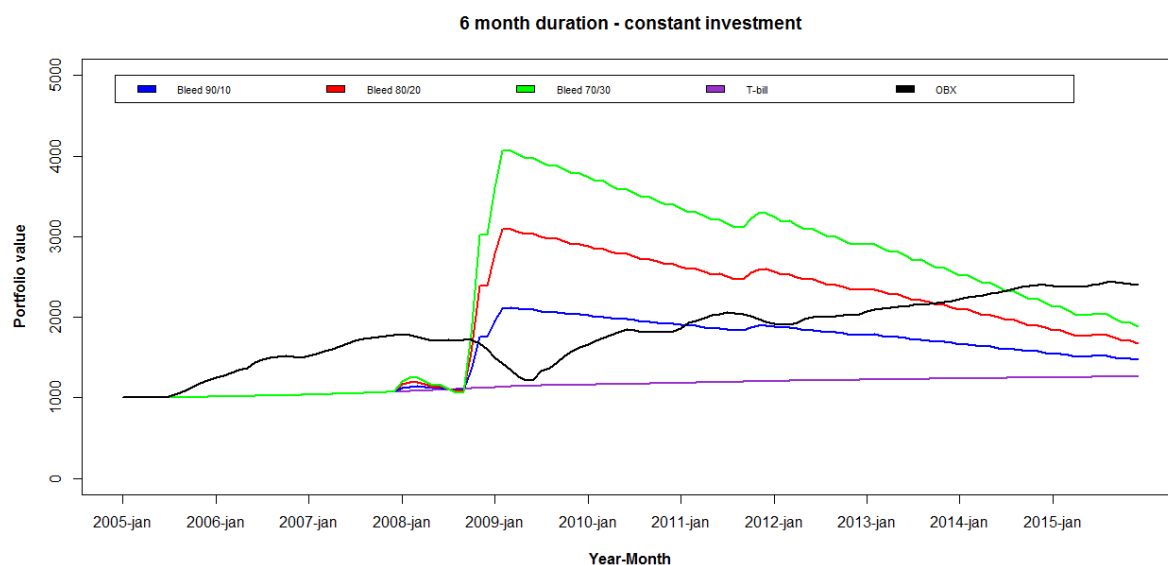


Figure 14: 6-month horizon portfolios with constant investments

Table 10: 6-month horizon portfolios with constant investments. Monetary returns.

	90/10	80/20	70/30	OBX	T-bills
Lifetime returns	936.19	1352.06	1767.94	2809.77	520.32
Lifetime % returns	93.62 %	135.21 %	176.79 %	280.98 %	52.03 %
Obs. w/ return > 0	73	73	73	91	126
Obs. w/ option	61	61	61	0	0
Total obs.	132	132	132	132	132

The lifetime returns for all the portfolios appears to be decent in monetary terms. However, the OBX-portfolio clearly outperforms the bleed portfolios.

During the successful period from month 45 to 50 the bleed portfolios increased by 269%, 442% and 622% respectively. The monetary ascents are 2025, 4002 and 5979 NOK.

The remaining period is dominated by bleeding and in the end the bleed portfolios fell far from their peaks of 3230, 5181 and 7132 NOK to 1936, 1352 and 1768 NOK respectively.

Table 11: 6-month horizon portfolios with constant investments. Descriptive statistics.

	90/10	80/20	70/30	OBX	T-bills
Arithmetic mean	0.0213	0.0307	0.0402	0.0639	0.0118
Mean: Standard error	0.0253	0.0503	0.0754	0.0155	0.0006
Geometric mean	0.0317	0.0417	0.0501	0.0631	0.0191
Std. Dev.	0.2906	0.5782	0.8658	0.1782	0.0070
Downside deviation*	0.0651	0.1291	0.1931	0.1161	0.0040
Skewness	6.3896	6.4089	6.4149	-0.8661	0.9610
Kurtosis	45.1237	45.3909	45.4759	2.9585	0.2903
Sharpe ratio	0.0327	0.0327	0.0328	0.2924	0.0000
Omega*	1.2217	1.2272	1.2291	2.2401	1.0000
Sortino ratio*	0.1451	0.1465	0.1469	0.4482	0.0000
Kappa 3rd moment*	0.1246	0.1258	0.1262	0.2871	0.0000
Min.	-0.0965	-0.1969	-0.2973	-0.5979	0.0000
Max.	2.4754	4.9222	7.3690	0.5809	0.0297

The arithmetic mean returns for all the bleed portfolios are higher than the mean T-bill returns. However, they are all lower than the OBX-portfolio mean return. The geometric means show slightly higher returns than arithmetic means. The uncertainty around the means are still large as shown by the standard errors.

The standard deviations are quite high for the bleed portfolios, whilst the downside deviations show less risk compared to the OBX-portfolio. The “worst” and “best case”-scenarios are in the bleed portfolios favor. Also for this horizon and investment strategy.

Risk-adjusted performance measures are higher for the OBX-portfolio than the bleed portfolios.

5.4 Analysis of chosen options

To review the significance of the assumptions and option choice-rules. The consequences are reviewed and potential weaknesses discussed.

5.4.1 3-month options

For the 3-month options 112 options were chosen for the bleed portfolios. If the spread rule was ignored, 112 options would again be chosen, however 83 would be different and thus had a spread of less than or equal to 0.8. This would cause slightly weaker performance. This is illustrated by the 90/10-portfolio:

Table 12: No spread rule. 3-month horizon.

Variable investments	Spread rule	No spread rule
Lifetime returns	-836.11	-893.01
Arithmetic mean	-0.0078	-0.0106
Downside deviation*	0.0293	0.0303
Sortino ratio*	-0.3338	-0.4132
Constant investments	Spread rule	No spread rule
Lifetime returns	-1104.54	-1361.57
Arithmetic mean	-0.0234	-0.0951
Downside deviation*	0.0879	0.0909
Sortino ratio*	-0.3338	-0.4132

Among the 112 chosen options there are 11 pairs of options that have the same expiration date. This means that if the strike prices are equal, the same option has been purchased at different times, although with a difference in duration from 1 to 20 days. This due to the assumption that an option can be bought at the beginning of each month. Albeit, some of the have slightly different strike prices. However, the payoffs between them will be dependent of each other, which can be unfortunate in portfolio management context as it will impair diversification.

5.4.2 6-month options

The 6-month horizon portfolios only chose 61 months with option investments and in the first 30 months none were chosen. Disregarding the spread rule does not improve this and there are options chosen for the exact same 61 months. However, 26 of the options has been changed

for others with lower spread and moneyness. The performance is slightly weaker without the spread rule. They are illustrated by the 90/10-portfolio:

Table 13: No spread rule. 6-month horizon.

Variable investments	Spread rule	No spread rule
Lifetime returns	143.97	102.15
Arithmetic mean	0.0035	0.0033
Downside deviation*	0.0109	0.0109
Sortino ratio*	0.1451	0.1189
Constant investments	Spread rule	No spread rule
Lifetime returns	936.19	861.93
Arithmetic mean	0.0213	0.0196
Downside deviation*	0.0651	0.0653
Sortino ratio*	0.1451	0.1189

A big weakness with the study of the 6-month horizon is that the options appears to be chosen in pairs. Among the 61 options, 60 are options without unique expiration dates. Of these 30 pairs, 16 of them have the exact same strike price. In other words, they are basically the same option bought in different months, albeit with a slightly different duration. This suggests that the data for the options of 6-month duration are weak due to the low liquidity in the Norwegian out-of-the-money put option market.

6 DISCUSSION/CONCLUSION

The purpose of this work was to analyze highly positively skewed portfolios performances in the Norwegian financial markets. Furthermore, it was intended to present an overview of relevant theories that helps understand characteristics of bleed derivatives. Hereunder how to evaluate their performances. The aim of this chapter is to discuss the empirical findings in conjunction with relevant theories and to what extent the thesis has succeeded in answering the research question. Moreover, weaknesses of the empirical analysis will be discussed and interesting potential future research topics will be suggested.

6.1 Discussion

Bleed derivatives have some interesting characteristics that carry consequences for their performance evaluations. Their variances are typically large, while much of this variance is caused by large positive observations that an investor would not consider as risk. Therefore, variance or volatility might be an inaccurate measure of the derivatives risk. To address this issue, it has been attempted to calculate downside deviation, which is a measurement of only “unwanted” deviation below a threshold. The downside deviations have given a soberer risk measure when compared to the alternative investment-portfolio. Moreover, it has been noted that positive skewness itself has been found to be preferred among investors according to previous research(Kraus & Litzenberger, 1976). Whilst kurtosis is viewed as a negative trait for a portfolio. The bleed derivatives are positively skewed, whilst this also appears to bring along high kurtosis. Furthermore, the bleed portfolios have a higher “floor” of negative returns. Leading to better “worst case”-observations than for the OBX-portfolio. This might be considered as an aspect of the bleed portfolios that are advantageous compared to the OBX-portfolio in a risk context.

One of the main problems when analyzing performance of bleed derivatives relates to the Law of large numbers. Due to the bleed derivative’s distribution, many observations are needed to make accurate estimations of its properties. In real life phenomena, finance included, such a large number of observations are hard to obtain as they often do not exist. This fact, combined with the short period of research and few out-of-the-money put options, makes the results obtained extremely uncertain.

Several performance measurements were calculated, some of them which were risk-adjusted. The arithmetic mean gave bad results for the bleed portfolios compared to the OBX-portfolio. They were however better than the geometric means, which were disadvantageous due to the high variance as discussed. Even though the downside deviation is a better representation to calculate “unwanted risk” due to occasional returns far above the mean. Furthermore, the traditional Sharpe ratios were calculated and were found to evaluate bad performance for the bleed portfolios. The Sortino ratio, Omega and third moment Kappa were found to be more accurate suitable risk-adjusted performance measurements. However, the results were still in the OBX-portfolios favor.

6.2 Weaknesses of the thesis

6.2.1 Few observations

One of the main difficulties of examining highly skewed distributions is that, according to the law of large numbers, a large number of observations are needed for confident results. Large enough numbers of observations are infeasible for many phenomena. The Norwegian financial markets are no different. The behaviors of the bleed portfolios have been examined for a relatively short time period and the confidence in predicting future behavior of bleed portfolios is still low.

6.2.2 Illiquid out-of-the-money put option market

The Norwegian financial markets are much smaller in scale than for example the American. And the trading volume is much lower. This is especially seen for the less traditional derivatives, like far-out-of-the-money put options. This led to a shorter time frame, due to the lack of available options before 2005, and to high bid/ask spreads. This might have led to the use of unrealistically high prices, as this thesis assumed the best Ask-price as option price. This might lead to worse results than if a bleed strategy was conducted in real life.

6.2.3 Logarithmic or simple returns

As the variance of highly positively skewed distributions might be a measurement to avoid in financial context, there is high uncertainty regarding what types of returns and mean returns to use. A high variance leads to a large difference in arithmetic and geometric mean, which means that they might evaluate performance very differently. This thesis assumed that simple

returns and arithmetic return would yield the most accurate results. However, this is not certain. The geometric means have been presented. If they were to be weighted the most, the conclusion would be more critical of the bleed portfolios performance.

6.3 Future research

The aim of research is to answer questions through scientific procedure. However, a research might lead to more questions than it answers. And subsequently interesting new ideas or topics for research. The aim of this sub-chapter is to present some potential future research ideas.

6.3.1 Relaxing some assumptions

When making assumptions. It is interesting to examine what happens if some of the are relaxed or removed completely. The main assumptions that was made during this research was that options are assumed to be bought at the beginning of each month and the durations has been assumed to be 3 or 6 months, for durations in the interval 80-100 and 160-200 respectively.

6.3.2 Choosing options differently

When the options were chosen, the choices were made by simply taking the option with the least moneyness while it fulfilled a spread restriction. Further research could examine different methods to choose options, or perhaps even make a portfolio of several of them. An interesting idea would be to look at put options that were less out-of-the-money, meaning that the skewness would be lower. But perhaps such a strategy would yield better results. Another benefit would be that less out-of-the-money options tend to have higher trading volume, making prices more accurate, and the law of large numbers would demand less observations.

6.3.3 Derivatives with more available observations

The put options proved to be of questionable quality when considering liquidity and number of observations. It would be interesting to research other positively skewed derivatives that perhaps are more suitable with regards to available data. An example could be currency trading. This might be done by investing in a “risky” currency, while shorting a “lucrative” currency. This is called carry trading.

6.3.4 Logarithmic returns

This thesis assumed that simple returns and arithmetic mean would yield the most accurate results for skewed portfolios. However, this is not necessarily the case and further research on skewed portfolios where logarithmic returns are calculated might be interesting.

7 CONCLUSION

This thesis aimed to answer the research question:

Could a barbell portfolio with extremely positively skewed derivatives create risk-adjusted excess returns in the Norwegian financial market between 2005-2015 compared to alternative investments?

This question was researched in the context of theories relevant for positively skewed portfolios. It was found that the highly skewed distributions variance could represent an unrealistic view of “unwanted risk”. Furthermore, this implies uncertainty of results achieved from the Sharpe ratio and gives relatively large differences in arithmetic and geometric mean. In an attempt to avoid the weaknesses of variance and volatilities ability to assess risk and for Sharpe ratio to assess risk-adjusted performance other measurements were introduced. Downside deviation to evaluate risk, and Sortino ratio, Omega and third moment Kappa to evaluate risk-adjusted performance. After a comprehensive assessment of the measurements, the bleed portfolio can be said to have performed worse than the benchmark portfolio for the period in question. However, it is noteworthy that the bleed portfolios carried some properties that are preferred by investors. Namely skewness, low “worst case”- and high “best case”- returns.

Due to the data quality problems, the law of large numbers and subsequently few observations the conclusion is uncertain and cannot be generalized. Taking this into account, it is of the authors opinion that highly skewed portfolios remains an interesting area for future research in finance.

8 REFERENCE LIST

- "Finkin". Openclipart. Retrieved 20.05.2017, from <https://openclipart.org/detail/247308/barbell>
- Chaudhry, A., & Johnson, H. L. (2008). The efficacy of the Sortino ratio and other benchmarked performance measures under skewed return distributions. *Australian Journal of Management*, 32(3), 485-502.
- Constantinides, G. M., & Malliaris, A. G. (1995). Portfolio theory. *Handbooks in operations research and management science*, 9, 1-30.
- DeCarlo, L. T. (1997). On the meaning and use of kurtosis. *Psychological methods*, 2(3), 292.
- Fooladi, I., & Roberts, G. S. (1992). Bond portfolio immunization: Canadian tests. *Journal of economics and business*, 44(1), 3-17.
- Goulding, C. (2015). *Opposite sides of a skewed bet: Implications and evidence for forecast dispersion and returns*. (Doctorate Dissertation in finance), University of Pennsylvania.
- Hudson, R., & Gregoriou, A. (2010). Calculating and comparing security returns is harder than you think: A comparison between logarithmic and simple returns.
- Hull, J. C. (2015). *Options, futures, and other derivatives* (Ninth ed.): Pearson Education Inc.
- Jacobsen, D. I. (2005). Hvordan gjennomføre undersøkelser?: innføring i samfunnsvitenskapelig metode (2. utg. utg.). *Kristiansand: Høyskoleforl.*
- Kaplan, P. D., & Knowles, J. A. (2004). Kappa: A generalized downside risk-adjusted performance measure. *JOURNAL OF PERFORMANCE MEASUREMENT.*, 8, 42-54.
- Keating, C., & Shadwick, W. F. (2002). An introduction to omega. *AIMA Newsletter*.
- Klenke, A. (2013). *Probability theory: a comprehensive course*: Springer Science & Business Media.
- Kothari, C. R. (2004). *Research methodology: Methods and techniques*: New Age International.
- Kraus, A., & Litzenberger, R. H. (1976). Skewness preference and the valuation of risk assets. *The Journal of Finance*, 31(4), 1085-1100.
- Nawrocki, D. N. (1999). A brief history of downside risk measures. *The Journal of Investing*, 8(3), 9-25.
- Naylor, M., Wongchoti, U., & Chen, J. (2011). *Profiting from Black Swan Trades: Evidence from S&P 500 puts*. Working paper.
- Norges Bank. Retrieved 20.03.2017, from <http://www.norges-bank.no/Statistikk/Rentestatistikk/Statsobligasjoner-Rente-Manedsgjennomsnitt-av-daglige-noteringer/>
- Rollinger, T., & Hoffman, S. (2013). Sortino ratio: A better measure of risk. *Futures Magazine*, 1(02).
- Scott, R. C., & Horvath, P. A. (1980). On the direction of preference for moments of higher order than the variance. *The Journal of Finance*, 35(4), 915-919.
- Taleb, N. (2007). *The Black Swan*: Random House.
- Titlon. Retrieved 20.03.2017, from <https://titlon.uit.no/>
- Tversky, A., & Kahneman, D. (1971). Belief in the law of small numbers. *Psychological bulletin*, 76(2), 105.
- Weinberg, A. I. Investors Reach for a New Kind of 'Barbell'. Retrieved 17.05.2017, from <https://www.wsj.com/articles/more-investors-use-a-barbell-strategy-to-structure-their-portfolios-1410120110>

Weisstein, E. W. Skewness. Retrieved 17.05.2017, from <http://mathworld.wolfram.com/Skewness.htm>

APPENDICES

Appendix 1: R-code for Treasury bills and OBX

```
rm(list=ls())
suppressPackageStartupMessages(require(PerformanceAnalytics) || {install.packages("PerformanceAnalytics");require(PerformanceAnalytics)})

## [1] TRUE

obx <- read.csv(file="c:/Users/Espen/Documents/Masteroppg/obxlon.csv", header=TRUE, sep=";");head(obx)

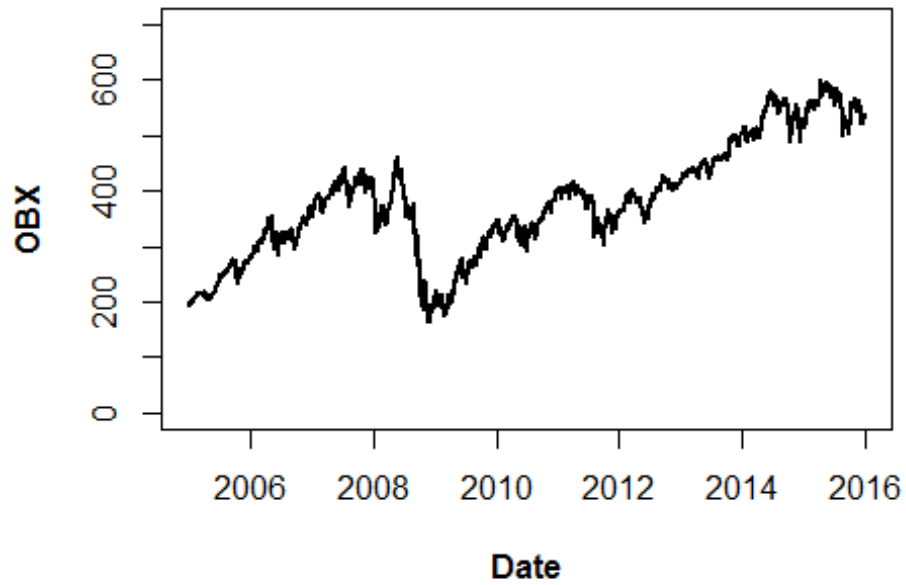
##           Date  Index Month Year MonthIDX
## 1 30.12.2015 538.98    12 2015     132
## 2 29.12.2015 539.11    12 2015     132
## 3 28.12.2015 533.00    12 2015     132
## 4 23.12.2015 536.86    12 2015     132
## 5 22.12.2015 521.10    12 2015     132
## 6 21.12.2015 523.20    12 2015     132

tbill <- read.csv(file="c:/Users/Espen/Documents/Masteroppg/tbill.csv", header=TRUE, sep=";")
obx$Date <- as.Date(obx$Date, format="%d.%m.%Y")
obx$Change <- 0
numb <- nrow(obx)

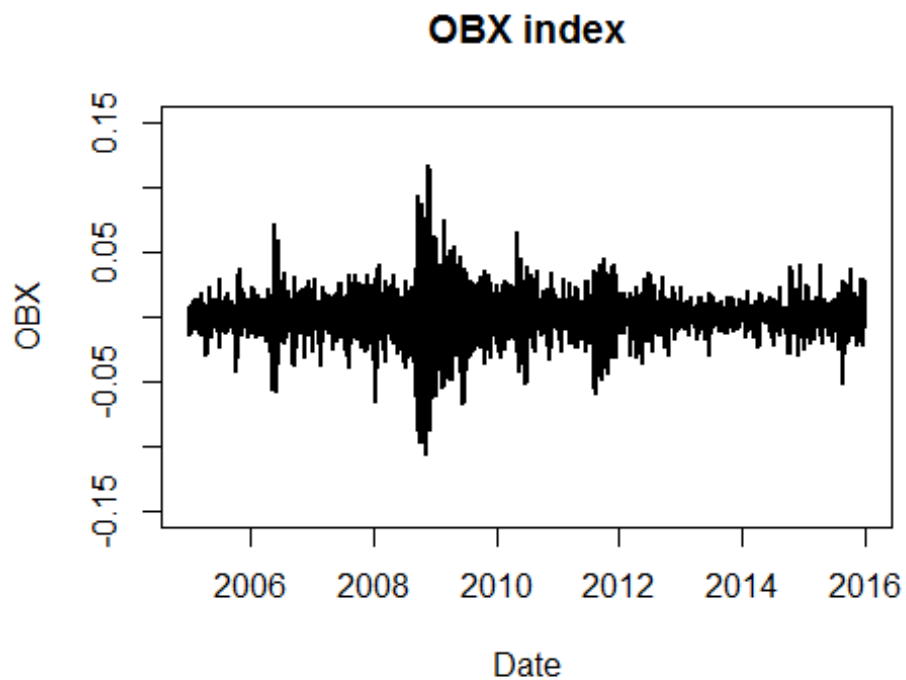
for(i in 2:numb){
  obx$Change[i] <- (obx$Index[i]-obx$Index[(i+1)])/obx$Index[(i+1)]
}

plot(obx$Index~obx$Date,xlim=c(as.Date("01.01.2005",format="%d.%m.%Y"),as.Date("31.12.2015",format="%d.%m.%Y")), ylim=c(0,700), type="l", ylab="OBX", xlab="Date", main="OBX - Total return index", lwd=2, font.lab=2)
```

OBX - Total return index



```
plot(obx$Change~obx$Date,xlim=c(as.Date("01.01.2005",format="%d.%m.%Y"),as
.Date("31.12.2015",format="%d.%m.%Y")), ylim=c(-0.15,0.15), type="l", ylab
="OBX", xlab="Date", main="OBX index", lwd=2)
```



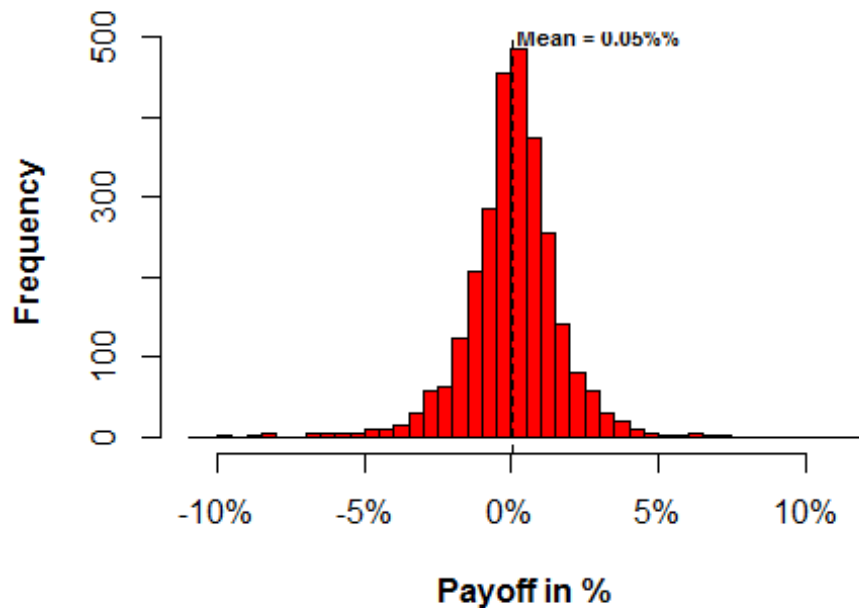
```

a <- mean(obx$Change, na.rm=TRUE)

hist(obx$Change, breaks=50, main="Daily returns of OBX 2005-2015",xlab="Pa
yoff in %" , font.lab=2, xaxt="n", col="red")
axis(1, at=c(-0.10,-0.05,0,0.05,0.10), labels=c("-10%","-5%","0%","5%","10
%"))
abline(v=a, col="black", lty=2)
text(0.03, 500, "Mean = 0.05%", col = "black", cex=0.7, font=2)

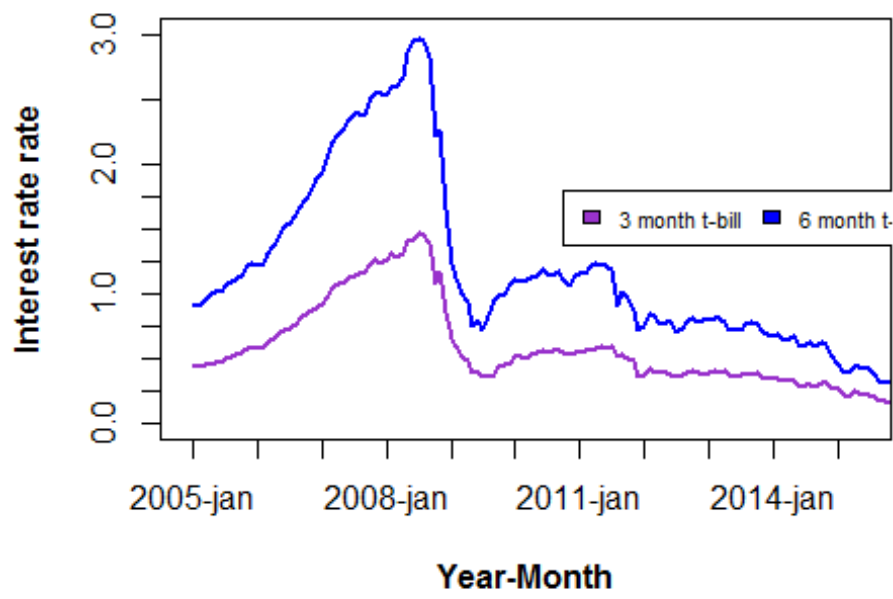
```

Daily returns of OBX 2005-2015



```
PerformanceAnalytics::skewness(obx$Change,method=c("sample"))  
  
## [1] -0.3292193  
  
plot(tbill$X3.month.t.bill~tbill$MonthIDX, type="l", col="darkorchid", ylim=c(0,3),xlim=c(0,126), ylab="Interest rate rate",xlab="Year-Month",main="Treasury bills - monthly average", lwd=2, xaxt="n", font.lab=2)  
par(new=T)  
plot(tbill$X6.month.t.bill~tbill$MonthIDX, type="l", ylim=c(0,3),xlim=c(0,126),col="blue",xaxt="n",yaxt="n", ylab="", xlab="", lwd=2)  
par(new=T)  
legend(x=70, y=1.8,cex=0.7, c("3 month t-bill","6 month t-bill"), fill=c("darkorchid", "blue"), horiz=TRUE)  
par(font=2)  
axis(1, at=c(1,13,25,37,49,61,73,85,97,109,121), labels=c("2005-jan","2006-jan","2007-jan","2008-jan","2009-jan","2010-jan","2011-jan","2012-jan","2013-jan","2014-jan","2015-jan"))  
axis(2,at=c(0.25,0.75,1.25,1.75),labels=c("", "", "", ""))
```


Treasury bills - monthly average



Appendix 2: R-code for 6-month portfolios with variable investments

Similar code were used when calculating other durations and investments strategies.

```
rm(list=ls())

suppressPackageStartupMessages(require(pacman) || {install.packages("pacman");require(pacman)})

## [1] TRUE

suppressPackageStartupMessages(require(purrr) || {install.packages("purrr");require(purrr)})

## [1] TRUE

suppressPackageStartupMessages(require(dplyr) || {install.packages("dplyr");require(dplyr)})

## [1] TRUE

suppressPackageStartupMessages(require(psych) || {install.packages("psych");require(psych)})

## [1] TRUE

suppressPackageStartupMessages(require(pastecs) || {install.packages("pastecs");require(pastecs)})

## [1] TRUE

suppressPackageStartupMessages(require(moments) || {install.packages("moments");require(moments)})

## [1] TRUE

suppressPackageStartupMessages(require(PerformanceAnalytics) || {install.packages("PerformanceAnalytics");require(PerformanceAnalytics)})

## [1] TRUE
```

```

p_load(tidyverse)

obx <- read.csv(file="c:/Users/Espen/Documents/Masteroppg/obxall15.csv", header=TRUE, sep=";")
obxm <- read.csv(file="c:/Users/Espen/Documents/Masteroppg/obxmonths2.csv", header=TRUE, sep=";")
tbill <- read.csv(file="c:/Users/Espen/Documents/Masteroppg/tbill.csv", header=TRUE, sep=";")
index <- read.csv(file="c:/Users/Espen/Documents/Masteroppg/obxlon.csv", header=TRUE, sep=";")
obx <- as_data_frame(obx)
obx <- subset(obx, obx$Moneyiness < 1 & obx$Duration >= 160 & obx$Duration <= 200)
obx$Date <- as.Date(obx$Date, format="%d.%m.%Y")
index$Date <- as.Date(index$Date, format="%d.%m.%Y")

obx <- obx %>% mutate(PercPay = Payoff/BestAskPrice*100,
                     spread = BestBidPrice/BestAskPrice,
                     PortefolioMoneySpread = 0,
                     Date = as.Date(Date, format="%d.%m.%Y"),
                     ExpirationDate = as.Date(ExpirationDate, format="%d.%m.%Y"))

obxm <- obxm %>% mutate(PercPay = Payoff/BestAskPrice*100,
                       spread = BestBidPrice/BestAskPrice,
                       PortefolioMoneySpread = 0,
                       Date. = as.Date(Date, format="%d.%m.%Y"),
                       ExpirationDate = as.Date(ExpirationDate, format="%d.%m.%Y"))

obx <- subset(obx, obx$spread > 0.8 & obx$Moneyiness > 0)

vec <- as.vector(rep(0, nrow(obx)))
for (i in unique(obx$MonthIDX)) {

```

```

tmp <- subset(obx, obx$MonthIDX == i)
t <- which(iffelse(obx$MonthIDX == i,obx$Moneyiness, FALSE) == min(tmp$Moneyiness))[1]
vec[t] <- 1
}
obx$PortefolioMoneySpread <- vec
obxport <- subset(obx, obx$PortefolioMoneySpread == 1)

for(j in 1:nrow(obxport)){
  m <- obxport$MonthIDX[j]
  obxm[m,] <- obxport[j,]
}

vec2 <- obxm$Interest.rate
for (i in unique(obxm$MonthIDX)) {
  rf <- subset(tbill, MonthIDX == i)$X6.month.t.bill
  vec2 <- iffelse(obxm$MonthIDX == i, rf, vec2)
}
obxm$Interest.rate <- vec2

obxm$PortPay10 <- 0;obxm$PortPay20 <- 0;obxm$PortPay30 <- 0

for(v in 1:nrow(obxm)){
  obxm$PortPay10[v] <- iffelse(is.na(obxm$Name[v]),(1+obxm$Interest.rate[v]/100),0.1*(1+(obxm$PercPay[v]/100))+0.9*(1+obxm$Interest.rate[v]/100))
  obxm$PortPay20[v] <- iffelse(is.na(obxm$Name[v]),(1+obxm$Interest.rate[v]/100),0.2*(1+(obxm$PercPay[v]/100))+0.8*(1+obxm$Interest.rate[v]/100))
  obxm$PortPay30[v] <- iffelse(is.na(obxm$Name[v]),(1+obxm$Interest.rate[v]/100),0.3*(1+(obxm$PercPay[v]/100))+0.7*(1+obxm$Interest.rate[v]/100))
}

vec3 <- as.vector(rep(0,nrow(index)))
for (h in unique(index$MonthIDX)) {
  tmp2 <- subset(index, index$MonthIDX == h)
  t2 <- which(iffelse(index$MonthIDX == h,index$Date, FALSE) == min(tmp2$Da

```

```

te))[1]
  vec3[t2] <- 1
}
index$first <- vec3; index <- subset(index, index$first == 1)
index$IndexRet <- 0; index$IndexRet[1] <- 0
index <- index[order(index$MonthIDX),]
for(l in 2:nrow(index)){
  index$IndexRet[l] <- (index$Index[l]-index$Index[(l-1)])/index$Index[(l-1)]
}
obxm$obxPay <- index$IndexRet

obxm$PortValue10 <- 0; obxm$PortValue20 <- 0; obxm$PortValue30 <- 0; obxm$tbill <- 0
obxm$PortValue10[1] <- 1000; obxm$PortValue10[2] <- 1000; obxm$PortValue10[3] <- 1000; obxm$PortValue10[4] <- 1000; obxm$PortValue10[5] <- 1000; obxm$PortValue10[6] <- 1000
obxm$PortValue20[1] <- 1000; obxm$PortValue20[2] <- 1000; obxm$PortValue20[3] <- 1000; obxm$PortValue20[4] <- 1000; obxm$PortValue20[5] <- 1000; obxm$PortValue20[6] <- 1000
obxm$PortValue30[1] <- 1000; obxm$PortValue30[2] <- 1000; obxm$PortValue30[3] <- 1000; obxm$PortValue30[4] <- 1000; obxm$PortValue30[5] <- 1000; obxm$PortValue30[6] <- 1000
obxm$tbill[1] <- 1000; obxm$tbill[2] <- 1000; obxm$tbill[3] <- 1000; obxm$tbill[4] <- 1000; obxm$tbill[5] <- 1000; obxm$tbill[6] <- 1000
obxm$obx[1] <- 1000; obxm$obx[2] <- 1000; obxm$obx[3] <- 1000; obxm$obx[4] <- 1000; obxm$obx[5] <- 1000; obxm$obx[6] <- 1000

for(n in 7:nrow(obxm)){
  obxm$PortValue10[n] <- (obxm$PortValue10[(n-1)]+((obxm$PortValue10[(n-6)]/6)*obxm$PortPay10[(n-6)]-obxm$PortValue10[(n-6)]/6)
  obxm$PortValue20[n] <- (obxm$PortValue20[(n-1)]+((obxm$PortValue20[(n-6)]/6)*obxm$PortPay20[(n-6)]-obxm$PortValue20[(n-6)]/6)
  obxm$PortValue30[n] <- (obxm$PortValue30[(n-1)]+((obxm$PortValue30[(n-6)]/6)*obxm$PortPay30[(n-6)]-obxm$PortValue30[(n-6)]/6)

```

```

obxm$tbill[n] <- obxm$tbill[(n-1)]+((obxm$tbill[(n-6)]/6)*(1+obxm$Intere
st.rate[(n-6)]/100))-(obxm$tbill[(n-6)]/6)
obxm$obx[n] <- obxm$obx[(n-1)]+((obxm$obx[(n-6)]/6)*(1+obxm$obxPay[(n-6)
])*(1+obxm$obxPay[(n-5)])*(1+obxm$obxPay[(n-4)])*(1+obxm$obxPay[(n-3)])*(1
+obxm$obxPay[(n-2)])*(1+obxm$obxPay[(n-1)]))-(obxm$obx[(n-6)]/6)
}

mean(obxm$PortPay10,na.rm=TRUE)

## [1] 1.021428

mean(obxm$PortPay20,na.rm=TRUE)

## [1] 1.03088

mean(obxm$PortPay30,na.rm=TRUE)

## [1] 1.040332

mean(obxm$spread,na.rm=TRUE)

## [1] 0.8548855

mean(obxm$PercPay,na.rm=TRUE)

## [1] 21.65387

mean(obxm$Moneyiness,na.rm=TRUE)

## [1] 0.8557377

mean(1+(obxm$Interest.rate/100))

## [1] 1.011977

plot(obxm$tbill~obxm$MonthIDX, type="l", col="darkorchid", ylim=c(0,5000),
ylab="Portfolio value",xlab="Year-Month",main="6 month duration - variabl
e investment", lwd=2, xaxt="n", font.lab=2)
par(new=T)
plot(obxm$PortValue10~obxm$MonthIDX, type="l", col="blue", ylim=c(0,5000),
xaxt="n",yaxt="n", ylab="", xlab="", lwd=2)

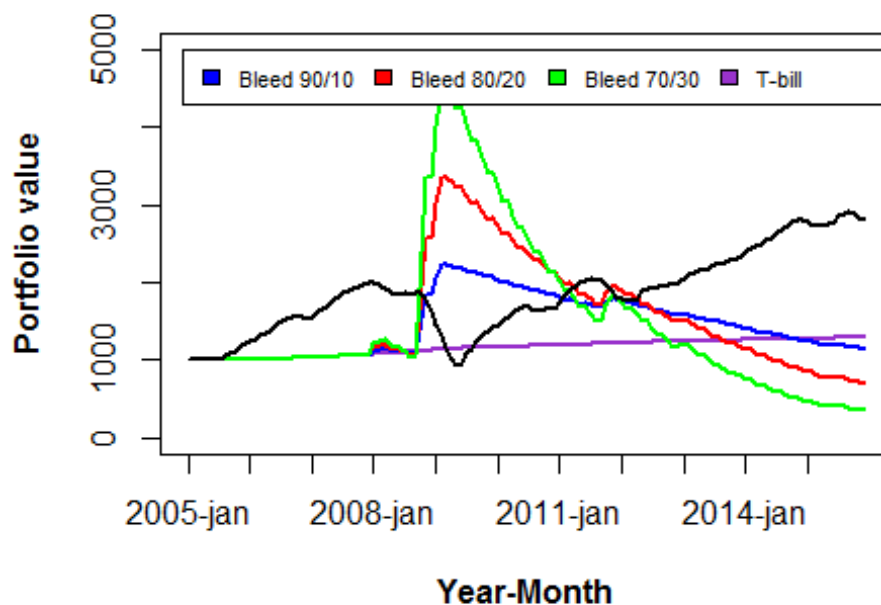
```

```

par(new=T)
plot(obxm$PortValue20~obxm$MonthIDX, type="l", col="red", ylim=c(0,5000),x
axt="n",yaxt="n", ylab="", xlab="", lwd=2)
par(new=T)
plot(obxm$PortValue30~obxm$MonthIDX, type="l", col="green", ylim=c(0,5000)
,xaxt="n",yaxt="n", ylab="", xlab="", lwd=2)
par(new=T)
plot(obxm$obx~obxm$MonthIDX, type="l", col="black", ylim=c(0,5000),xaxt="n
",yaxt="n", ylab="", xlab="", lwd=2)
legend(x=0, y=5000,cex=0.7, c("Bleed 90/10","Bleed 80/20", "Bleed 70/30","
T-bill", "OBX"), fill=c("blue","red","green","darkorchid", "black"), horiz
=TRUE)
par(font=2)
axis(1, at=c(1,13,25,37,49,61,73,85,97,109,121), labels=c("2005-jan","2006
-jan","2007-jan","2008-jan","2009-jan","2010-jan","2011-jan","2012-jan","2
013-jan","2014-jan","2015-jan"))

```

6 month duration - variable investment



```

obxm$Port10 <- 0
obxm$Port20 <- 0

```

```

obxm$Port30 <- 0
obxm$obxPay2 <- 0
obxm$tbill2 <- 0

obxm$Port10Geo <- 1
obxm$Port20Geo <- 1
obxm$Port30Geo <- 1
obxm$obxGeo <- 1
obxm$tbill2Geo <- 1

for(n in 7:nrow(obxm)){
  obxm$obxPay2[n] <- ((obxm$obx[n]-obxm$obx[(n-1)])/obxm$obx[(n-6)])
  obxm$Port10[n] <- ((obxm$PortValue10[n]-obxm$PortValue10[(n-1)])/obxm$PortValue10[(n-6)])
  obxm$Port20[n] <- ((obxm$PortValue20[n]-obxm$PortValue20[(n-1)])/obxm$PortValue20[(n-6)])
  obxm$Port30[n] <- ((obxm$PortValue30[n]-obxm$PortValue30[(n-1)])/obxm$PortValue30[(n-6)])
  obxm$tbill2[n] <- ((obxm$tbill[n]-obxm$tbill[(n-1)])/obxm$tbill[(n-6)])
  obxm$obxGeo[n] <- obxm$obx[n]/obxm$obx[(n-6)]
  obxm$Port10Geo[n] <- obxm$PortValue10[n]/obxm$PortValue10[(n-6)]
  obxm$Port20Geo[n] <- obxm$PortValue20[n]/obxm$PortValue20[(n-6)]
  obxm$Port30Geo[n] <- obxm$PortValue30[n]/obxm$PortValue30[(n-6)]
  obxm$tbill2Geo[n] <- obxm$tbill[n]/obxm$tbill[(n-6)]
}

desc <- (obxm[1:132,31:35])
desc1 <- stat.desc(desc)
format(round(desc1,4),nsmall=4)

##           Port10   Port20   Port30  obxPay2   tbill2
## nbr.val      132.0000  132.0000  132.0000  132.0000  132.0000
## nbr.null       6.0000   6.0000   6.0000   6.0000   6.0000
## nbr.na         0.0000   0.0000   0.0000   0.0000   0.0000
## min          -0.0161  -0.0328  -0.0495  -0.0997   0.0000

```



```

## max          0.4126   0.8204   1.2282   0.0968   0.0050
## range        0.4287   0.8532   1.2777   0.1965   0.0050
## sum          0.4681   0.6760   0.8840   1.4049   0.2602
## median       0.0011   0.0011   0.0011   0.0118   0.0017
## mean         0.0035   0.0051   0.0067   0.0106   0.0020
## SE.mean      0.0042   0.0084   0.0126   0.0026   0.0001
## CI.mean.0.95 0.0083   0.0166   0.0248   0.0051   0.0002
## var          0.0023   0.0093   0.0208   0.0009   0.0000
## std.dev      0.0484   0.0964   0.1443   0.0297   0.0012
## coef.var     13.6596  18.8168  21.5482  2.7910   0.5954

```

```
tb <- mean(obxm$tbill12);tb
```

```
## [1] 0.00197089
```

```
geometric.mean(obxm$Port10Geo)-1
```

```
## [1] 0.00736477
```

```
geometric.mean(obxm$Port20Geo)-1
```

```
## [1] -0.01268168
```

```
geometric.mean(obxm$Port30Geo)-1
```

```
## [1] -0.04275101
```

```
geometric.mean(obxm$obxGeo)-1
```

```
## [1] 0.04904387
```

```
geometric.mean(obxm$tbill12Geo)-1
```

```
## [1] 0.01164749
```

```
SortinoRatio(desc$Port10, MAR = tb)
```

```
##                                     [,1]
```

```
## Sortino Ratio (MAR = 0.197%) 0.1450914
```

```
SortinoRatio(desc$Port20, MAR = tb)
```

```

##                                     [,1]
## Sortino Ratio (MAR = 0.197%) 0.1464526

SortinoRatio(desc$Port30, MAR = tb)

##                                     [,1]
## Sortino Ratio (MAR = 0.197%) 0.1468683

SortinoRatio(desc$obxPay2, MAR = tb)

##                                     [,1]
## Sortino Ratio (MAR = 0.197%) 0.4481732

sum(obxm$Port10 > 0);sum(obxm$Port20 > 0);sum(obxm$Port30 > 0);sum(obxm$ob
xPay2 > 0);sum(obxm$tbill12 > 0)

## [1] 73
## [1] 73
## [1] 73
## [1] 91
## [1] 126

print(obxm$PortValue10)

## [1] 1000.000 1000.000 1000.000 1000.000 1000.000 1000.000 1001.514
## [8] 1003.034 1004.624 1006.260 1007.944 1009.642 1011.401 1013.225
## [15] 1015.067 1016.976 1019.033 1021.095 1023.140 1025.235 1027.466
## [22] 1029.785 1032.251 1034.821 1037.466 1040.221 1043.089 1046.067
## [29] 1049.187 1052.446 1055.798 1059.386 1063.169 1067.066 1071.040
## [36] 1075.135 1124.823 1142.036 1146.238 1132.469 1118.713 1123.281
## [43] 1108.812 1094.228 1099.218 1373.630 1835.181 1840.699 2060.035
## [50] 2229.541 2234.728 2216.397 2192.035 2197.134 2166.631 2133.211
## [57] 2136.913 2103.086 2068.999 2071.912 2038.152 2005.255 2008.654
## [64] 1976.706 1945.307 1949.005 1918.438 1888.321 1892.052 1862.457
## [71] 1833.342 1837.190 1808.517 1780.257 1783.964 1756.020 1728.378
## [78] 1731.882 1704.859 1678.289 1681.907 1749.929 1787.705 1791.226

```

```

## [85] 1765.832 1740.149 1742.992 1716.352 1688.888 1691.018 1663.589
## [92] 1636.780 1639.168 1612.538 1586.350 1588.543 1590.486 1592.487
## [99] 1567.075 1542.152 1544.233 1519.634 1495.007 1497.156 1472.933
## [106] 1449.039 1450.893 1427.198 1403.906 1405.795 1382.962 1360.487
## [113] 1362.236 1339.935 1317.954 1319.562 1297.838 1276.485 1278.018
## [120] 1256.890 1236.124 1237.485 1217.007 1196.925 1198.228 1199.300
## [127] 1200.227 1201.063 1181.492 1162.317 1163.200 1143.966

a <- (obxm$PortValue10[50]-obxm$PortValue10[45])/obxm$PortValue10[43];a
## [1] 1.019401

b <- (obxm$PortValue20[50]-obxm$PortValue20[45])/obxm$PortValue20[43];b
## [1] 2.040916

c <- (obxm$PortValue30[50]-obxm$PortValue30[45])/obxm$PortValue30[43];c
## [1] 3.090409

a1 <- obxm$PortValue10[50]-obxm$PortValue10[45];a1
## [1] 1130.324

b1 <- obxm$PortValue20[50]-obxm$PortValue20[45];b1
## [1] 2265.756

c1 <- obxm$PortValue30[50]-obxm$PortValue30[45];c1
## [1] 3430.25

d1 <- obxm$PortValue10[132]-obxm$PortValue10[1];d1
## [1] 143.9659

d2 <- obxm$PortValue20[132]-obxm$PortValue20[1];d2
## [1] -287.1889

d3 <- obxm$PortValue30[132]-obxm$PortValue30[1];d3
## [1] -650.8834

```

```

d4 <- obxm$obx[132]-obxm$obx[1];d4

## [1] 1821.657

d5 <- obxm$tbill[132]-obxm$tbill[1];d5

## [1] 292.4231

PerformanceAnalytics::skewness(obxm$Port10,method=c("sample"));kurtosis(ob
xm$Port10,method="sample_excess")

## [1] 6.389573

## [1] 45.12366

PerformanceAnalytics::skewness(obxm$Port20,method=c("sample"));kurtosis(ob
xm$Port20,method="sample_excess")

## [1] 6.40893

## [1] 45.3909

PerformanceAnalytics::skewness(obxm$Port30,method=c("sample"));kurtosis(ob
xm$Port30,method="sample_excess")

## [1] 6.414926

## [1] 45.47587

PerformanceAnalytics::skewness(obxm$obxPay2,method=c("sample"));kurtosis(o
bxm$obxPay2,method="sample_excess")

## [1] -0.866056

## [1] 2.958542

PerformanceAnalytics::skewness(obxm$tbill2,method=c("sample"));kurtosis(ob
xm$tbill2,method="sample_excess")

## [1] 0.9609825

## [1] 0.2902797

DownsideDeviation(desc$Port10, MAR = tb)

```

```
## [1] 0.01085714
DownsideDeviation(desc$Port20, MAR = tb)
## [1] 0.02151245
DownsideDeviation(desc$Port30, MAR = tb)
## [1] 0.03217735
DownsideDeviation(desc$obxPay2, MAR = tb)
## [1] 0.01935007
DownsideDeviation(desc$tbill12, MAR = tb)
## [1] 0.0006738862
Kappa(desc$Port10, MAR = tb,3)
## [1] 0.1246346
Kappa(desc$Port20, MAR = tb,3)
## [1] 0.1258013
Kappa(desc$Port30, MAR = tb,3)
## [1] 0.1261553
Kappa(desc$obxPay2, MAR = tb,3)
## [1] 0.2870778
Kappa(desc$tbill12, MAR = tb,3)
## [1] 0
Kappa(desc$Port10, MAR = tb,1)+1
## [1] 1.221665
Kappa(desc$Port20, MAR = tb,1)+1
## [1] 1.227191
```

```
Kappa(desc$Port30, MAR = tb,1)+1
## [1] 1.229095

Kappa(desc$obxPay2, MAR = tb,1)+1
## [1] 2.240144

Kappa(desc$tbill12, MAR = tb,1)+1
## [1] 1

test <- subset(obxm, obxm$Name != "NA" & obxm$spread >= 0.8)
```