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Modern climate-economic models and climate policies

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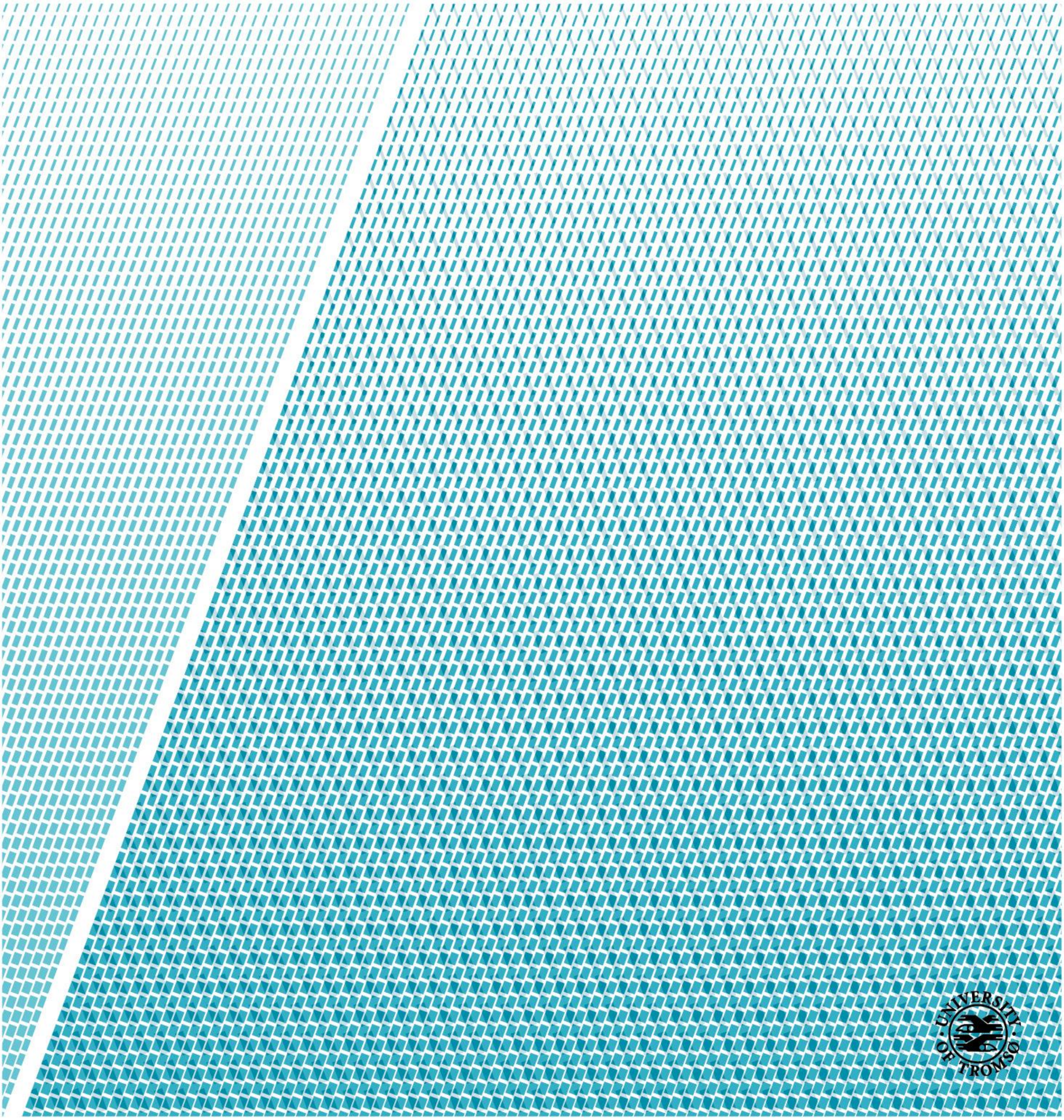

Abstract

The problem of climate change is one of the most discussed problems nowadays. The global warming has an unquestionable influence on the economic growth of the different countries, and, consequently, on the whole world economics. The climate economics thus is an actual topic to study. Moreover, it is important to predict how the climate will change over the next century and which resulting outcomes are possible.

Climate is changing both because of the natural effects and because of the human activity. Emissions of the greenhouse gases, especially the dioxide of carbon (CO_2), are considered as the main cause of climate change. The emissions, obviously, cannot be absolutely stopped right in the moment, because it will stop the economic growth as well.

The main goal of this thesis is to analyze the ways and costs of emission reduction, concepts of the Integrated Assessment Models (IAMs) and damage functions, which are crucial for creating the future emission scenarios. In this thesis we will also explore the modern climate policies and targets of those.

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Sofiia Grabovskaia

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Introduction

What is climate change

Climate change (global warming) is the observed rise in the average temperature of the Earth´s system of climate, and all the relevant effects. The fact is that the temperature and sea level have risen over the last 250 years, and snow cover has declined.

Climate change is caused by the several factors.

One of the important explanations is the greenhouse effect. It happens as following. Solar radiation goes through the atmosphere. Passing radiation is 343 watts per square meter. The net of passing solar radiation is 240 watts per square meter. Solar radiation is partly reflected by the atmosphere and the earth's surface. The reflected radiation is 103 watts per square meter. Solar energy is partly absorbed by the earth's surface and heats it. The amount of the absorbed solar energy is 168 watts per square meter. Then it is converted into heat, causing the emission of a long-wave (infrared) radiation into the atmosphere. Infrared radiation is partly reflected and absorbed back by greenhouse gas molecules. A direct effect of this is the heating of the Earth's surface and the troposphere. Other part of the infrared radiation passes through the atmosphere and goes out to space. Net of the outgoing radiation is 240 watts per square meter. The surface of the Earth gets more heat, and infrared radiation is emitted again.

The carbon dioxide is one of the main anthropogenic gases. Fossil fuels emissions highly contribute to the climate change. Other important factors are aerosols (especially sulfates), cement production, deforestation and agriculture, cattle breeding.

There are also some non-anthropogenic factors, for example, tectonic movements of plates, volcanic eruptions, changes in the Earth's orbit.

Thesis structure

There is clearly a link between the climate and economics. The climate change may have either beneficial or detrimental effect on the economics in the certain country, and in the modern world it is crucial to make predictions about possible economic outcomes related to these changes and, furthermore, take some actions.

Thus, the objectives of this thesis are:

- To explore the ways of the emission reduction and related concepts.

- To analyze the economic impacts of climate change.

- To study the modern climate policies.

The thesis is structured as following.

In Chapter 1 we review the possible emission scenarios, ways of the reduction of emissions, and abatement costs. We also explain which approaches the climate policy may use to achieve the emission reduction, and the concept of the cost-effective solution.

In the Chapter 2 we look at approaches which can be applied to project the market impacts of the climate change. We mainly concentrate on the top-down approach and explain the core and framework of it. We also explain what are the damage (impact) functions and how they are used. Then, we analyze the concept of the Integrated Assessment models on example of three models - Dynamic Integrated model of Climate and the Economy (DICE), Climate Framework for Uncertainty, Negotiation and Distribution (FUND) and Policy Analysis of the Greenhouse Effect (PAGE). Finally, we build the climate and economic models using the suitable parameters.

Chapter 3 is concerned with analysis of the climate policies. In this chapter we explain the concept of the benefit-cost analysis. Then, we discuss the most important environmental agreements, namely Kyoto protocol and Paris agreement, and the relevant complications. Finally, as an example for a certain country, we review how Norway contributes to the solution of the climate change problem and which climate policy Norway follows.

1 Emission reduction

1.1 Emission scenarios

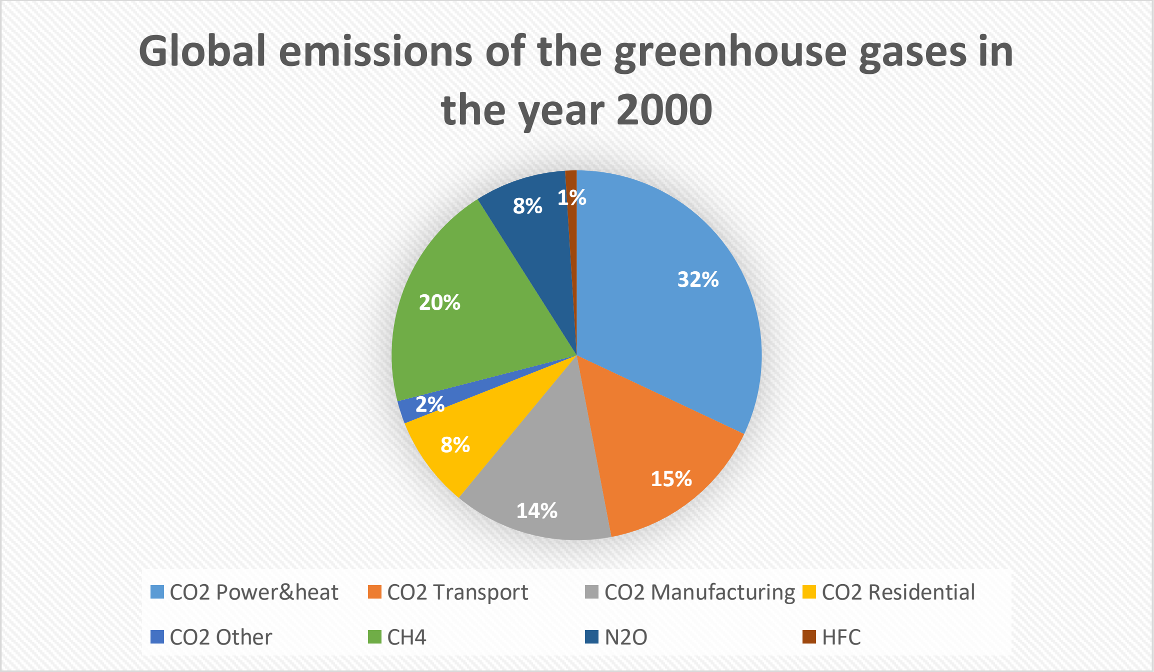
1.1.1 Greenhouse emissions and causes of these

Greenhouse gas is the gas that have high transparency in the visible range and high absorption in the far infrared range.

Figure 1-1 shows the relative contributions of different greenhouse gases in the year 2000.

Figure 1-1 Global emissions of the greenhouse gases in the year 2000.

Source: Richard S.J. Tol: "Climate Economics: Economic Analysis of Climate, Climate Change", 2014.



The global emissions of greenhouse gases and source (only for CO_2) in 2000:

Power and heat CO_2 – 32%

Transport CO_2 – 15%

Manufacturing CO_2 – 14%

Residential CO_2 – 8%

Other sources of CO_2 – 2%

⟹ Total CO_2 – 71%

CH_4 – 20%

N_2O – 8%

HFC (Hydrofluorocarbons), used in refrigerants and air conditioning - 1%

The most important anthropogenic greenhouse gas is the carbon dioxide (CO_2), and its main source is combustion of fossil fuels. We need to get energy out of the fossil fuel and we cannot do it without forming the CO_2 emissions. This is because the fossil fuel is a carbohydrate. The chemical carbon-hydrogen bond breaks when the fossil fuels are burning. Then carbon oxidizes to CO_2 and hydrogen to H2O and the energy is getting out during this process.

Another source of CO_2 is the land use change, because plants contain carbohydrates as well. In fact, the taller the tree, the more carbon is stored in the terrestrial vegetation. Many woods were burned and that is why CO_2 was formed.

One more CO_2 source is the cement production, however, it does not have such a great importance.

Methane is the influential anthropogenic greenhouse gas as well. The main source of methane are animals that eat grass and other plant-based food (for example, cows, goats etc.). These animals are called the ruminants. Meat and grass are carbohydrates, that is why these animals are forming relations with methanogenic bacteria, and the one carbon atom is sacrificed to move away four hydrogen atoms. The result is that methane is getting out. The marsupials (for example, kangaroos) are the source of acetate rather than methane, however, we cannot use these animals to get milk. This example shows that methane emissions are necessary to achieve the milk production. The meat production, in its turn, would not stop, but could be different without the methane emissions.

The high methane emissions are also the result of landfills, because a lot of organic material is stored there. However, can use incineration and composting to solve this problem. It is also possible to cap the landfill and capture the methane, and then use it to substitute the natural gas.

In fact, when natural gas is exploited and transported, methane leaks into the atmosphere. Gas is also emitted from the oil and coal exploitation too.

Another source of methane is paddy rice. CH_4 also forms, when the plant material rots without oxygen. To get less methane emissions, we need to use the alternatives to paddy rice. But, if we switch to another crops, it will reduce the production of food, because this rice is one of the most productive crops.

The third most important anthropogenic greenhouse gas is the nitrous oxide. It is difficult to reduce its emissions without affecting the production of food. This is because the main source of such emissions are agricultural soils that are treated with the nitrogenous fertilizers.

There are also some industrial (non-anthropogenic) greenhouse gases, most of them do not occur naturally. Some of these gases are products of industrial processes, some were invented for specific purposes after World War II. These artificial gases have an atmospheric lifetime that can last thousands of years, even though the volumes of such emissions are relatively small. To achieve the emission reduction in this case, some alternatives to the processes should be applied. It would be possible to achieve this through improved waste management as well.

1.1.2 The Kaya identity and its features

The following equation is named the Kaya identity.

$$M = P * (\frac{Y * E * M}{P * Y * E})$$

This identity is applied to the fossil fuel combustion's carbon dioxide. It allows us to see how the emissions can be cut and, furthermore, how to project the future emissions.

Here, M denotes emissions, P – population, Y – the Gross Domestic Product and E denotes the primary energy use.

So, we see that the emissions are equal the population (number of people) times per income per capita times energy used per unit of economic activity times the emissions per unit of energy use (intensity of carbon). The terms can be also broken down into some subcomponents. For example, it is possible to break the energy component down into fossil and non-fossil shares.

All the terms on the right-hand side of the equation above, except M, are canceled out, so we have an identity M=M. This can be also expressed in proportional growth rates, to get more use from the equation. To obtain this expression, we need to take logs on both sides of the Kaya identity and the first partial derivative with respect to time.

The growth rate of the emissions equals the growth rate of the number of people plus the growth rate of income per capita plus the growth rate of energy per unit plus the growth rate of emissions (carbon intensity).

One meaningful property of the multiplicative identity - growth rates of the components are additive. For example, CO_2 emissions related to the global energy were estimated as increased by approximately 1.7% per year since, say, the year 1850. This growth rate can be decomposed roughly into a 3% growth in gross world product (the sum of a 1% growth in population and 2% growth in income per capita) minus 1% decline (per year) in the intensity of energy of the world GDP and 0.3% decline (per year) in the intensity of carbon of primary energy.

However, there are some important caveats. At first, the four right-hand side terms of the equation should be considered neither as independent from each other nor as the main driving forces in themselves. At second, should mention that sometimes the global analysis is not that instructive, because the heterogeneity among the different populations needs to be considered as well. For instance, there are large differences between the amount of CO_2 emitted in the rich and poor countries. Some level of aggregation is important, but the heterogeneity of the emission growth is thus hidden.

The population growth in the industrial countries has been tempered, so their emissions are in line with the increases and declines in their economic activities. But for the developing countries both population and income growth are the main indicators. Often there is also a regional heterogeneity in the developing countries and it also becomes hidden in the aggregate analysis.

1.1.3 Methods for emission reduction

The Kaya identity allows to see how the emissions can be cut. Thus, either the income or the population should be reduced. Another way is to improve either the energy or the carbon efficiency.

However, there are not that many countries the governments of which are willing to do something to reduce the population. As exception, there is an example of China – this country contributes to the case above through one-child policy. Also, some African countries have murderous regimes that reduce the population.

Reducing the income means the reduced economic growth. This is probably an effective way for reducing the greenhouse emissions, however, it is obviously not generally good idea to promote the slow economic growth.

So, the only alternative left is to deal with two other components of the Kaya identity.

The energy is costly, so the emissions of the carbon dioxide have risen because of the improvement of energy efficiency. Companies that produce gadgets know that the devices that use less energy are more attractive for customers to buy.

But the improvement of efficiency does not necessarily imply that the energy use will be reduced. Sometimes the gains are rather to increase comfort than energy reducing. So, what we get is called the rebound effect. If the efficiency of energy gets better, then costs of energy get lower and then energy use gets higher.

There are, although, some alternative options that are not provided by the analysis of the Kaya identity.

The carbon dioxide is released by the land use as well. To reduce this kind of emissions, the deforestation should be slowed down or even stopped. Moreover, there are many other reasons against deforestation. For example, some forests give shelter for animals, protect coasts from wind, waves, floods. Agriculture is also useful for the conservation of soils. There were lots of trials to make a deforestation slower, however, it still happens, so these attempts appeared to be too expensive and difficult. So, deforestation will probably continue and, moreover, the climate policy can even speed it up, because the land use is needed to get more bioenergy.

The methane emissions can be significantly reduced only by cutting the production of the rice, dairy and some types of meat that causes these emissions. Almost all emissions can be reduced if the investments in this are very high. Instead of industrial gases, other types of substances could be put into practice. The only problem is that these substances can perform not that good or be much more expensive.

One more way out is to reduce emissions by geoengineering. It is aimed to slow down the climate change or even reverse it, and this can be achieved by reducing the amount of carbon dioxide in the atmosphere and by reducing the solar energy that comes to Earth. To block some solar energy, we can, for instance, put mirrors in space into practice. The special aerosols can be used to reduce the atmospheric CO_2. Geoengineering is not expensive, however, it is risky, because of the uncertainty connected with climate change – no one really knows to which consequences it may lead. Moreover, it requires constant prospective investments.

Basically, there are defined two ways to reduce the emissions. At first, use the environmentally friendly sources of energy - the sun, wind, tides, geothermal waters, etc. At second, reducing the amount of natural raw materials (oil, gas, coal) consumed. So, for example, the fuel consumed by cars can be reduced by about 10% only due to improved transmission and adjustment of engine. Elimination of the traffic jams on roads will also reduce the consumption of gas and emissions into the atmosphere.

1.1.4 Future emission scenarios

The Kaya identity allows us to project the future emissions. These are not predictions, because we cannot have any confidence in the forecasts over a century or longer. These are rather scenarios of what can happen, which are based on the independent of each other assumptions about economy, population and technologies.

Besides the number of people and their income, emission scenarios can also include the data about their age distribution, their education, urbanization etc. Based on these additional assumptions we can make effective decisions about the economic growth and energy use. For instance, if urbanization is high, people use transport more, and this has an impact on the indicator of the energy use. Emission scenarios may also include an information about how much each type of the energy source is applied. Some of the sources are, obviously, more environmentally friendly than others. Emission scenarios must include the intensity of carbon in different areas, and might also include the sectors of economy emitting greenhouse gases, for example, agricultural.

There are two types of the climate change scenarios. The first one is without the climate policy. This is still relevant for some countries, although the climate policy has been actual for nearly 20 years now. The other type of scenario implies the climate policy, and this will be discussed in the part 1.3.

1.2 The costs of abatement

1.2.1 Emission reduction costs in general

Climate policy requires more investments in savings of energy, so humans and companies need to practice more expensive energy sources instead of the regular ones. If there are no climate policy, the common technologies are available and the emissions of the greenhouse gases are free. With climate policy the emissions are not free and that explains why the emission reduction is costly. Mathematically, if we have an optimization problem, then the climate policy puts a new constraint on it.

The emission reduction is cheaper in the medium and long run than in a short one. If the objectives of the emission reduction are not strict for the first time, there are some reasons why money can be saved. To reduce the emissions, people and companies need to change their habits, behavior and technologies, which are constrained by the lasting consuming of goods and investments. If the humans and companies continue to do the same things, for example, work in the same place and use the same equipment, the emissions cannot be reduced by the carbon tax. So, this tax imposes a fine on the investments made before the climate policy have started. From economic point of view this is considered as a deadweight loss which is diminishing with the capital turnover. If the excessive costs increase, the tax on carbon can be increased as well.

One more reason why the emission reduction costs more in the short run is the change of technology. The carbon-neutral technology is still not fully developed, however, the fossil fuel technology is well-developed and continues improving. Some reserves of gas and oil can be unlocked, but it is quite expensive, despite the progress in this industry. Moreover, those sources of fossil fuels that can be accessed easily are getting empty. But the findings connected with bioenergy and solar energy still can be expected. So, the suggestion is that, after a while, the renewable sources will cost less and the fossil fuels more. Consequently, the abatement costs will fall, because these costs are the difference between the fossil fuel and renewables costs.

One more point is that the costs of emission reduction are going to decrease in the future. If the emission reduction is delayed, there will be a fall in the costs' present value. Emissions are also decomposed in the atmosphere. Normally, the climate policy set targets for a long term. If, for instance, we need to achieve the target in the year 2100, so the emissions in 2090 will play more important role in influencing what will happen in 2100, than emissions in, say, 2018. The emissions should be reduced later, because the atmospheric degradation here works as a discount rate.

1.2.2 Estimates for the costs

It is usually relatively difficult to estimate the costs of any policy and the climate policy is not an exception. The climate policy is commonly analyzed in the following way. We need to observe two hypothetical situations: see how different welfare will be with and without this policy, and calculate a cost estimate. We can study the past policy impacts, but then we evaluate only one side. So, the cost estimates must rely on some models. In fact, not all models are equally good. However, if the model is good, the cost estimate will be good as well.

The estimates of emission reduction costs are different, because every estimate relies on a certain model and because there exists a little climate policy for calibrating the models. But most studies agree that if policies are smart enough, it is possible to achieve the economy decarbonization at reasonable costs.

The existing models disagree on the costs of the emission reduction. The rate of technological change is applicable to determine the costs of the emission reduction in future. For example, there is a difference in the costs of carbon emitting energy (like oil, coal, gas) and carbon-neutral (wind, solar, nuclear energy). Emission reduction will not be expensive if solar energy costs just a little bit more than coal. The difference in costs for the present and past is known for the present and past time, but we also should make assumptions for the future. The abatement costs will, of course, be lower, if there is more progress in the carbon-neutral technology, than in the carbon-emitting technology. Because in this case solar energy is going to get cheaper faster than coal.

Different models show different points of view about the technological progress rates. Some models, for instance, assume there are opportunity costs to stimulate the technological progress in energy, other do not include these costs. Some models say that if we use the climate policy, then progress in the field of carbon-neutral technologies accelerates. However, other models do not agree with this. That is why the cost estimates differ.

The cost estimates depend on price- and substitution- elasticities. If these indicators are high and capital depreciates rapidly, the cost estimates will be lower, than those of the model which assumes that above indicators are low. If the model considers low elasticities, it assumes that the energy is measured in its carbon-intensive ways, and thus it is going to be expensive and difficult to change the course.

Also, some models assume that greenhouse gas emission will rise very fast without the climate policy, so we need to make a large effort to meet the emission targets. But other models tell us that these emissions will not rise rapidly, so it is easy to reach the emission targets.

Consider the tables that observe equilibrium and optimization models. Table 1-1 shows results for the policy scenarios of 11 different models. There are also different stabilization targets (measured in parts per million of carbon dioxide equivalent) and different approaches to the targets. The first one is the approach from above – it means that the target holds for only the final year (2100). The second approach is the one from below. It means that the target caps concentrations at all years. In most cases, the approach from below is optimal. The "above" approach has not that many constraints as "below" approach, however, there are much momentum in energy system and carbon cycle, so the actual difference is small.

Scenarios also differ by the participation of the poorer countries. In some of the models, every country begins to reduce emissions from 2015. But in other models only rich countries do it, poor countries start to reduce emissions later. This influences the estimated emission reduction costs. If a part of emissions is not counted within the abatement, another part should be reduced more, to meet the same target. The total costs will rise for sure, because of the linearity of emission reduction costs. Moreover, we can find a lot of cheaper options for emission reduction in poorer countries. This is because the economies of these countries often rely on the old technologies that are not efficient enough.

Policies in table 1-1 have different concentration targets – 650 ppm, 550 ppm and 450 ppm. The cost is higher if the concentration target is more stringent. Some models do not show results for the targets that are the most stringent. There can be different reasons for this. For instance, the model may not meet the target, because its carbon cycle or emission reduction descriptions do not allow to do this. There can also take place some political reasons – the model meets the target, but the person who worked on the model do not report the result, because of the excessive costs that are required for this.

Table 1-2 contains results for the same models as in Table 1-1, but for the marginal abatement costs. That is, the table shows how much energy prices are increased (in dollars per ton of CO_2), also the carbon tax that will be required in 2020. It is assumed that the carbon tax is imposed on all emissions of greenhouse gases in participating countries in 2015. The carbon tax increases with time, even if it was not that large in the beginning. However, the models disagree in results.

Table 1-1 Total costs of greenhouse gas emission reduction.

Source: Clarke et al.

Target	**650 ppm**		**550 ppm**				**450 ppm**			
Approach	**below**		**above**		**below**		**above**		**below**	
Non-OECD	**now**	**later**	**now**	**later**	**now**	**later**	**now**	**later**	**now**	**later**
Model 1	-0.2	0.5	4.8	6.4	5.1	7.4	36.2	78.6	54.4	X
Model 2	13.4	18.8	30.4	48.2	30.9	64.1	123.4	X	X	X
Model 3	23.8	18.9	33.9	26.3	38.0	X	56.7	X	X	X
Model 4	1.4	1.2	3.8	5.1	5.1	10.2	X	X	X	X
Model 5	15.6	17.3	29.7	X	32.7	X	X	X	X	X
Model 6	7.2	7.8	16.2	29.8	18.8	35.7	X	X	X	X
Model 7	2.2	6.5	4.4	9.1	10.9	X	11.9	X	X	X
Model 8	2.2	Na	5.9	Na	12.4	Na	27.9	X	X	X
Model 9	2.4	3.1	5.3	6.7	6.5	X	15.5	32.8	25.7	X
Model 10	13.0	12.8	44.3	59.8	44.3	59.8	X	X	X	X
Model 11	1.9	2.6	27.9	39.7	32.1	64.5	X	X	X	X

Costs (given in trillions of dollar) - the net present value of the abatement costs over the 21st century.

X - infeasible results.

OECD - Organization for Economic Co-operation and Development. The non-OECD countries begin emission reduction either now (near future) or later.

Na - Not available.

Table 1-2 Marginal costs of greenhouse gas emission reduction for 2020.

Source: Clarke et al.

Target	**650 ppm**		**550 ppm**				**450 ppm**			
Approach	**below**		**above**		**below**		**above**		**below**	
Non-OECD	**now**	**later**	**now**	**later**	**now**	**later**	**now**	**later**	**now**	**later**
Model 1	3	5	8	13	10	24	77	214	1297	X
Model 2	20	43	51	147	52	239	260	X	X	X
Model 3	14	16	27	28	27	X	28	X	X	X
Model 4	1	1	11	12	16	92	X	X	X	X
Model 5	13	27	43	X	52	X	X	X	X	X
Model 6	9	13	29	154	35	256	X	X	X	X
Model 7	6	35	7	35	26	X	15	X	X	X
Model 8	6	Na	12	Na	27	Na	70	X	X	X
Model 9	4	7	8	10	14	X	20	53	101	X
Model 10	10	11	40	67	30	67	X	X	X	X
Model 11	3	6	4	36	22	131	X	X	X	X

Marginal costs for 2020 (given in dollars per ton of CO_2 equivalent) – apply only for participating countries.

X - infeasible results.

OECD - Organization for Economic Co-operation and Development. The non-OECD countries begin emission reduction either now (near future) or later.

Na - Not available.

Table 1-3 shows how much carbon tax will add to a price of liter of different sources. It therefore does the translation from $/$tCO_2$ to specific currency (per unit of energy use).

Table 1-3 Emissions of CO_2 per unit of energy use. Increase in price that is caused by \$100/tC carbon tax.

Source: Richard S.J. Tol: “Climate Economics: Economic Analysis of Climate, Climate Change”, 2014.

Fuel	Unit	Brazil	China	Germany	France	India	Japan	UK	USA
Emissions per unit									
Petrol	kgCO_2/1	2.312	2.312	2.312	2.312	2.312	2.312	2.312	2.312
Diesel	kgCO_2/1	2.668	2.668	2.668	2.668	2.668	2.668	2.668	2.668
Gas	kgCO_2/kWh	0.184	0.184	0.184	0.184	0.184	0.184	0.184	0.184
Coal	kgCO_2/kg	2.383	2.383	2.383	2.383	2.383	2.383	2.383	2.383
Power	kgCO_2/kWh	0.076	0.794	0.451	0.097	1.239	0.437	0.487	0.544
Carbon tax^a									
Carbon tax	LC/tCO_2	64	168	21	21	1784	2715	17	27
Carbon tax	LC/tC	235	617	76	76	6540	9955	64	100
Price increase per unit^a									
Petrol	LC/1	0.148	0.389	0.048	0.048	4.123	6.276	0.040	0.063
Diesel	LC/1	0.171	0.449	0.055	0.055	4.758	7.243	0.047	0.073
Gas	LC/kWh	0.012	0.031	0.004	0.004	0.327	0.498	0.003	0.005
Coal	LC/kg	0.153	0.401	0.049	0.049	4.250	6.470	0.042	0.065
Power	LC/kWh	0.004	0.125	0.009	0,002	1.697	1.126	0.008	0,014

^{a}**LC** – local currency: Brazil – real, China – renminbi, Germany – euro, France – euro, India – rupiah, Japan – yen, UK – pound sterling, USA – dollar.

Figure 1-2 shows us the marginal costs of emission reduction which were needed to meet in 2015. Full participation, CO_2eq target – 650 ppm in 2100 according to models.

In the Figure 1-3 we can see the marginal costs of emission reduction in 2015, averaged across models. The alternative targets should be achieved in 2100. Rates of participation are different here.

Figure 1-2 Marginal costs of emission reduction in 2015 according to different models.

Source: Richard S.J. Tol: "Climate Economics: Economic Analysis of Climate, Climate Change", 2014.

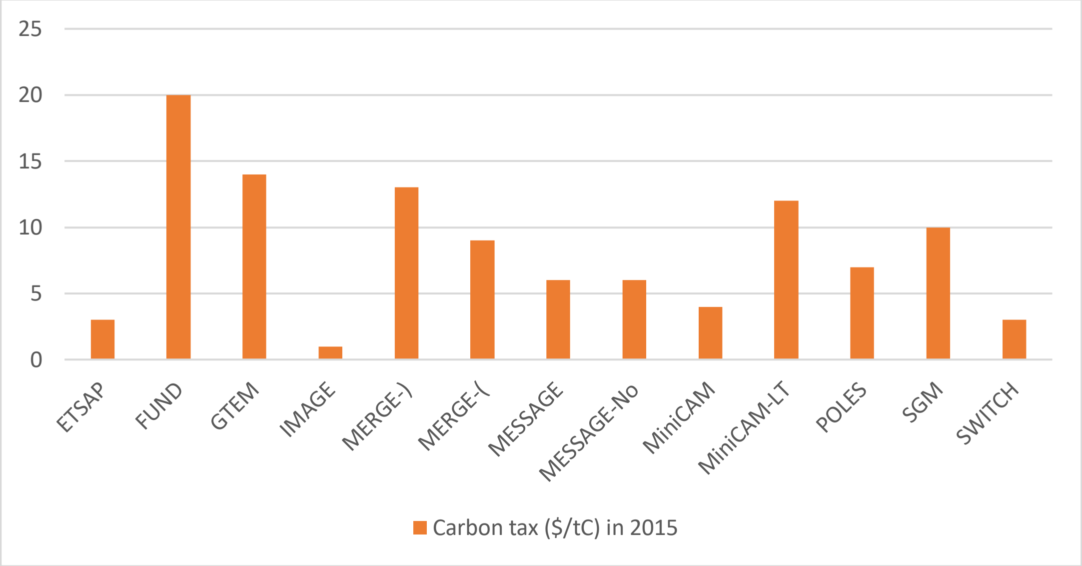
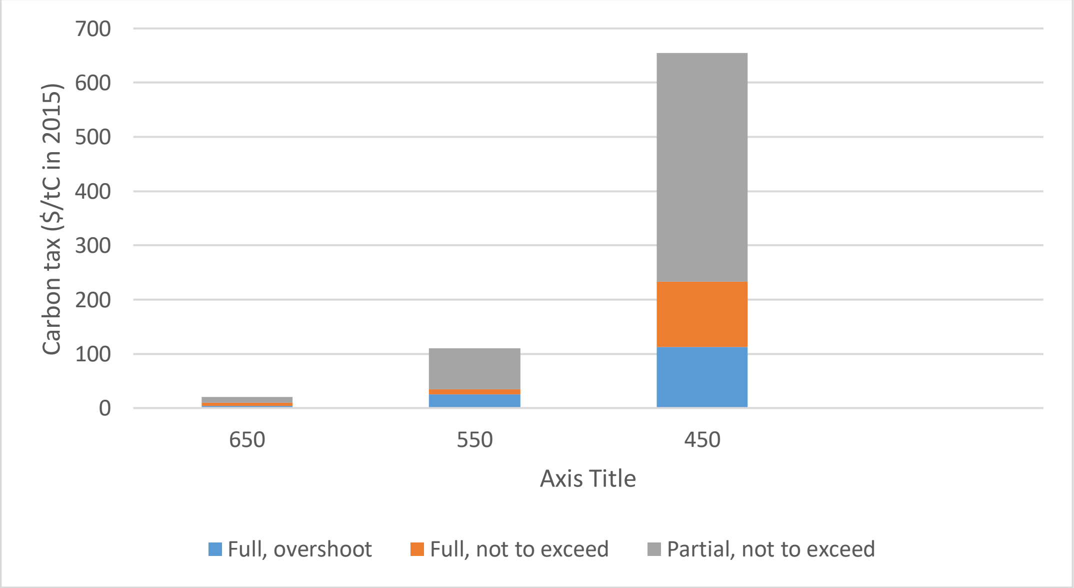


Figure 1-3 Marginal, averaged across models, costs of emission reduction in 2015.

Source: Richard S.J. Tol: "Climate Economics: Economic Analysis of Climate, Climate Change", 2014.



1.2.3 Negative costs of abatement

As a result of the imperfect accounting, there are some claims that the emissions and money can be saved in the same time, which means that the emission reduction costs are negative. The following are common mistakes. People often confuse the changes in technology in case when we have a climate policy and in case we do not. There are lots of activities within the non-climate policy scenario that lead to emission reduction and are also commercially applicable, and the efficiency of energy progresses over time. These actions do not need the policy help, because of the commerce, so it is completely wrong to relate them to climate policy.

One more mistake people make is undervaluation of the investment costs. For instance, the public interest rates are usually lower than the private ones, and that is why investment is not so attractive, however, some specialists still suppose that companies and government borrow money with identical interest rates. Also, for example, the new technologies do not have the good reputation which characterizes old and already established technologies.

Another claim is that cut of the imports of the fossil fuel will support economy, but the substitution of the cheap imported with the more expensive domestic energy leads actually to the slow economic growth. A long time ago there was a theory called mercantilism, which promoted import substitution, but it was defaced later. In the 1980s such strategies were abandoned. It was shown that such policy only increases the number of the lobbying companies rather than create the competitive enterprises. Reduced imports mean also reduced foreign investments, reduced exports etc.

In reality there are lots of imperfections and policy misinterpretations within the no-climate policy scenario. Some of these can be solved by the climate policy, but some preceding distortions can affect climate policy. In the first case the costs would decrease, but in the second one they will increase.

A carbon tax is one of the ways to carry out the climate policy. It is distortionary and affects the choices people make, so there is no Pareto optimum that can be found in a market. This gets the market to equilibrium with the lower welfare and the loss in welfare shows us how much the tax is distorted. But we can also get a revenue of the carbon tax and this revenue can be used for reducing other taxes, which are even more distortionary and take people, companies and their behavior away from the optimum. The higher is tax, the more distortionary it is. Also, the higher are price elasticities and the narrower is the tax base (number of people), the more distortionary tax is. For the carbon tax we observe low price elasticities and a large base, so it is not highly distortionary. We can get a benefit, if the revenue from the carbon tax is used to reduce another type of tax. This benefit may compensate the initial emission reduction costs.

To sum up, the revenue of the tax on carbon can bring benefits and at least partly compensate the abatement costs. This benefit may be significant if the tax reform is well-prepared to influence of some fiscal circumstances. But it is not a guarantee that any use of revenue and any tax reform can be equally beneficial, because some theoretical benefits may not be practically realized.

1.3 Emission reduction policy and its instruments

1.3.1 The public policy

According to the First Theorem of Welfare economics, the competitive equilibrium is a weakly Pareto optimum. A willing exchange is Pareto improving, because both sides are at least as well off as without any exchange. A continuance of such exchanges improves welfare and if there is no supplementary exchange that is possible to satisfy all sides, then we are in a Pareto optimum. But there should be an equilibrium on a market as well to make no more exchanges happen.

We can use the First Welfare Theorem to explain that any intervention of the government to market will be Pareto inferior. However, there are some exceptions. For example, if there are external factors, the market equilibrium will not be a Pareto optimal, because the externality is an unexpected impact that is also uncompensated. If the willing exchange of two parties accidentally harms a third one and this exchange is not cancelled, it is no longer Pareto optimal. So, the sequence of these exchanges will not take us to a Pareto optimum.

We attribute carbon dioxide to externalities, because the fossil fuel is obviously burnt to set up the electricity and drive cars, but not to emit the carbon dioxide – emissions are not intentional. The welfare of all the people is affected by the climate change, and humans cannot be compensated by the carbon dioxide emitters. The intervention of government is justified, if there are some externalities, because the welfare can be improved. The one of the interventions is called the Pigou tax (named after English economist Arthur Pigou) and does the following three things. At first, it taxes the activities that generate externalities. At second, it uses the revenue of the tax to compensate the sufferers of externality. At third, this compensation neutralizes the welfare loss at the margin.

1.3.2 The first approach: direct regulation

There are many ways for regulator to influence the emissions and each of these ways has different features, which make them more applicable to solve specific kinds of problems rather than others. The most common form of the policy is a direct regulation. It had success in the OECD countries: during the 1960-70s the environment in the North America and Europe was polluted, but now the situation changed, because the direct regulation of environmental policy did the clean-up.

Direct regulation is characterized as command-control, it means the regulator tells companies and households what to do, what not to do and how to do these things. This regulation is homogeneous, because the regulator has the capacity constraints, and everyone should be fairy treated the same way. If there is no considerable heterogeneity between the regulated, it is good.

The direct regulation can take different forms:

- Some inputs into the process of production can be forbidden by the regulator, also some input amount standards may be set up.
- Some types of technologies used to produce things may be forbidden by the regulator as well.
- The regulator can also set up the limits on some outputs of the produced or put requirements on these.

- There can be time limits or location requirements for holding the certain activities.

For example, the government can say that car engines need to meet the standards of fuel efficiency. Power plants should emit a certain Sulphur amount. Planes may land or take off all hours except the time between 12 pm and 6 am. And there are many other examples.

1.3.3 Another approach: market-based instruments

The main alternative to direct regulation are market-based instruments, the oldest of these are taxes and subsidies. A tax means a charge for every unit of the harmful substance used (or emitted). A subsidy is a financial reward for every unit of the harmful substance not used.

Both subsidies and taxes will have the same effect on emissions in a short run. In case we consider subsidy, will get a reward for every ton of emissions avoided, and in case we consider tax, the tax burden will be reduced by any ton of emissions avoided, so will get a reward as well.

The difference between taxes and subsidies is the distributional effect they have. With a tax, enterprises and households must give money to government. Subsidy means that money flow from government to enterprises and households.

That is why the effects that taxes and subsidies have on emissions in the medium run are different as well. A tax on emission increases the average costs of producing something in a certain sector. So, the emitting sector diminishes compared to how large it could be without a tax, and investment flows to another place. But a subsidy to prevent the emission reduces the average costs of producing something in this sector. The extra investment flows there, so this sector spreads (compared how large it could be without a subsidy).

Besides the taxes and subsidies there is more recent instrument that the regulator can use. This one is called tradable permits. The regulator thus can set the general limit for emissions, production or consumption. The limit for emissions is then divided into units, and a certain number of permits to emit is given to every emitter. This is a direct regulation. But if an enterprise will find out that the amount of permits it has is too small, it may buy some additional permits from an enterprise that has many.

The price for the permits to emit works the same way as a tax: for each unit of extra emissions, an enterprise must pay for an additional permit, otherwise enterprise can sell fewer of the permits it has (it costs money as well). To get a benefit, company can sell more permits or buy fewer permits in the market for every unit of emissions avoided.

The regulator does not point out how emissions are reduced, and this is the advantage of the market-based instruments. Enterprises and households decide it by themselves. The regulator only says that emissions must be cut.

1.3.4 Cost-effectiveness, static efficiency

It is important to know that the costs of the emission reduction are uniform at margin.

Consider the equation:

(1) $C = \sum_n C_n = \sum_n \alpha_n M_n + \beta_n M^2_n$

Here: C – social costs

C_n - costs per company n,

M_n - emission reduction efforts of company n,

α, β – parameters

M – desired total effort of emission reduction

So, the least-cost solution for emissions:

(2) $min_{M_n} \Sigma_n C_n$ so that $\Sigma_n M_n > M$

Next, we need to form a Lagrangian

(3) $L = \Sigma_n \alpha_n M_n + \beta_n \mu_n^2 - \lambda(\Sigma_n M_n - M)$

Then, take first partial derivative with respect to the emission reduction effort (policy instrument). Get the optimality first-order conditions:

(4) $\frac{\partial L}{\partial M_n} = \alpha_n + 2\beta_n M_n - \lambda = 0 \forall n$, so $\frac{\partial C_n}{\partial M_n} = \lambda \forall n$

So, we see that to obtain least-cost emission reduction, all emitters should have the same abatement cost at the margin. M is the shared constraint, so the shadow price of λ-constraint is set at the level of society and is equal for all the emitters.

A cost-effective solution is the solution that is supposed to meet a target and in the same time is least-cost. The optimum is cost-efficacy and the solution can be either cost-effective or not. In fact, the words "more cost effective" and "cheaper" are not the synonyms. Also, "cost-efficacy" is not the same as "cost-efficiency". Here we understand "cost-efficiency" as the dual for "production efficiency". Efficiency means quality of being efficient, producing something with minimum unnecessary effort. But efficacy is the power to produce a requested effect, this is more general concept.

Consider an enterprise that is faced with an emission tax and wants to minimize its costs:

(5) $min_{M_n} \alpha_n M_n + \beta_n M_n^2 - tM_n \forall n$

The cost function C is the same as above, but now for each unit of M (emission reduction effort), pays t less tax.

Equation (5) is an unconstrained optimization problem. Then first partial derivative must equal zero:

(6) $\alpha_n + 2\beta_n M_n - t = 0 \forall n$, so $\frac{\partial C_n}{\partial M_n} = t \forall n$

Equation (6) is the same as equation (4) if t=λ

Equation (5), if regulator uses tradable permits:

(7) $min_{M_n} \alpha_n M_n + \beta_n M_n^2 - pM_n \forall n$

Here p is the permit price.

Equation (5), if regulator uses subsidies:

(8) $min_{M_n} \alpha_n M_n + \beta_n M_n^2 + sM_n \forall n$

In this equation s is the subsidy.

We see that uniform emission tax, tradable permits with uniform price, uniform subsidy to avoid emission lead to the uniform marginal abatement costs. So, there is a guarantee for effectiveness of costs.

For the direct regulation there exists no such guarantee. To achieve the cost-efficacy in this case, the regulator needs to know the marginal abatement cost functions of all the households and enterprises. But this is not realistic, because normally everyone applies different technologies.

1.3.5 Dynamic efficiency

We have just derived the static efficiency condition – a uniform price for carbon. Now we are going to obtain the conditions for dynamic efficiency. Let us observe three alternative cases.

1. Emission reduction as a resource problem.

At first, we can consider the climate policy as a waste clearance problem. In fact, there is some disposal capacity and emissions reduce this capacity. But if we cut the emissions, it will affect output.

Do let maximize net present welfare:

$$(9)\quad \max_{C(t),E(t)} W = \int U(C(t))e^{-\rho t} \text{ for } M > 0 \text{ and } U = \underline{U} \text{ for } M \leq 0$$

Subject to

$$(10)\quad K = Y(t) - C(t) = Y\big(K(t), E(t)\big) - C(t) \text{ and } (11)\quad M = \delta M(t) - E(t)$$

Here W – net present welfare

U – Instantaneous utility

C – Consumption,

Y – Output

K – Capital

E – Emissions

M – Total emissions allowed

Then, observe the Hamiltonian for the current value:

$$H = U\big(C(t)\big) + \kappa K + \mu M = U\big(C(t)\big) + \kappa(Y\big(K(t), E(t)\big) - C(t) + \mu(\delta M(t) - E(t)) \tag{12}$$

Then, find the first order conditions:

Marginal utility equals the return on savings or the capital shadow price:

$$\frac{\partial H}{\partial C} = \frac{\partial U}{\partial C} - \kappa = 0, \text{ then } U_c = \kappa \tag{13}$$

Marginal cost of emission reduction equals the emission allowance shadow price, measured in utils and normalized by marginal utility for converting to money:

$$\frac{\partial H}{\partial E} = \kappa \frac{\partial Y}{\partial E} - \mu = 0, \text{ then } Y_E = \frac{\mu}{\kappa} \tag{14}$$

Growth rate of the capital shadow price equals the time preference pure rate minus return to capital:

$$\kappa = \rho\kappa - \frac{\partial H}{\partial K} = \rho\kappa - \kappa \frac{\partial Y}{\partial K} \text{ , then } \frac{\kappa}{\kappa} = \rho - Y_K \tag{15}$$

Growth rate of the emission allowance shadow price equals the difference between the discount rate and the rate with waste of disposal capacity added:

$$\mu = \rho\mu - \frac{\partial H}{\partial \mu} = \rho\mu - \mu\delta \text{ , then } \frac{\mu}{\mu} = \rho - \delta \tag{16}$$

So. the carbon tax should increase at the rate that is difference between the discount rate and improvement rate.

2. Emission reduction as an efficiency problem

At second, we look at climate policy as to an efficiency problem. Emissions also add to concentrations of the atmospheric greenhouse gases, and welfare depends on these. But again, cutting emissions will affect output.

Maximize the net present welfare:

$$\max_{C(t),E(t)} W = \int U\big(C(t), M(t)\big)dt \text{ , subject to} \tag{17}$$

(18) $K = Y(t) - C(t) = Y\big(K(t), E(t)\big) - C(t)$ and (19) $M = E(t) - \delta M(t)$

The Hamiltonian is:

20) $H = U\big(C(t)\big) + \kappa K + \mu M =$

$= U(C(t), M(t) + \kappa\Big(Y\big(K(t), E(t)\big) - C(t)\Big) + \mu(E(t) - \delta M(t))$

Find first-order conditions:

Marginal utility equals the return on savings or the capital shadow price, as above:

(13) $\frac{\partial H}{\partial C} = \frac{\partial U}{\partial C} - \kappa = 0$, then $U_c = \kappa$

Growth rate of the capital shadow price equals the time preference pure rate minus return to capital, as above:

(15) $\kappa = \rho\kappa - \frac{\partial H}{\partial K} = \rho\kappa - \kappa\frac{\partial Y}{\partial K}$, then $\frac{\kappa}{\kappa} = \rho - Y_K$

Marginal cost of emission reduction equals the emission allowance shadow price, measured in utils and normalized by marginal utility for converting to money. But now, the signed flipped, because we changed the interpretation of the stock equation.

(21) $\frac{\partial H}{\partial E} = \kappa\frac{\partial Y}{\partial E} + \mu = 0$, then $Y_E = -\frac{\mu}{\kappa}$

Growth rate of the emissions' shadow price is the sum of discount rate and the rate of the degradation of the atmosphere, and minus the climate change marginal damage (in utils, per concentration) over the shadow price (in utils, per emission):

(22) $\mu = \rho\mu - \frac{\partial H}{\partial M} = \rho\mu - \frac{\partial U}{\partial M} + \mu\delta$, then $\frac{\mu}{\mu} = \rho + \mu - \frac{U_M}{\mu}$

The right term is measured in emissions per concentration (rate like ρ and δ). It shows us how fast the problem of climate change gets worse.

The emissions' shadow price becomes higher if the welfare effects increase less fast, if, because of the fewer emissions, future is less problematic, and if we assumed to care less about the future.

3. Emission reduction as a cost-effectiveness problem.

Third, we will consider climate policy as a problem of a cost-effectiveness. In fact, there is an agreed upper limit on the atmospheric greenhouse gases concentration. Say, the damages above the certain point are summarily high and the damages are zero below this point. The output, again, will be affected of emissions.

Do let maximize net present welfare:

(23) $\max_{C(t),E(t)} W = \int U\big(C(t), M(t)\big)dt$ with $\frac{\partial U}{\partial M} = 0$ for $M \leq \bar{M}$ and U = $\underline{\text{U}}$

subject to:

(18) $K = Y(t) - C(t) = Y\big(K(t), E(t)\big) - C(t)$ and (19) $M = E(t) - \delta M(t)$

The Hamiltonian is the same as in the second case:

(20) $H = U\big(C(t)\big) + \kappa K + \mu M =$

$= U(C(t), M(t) + \kappa\Big(Y\big(K(t), E(t)\big) - C(t)\Big) + \mu(E(t) - \delta M(t))$

First-order conditions:

(13) $\frac{\partial H}{\partial C} = \frac{\partial U}{\partial C} - \kappa = 0$, then $U_c = \kappa$ (same as above)

(15) $\kappa = \rho\kappa - \frac{\partial H}{\partial K} = \rho\kappa - \kappa\frac{\partial Y}{\partial K}$, then $\frac{\kappa}{\kappa} = \rho - Y_K$ (same as above)

(21) $\frac{\partial H}{\partial E} = \kappa\frac{\partial Y}{\partial E} + \mu = 0$, then $Y_E = -\frac{\mu}{\kappa}$

Growth rate of the shadow price of emission allowance is the sum of the discount rate and the atmospheric degradation rate (without taking into account the marginal damage).

(24) $\dot{\mu} = \rho\mu - \frac{\partial H}{\partial M} = \rho\mu + \mu\delta \Longrightarrow \frac{\dot{\mu}}{\mu} = \rho + \delta$ for $M \leq \bar{M}$

If we compare the above results, we can see the following. If we set a constraint on concentrations, so the price of carbon is supposed to increase at the sum of the discount rate and the atmospheric degradation rate (until we meet the target). We have the same result if we observe emission of greenhouse gases as a waste clearance problem with a certain capacity. The price of carbon should increase at the rate that is equal to the difference between the rate of discount and the rate of addition to the clearance capacity. The latter is minus rate of atmospheric removal. If we need to maximize the welfare, the price of carbon should increase at the discount rate plus the rate of removal from the atmosphere, but minus the rate at which the problem of climate change becomes worse.

2 Climate and economic models

2.1 Projections of the market effects of climate change

2.1.1 Approaches

To evaluate the policies for reducing greenhouse gases emissions, we need economic models. For these models we need some estimates of how the will-being will be affected of the future change in climate. Almost all the estimates of the future warming are developed by combining several estimates from individual sectors of the economy. A warming variation over time and space has been used to get top-down estimates of how the economic output was affected by past climate shocks. The statistical framework has been used to convert the top-down estimates of past data into future projections of the global warming costs. The results tell us that future warming can reduce the expected economic growth in poor countries and increase it in richer countries.

Cost-benefit IAMs (integrated assessment models) make connections between the economy and climate, in order to find the social cost of carbon. The social cost of carbon (SCC) – the monetary estimate of all the social damages over time from an extra ton of carbon dioxide, caused by climate change. For instance, there can have place damages of the infrastructure, agricultural productivity, human health and ecosystems. The SCC is used to inform the decisions of policy.

The damage function (impact function) – is the link that translates future warming into the economic results. When modelers derive this relation, they assume that cumulative warming reduces economic output, and that functional form relates this output loss to global mean temperature of surface. They derive this relation by calibrating this function to estimates of impacts in several economic sectors at low to moderate the warming levels. We will return to the concept of damage functions in part 2.2 of this thesis.

The first approach is the ‘Bottom-up’ approach and it assumes constructing a damage function from sectoral estimates of climate impacts. However, some work nowadays has shown that basic assumptions about the form of the function are very important to policy evaluations. Some economists think that integrated assessment models have uncertain underpinnings, and not sure that these models are relevant to policy.

A new ‘Top-down’ approach is microeconomic-based. This approach constructs an impact (damage) function from historical relationships between climate and economy plus climate models’ simulations of the future outcomes. The history of warming is limited, so it prevents us from estimating those economic replies that are nonlinear. The economic results are linked with global mean surface temperature by using physical climate models, to quicken the spatially heterogeneous implications of global climate change in future.

2.1.2 The core of the top-down approach

There is a multidisciplinary framework, which converts historical estimates into probability distributions for economic effects of the future change of climate.

This framework is the following (see Figure 2-1):

A. Time series of climate variables, economic variables and population variables by country, over the latter half of the XX century.
B. Physical projections of future temperature and precipitation for the climate models.
C. Benchmark socioeconomic projections for economic variables and population variables.
D. Probability distributions for the climate impacts in the future (in each country).
E. Ethical criteria, which might have an impact.
F. Mean global temperature for each time step.
G. Regional impacts. Probability distributions for the parameters that tell us how average global growth and its year-to-year variance change with global warming.

In the Figure 2-1 colored boxes are inputs (each color corresponds to each source), and white boxes are outputs (results calculated with help of the framework). Solid borders frame the input-boxes with the future variables projections, dotted borders – the input-boxes with the past data, dashed borders – the input-boxes with the preference (ethical) parameters.

Figure 2-1 Methodology for calculating regional impacts of climate change.

Source: Derek Lemoine & Sarah Kapnick: “A top-down approach to projecting market impacts of climate change” (2015).

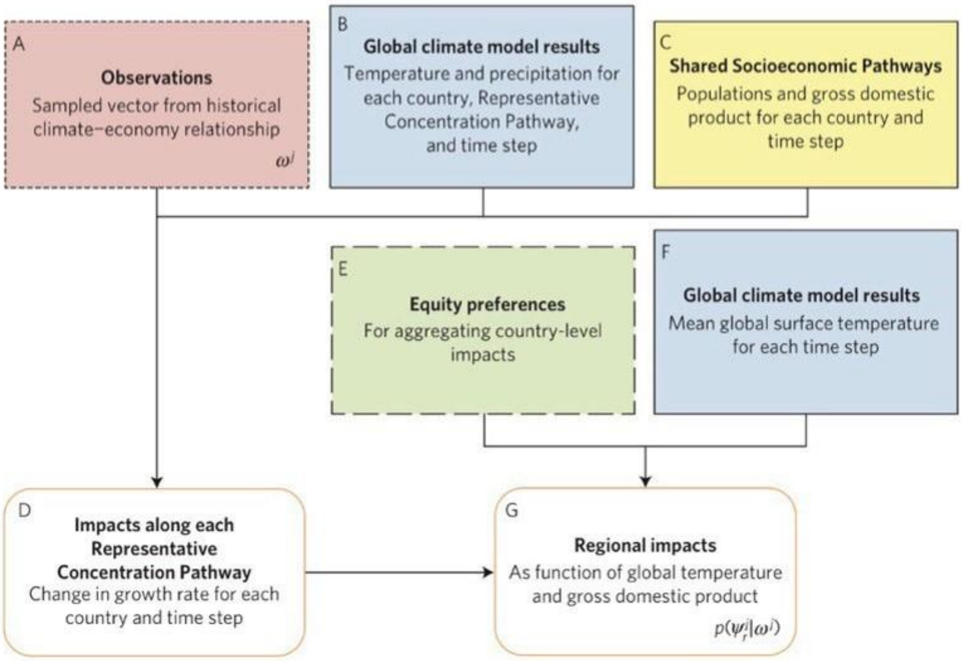


Figure 2-2 a, b shows the expected value of each country’s distribution for the:

a) Medium-run effects of global warming on the average economic growth of each country.

b) Short-run effects of global warming on the year-to-year variance of the economic growth of each country. Those are calculated holding population and GDP fixed at year 2010 values. Strongly positive values in case a) signify that warming increases the average growth and in case b) that warming increases the variance of growth.

Figure 2-2 c, d, in its turn, shows the standardized variables produced by dividing each country’s expected value by its standard deviation in c) Medium run and d) Short run Values greater than 1/ less than -1 in figure assume that the main part of the estimated distributions is, respectively, above/ below zero.

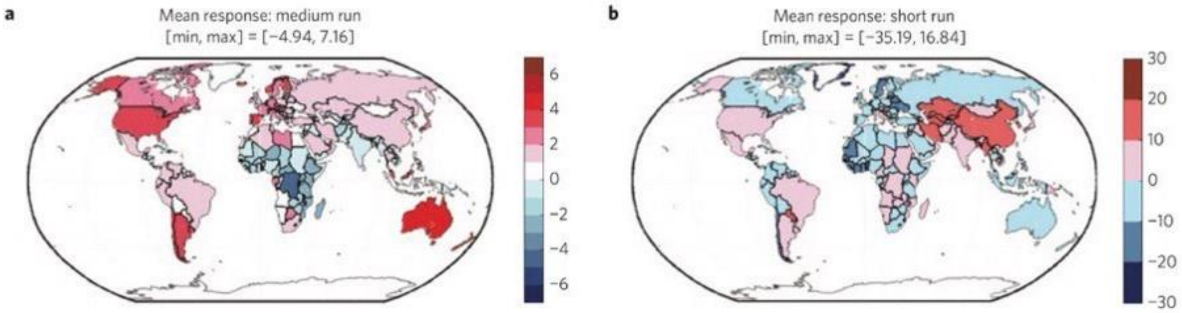
So, if we hold income fixed at year 2010 GDP per capita, the results are the following.

In a short run, a degree of total warming around the globe contributes to increasing of the growth variability in many countries of central and eastern Asia by 10-20%, in many countries of America and central Africa by up to 10%. An additional degree of global mean temperature of surface contributes to decreasing of the growth variability in West Africa by 10-20%, in Mediterranean countries, typically, by up to 10%. In the short run, the precipitation in different countries affects the variability of growth as much as the temperature does.

In a medium run, the additional degree of warming over the course of ten years contributes to the 1-3 % growth increase in much of the world. The chance of the negative impacts- climate damages is often nearly 1-2 standard deviations below the expected value. There are, however, some exceptions. An extra degree of warming is likely to reduce the growth up to 2% in many parts of sub-Saharan Africa and south Asia. In the medium run, there is an interaction between temperature and GDP per capita. It means the economies of richer countries can benefit from warming, but poorer countries' economies are damaged.
In contrast to short-run case, the precipitation does not affect the growth variability as strong as temperature does.

Figure 2-2 Effects of climate on economic growth through changes in averages and variability.

Source: Derek Lemoine & Sarah Kapnick: "A top-down approach to projecting market impacts of climate change" (2015).



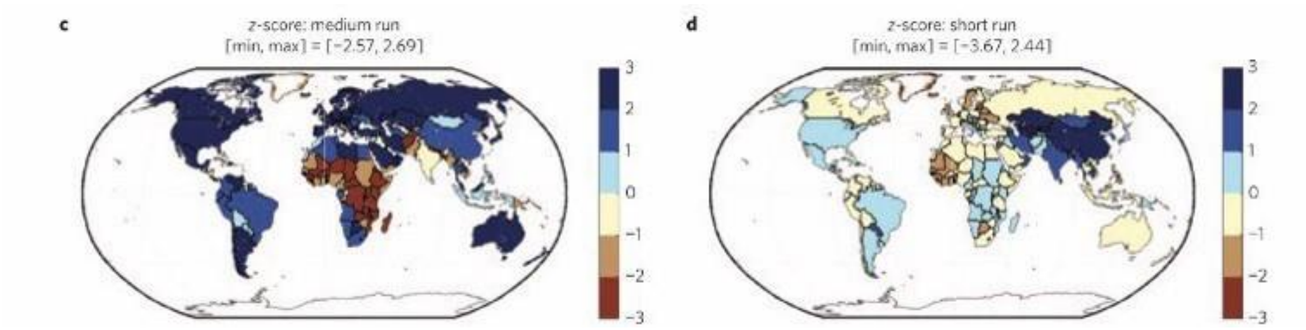


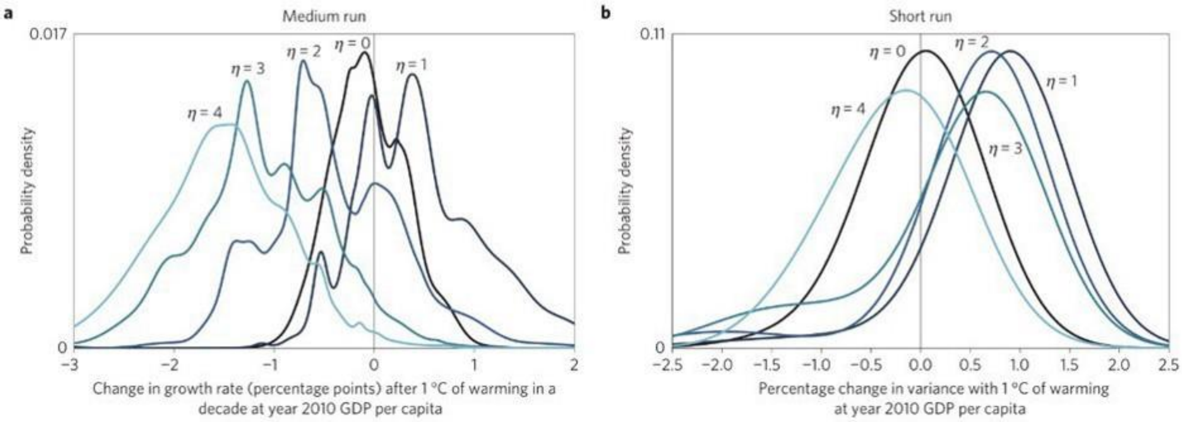
Figure 2-3 depicts the estimated distributions for those parameters that link global growth of output to global warming using η - a value that displays inequality aversion.
Figure 2-3a shows the marginal distribution for the warming effect at year 2010 global GDP per capita (medium run). Figure 2-3b shows the warming effects on the variability of the growth of global output at year 2010 GDP per capita (short run). One suggestion is that preferences over consumption inequality have a strong impact on the estimated global relations.

We can see a slight decline in growth variability for the extreme η values in the short run. This reduction corresponds to the negative effects on variability in many countries. There is nearly 0.5-1% increase in variability for more common values of η. The effects that climate change has on the growth variability are lighten if the regions insure one another (directly or through trade).

Now let us consider the medium-run case. If there are no equality weighting (η=0), the effects are concentrated around zero, balancing the strain between the slower growth of the poor countries and the faster growth of the rich ones. If η=1, have the log utility, and extra warming is probably advantageous at year 2010 global per capita income. If η=2, the extra warming can be detrimental. The suggestion here is that additional degree of warming diminishes global growth by approximately a full percentage point. See that larger values of η lead to the detrimental effects. The interaction term becomes smaller as η increases, saying that future growth cannot quickly convert the warming from a harmful to a beneficial for η>=2.

Figure 2-3 Distribution of the growth rate change as a function of inequality aversion.

Source: Derek Lemoine & Sarah Kapnick: “A top-down approach to projecting market impacts of climate change” (2015).



2.1.3 Methods for statistical framework of the top-down approach

Now, let us consider the methodology in the more precise way to understand how the above results have been found.

The GDP per capita input is the data from the World Bank’s data set (constant-dollars, purchasing power parity-adjusted) in year 2015-dollars. The set is then rescaled to year 2000-dollars. The current-dollars set is used to match the historical regression units. The population in year 2010 comes from the World Bank data set as well. Initial log GDP is 9.0642 per capita.

For a short run, to not to merge intermodal variability with inter annual one, there is used a population-weighted precipitation and temperature from five simulations of the model called NOAA-GFDL CM2.5, following the RCP8.5 pathway.

For a medium run, is used each country's population-weighted precipitation and temperature from 17 global climate models from IPCC (Intergovernmental Panel on Climate Change) Fifth Assessment report.

One more aspect is finding the distributions for the ψ coefficients in the following two relations, which describe the future impacts. These are not the regression equations which can be applied to the data from past.

$$I_r^M\left(\Delta T_t^g\right) = \psi_{r,T}^M \Delta T_t^g + \psi_{r,Ty}^M \Delta T_t^g \ln\left(\frac{y_{rt}}{y_{r0}}\right)$$

$$var\left(I_r^S\left(T_t^g\right)\right) = \exp\left[\psi_{r,T}^S T_t^g + \psi_{r,Ty}^S T_t^g \ln\left(\frac{y_{rt}}{y_{r0}}\right)\right]$$

I_r^M and I_r^S denote the changes in growth rates in region r due to time t in the medium- and short run.

T_t^g is global mean surface temperature.

y_{rt} is the economic output (in per-capita GDP, conditional on the log change) between the initial time and time t.

ΔT_t^g is the change in global mean temperature of surface between t −1 and t times.

y_{rt} is per-capita GDP in region r at time t.

y_{r0} is per-capita GDP in region r in the initial period.

The region impacts are not projected as a function of regional climate, but as a function of global mean temperature of surface. This is because the impact function here is supposed to be applicable for the climate-economy integrated assessment models and these models typically simulate only one temperature index.

Coefficient $\psi_{r,T}^{M}$ denotes the effect of one degree increase in decadal global mean temperature on growth in a medium run. Coefficient $\psi_{r,T}^{S}$, in its turn, shows the effect of a one degree increase in global mean temperature on the variance of the growth in a short run.
$\psi_{r,Ty}^{M}$ and $\psi_{r,Ty}^{S}$ are the coefficients that show how the temperature connects with a nearly 2.7-fold increase in GDP per capita.

To get probability distributions for every vector of ψ_r^j- coefficients in the equations above, use the following relation, namely the law of conditional probability.

$$p(\psi_r^j) = \int p(\psi_r^j \,|\omega^j)p(\omega^j)d\omega^j$$

Here $p(\omega^j)$ is probability of any set of historical relationships, which is denoted by the vector ω^j. If we combine $p(\omega^j)$ with conditional probability $p(\psi_r^j|\omega^j)$, can then calculate the unconditioned distribution for ψ_r^j. This distribution, in its turn, contains the economic uncertainty about historical relations between climate and economy via $p(\omega^j)$ and scientific uncertainty via $p(\psi_r^j|\omega^j)$: how the global mean surface temperature will relate to climatic outcomes in a country-level in the future.

The conditional probability $p(\psi_r^j|\omega^j)$ is calculated by combining the physical simulations of climate and socioeconomic projections to get the information about spatially heterogeneous implications of the change in the global temperature and uncertainty connected with those implications.

As discussed above, the box A in a picture 2 contains a sampled vector ω^j, box B – temperature and precipitation simulations for the physical climate models, box C – projections for GDP and population from the newly developed Shared Socioeconomic Pathways. Combining the three boxes we will get the box D – projected impacts for every country at each year (short-run case) or at every 10 years (medium-run case).

The country-level impacts for the short run and medium run are calculated in different ways.

For estimating the short-run effects of changing the weather variability, use forecast errors of actual changes in precipitation and temperature. The assumption is that unexpected weather shocks are harmful for agents [Dell, M., Jones, B. F. & Olken, B. A. Temperature shocks and economic growth: Evidence from the last half century. Am. Econ. J. 4, 66–95 (2012)].
It is also assumed that agents accurately predict the global mean surface temperature for the next year, this is done to separate uncertainty about the warming in future from weather that is unexpected conditional on global warming.

For estimating the effects of the change in average weather outcomes in a medium run, need to make a translation of every sampled vector ω^j to a sampled impacts trajectory. This is calculated by multiplying vector ω^j by changes in average precipitation and temperature of every time step and by interactions between this changes and log GDP per capita.

The final calculations are aimed to get the conditional probability $p(\psi_r^j|\omega^j)$ and converting the impacts on country level into regional impacts and then estimating these regional impacts as a function of global mean temperature of the surface. The regions of interest can include more then only one country. In this case the country-level impacts are aggregated using the function of social welfare that can show aversion to not equal consuming over space (box E). The parameter η shows the degree of inequality aversion in standard integrated assessment models. To aggregate consuming over time in these models there, used the same type of power utility function of social welfare. The value of the parameter η is alter between 0 (no inequality aversion at all) and 4 (high inequality aversion). Standard integrated assessment models work with values between 1 and 2. Values between 2 and 4 are considered as reasonable as well.

The parameter vector ψ_r^j is estimated such that it most fits a sampled vector ω^j. To get this, the simulated regional impacts are aggregated with the simulations of global mean temperature of surface in global climate models (box F). There coefficients and standard errors that are result of this estimation determine the distribution for the desired impact coefficient ψ_r^j for every region (box G). This impact coefficient, in its turn, shows the conditional probability $p(\psi_r^j|\omega^j)$ of any value of ψ_r^j, if the sampled value of ω^j is given.

This conditional probability catches the uncertainty, namely how the global warming affects the precipitation and temperature on the country level. $p(\omega^j)$ captures the uncertainty about the relations between future climate on the country level and growth. $p(\omega^j)$ shows variability in data of the late XX century, but does not show the uncertainty about how the historical relations can change if the warming is moved to the higher level. If $p(\psi_r^j|\omega^j)$ and $p(\omega^j)$ are aggregated, is creates a distribution for the coefficients in the relation characterizing impacts for region r as a global temperature function. The uncertainty about historical relations between the climate (on the country level) and economy is displayed there, as well as the uncertainty about the future relations between the climate (on the country level) and global warming.

2.2 Measuring the economic risks of the climate change

2.2.1 Damage functions

The aim of the damage (impact) functions is to specify simplified relations between such climate variables as change in temperature and losses in economy.

It is important to estimate the economic damages resulted by climate change. This knowledge tells about relationship between the benefits of reducing the greenhouse gas emissions and costs. It also shows the value that is spent on moderation of climate relative to other social financing.

The application of the social cost of carbon (SCC) have been increased, and the analysis of the different modelling approaches, especially integrated assessment models (IAMs), has become more popular. Integrated assessment models show the main components of the Earth and human system and are aimed to monetize impacts of climate.

The IAMs represent the damages of climate change through the damage function. This function relates temperature, rise of the sea level, concentrations of the carbon dioxide and other climate variables to economic welfare. The damage function can have a simple form, however, it is able to parametrize complex socioeconomic and physical relations to combine the net effects of climate change in a certain region or sector.

Figure 2-4 represents complex series of socioeconomic and physical processes and relationships contained by damage function.

The stages on the figure 1 are used to determine the damages. Here, Δ is the change in parameter. 1 – is the biophysical sensitivity to driver of climate, 2 – is the adaptation effectiveness, 3 – is the effects of the general equilibrium, 4 – is the economic preferences.

Figure 2-5 represents an example for the agriculture sector.

Figure 2-4 Processes contained by the damage function.

Source: Delavane Diaz and Frances Moore: "Quantifying the economic risks of climate change" (2017).

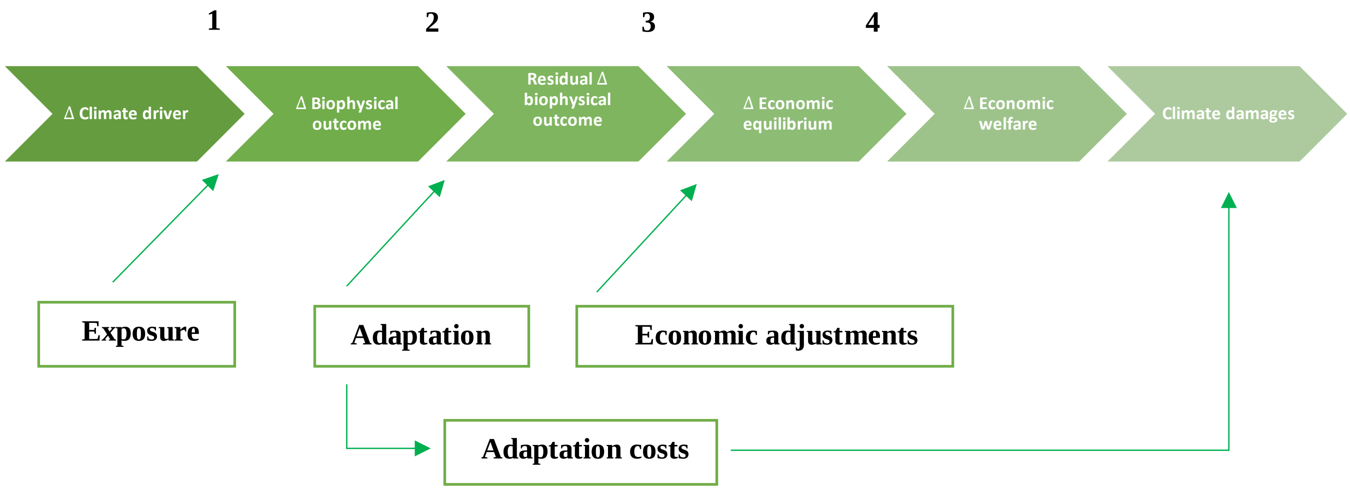
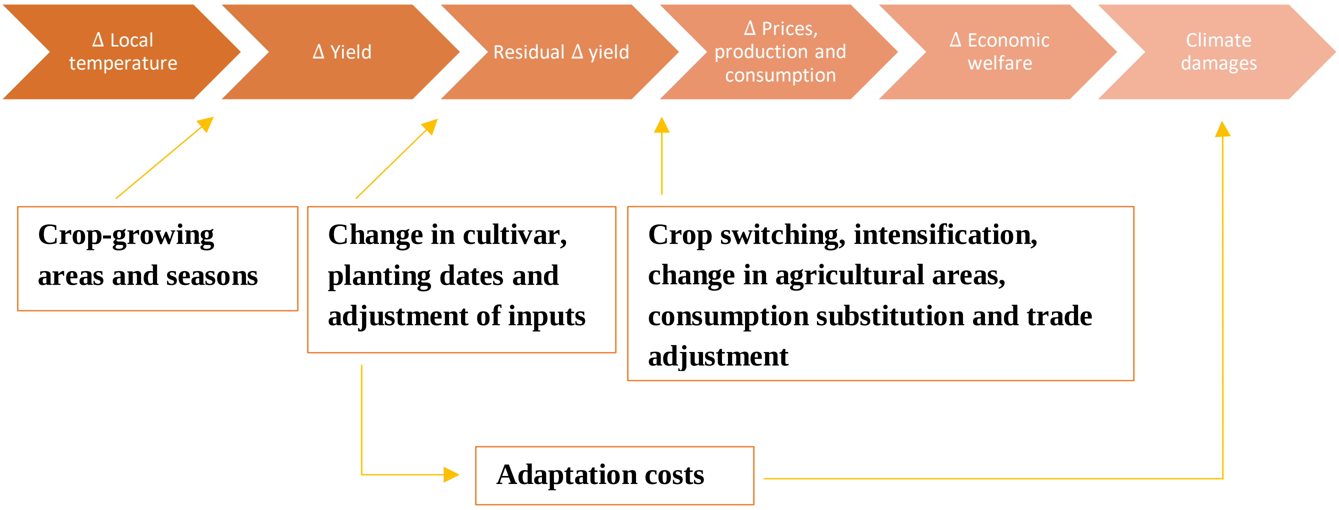


Figure 2-5 Processes contained by the damage function, example for the agriculture sector.

Source: Delavane Diaz and Frances Moore: “Quantifying the economic risks of climate change” (2017).



The important point is that there are criteria that the studies of the damage functions should ideally meet. These are the following.

1. Should use a general framework that have consistent assumptions about changes in technology, economic and population growth.
2. Should present impacts with respect to some physical driver of climate, for instance SLR, change in temperature, carbon dioxide concentration, socioeconomics, rather than (or in addition to) time-courses that are based on a specific scenario of emissions or RCP (representative concentration pathway).
3. Should have the global coverage.
4. Should combine the effects of climate change’ benefits and costs in a specific sector.
5. Should look at interactions between regions and sectors.
6. Should involve benefits and costs of the cost-effective adaptations.

7. If possible, should consider biophysical impacts' general-equilibrium or partial-equilibrium economic regulations.
8. Should be shown in economic units (ideally as changes in welfare).
9. Should measure the uncertainty in impacts.

2.2.2 DICE, FUND and PAGE Integrated Assessment Models

Now let us concentrate on the damage functions in the IAMs used by the government of US to estimate the social cost of carbon. Consider three models: Dynamic Integrated model of Climate and the Economy (DICE), Climate Framework for Uncertainty, Negotiation and Distribution (FUND) and Policy Analysis of the Greenhouse Effect (PAGE).

The DICE model is normally run as an intertemporal optimization with objective function that is nonlinear. So, this model chooses several decision variables to maximize the welfare of society over some horizon of time, usually a century or more. DICE forecast most damages in the non-SLR (non-sea level rise) classification. Sea level rise contributes to less than one fifth of all the damages over this century.

FUND and PAGE models typically do the analysis of parametric uncertainty and use a lot of runs (tens of thousands) in a Monte Carlo simulation.

The FUND model forecasts important net advantages from increase of the agricultural productivity and avoided costs on energy (as a result of diminished heating demand). The main detrimental effects are caused by damages of water resources and increasing cooling costs.

According the PAGE model, damages at low warming levels are typically caused by sea level rise and economic damages, however, the economic damages can be prevented by higher ability of adaptation. If the threshold of 3 degrees is crossed, the damages are discontinuous. The adaptation costs in this model are small, constant and insensitive to temperature.

The characteristics of the DICE, FUND and PAGE are summed up in Table 2-1. These characteristics define which damage outcome the model will forecast for the same climate and socioeconomic conditions.

Data presented in the Table 2-1 is for the three IAMs7–9 that are used to determine the newest US government SCC estimates.

Table 2-1 Characteristics of DICE, FUND and PAGE models.

Source: Delavane Diaz and Frances Moore: “Quantifying the economic risks of climate change” (2017).

Adopted from: Rose, S. K., Diaz, D. B. & Blanford, G. J. Understanding the social cost of carbon: a model diagnostic and inter-comparison study. Clim. Chang. Econ.
8, 1750009 (2017).

Model details	**Dynamic Integrated model of Climate and the Economy (DICE2010)**	**Climate Framework for Uncertainty, Negotiation and Distribution (FUND v.3.8)**	**Policy Analysis of the Greenhouse Effect (PAGE09)**
Regions	One region (world)	Sixteen regions	Eight regions
Sectors	Two sectors Market: SLR, aggregate non-SLR Non-market: aggregate non-SLR	Fourteen sectors Market: SLR, agriculture, forests, heating, cooling, water resources, tropical and extratropical storm damages Non-market: biodiversity, cardiovascular/ respiratory, vector-borne diseases,	Four sectors Market: SLR, economic, discontinuity (for example, abrupt change or catastrophe) Non-market: non-economic

		diarrhoea, morbidity, tropical and extratropical storm deaths, migration	
Damage functional form	Estimates damages *D* as a percent loss of global GDP Quadratic function of climate variable, for example: D = Estimates damages *D* as a percent loss of global GDP Quadratic function of climate variable, for example: $\mathbf{D = \delta 1\ \Delta T + \delta q\ \Delta T^2}$ Where δl and δq are linear and quadratic damage coefficients and Δ*T* is temperature change	Estimates damages *D* as a per cent change in regional productivity. Uniquely formulated by sector, with damage function *f* scaled by a dynamic vulnerability term, for example: $\mathbf{D = f\ (\Delta T^{x})\ (YPC_t / YPC_0)^{-\varepsilon}}$ Where *x* is the climate variable exponent, YPC is per capita income, *t* and 0 are the current and reference time periods, and ε is income elasticity	Estimates residual damages *D* after adaptation as a percent loss of regional GDP. Power function of residual climate variable plus adaptation costs *C*, for example: $\mathbf{D = \delta\Delta(T_r - T_{adapt})^x + C_{adapt}}$
Climate variable	Global mean temperature change, global mean SLR	Global mean or regional temperature change	Regional mean temperature change, global mean SLR

		(all), rate of warming (agriculture), CO_2 concentrations (agriculture, forestry, storms), global mean sea-level change (SLR), ocean temperature (storms)	
Socioeconomic drivers	Global income	Population, income, per capita income, population density, technological change, production cost, land value 'Dynamic vulnerability' allows climate resiliency or exposure to change over time in response to income growth or technological change	Income, per capita income, regional adaptation capacity and costs, regional scaling factor relative to European Union, modest equity weights
Calibration	DICE2010 is loosely calibrated to the IPCC and a metaanalysis of net damages for	Calibrated to sector-specific impact studies,	Economic and non-economic calibrated to a review of damage estimates for 3 °C (from

	1–3 °C via RICE-2010	mostly published between 1992 and 1998	four IAMs, including DICE, FUND and PAGE)
Adaptation	Implicit: calibrated to estimates of damages net of adaptation	Explicit: agriculture includes lagged rate component that fades with adaptation, SLR assumes cost-effective adaptation with sea-walls or retreat Implicit otherwise: calibrated to econometric studies of net response to warming	Explicit: two types of exogenous adaptation, modelled as fixed regional policies (constant regardless of climate change and socioeconomics) that reduce impacts for a cost
Uncertainty representation	Deterministic design	Probabilistic design represents parametric uncertainty with thin-tailed (for example normal) distributions	Probabilistic design represents parametric uncertainty with triangular distributions
Catastrophic risk	Implicit: net damage includes the expected value of catastrophic loss	Potential for catastrophic outcome through tails	‘Discontinuity’ impact occurs with a positive probability linked to temperatures over 3°C

	per Nordhaus expert survey	of parameter distributions	

We see that the DICE model has more simple form of the damage function and less input variables, compared to other two models. In the part 2.3 we will build the economic model of the DICE-type.

2.3 Building climate and economic models

Now we are going to see how the climate-economic modelling work.

To plot a climate model's run that is going to be used to determine the suitable parameters, we take data from the IPCC. This allows us to build the relation between the time (measured in years) and change in temperature (measured in degrees Celsius), Figure 2-6a. See Appendix.

The following expression allows us to fit the data into the curve. The time t range is between 0 and 5000 years.

$$(1)\quad \frac{\tau_1}{c_1} * \left(1 - \exp\left[\frac{-t}{\tau_1}\right]\right) + \frac{\tau_2}{c_2} * \left(1 - \exp\left[\frac{-t}{\tau_2}\right]\right) + \frac{\tau_3}{c_3} * \left(1 - \exp\left[\frac{-t}{\tau_3}\right]\right), 0 \ll t \ll 5000$$

We determine the suitable parameters as:

$c_1 = 3.1$

$c_2 = 400$

$c_3 = 420$

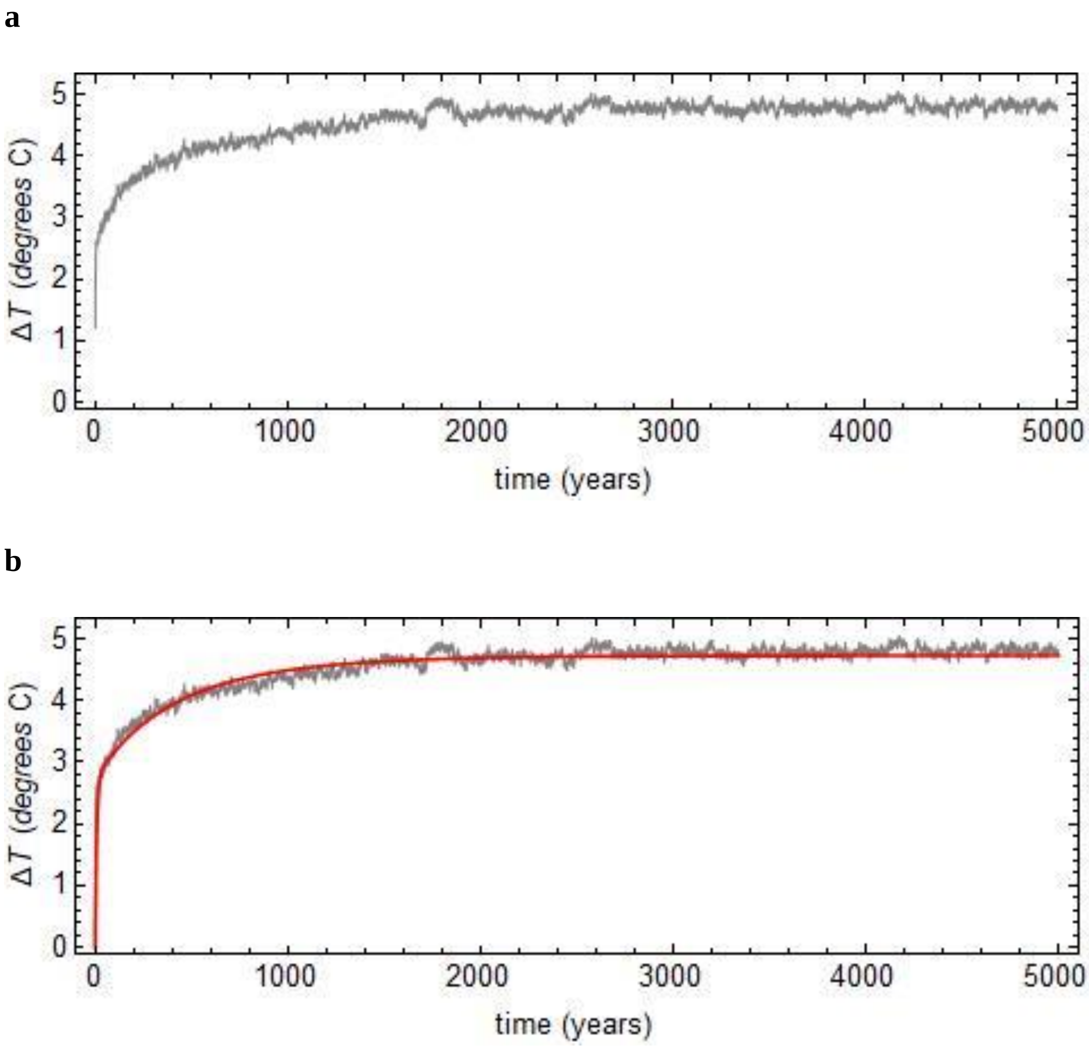
$\tau_1 = 8.5$

$\tau_2 = 261$

$\tau_3 = 560$

Inserting those into the (1) helps us to plot the red curve in the Figure 2-6b. See Appendix.

Figure 2-6 Relation between the time and temperature change without and with the fitted parameters.



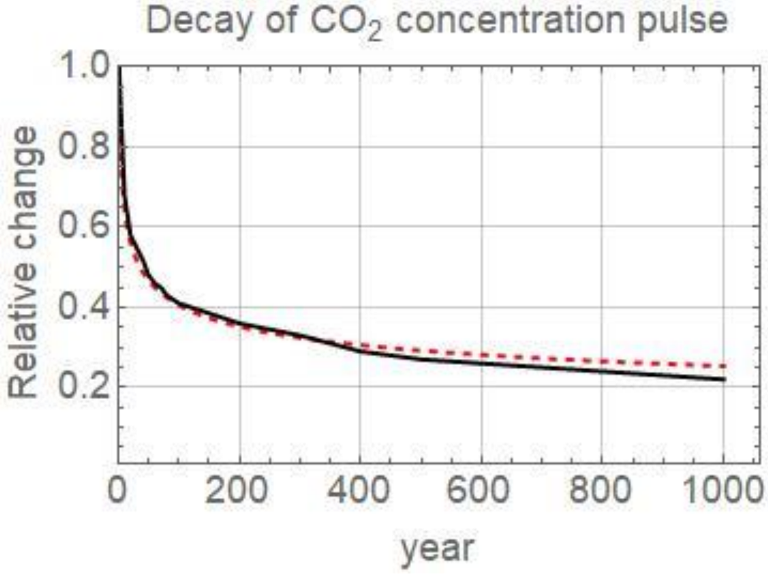
The next important thing to consider is the emissions of carbon dioxide and the response of the CO_2 concentration on the changes in emissions. To plot the mean-response curve, we use a data source (Figure 1 in Joos et al. (2013)). This curve is the black one on the Figure 2-7.

Now we are going to model the breakdown of the CO_2. The value of x in the following expression is ranged between 0 and 1000 years.

$$1.0394376335892603^{`} * x^{-0.20426475506502825^{`}}, 0 \ll x \ll 1000 \tag{2}$$

The red dashed curve on the Figure 2-7 shows the result. We see that if, for instance, the relative change in the emissions is equal 0.2, the decay of CO_2 concentration will take around 1000 years.

Figure 2-7 Breakdown of the CO_2.



Next, we construct the scenarios for emissions. Let the start point be the year 1880. In this year the time t is equal zero. Here, t_1 will denote the time when some action is taken. $S(t)$ is the emissions rate at time t. In the year 1880, $S(t) = S(0) = S_0$. Besides t, t_1, and S_0, the emissions rate $S(t)$ is depending on the values of two more parameters, namely g - the exponential growth rate of the scenario without mitigations, and m, the exponential rate of mitigation.

$$S[t, t_1, S_0, g, m] \tag{3}$$

We fit the following function through the two points: (80,4) and (130,11)

$$S(t) = S_0 * \exp[g * t] \tag{4}$$

So, S_0 = 0.8 GtC/yr and g = 0.02 1/yr (the emissions growth is about 2% per year).

We will look at three types of emission scenarios:

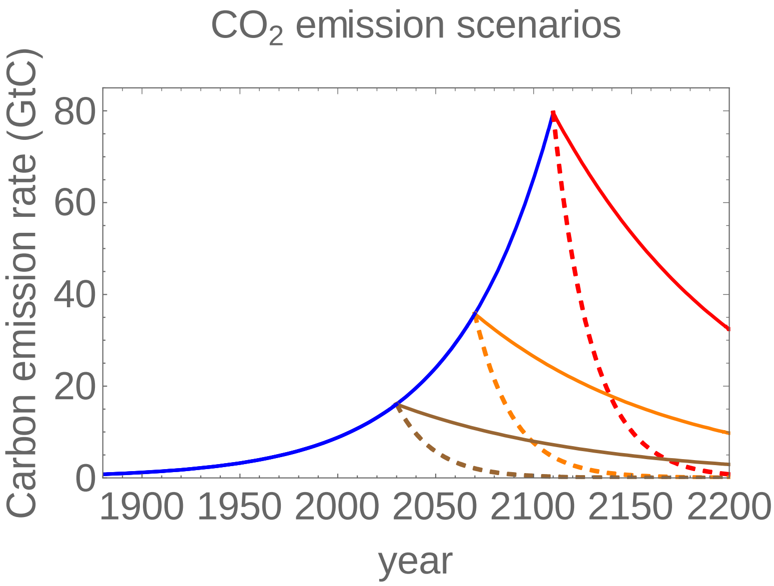
1. $t_1 = 150$, the year of the action is thus 2030.
2. $t_1 = 190$, the year of the action is 2070.

3. $t_1 = 230$, the year of the action is 2110.

For each scenario we apply two different rates of mitigation: m = 1% and m = 5%. This rate shows by how much the emissions are reduced. So, basically, we have six different scenarios. We plot the emission scenarios and the curve of the historical emissions (see Appendix).

The brown curves in the Figure 2-8 are the type one of the emission scenarios, the orange curves – type two, the red curves – type three. The solid lines are the scenarios with rate m = 1% and the dashed lines are the scenarios with m = 5%. The blue curve is the curve of the historical emissions.

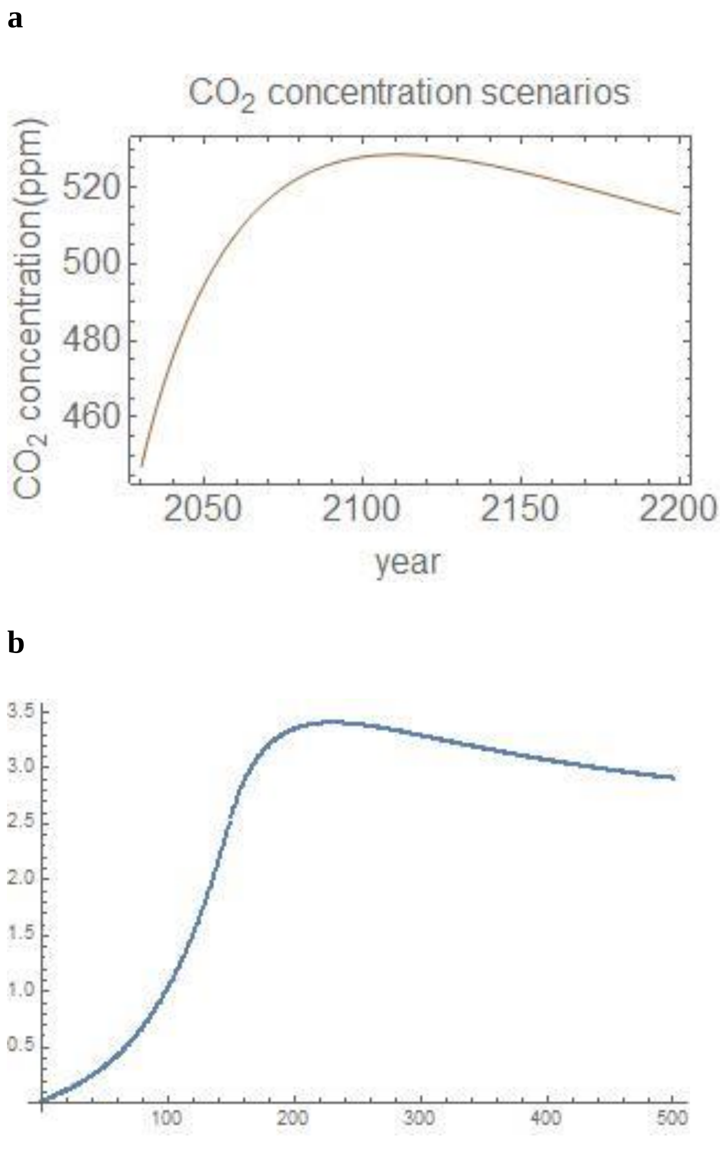
Figure 2-8 Emission scenarios and historical emissions.



The earlier we start to make actions, the faster the carbon emissions rate will approach zero. The peak of the emissions will be, obviously, in the year 2110, if we will not take any actions before this time point. In this case, the carbon emissions rate will be approximately equal 80.

Then, we plot the curve for the concentration scenarios for CO$_2$, within the time range from 2030 to 2200 (see Appendix). On the Figure 2-9a we see that the CO$_2$ concentration in the year 2110 (maximum point on the graph) is equal almost 530 ppm. Figure 2-9b shows the climate forcing corresponding to this scenario. On this graph we can also see that the peak point is in 230 years from the start point (1880), so the maximum corresponds to year 2110.

Figure 2-9 CO₂ concentration scenarios and climate forcing.



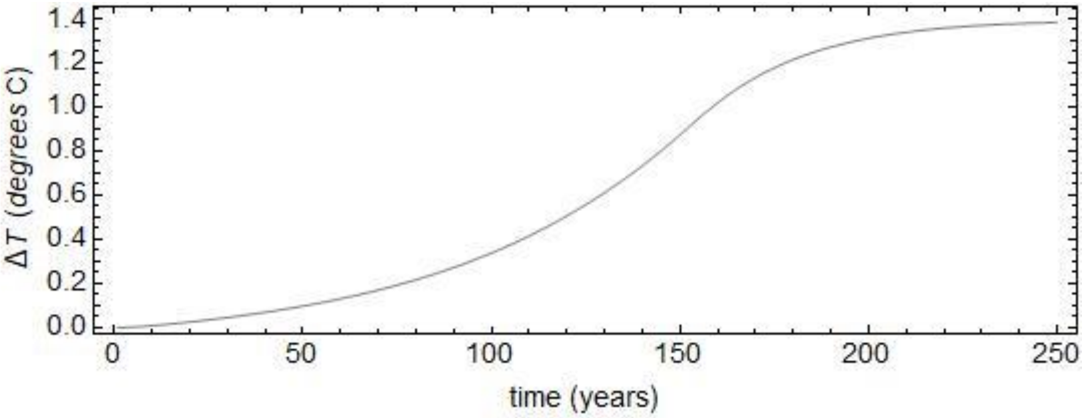
Then, we build the climate model itself (see Appendix). For the scenario without mitigations, the growth rate is:

$$g = \frac{1}{c_1} * \left(1 - \exp\left[\frac{\#}{\tau_1}\right]\right) + \frac{1}{c_2} * \left(1 - \exp\left[\frac{\#}{\tau_2}\right]\right) + \frac{1}{c_3} * \left(1 - \exp\left[\frac{\#}{\tau_3}\right]\right) \tag{5}$$

Here, we use the same values for parameters c- and τ-parameters, which were given above.

On the figure 2-10 we see that if there is no mitigation, the change in temperature will equal 1,4 Celsius degrees in the year 2130 (250 years from the start point).

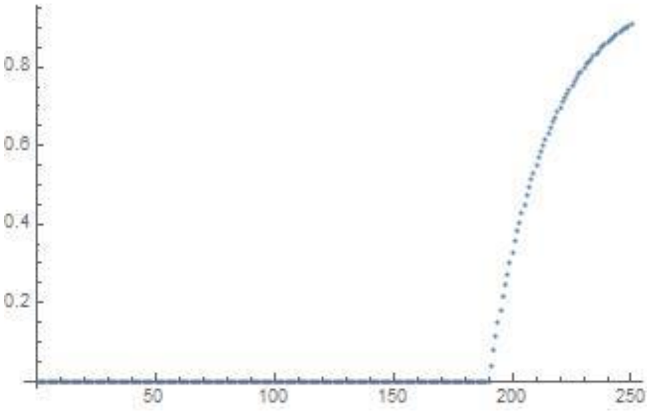
Figure 2-10 Climate model.



Now, based on the data above, we construct the economic model.

Figure 2-11 shows us the emission reduction rate. We see that it is on the zero level until the year 2070.

Figure 2-11 Rate of the emission reduction.

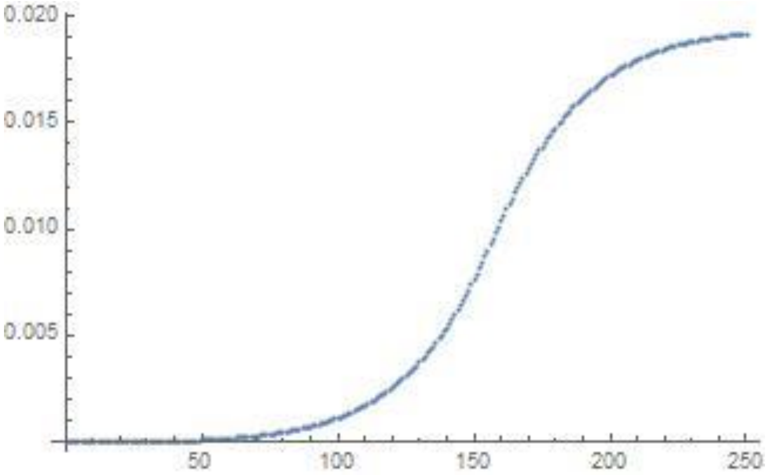


Then, we construct a damage function (See Appendix). This damage function is of the form for the DICE economic model (Dynamic Integrated model of Climate and the Economy).

$$D = \psi_1 * \Delta T + \psi_2 * \Delta T^2 + \psi_6 * \Delta T^6 \quad (6)$$

Here D denotes damages, ψ_1, ψ_2, ψ_6 are linear and non-linear damage coefficients, ΔT- temperature change.

Figure 2-12 Damage function.



We choose some parameters for our economic model. Let us try different sets of parameters to compare the results. We also choose the rate of the emission reduction (mitigation rate), which we apply to our code (see Appendix). Then, we plot the graph, which shows a dependence of the welfare function on the year when the mitigation starts.

First, we use 2% (per year since the mitigation starts) mitigation rate for the following set of the parameters:

Set 1:

$s = 0.3$
$\alpha = 0.1$
$\delta = 0.2$
$A = 0.5$
$\beta = 0.1$
$\rho = 0.04$
$\tau = 400$

As a result, we see that mitigation should start in the year 2095, if we want to maximize the welfare (see Figure 2-13a).

Then, we slightly change the Set 1 by diminishing some parameters, but still apply the same mitigation rate, *m* = 2% (per year since the mitigation starts).

Set 2:

$s = 0.1$
$\alpha = 0.1$
$\delta = 0.05$
$A = 0.5$
$\beta = 0.1$
$\rho = 0.02$
$\tau = 400$

Now, to maximize the welfare function, we should start the mitigation earlier, in the year 2040 (see Figure 2-13b).

The next step is trying to apply the different emission reduction rate, 1% per year, for the same two sets of the parameters. After doing this we see that, if we use Set 1, the mitigation should start in 2085, and, if we use Set 2, in 2030 (See Figure 2-13 c, d). It is interesting that in the both cases we should start the mitigation ten years earlier than if we would use the rate equal to 2% per year.

Now, let us try to change the Set 1 one more time, but now some of the parameters take larger values. Let the mitigation rate again be equal to 2% per year.

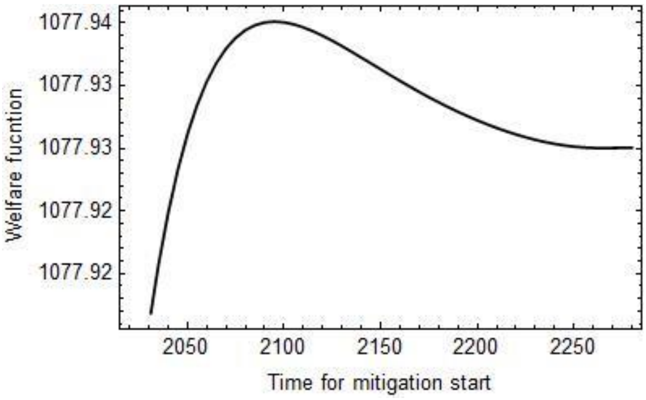
Set 3:

$s = 0.3$
$\alpha = 0.3$
$\delta = 0.2$
$A = 2$
$\beta = 0.3$
$\rho = 0.04$
$\tau = 500$

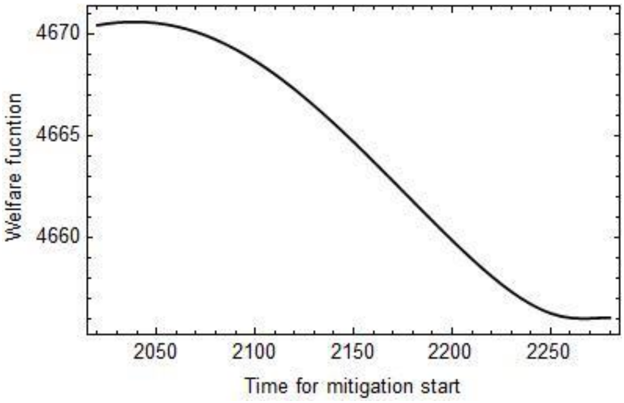
On the Figure 2-13e we see that to maximize welfare in this case, the mitigation should start in 2170, even later than in the first case.

Figure 2-13 Economic model with different sets of parameters.

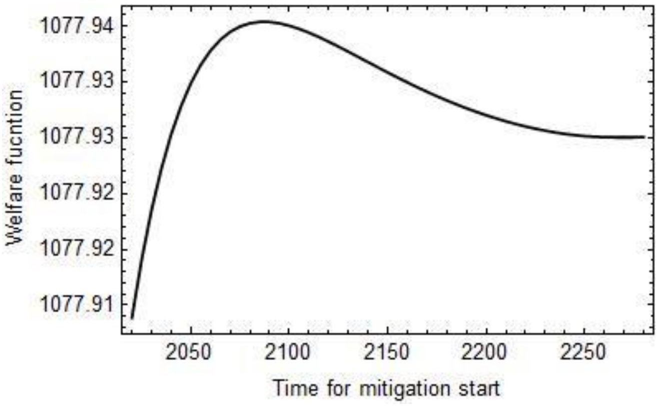
a



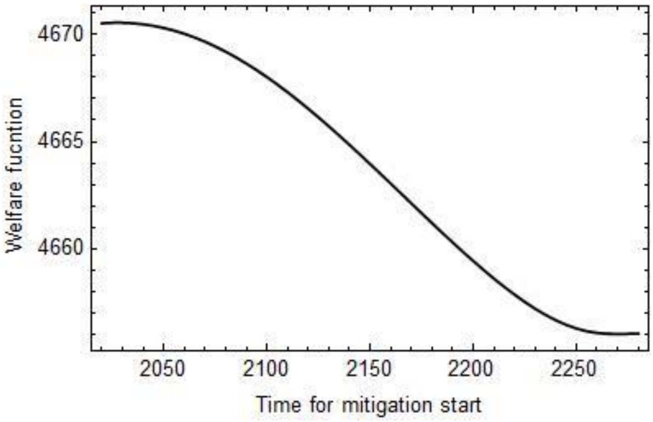
b



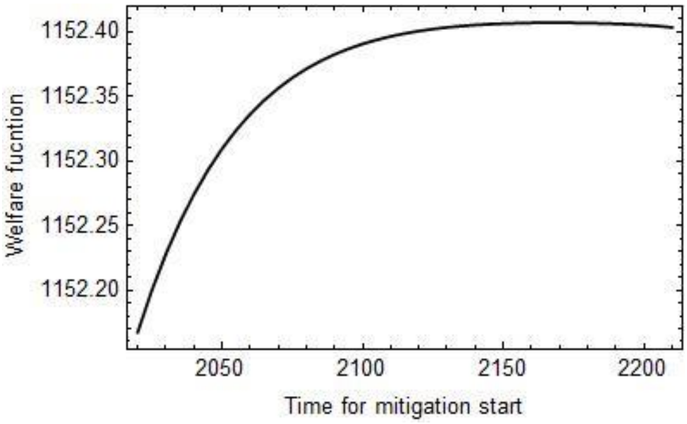
c



d



e



One more comment here is that the earlier we start the mitigation, the higher are the values which take the welfare function. In the Figure 2-13e, the values on the welfare axis are the lowest. It means that the emissions will mostly have a detrimental impact on the economics, and the emission reduction should start as soon as possible.

3 Modern climate policies

3.1 The optimal policy

3.1.1 The best target

According to the Article 2 of the United Nations Framework Convention on Climate Change (UNFCCC), "*the ultimate objective of [...] is to achieve [...] stabilization of greenhouse gas concentrations in the atmosphere at a level that would prevent dangerous anthropogenic interference with the climate system. Such a level should be achieved within a time-frame sufficient to allow ecosystems to adapt naturally, to ensure that food production is not threatened and to enable economic development to proceed in a sustainable manner*".

Let us analyze this statement.

This is the simple stock model:

$$S_t = (1-\delta)S_{t-1} + E_{t-1} \tag{1}$$

This one is a first-order difference equation, which represents a geometric process. Here, S_t denotes the stock at time t. E denotes emissions. δ - the rate of degradation.

$S_t = S_{t-1}$ is the condition that shows that stock does not change. This is necessary for the stabilization.

We have:

$$S = (1-\delta)S + E \leftrightarrow \delta S = E \leftrightarrow S = \frac{E}{\delta} \tag{2}$$

See that if we stabilize the emissions $E_t = E_{t-1}$, concentrations will be stabilized as well in such a level that is proportional to the degradation rate of atmospheric emissions. Here, the fact that the concentrations are stabilized implies stabilized emissions at all the levels. Is the Article 2 thus nonviable?

The CO_2 is cleared away from the atmosphere by some processes.

Mathematically, can represent the atmospheric carbon dioxide as five separate stocks.

(3) $S_t = \sum_{i=1}^{5} S_{i,t}$

(4) $S_t = (1 - \delta_i)S_{i,t-1} + \alpha_i E_{t-1}; \sum_{i=1}^{5} \alpha_i = 1$

All the five separate sub-concentrations should stabilize to get the stabilization in the atmosphere. But in one of the five “stocks” the atmospheric degradation happens by a rock weathering (which is a geological process at a geological time measure). For human time measure there is no atmospheric degradation, that is δi=0 for i=5 and αi=0.13. So, nearly 13 percent of emissions of anthropogenic CO_2 stay in the atmosphere permanently.

There is one problem caused by these permanent emissions. If we look at equation (2), the stable concentration and rate of degradation make an inversed proportion. It is not allowed to divide by zero, but the solution for δ=0 exists. The concentrations stabilize if E is equal zero.

The conclusion is that the first sentence in Article 2 is not meaningless. It radically tells us that emissions should be on the zero level to stabilize the concentration of the atmospheric CO_2. Most of the countries are legally obliged to cut their emissions by 100%. It is not known whether those who prepared the Article 2 knew this or not. In fact, politicians attribute an 80% emission reduction goal (for the long run), and the greener ones – 90%. In the international law the goal is 100%.

3.1.2 Benefit-cost analysis

Benefit-cost analysis is applied to determine the best way of actions. The algorithm is the following. We have a finite number of alternatives and first should estimate benefits and costs of each one. Then, eliminate the alternatives which costs are greater than benefits. Then, rank the rest of the alternatives by the size of the benefit-cost ratio. After this, should invest in alternatives that have highest benefit-cost ratio until there is no money left. The money can also be borrowed, if we want to invest in more existing options.

If there is a sequence of actions, the benefit-cost analysis works in another way. If, for instance, we take into account a carbon tax, this tax can be of any size. Here, benefit-cost analysis is aimed to maximize the objective function (benefits minus costs – net benefits). We find the maximum using the differentiation and then, the first partial derivative of the net benefits with respect to control variable should be equal to zero. If we reorganize the terms, marginal cost and marginal benefits should be equal.

Let us consider the objective function:

$$\text{(5)} \quad max_E G = D(E) - B(E)$$

Here, G are net gains, E – emissions, D – damages of emissions, B – benefits of emissions. The control variable is emissions, so the emissions should maximize the net gains.

Set the first derivatives equal to zero:

$$\text{(6)} \quad \frac{\partial G}{\partial E} = 0 \leftrightarrow \frac{\partial D}{\partial E} - \frac{\partial B}{\partial E} = 0 \leftrightarrow \frac{\partial D}{\partial E} = \frac{\partial B}{\partial E}$$

See that the marginal damages and the marginal benefits of emissions should be equal.

But, in fact, the climate change is a dynamic problem. That is why we should replace the equation (5) with the equation (7).

$$\text{(7)} \quad max_{E_0,E_1,\dots} \sum_t \frac{G_t}{(1+r)^t} = \sum_t \frac{D_t(E_t,E_{t-1},\dots,E_0) - B_t(E_t)}{(1+r)^t}$$

r denotes the discount rate. Here is maximization of the net present value of gains instead of net gains. Assume that benefits of emissions are immediate. It means that benefits at time t are only dependent on the emissions at time t. However, the emissions' damages depend on all of the preceding emissions.

In equation (6) we chose the emissions' level. But now we have a dynamic problem and should choose the emissions' level simultaneously for every time point.

We get many first order conditions:

$$\text{(8)} \quad \sum_S \frac{1}{(1+r)^S} \frac{\partial D_{t+S}}{\partial E_t} = \frac{\partial B_t}{\partial E_t} \forall t$$

So, at each time point, the net present value of the emissions' marginal damages should equal the emissions' marginal benefits.

There are, basically, two main observations from the benefit-cost analysis. At first, emissions should optimally be cut, if there are some damages caused by emissions. Benefit-cost analysis requires more actions than just the indicated reduction of emissions. This is because it is relatively not expensive to cut emissions in the beginning, and at the same time the benefits of such a reduction are relatively significant. At second, benefit-cost analysis not often requires the complete removal of emissions, because, at the final point, emission reduction is relatively expensive, and the benefits of this are relatively low.

The emissions of a greenhouse gas can be cut for a little bit. But removal of all the emissions is troublesome, because many alternative energy sources are costly or inconvenient in use at a large scale.

The climate change can do lots of damages, if we do not take it under control. So, the emission reduction will bring significant benefits at the first time. If climate change is constantly diminished, these benefits reduce. Benefit-cost analysis does not give an advice to reduce emissions by 100 percent, because if we reduce global warming from 0.1ºC per century to 0.01ºC, the benefits will not be large. Despite this, international law requires the 100% reduction of emissions.

3.2 International environmental agreements

3.2.1 Types of abatement

Above we considered climate policy from the global social planner point of view. Global social planner means the optimal climate policy, because can maximize the global welfare. But such a policy has no realistic prospects. There exists no organization that can force the sovereign countries to accept the policies aimed to reduce emissions, so it is hard to put the global social planner into practice. That is why more rational images of climate policy must lead to welfare which is lower than in the global optimum.

The climate policy can either be cooperative or not cooperative and there is a big difference between two those. Of course, the world is not completely cooperative. However, it is not isolative as well. So, there is a question – how much climate policy cooperation is possible to achieve, if the countries are self-centered?

3.2.2 Free-riding

One of the ways of studying the public goods supply is free-riding. In the beginning, there is a full cooperation between the countries. Then, every country has to make a decision if it wants carry on the cooperation. Each country does not consider the reasons of other countries to cooperate. Such an indifference for the others’ intentions is called the Nash behavior. It means that the country assumes that all other countries will continue cooperation, while making a decision about the further cooperation. There is then a compromise between the savings in costs (because of the diminished emission reduction) and extra damages.

Let us write the emission reduction’s (R) costs C for country i as:

$$C_i = \alpha_i R_i^2 \tag{1}$$

Here α denotes unit cost of the reduction of emission. It is assumed that the emission reduction’ costs of the country *i* are dependent only on reduction of emissions in country *i*, and the other countries’ emission reduction does not matter.
Benefits of emission reduction:

$$B_i = \beta_i \sum_j R_j E_j \tag{2}$$

Here, β denotes the social carbon cost, E – emissions when there is no climate policy (initial emissions), R – again the emission reduction effort. The emission reduction’ benefits thus depend on the behavior of all the countries based on their E and R.

If there is a cooperation, all the countries will join and maximize their accumulated benefits by balancing marginal benefits and costs:

$$\frac{\partial \sum_j B_j}{\partial R_i} = E_i \sum_j \beta_j = 2\alpha_i R_i = \frac{\partial C_i}{\partial R_i} \forall i \rightarrow R_i' = \frac{\sum_j \beta_j E_i}{2\alpha_i} =: \frac{\beta E_i}{2\alpha_i} \forall i \tag{3}$$

If there is no cooperation, every country will maximize its own utility (net benefits) by balancing the marginal benefits and costs:

$$\frac{\partial B_i}{\partial R_i} = \beta_i E_i = 2\alpha_i R_i = \frac{\partial C_i}{\partial R_i} \forall i \rightarrow R_i^* = \frac{\beta_i E_i}{2\alpha_i} \forall i \tag{4}$$

In both equations, the optimal reduction of emission equals the marginal benefits multiplied by own emissions over two multiplied by the unit cost of abatement. But there is one important difference. In the case of cooperation, we look at all countries' marginal benefits. In the case of non-cooperation, only own country's marginal benefits are considered.

If we substitute (3) and (4) into (1), will get the costs' difference:

$$(5)\quad C_i' - C_i^* = \alpha_i\left(\frac{\beta E_i}{2\alpha_i}\right)^2 - \alpha_i\left(\frac{\beta_i E_i}{2\alpha_i}\right)^2 = (\beta^2 - \beta_i^2)\frac{E_i^2}{4\alpha_i}\forall i$$

The difference in benefits is:

$$(6)\quad B_i' - B_i^* = \beta_i\left(\frac{\beta E_i}{2\alpha_i} - \frac{\beta_i E_i}{2\alpha_i}\right)E_i = (\beta\beta_i - \beta_i^2)\frac{E_i^2}{4\alpha_i}\forall i$$

We have the case of free-riding, so this benefits' difference comes from the change in emission reduction by the own country.

For free-riding it is better if the additional damages in equation are lower than savings of costs in equation 5. And it is true for the most cases.

All the countries in the following table have incentives for free-riding. Emission are equal 1, this is done to make things more convenient.

Table 3-1 Illustration of free-riding.

Source: Richard S.J. Tol: "Climate Economics: Economic Analysis of Climate, Climate Change", 2014.

$\boldsymbol{\beta}$	$\boldsymbol{\beta_i}$	$\boldsymbol{A}$	$\boldsymbol{E}$	$\boldsymbol{\Delta C}$	$\boldsymbol{\Delta B}$	$\boldsymbol{\Delta C - \Delta B}$
4	1	0.5	1	7.5	3	3.5
10	1	0.5	1	49.5	9	40.5
2	1	0.5	1	1.5	1	0.5

Let us, for instance, consider the first row of the Table 3-1. Here, the national carbon cost is one-fourth of the global social cost. The savings on cost equal 7.5 and additional damages equal 3 – it is good for the case of free-riding.

In the second row of the table, the national carbon cost is the one-tenth part of global social cost. Cost-savings are higher and now equal 49.5 – as a country that is smaller should do much more for other countries of the world. The additional benefits here are higher as well and equal 9. In this case, free-riding is preferable as well, because the costs of the emission reduction are linear and in the same time the benefits of emission reduction are linear.

If we look at the third row, the national carbon cost is the half of the global social cost, cost savings equal 1.5 (diminished) and the extra damages equal 1. So, free-riding is still in preference.

If almost every country would free-ride, the cooperation, of course, is not possible.

3.2.3 Cartels

Besides the cases when countries fully cooperate and when they do not cooperate at all, there are some intermediate situations as well. It means there is collaboration between some countries, but no collaboration between other. Cartel formation games can help us to study such situations. A cartel is a group of differently independent countries or businesses that make some actions together and thus can establish prices for their goods and services without any competition. So, the cartel information games are created in industrial institution.

An allied group is considered as stable if only it is stable both externally and internally and brings profit. It is externally stable if no one of those who are not in the group would be better off inside the group. Internally stable means that no one of the group members would be better off outside the group. Basically, if no one wants to join or leave, we can say that the group is stable. Such coalition is called profitable if every its member is at least as well off as in the case when there is no cooperation at all. So, the starting point of the cartel theory is the non-cooperative case. The coalition which consists of all agents is logically always externally stable, because there exist no non-members. Such a coalition is named the grand coalition.

Consider the formation of cartel as the linear-quadratic game. Let us assume there are only two agents, just for simplicity.

So, the costs of emission reduction are:

(7)

$$C_1 = \alpha_1 R_1^2$$

$$C_2 = \alpha_2 R_2^2$$

The emission reduction’ benefits:

(8)

$$B_1 = \beta_1(R_1E_1 + R_2E_2)$$

$$B_2 = \beta_2(R_1E_1 + R_2E_2)$$

If there is cooperation, the solution is:

(9)

$$R_1' = \frac{(\beta_1 + \beta_2)E_1}{2\alpha_1}$$

$$R_2' = \frac{(\beta_1 + \beta_2)E_2}{2\alpha_2}$$

If there is no cooperation, the solution is:

(10)

$$R_1^* = \frac{\beta_1 E_1}{2\alpha_1}$$

$$R_2^* = \frac{\beta_2 E_2}{2\alpha_2}$$

If we subtract equation (8) from the equation (9):

(11)

$$\Delta R_1 = \frac{\beta_2 E_1}{2\alpha_1}$$

$$\Delta R_2 = \frac{\beta_1 E_2}{2\alpha_2}$$

Cooperation happens in the following way. Agent 1 (at his costs) further reduces his emissions, so agent 2 responds with reducing his emissions too. This benefits the first agent. Agent 1 would like to cooperate if benefits are larger than costs. He wants to increase his abatement by a little amount to get many extra abatement by agent 2 in return. But the second agent wants to do exactly the same thing. The agents can collaborate only if both of them will be better off.

For the stable collaboration, the following should be true:

(12) $\Delta C_1 < \Delta B_1 \wedge \Delta C_1 < \Delta B_1$

If we have not only two, but N agents:

(13) $\Delta C_1 < \Delta B_1 \wedge \Delta C_1 < \Delta B_1 \wedge \dots \wedge \Delta C_N < \Delta B_N$

The more agents are in the coalition, the more every member need to do at increasing costs. The benefits increase as well, however, both members and non-members are getting these benefits. So, the cartel formation game can be solved by those stable international environmental agreements, which are signed by few agents. Another solution is to have lots of signatures, but each of the agents should do a little bit more that they would have done anyway. The international environmental agreements can thus be either with a lot of signatures and not that much to do, or with few signatures that do much. The effect on the global emission will be limited anyway.

Above we discussed how the single coalition is formed and which factors are required for its stability. The stability condition for multiple coalitions are almost the same. The set of coalitions should bring profit. Also, no member wants to leave the coalition, no non-member wants to join and no one of the members wants to switch to another coalition. The benefit of the multiple coalitions is that there are more choice and welfare might improve. The drawback is that there is one more constraint added – the stability between different coalitions. The more coalitions we have, the more difficult is to find the solution. It means that multiple coalitions are not able to reduce emissions more than only one single coalition.

One paradoxical result is that multiple coalitions are more important for the negotiations with few agents than for negotiations with many agents. This is because if we have small number of agents and some coalition forms, it will have a relatively large impact on the non-members. If one more coalition forms, it can change the situation and those two coalitions can cooperate and have a large influence. If we have large number of agents, all the coalitions that form are relatively small and the number of non-members are relatively large. That is why the coalitions cannot have that much influence as in the case with small number of agents.

3.2.4 Kyoto Protocol and Paris agreement

Two environmental agreements – Kyoto Protocol and Paris agreement have the same aim which is to reduce the rise of the global temperature to the value below 2°C. The global temperature rise is mainly caused by increase of the extraordinary emissions of the greenhouse gases, so all the countries are obliged to do something to limit the increasing temperature.

Kyoto Protocol is an international agreement, the first one which has the goal of to take the rise in global temperature under control; a supplementary document to the UNFCCC (United Nations Framework Convention on Climate Change, 1992), adopted in Kyoto, Japan in December 1997.

The Paris Agreement is an agreement under the United Nations Framework Convention on Climate Change, adopted 12th December 2015 during the Climate Conference in Paris and signed 22th of April 2016. Regulates actions to reduce amount of CO_2 in the atmosphere from the year 2020. This agreement was prepared to take the place of the Kyoto Protocol.

There is, however, a difference between those two agreements. At first, it is important to know that that different countries are responsible different amount of greenhouse gas emissions, and the past (historical) emissions in all the countries were different as well. This indicates that the contribution each country made to the present increase in the global temperature was not the same. That is why the efforts which different countries are obliged to make for emission reduction are supposed to be adjusted according the emissions made by these countries.

The Kyoto Protocol and Paris Agreement suggest different approaches to hit the target of stabilizing the atmospheric greenhouse gases and keep the increase in temperature below 2°C. These agreements make developing and developed countries contribute to emission reduction differently.

According to the Kyoto Protocol, developed countries are legally obliged to cut their emissions. The list of these countries is indexed in the Annex I of UNFCCC. There are two commitment periods of Kyoto Protocol. During the first one (from 2008 to 2012) the requirement for developed countries was to cut their emissions by 5 percent below the level of year 1990. During the second one (from 2008 to 2013), the requirement changed to 18 percent cut. The developing countries were not obliged to reduce their greenhouse gas emissions.

We see that there is a stringent difference between the requirements for developed and developing countries in the Kyoto protocol. The situation is different for the Paris Agreement. According this agreement, all the nations should willingly perform actions on their targets of emission reduction.

195 countries adopted the Paris Agreement on the 21stConference of Parties (COP21) which was held in Paris in December 2015. Before that, on the COP19 in Warsaw (2013) all nations were asked about their domestic agreements and plans for reducing the detrimental effects of climate change. 180 nations, which in fact contribute 90% to the total world emissions, connected UNFCCC before Paris with their targets for emission reduction.

The Kyoto protocol covered not that much of the total emissions in the world, the planned positive effect was not reached. The reasons are that a few countries exit from this agreement and did not participate in the second period of commitment, some countries could not hit targets, and USA did not participate in the protocol at all. Paris agreement, in its turn, can make a lot of nations to reduce their emissions, but the developed countries are asked to take the leading positions.

3.2.5 The reasons for the complications in the solution of the global warming problem

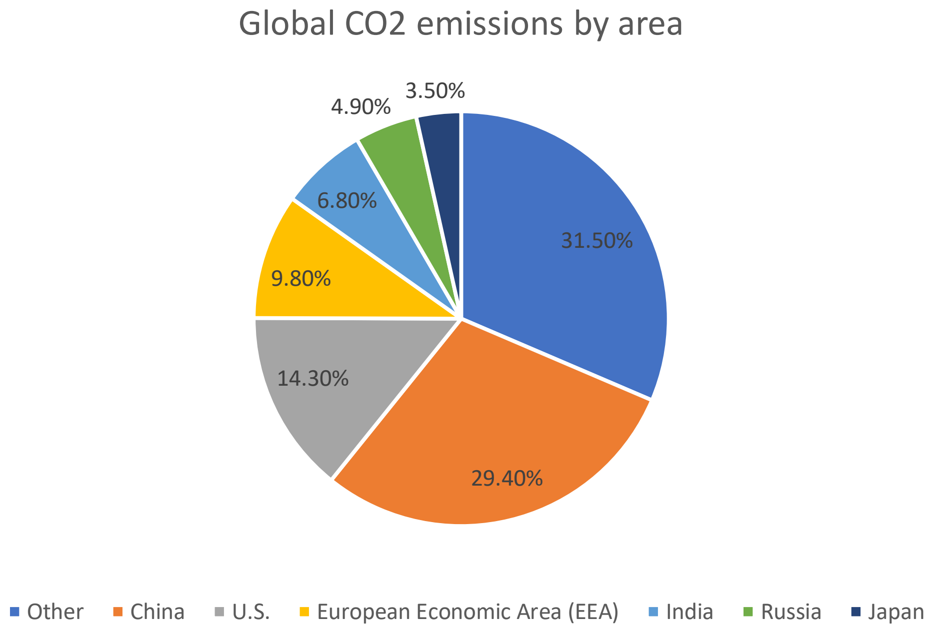
According to the data provided by European Commission (EC), the European Union countries reduced their carbon dioxide emissions by 20.8% during the period from 1990 to 2016.

However, in recent years this situation stopped improving. In the EC report on carbon dioxide emissions for 2017 it is indicated that European countries increased the emissions by 0.2%.

Looking at the Figure 3-1, we see that USA takes the second place (after China) in terms of greenhouse gas emissions. However, on 1st of June 2017, US President Donald Trump announced that USA withdraw from the Paris agreement. According the agreement, the earliest time USA can withdraw is November 2020.

Figure 3-1 Global CO_2 emissions by area.

Source: Emission Database for Global Atmospheric Research.



As it was already mentioned above, the main goal of the Paris Agreement was to prevent the rising of the global surface temperature by 2 ° C compared to the pre-industrial level.

Scientists believe this will lead to irreversible climatic changes. According to National Aeronautics and Space Administration (NASA), the global average temperature is +15 ° C. Many experts agree that the global surface temperature has risen by less than 1°C compared to pre-industrial level.

According to the United Nations (UN) forecasts, the average global temperature can jump at least by 6 ° C in the next 100 years. Scientists record a quick warming in the Arctic, an increase in the number of natural disasters and level of the World Ocean.

Experts think that not compromising policy of some countries and the reassessment of the renewable sources of energy are the main reasons of failures in fight against the climate change. The withdrawal of United States from the Paris Agreement has an unfavorable effect as well. Donald Trump prioritized the national interests over the common environmental interest, however some governors in U.S. do not support Trump and disagree with his environmental behavior.

The European Union took the leadership in the fight against the global warming. In the past 40 years European countries have adopted a lot of programs for the green economy. But the emission reduction incline in Europe has recently slowed. According the report of the European Commission (EC), European countries increased emissions by 0.2% in 2016.

The main alternatives to the nuclear power plants are the hydroelectric power stations, windmills and solar batteries. All of these alternatives are the renewable sources of energy (RES).

The European countries support the use of the renewable energy sources since 1980s. The plan is to make the share of RES equal 20% of the total energy consuming in 2020. This was approved by Council of Europe in March 2007. The goal of 20% RES was actually reached in 2012, because some small countries in the western Europe, for instance Iceland, Sweden, Denmark, Portugal achieved very good results. In Reykjavik, Iceland the geothermal keys provide the power. However, the situation worsened in 2013-2014. For example, Brussels has successfully developed RES, but then was forced to use coal and gas again, and this, of course, increases the CO_2 emissions. By 2030, the share of the renewable energy sources in European Union is expected to increase to only 27%. In November 2017 the agency BNE IntelliNews announced that it looks like Europe have exhausted the potential for development of the alternative energy.

China, which takes the first place in terms of emissions of the greenhouse gases, is into ambiguous situation. On the one hand, China has environmental problems during the last years and in October 2017 there was made a decision to fight with the air pollution. On the other hand, the large part of the Chinese exports (more than half) go to the western countries. China is chosen as the main place for the European and U.S. industry development - from the 1970s to the 1990s Europe and U.S. transferred lots of production abroad. This economic policy helped western countries to make the environmental situation on their territory better, while China was in a critical situation. The experts say that China, as well as India, would have to make a deep reorganization of the industry to accomplish the international obligations, and it would lead to the damage of their own economic growth.

3.3 Norwegian contribution to solution of the climate change problem

3.3.1 WWF Norway and its suggestions

WWF (World Wildlife Fund) in Norway works on fighting the climate change and global warming and providing the safety for people, animals and ecosystems all around the world. There are some main fields of WWF's activity.

At first, WWF works on the new climate strategies for Norway and these are the following. The own Norwegian climate law that ensures that Norway will achieve the goals that are set and puts Norway on a course to a society with zero emissions level in 2050. Norway's state-owned company must do its business in an environmentally sustainable manner, for example, by pulling Statoil out of the dirty tar sands located in Canada.

The second field is the global energy access and the renewable energy. The energy policy and industrial development of Norway must contribute to the 100% renewable-energy society by the year 2050. This goal should be reached partly through the plans for energy efficiency, development of network, electrification of the offshore installations. The increased availability of energy for the people who live in the developing countries need to be based on renewable energy rather than on fossil one.

Then, WWF also works on the global strategies for a safe and sustainable climate. The global climate agreement under the UN is binding and sets challenging targets for the turning off the extreme climate change. This pushes the vulnerable ecosystems and societies to adapt to the impossible changes. Norway mush carry on its good work in this field. Norway also should continue its contribution to saving of the tropical forests through the initiative of the Redd+. WFF also studies the development of the sustainable business in China, because this country is the largest world's emitter and has a great potential for contribution to the safe climate.

WWF has determined the six main preconditions to meet the world's increasing demand for the energy and, in the same time, avoid the most detrimental climate change.

At first, the WWF's model shows that it is possible to satisfy an increasing energy demand without the increasing of production of energy. The existing efficient energy solutions must be instantly applied in order to get some time for the future development and use of the renewable energy production.

Then, the WWF mentions the importance of hastening of low-emission technologies. To be ready to meet challenges in time, it is essential to invest in many technological solutions in the same time. Water-, solar-, wind- and bioenergy will help to improve the energy supply and can be used depending on the needed type of energy service and geographical conditions.

Then, there should be developed some flexible solutions for fuel. For instance, in the transport sector, will need solutions that enable energy storing and distributing from wind and sun. One option is to store electricity in the batteries, but will also need solutions that ask for the large adjustments of infrastructure. For example, can use hydrogen as the carrier of energy.

Natural gas can be used as a transitional solution. In order to avoid too large investments in coal power, before the alternative sources of energy will be fully used, the natural gas can help in this intermediate period. The electricity which is produced from the natural gas causes less CO_2 emissions than produced from the coal power.

WWF also thinks it is necessary to stop the deforestation, especially in tropics, because this is crucial for the climate stabilizing. If the deforestation continues. With continued deforestation, we will soon need the unthinkable large emissions reduction.

The last suggestion is the capture and storage of carbon. Not so soon, but by 2050, all the power plants need to be equipped with this technology.

To contribute to a global climate solution, Norway has to reduce its emissions by 40% by the year 2020 (compared with 1990 level). This is, in fact, 7.8 million tons more than it is suggested by the Climate agreement. The budget of carbon must be reduced to 29,9 million tons by 2020. This is 23,9 million tons less compared to greenhouse gas emissions in Norway in 2008. WWF point out that the longer we wait, the more each years' emissions must be cut to a higher cost.

3.3.2 Norwegian government's climate collaboration with the EU

Norway was one of the first countries who joined the Paris agreement. Vidar Helgesen, the Norwegian Minister of Climate and Environment, signed the agreement in New York 22th of April 2016. 174 other countries signed it the same day. Norway has ratified the Paris agreement 20th of June 2016 – the ratified agreement was delivered to headquarters of UN by Geir O. Pedersen, the Norway's UN ambassador.

Norway joined the EU on reductions of climate change not because it is the best way to save the climate, but because Norway still wants to pay for the reductions instead of taking them to the domestic territory.

The idea to link Norwegian climate policy closer to the EU, in fact, came from the government politicians in the Conservative party (Norway).

In spring 2015, surprisingly, the government announced that Norway should to join EU's climate plans in the sector which is not subjected to quota. The politicians were seeking for a solution which will keep the effectiveness of costs and flexibility that Norway had as a benefit through the Kyoto protocol, namely the ability to purchase cuts of emissions in the developing countries and thus meet Norwegian own climate targets. At that time point, it was known that the Paris agreement did not begin to work yet and that the Kyoto Protocol will end in 2020. There was an uncertainty about which kind of the international plan will be launched. That is why the Norwegian government looked toward the Europe with the goal of providing the cost-efficiency and flexibility to the Norwegian emission targets' approach. If there is cooperation with European Union, Norway can pay for the less expensive cuts there and do something at home as well (in a less extent).

However, the EU’s climate targets are not adapted to the real circumstances. Climate change already leads fishes to death, because there is not enough oxygen in the oceans. Also, coral reefs fade away and the level of ice in the Arctic regions is extremely low. Carbon Action Tracker determined that the EU’s goal is not actually sufficient to deliver on the climate goals of the Paris agreement. In fact, the EU target is little ambitious, and there is a risk to cut only 142 million tons of the 22300 in the period of 10 years (according to Sandbag, European community interest company). With such a level of ambitions, the EU’s goals will not bring changes to the European countries unless the countries-members set their own ambitions for the national emission reduction. There is a risk to lose ten years of possible changes while there is a very critical time for our planet.

Besides the low ambitions level, there is nothing that can prevent cooperation of Norway with EU’s climate policy, as long as this policy acts as a guiding principle. It should set a minimum goal and every country should do something to cut emissions and contribute to the fastest possible changeover to the low-emission society. There is also nothing on the way of the cost-effective policy, as long as it is not used as an alibi for not taking any measures that contribute to the necessary changeover in Norway.

The Norwegian government announced the goal for 2030 in 2015. Since this, in was stated many times that this target is directed to climate cuts inside the country instead of the paying for reductions outside as it was during the past decades. In January 2017 Prime Minister, Erna Solberg, stated that most of the 40% reduction in non-quota emissions must be taken inside the country. On the other hand, in the climate change report, as the Parliament processing now, it is not hidden that Norway wants to use the EU system’s flexibility to accomplish its own obligations. According to the platform Jeløya, Norway will achieve its climate goals by "Using the opportunity in the EU's quota system (ETS) and the possibility of EU’s framework to fulfill Norwegian climate commitments in non-quota sectors, while at the same time having the ambition to take as much as possible of the obligations nationally ". So it is impossible to tell how ambitious is the plan for the domestic emission reduction in 2030.

For now, Norway ensures that it is within the emission budget given by EU, and that is why EU does not help Norway to prepare a plan for emissions cut inside the country, or to create green changeover.

Norway must create this plan self, and the responsibility of the Parliament is to make sure that the Norwegian welfare, business and nature are ready for the time without human-made greenhouse gas emissions. The measures in climate report are therefore not only about how to reach EU's and Norway's climate goals by 2020 and 2030, but also about how the society in Norway will adapt to the low-emissions future. The Norwegian Parliament has a chance to create a plan as they process the climate report now by adopting measures needed to give emission reduction, changeover and competitiveness in every sector. After that, the elected representatives should use Climate Act and the climate section of the budget in order to measure the government's follow-up of the climate targets.

Business in Norway will benefit from a forward-looking climate policy inside the country. It will lead to innovation in technology, which, in its turn, gives competitive advantages in a global market, where low-emission solutions will be appreciated. Although Norway is a relatively small country, it can make a big difference globally. Three examples of climate policy that create innovation, global emissions reduction and new industries for Norway are carbon capture and storage in cement, waste management, development of the floating offshore wind turbines, electrification of marine vehicles. In these fields Norway has experience, expertise and resources for creation of new solutions.

Fortunately, the EU does not say that countries can do nothing more than achieve the minimum targets in the Union. Some countries (The Netherlands and Sweden, for instance) join their forces to reduce more emissions than they obliged by the Union. Norway should join the countries that use the flexibility the EU's offers to take extra cuts at their domestic territory as well, for the climate reasons, but also because of what Erna Solberg is more concerned with, the consideration of the competitiveness of Norway.

Conclusion

In this thesis we looked at the ways and methods for predicting and preventing the detrimental economic consequences, which climate change may cause. The objectives were to analyze how the emissions can be cut, how the modern models of the climate economics are applied, and how the existing climate policies work.

In the first chapter we analyzed the Kaya identity, the variables of which are useful to determine some of the ways for emission reduction. We found out that to cut the emissions, for instance, either carbon efficiency or energy should be improved, or either population or income should be reduced. But, obviously, not every country is willing to reduce income and population. Some other alternatives for emissions cut are stopped deforestation, reduced production of dairy and meat, geoengineering. The reasonable way is use of the environmentally friendly energy sources, for example, wind and solar energy.

We explored what are the costs of the abatement. We looked at the tables that compare different models, which estimate the abatement costs and then show different results. This is because these models assume different price- and substitution- elasticities, different rates of participation of the poor countries and different speed of the greenhouse gas emissions' rise. Here we also found out that abatement costs can be (at least partly) compensated by the revenue that we get from the imposed carbon tax. So, if this revenue is used to reduce another type of tax, we can clearly get benefits, which can compensate the costs of the emission reduction in the beginning.

We discussed the two approaches of the emission reduction' policy. The first one is direct regulation, which assumes that there is a regulator, who tells companies and people what to do (cut the emissions) and how to act. The second approach uses market-based instruments, namely taxes and subsidies. Here the regulator only tells that emissions need to be reduced, and companies and people decide how to achieve this goal by themselves. The latter approach is thus more advantageable.

In the end of the first chapter of the thesis we pointed out that the best solution is the cost-effective solution. Thus, the solution should meet the goal and in the same be the least-cost one. The emission should be reduced, and in the same time this should not lead to serious economic losses. We used the mathematical optimization problems, and derived the static- and dynamic- conditions for efficiency.

We devoted the second chapter to the climate-economic models. We first analyzed the top-down approach, which is applied to project the climate change impacts on market. This approach is effective, because it uses historical data, future projections and equity preferences to get the results. If we consider the long perspective, namely medium run, the main result is that the additional warming is detrimental for the economies of the poor countries located in South Asia and sub-Saharan Africa, while for the rich countries it is beneficial. One more result is that precipitation does not play as important role as temperature in affecting the variability of growth in the medium run. However, precipitation is important factor, if we consider the short run.

Then, we concentrated our attention on the Integrated Assessment Models (IAMs) of the climate economics. Every type of model uses the specific type of the damage function - the function, that links future warming and consequences for economics. In this thesis we compared three types of the IAMs, namely DICE, FUND, and PAGE.

We then constructed the climate and economic models with help of the Wolfram Mathematica code (see Appendix). We first created the climate model and looked at six different CO_2-emission scenarios. Then, we programmed the economic model of the DICE-type and used three different sets of parameters and two different rates of mitigation as inputs. The resulting five graphs showed us that the time points (years) when we should start the mitigation are different, because of the various inputs. The important result that we got from such analysis is that the emission reduction should start as soon as possible. This again proves that it is crucial to care about the climate change problem.

The third chapter of the thesis was devoted to the modern climate policies. There, we looked at the ways in which countries may cooperate. We discussed which climate policies are actual in the modern world. So, right now, the operative document is the Paris Agreement, which was signed in April 2016. According to this agreement, there is an aim to keep the global temperature increase this century below 2 Celsius degrees (above the pre-industrial levels).

However, the USA announced withdrawal from the Paris agreement. This is, probably, not the best move for the world climate policy, because the previous agreement, namely Kyoto protocol, did not achieve its targets, because some countries quit, and USA did not participate. If we talk about Norway, it appears to be one of the first countries, who joined Paris agreement. Right now, Norway is within the EU's emission budget.

All in all, finding the right, cost-effective solutions for the climate change problem is the complicated process. If we make scenarios about which effect global warming will have on the economics, the result will depend on the type of the applied model, type of the data sources, regions that are considered in the analysis and other parameters. But such researches have a high importance, because the future economics will be obviously affected by the climate change in some way, and preventing the disadvantageous consequences, which global warming may bring to our world, is only under our responsibility.

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Appendix, Wolfram Mathematica code

Run the first hidden cell under this line.

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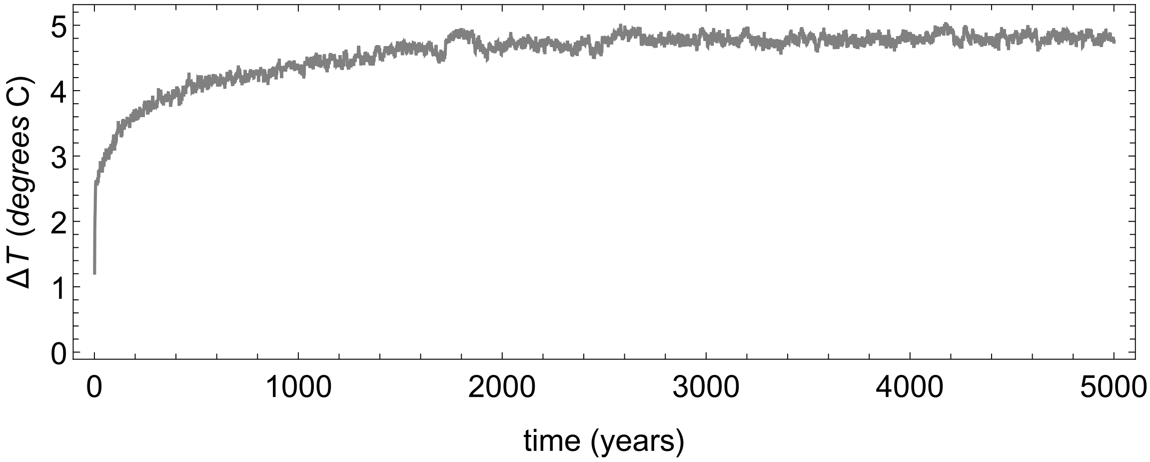
4.7033`, 4.7879`, 4.7292`, 4.7289`, 4.683`, 4.7944`, 4.7933`, 4.7937`, 4.7292`,
4.7477`, 4.821`, 4.7919`, 4.7261`, 4.7547`, 4.7027`, 4.6884`, 4.7788`, 4.7801`,
4.8863`, 4.8175`, 4.8005`, 4.782`, 4.7846`, 4.7666`, 4.76`, 4.7491`, 4.7835`,
4.7779`, 4.7708`, 4.7295`, 4.7607`, 4.7529`, 4.8318`, 4.8145`, 4.7869`, 4.7302`,
4.7323`, 4.6973`, 4.7689`, 4.7656`, 4.728`, 4.8044`, 4.7783`, 4.7371`, 4.7762`,
4.7416`, 4.8352`, 4.7545`, 4.8222`, 4.7192`, 4.6957`, 4.7941`, 4.6692`, 4.7307`,
4.6925`, 4.7606`, 4.7473`, 4.6904`, 4.7346`, 4.7259`, 4.6957`, 4.759`, 4.7096`,
4.6446`, 4.7524`, 4.794`, 4.8246`, 4.7655`, 4.8254`, 4.8534`, 4.775`, 4.8038`,
4.8328`, 4.8365`, 4.8766`, 4.815`, 4.7879`, 4.6779`, 4.7119`, 4.6743`, 4.612`,
4.7605`, 4.7383`, 4.7984`, 4.8536`, 4.8288`, 4.7944`, 4.8207`, 4.811`, 4.8921`,
4.8765`, 4.8799`, 4.8021`, 4.8607`, 4.9729`, 4.8902`, 4.8479`, 4.8959`, 4.852`,
4.8496`, 4.9015`, 4.8261`, 4.8524`, 4.8505`, 4.9346`, 4.8347`, 4.8707`, 4.9064`,
4.9403`, 4.8093`, 4.8352`, 4.834`, 4.7912`, 4.8504`, 4.8486`, 4.8022`, 4.8409`,
4.7818`, 4.813`, 4.7939`, 4.7906`, 4.7339`, 4.6817`, 4.7084`, 4.684`, 4.7017`,
4.7099`, 4.6894`, 4.6205`, 4.6998`, 4.7691`, 4.7608`, 4.8117`, 4.8347`, 4.8541`,
4.8152`, 4.7623`, 4.819`, 4.8476`, 4.7568`, 4.8217`, 4.861`, 4.8265`, 4.8553`,
4.7654`, 4.7841`, 4.8406`, 4.8652`, 4.7325`, 4.718`, 4.731`, 4.715`, 4.7823`,
4.7641`, 4.7777`, 4.8603`, 4.834`, 4.8625`, 4.8649`, 4.8708`, 4.8642`, 4.8815`,
4.8546`, 4.8781`, 4.9258`, 4.8507`, 4.9168`, 4.8365`, 4.8182`, 4.8075`, 4.8979`,
4.879`, 4.7824`, 4.8053`, 4.7614`, 4.7016`, 4.7788`, 4.7179`, 4.8094`, 4.7628`,
4.7922`, 4.8156`, 4.7379`, 4.7117`, 4.7652`, 4.7812`, 4.7233`, 4.8281`, 4.8186`,
4.8756`, 4.7259`, 4.8578`, 4.8839`, 4.8127`, 4.7822`, 4.8194`, 4.8198`, 4.899`,
4.94`, 4.8931`, 4.8173`, 4.817`, 4.8346`, 4.8458`, 4.8516`, 4.8611`, 4.8622`,
4.7957`, 4.9125`, 4.9006`, 4.8766`, 4.8373`, 4.8564`, 4.8519`, 4.9265`, 4.8297`,
4.8477`, 4.8659`, 4.842`, 4.932`, 4.9466`, 4.9049`, 4.9404`, 5.0006`, 4.9726`,
```

```
4.8147`, 4.9317`, 4.8622`, 4.8779`, 4.8943`, 4.8604`, 4.8928`, 4.9572`, 4.9395`,
4.882`, 4.8208`, 4.822`, 4.8646`, 4.8379`, 4.8205`, 4.7859`, 4.7804`, 4.7917`,
4.796`, 4.791`, 4.8105`, 4.8852`, 4.8617`, 4.8793`, 4.8595`, 4.8881`, 4.7953`,
4.8269`, 4.8218`, 4.875`, 4.9032`, 4.8674`, 4.8805`, 4.9053`, 4.909`, 4.9331`,
4.9341`, 4.8472`, 4.9037`, 4.9295`, 4.8625`, 4.8234`, 4.8085`, 4.6916`, 4.7324`,
4.7671`, 4.6876`, 4.6`, 4.6628`, 4.6579`, 4.6906`, 4.6442`, 4.6954`, 4.7139`,
4.6466`, 4.661`, 4.6305`, 4.7409`, 4.7537`, 4.6839`, 4.6907`, 4.7453`, 4.7169`,
4.7335`, 4.7101`, 4.8201`, 4.8076`, 4.7788`, 4.7428`, 4.7941`, 4.793`, 4.7424`,
4.75`, 4.7351`, 4.7656`, 4.8158`, 4.7384`, 4.7334`, 4.7626`, 4.8063`, 4.8248`,
4.7989`, 4.7783`, 4.8015`, 4.7414`, 4.7866`, 4.7821`, 4.8092`, 4.7294`, 4.7357`,
4.7841`, 4.7608`, 4.7625`, 4.7748`, 4.6974`, 4.7857`, 4.881`, 4.787`, 4.8405`,
4.9017`, 4.9082`, 4.8477`, 4.8018`, 4.8006`, 4.8035`, 4.7846`, 4.7027`, 4.7734`,
4.7816`, 4.8457`, 4.8543`, 4.815`, 4.7876`, 4.8658`, 4.8241`, 4.855`, 4.8341`,
4.9192`, 4.901`, 4.8858`, 4.8874`, 4.9147`, 4.8434`, 4.8835`, 4.9532`, 4.9354`,
4.8602`, 4.8632`, 4.9224`, 4.863`, 4.829`, 4.873`, 4.8301`, 4.8157`, 4.7652`,
4.9215`, 4.8265`, 4.8428`, 4.9143`, 4.8495`, 4.7421`, 4.794`, 4.8415`, 4.8404`,
4.7992`, 4.8122`, 4.7971`, 4.8477`, 4.7715`, 4.7775`, 4.7039`, 4.7976`, 4.7675`,
4.7727`, 4.8198`, 4.8994`, 4.7956`, 4.8092`, 4.8569`, 4.8681`, 4.8471`, 4.8055`,
4.7392`, 4.794`, 4.7633`, 4.7608`, 4.8215`, 4.7466`, 4.8593`, 4.9165`, 4.8727`,
4.8846`, 4.9649`, 4.9027`, 4.8205`, 4.826`, 4.8541`, 4.825`, 4.8308`, 4.7843`,
4.7823`, 4.773`, 4.914`, 4.9493`, 4.7821`, 4.7915`, 4.7882`, 4.8204`, 4.8341`,
4.9257`, 4.8363`, 4.7938`, 4.844`, 4.9221`, 4.9172`, 4.9063`, 4.8891`, 4.8932`,
4.892`, 4.8247`, 4.8486`, 4.7748`, 4.8274`, 4.7653`, 4.7735`, 4.88`, 4.8163`,
4.7562`, 4.6971`, 4.7292`, 4.8194`, 4.7569`, 4.7137`, 4.7247`, 4.7062`, 4.783`,
4.7371`, 4.7187`, 4.7952`, 4.889`, 4.7711`, 4.846`, 4.8256`, 4.7958`, 4.837`,
4.863`, 4.7866`, 4.8526`, 4.8071`, 4.7619`, 4.7926`, 4.7789`, 4.7461`, 4.7693`,
4.7395`, 4.6719`, 4.6786`, 4.8235`, 4.8221`, 4.7775`, 4.7824`, 4.7838`, 4.7586`,
4.839`, 4.8248`, 4.79`, 4.8276`, 4.8403`, 4.7989`, 4.8242`, 4.9039`, 4.8058`,
4.8063`, 4.797`, 4.7995`, 4.8914`, 4.7794`, 4.7866`, 4.8061`, 4.7784`, 4.8081`,
4.8222`, 4.9228`, 4.839`, 4.793`, 4.7883`, 4.665`, 4.6684`, 4.6904`, 4.7223`,
4.7447`, 4.75`, 4.7541`, 4.8395`, 4.7587`, 4.8184`, 4.8166`, 4.8357`, 4.833`,
4.7891`, 4.7556`, 4.7634`, 4.8286`, 4.7902`, 4.8278`, 4.833`, 4.9479`, 4.8091`,
4.8245`, 4.8798`, 4.8127`, 4.8321`, 4.7918`, 4.8837`, 4.8596`, 4.8205`, 4.8871`,
4.8487`, 4.9177`, 4.9155`, 4.8941`, 4.8118`, 4.8641`, 4.8396`, 4.9453`, 4.8227`,
4.8756`, 4.8859`, 4.9003`, 4.9316`, 4.8299`, 4.801`, 4.8374`, 4.8342`, 4.807`,
4.7516`, 4.7771`, 4.8519`, 4.7843`, 4.7599`, 4.7584`, 4.732`, 4.7432`, 4.7206`,
4.6834`, 4.7637`, 4.7804`, 4.7609`, 4.7538`, 4.7307`, 4.7165`, 4.7156`, 4.7253`,
4.7388`, 4.7936`, 4.8069`, 4.7379`, 4.8187`, 4.7898`, 4.7571`, 4.7858`, 4.7161`,
4.6904`, 4.6967`, 4.841`, 4.8609`, 4.8735`, 4.8068`, 4.8787`, 4.7853`, 4.7414`,
4.7723`, 4.7569`, 4.7914`, 4.8038`, 4.8014`, 4.7656`, 4.8363`, 4.6898`, 4.6901`,
4.7714`, 4.856`, 4.8753`, 4.8328`, 4.8609`, 4.7674`, 4.7631`, 4.7708`, 4.8051`,
4.7684`, 4.7921`, 4.9053`, 4.8275`, 4.7507`, 4.8852`, 4.81`, 4.844`, 4.8988`,
4.8489`, 4.858`, 4.9434`, 4.9378`, 4.8573`, 4.8765`, 4.8612`, 4.7902`, 4.7951`,
4.8419`, 4.8523`, 4.838`, 4.8249`, 4.7724`, 4.8003`, 4.8207`, 4.7601`, 4.8223`,
4.8109`, 4.806`, 4.8228`, 4.7648`, 4.8712`, 4.8707`, 4.8862`, 4.8246`, 4.7666`,
4.8232`, 4.7829`, 4.7808`, 4.79`, 4.7463`, 4.8064`, 4.8147`, 4.7475`, 4.8088`,
4.8259`, 4.7707`, 4.7556`, 4.8133`, 4.7556`, 4.7818`, 4.7203`, 4.7433`};
```

This is a climate model run that we will use to determine parameters in our climate module:

```
In[∘]:= PL1 = ListPlot[x, PlotRange → All, Joined → True, Axes → False , Frame → True,
          AspectRatio → 1 / 3, ImageSize → 600, FrameStyle → Directive[Black, 16],
          FrameLabel → {"time (years)", "ΔT (degrees C)"}, PlotStyle → Gray]
```

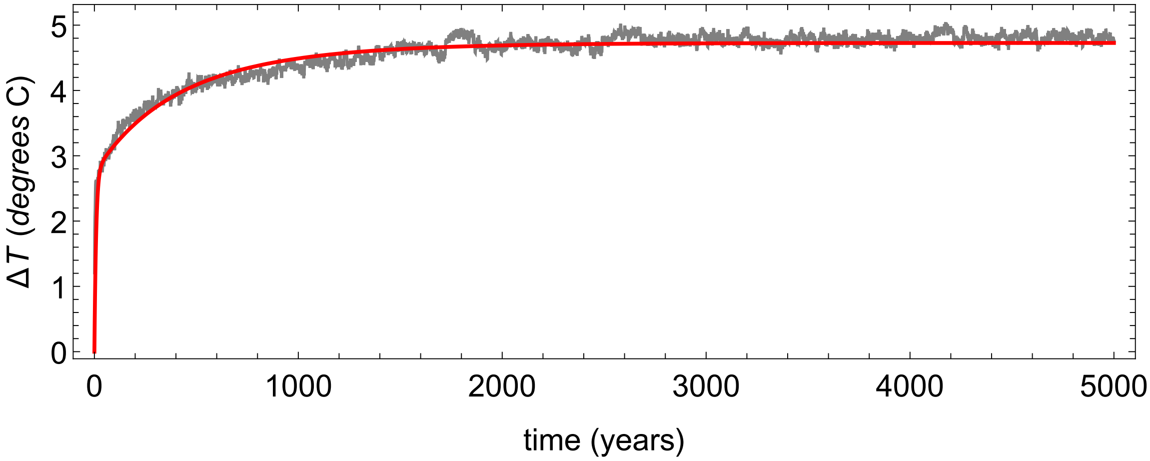
Out[∘]=



Here are parameters that work well.

```
In[∘]:= c1 = 3.1;
        c2 = 400;
        c3 = 420;
        τ1 = 8.5;
        τ2 = 261;
        τ3 = 560;
        Show[PL1,
         Plot[τ1 / c1 (1 - Exp[-t / τ1]) + τ2 / c2 * (1 - Exp[-t / τ2]) + τ3 / c3 * (1 - Exp[-t / τ3]),
          {t, 0, 5000}, PlotRange → {0, 5}, PlotStyle → {Red, Thick}]]
```

Out[∘]=



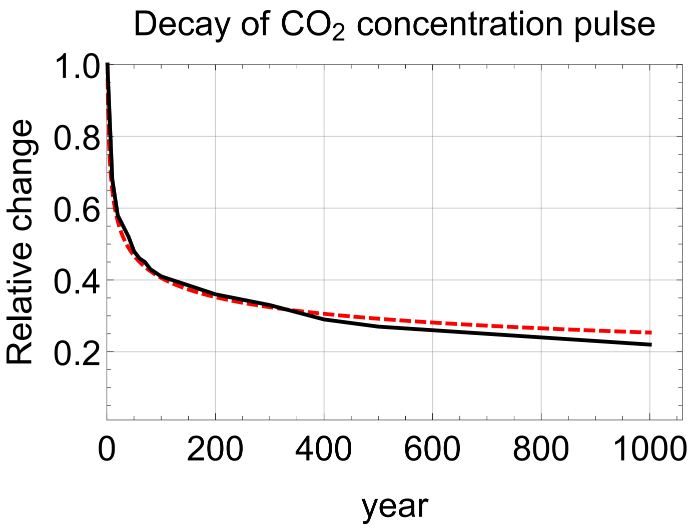
CO2 emissions and CO2 concentration response

From Fig. 1 in Joos et al. (2013) I create and plot the model-mean response curve:

```
In[◦]:= H = {{1, 1}, {10, 0.68}, {20, 0.58}, {30, 0.55}, {40, 0.52}, {50, 0.48}, {60, 0.46},
          {70, 0.45}, {80, 0.43}, {90, 0.42}, {100, 0.41}, {200, 0.36}, {300, 0.33}, {400, 0.29},
          {500, 0.27}, {600, 0.26}, {700, 0.25}, {800, 0.24}, {900, 0.23}, {1000, 0.22}};
        p1 = ListPlot[H, Joined → True, PlotStyle → {Black, Thick}];
```

```
In[◦]:= p2 = Plot[1.0394376335892603` * x^ (-0.20426475506502825`),
          {x, 1, 1000}, Axes → True, PlotRange →
           {{0, 1060}, {0.01, 1}}, GridLines → Automatic, Frame → True,
          PlotStyle → {Red, Dashed, Thick}, FrameLabel → {"year", "Relative change"},
          LabelStyle → {18}, PlotLabel -> Style["Decay of CO₂ concentration pulse", 18]];
        decayplot = Show[p2, p1]
```

Out[◦]=



Emission scenarios with action at t=150, 190, and 230 (t=0 in 1880)

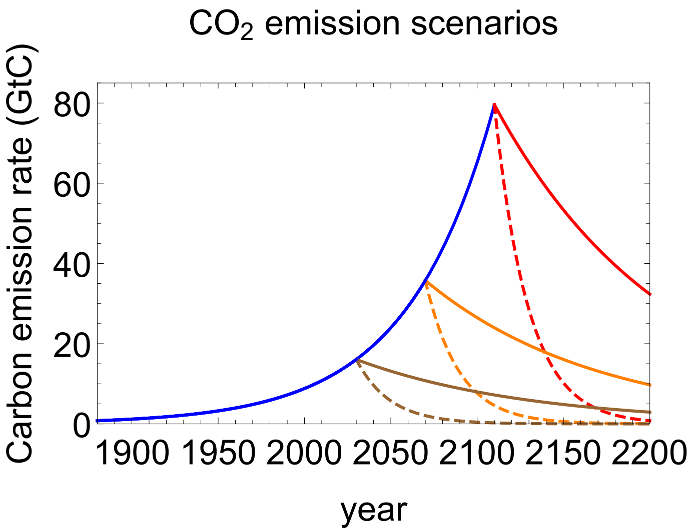
We construct a function for S(t) where the time origin is 1880:

```
In[◦]:= S[t_, t1_, S0_, g_, m_] := If[t < t1, S0 * Exp[g * t], S0 Exp[g * t1 - m * (t - t1)]]
```

Here t1 is the time of action, S0 is emission rate at t=0 (1880), g is the exponential growth rate of the unmitigated scenario, and m is the exponential mitigation rate. We have fitted an exponential S(t)=S0 exp[g t] through the points (80,4) and (130,11), which yields S0=0.8 GtC/yr and g=0.02 1/yr (a growth in emissions of about 2% per yr. Stocker use 1.8% per yr). The emission scenarios are given below:

```
In[•]:= S1onepercent = Plot[S[t - 1880, 150, 0.8, 0.02, 0.01],
           {t, 2030, 2200}, Frame → True, PlotRange → {{1880, 2200}, {0, 18}},
           FrameLabel → {"year", "Carbon emission rate (GtC)"}, LabelStyle → {18},
           PlotStyle → Brown, PlotLabel → Style["CO2 emission scenario", 18]];
        S1fivepercent = Plot[S[t - 1880, 150, 0.8, 0.02, 0.0513], {t, 2030, 2200},
           PlotRange → {{1880, 2200}, {0, 18}}, Frame → True,
           FrameLabel → {"year", "Carbon emission rate (GtC)"}, LabelStyle → {18},
           PlotStyle → {Brown, Dashed}, PlotLabel → Style["CO2 emission scenario", 18]];
        S1historic = Plot[S[t - 1880, 150, 0.8, 0.02, 0.0513], {t, 1880, 2030}, Frame → True,
           FrameLabel → {"year", "Carbon emission rate (GtC)"}, LabelStyle → {18},
           PlotStyle → {Blue}, PlotLabel → Style["CO2 emission scenario", 18]];
        S1 = Show[S1fivepercent, S1onepercent, S1historic];
        S2onepercent = Plot[S[t - 1880, 190, 0.8, 0.02, 0.01], {t, 2070, 2200}, Frame → True,
           FrameLabel → {"year", "Carbon emission rate (GtC)"}, LabelStyle → {18},
           PlotStyle → Orange, PlotLabel → Style["CO2 emission scenario", 18]];
        S2fivepercent = Plot[S[t - 1880, 190, 0.8, 0.02, 0.0513], {t, 2070, 2200},
           PlotRange → {{1880, 2200}, {0, 50}}, Frame → True,
           FrameLabel → {"year", "Carbon emission rate (GtC)"}, LabelStyle → {18},
           PlotStyle → {Orange, Dashed}, PlotLabel → Style["CO2 emission scenario", 18]];
        S2historic = Plot[S[t - 1880, 190, 0.8, 0.02, 0.0513], {t, 1880, 2070}, Frame → True,
           FrameLabel → {"year", "Carbon emission rate (GtC)"}, LabelStyle → {18},
           PlotStyle → {Blue}, PlotLabel → Style["CO2 emission scenario", 18]];
        S2 = Show[S2fivepercent, S2onepercent, S2historic];
        S3onepercent = Plot[S[t - 1880, 230, 0.8, 0.02, 0.01], {t, 2110, 2200}, Frame → True,
           FrameLabel → {"year", "Carbon emission rate (GtC)"}, LabelStyle → {18},
           PlotStyle → Red, PlotLabel → Style["CO2 emission scenario", 18]];
        S3fivepercent = Plot[S[t - 1880, 230, 0.8, 0.02, 0.0513], {t, 2110, 2200}, Frame → True,
           FrameLabel → {"year", "Carbon emission rate (GtC)"}, LabelStyle → {18},
           PlotStyle → {Red, Dashed}, PlotLabel → Style["CO2 emission scenario", 18]];
        S3historic = Plot[S[t - 1880, 230, 0.8, 0.02, 0.0513], {t, 1880, 2110},
           Frame → True, PlotRange → {{1880, 2200}, {0, 130}},
           FrameLabel → {"year", "Carbon emission rate (GtC)"}, LabelStyle → {18},
           PlotStyle → {Blue}, PlotLabel → Style["CO2 emission scenarios", 18]];
        S3 = Show[S3historic, S3fivepercent, S3onepercent];
        Show[S3, S2, S1, PlotRange → {0, 85}]
```

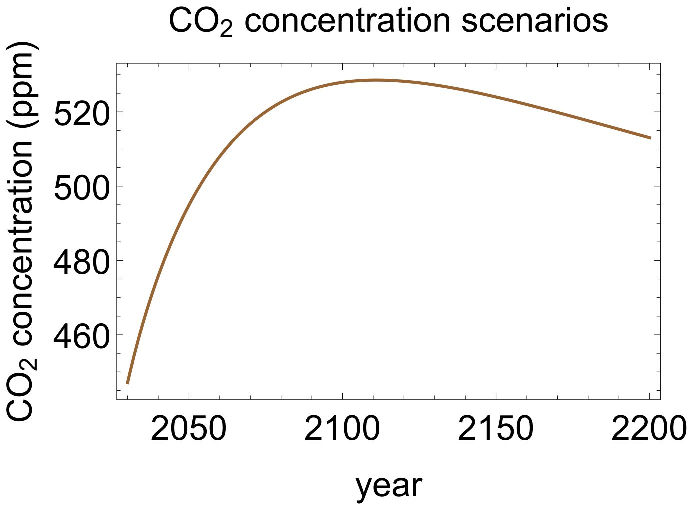
Out[∘]=


CO$_2$ emission scenarios

In[∘]:=

```
carb = Table[NIntegrate[(t - x)^-0.2 * S[x, 150, 0.8, 0.02, 0.02], {x, 0, t}], {t, 0, 500}];
carb = 280 + ((390 - 280) / carb[[130]]) * carb;
ListPlot[{Table[{1880 + i, carb[[i]]}, {i, 150, 320}]},
 Joined → True, PlotRange → All, PlotStyle → Brown, Frame → True,
 LabelStyle → {18}, FrameLabel → {"year", "CO₂ concentration (ppm)"},
 PlotLabel -> Style["CO₂ concentration scenarios", 18]]
```

Out[∘]=


CO$_2$ concentration scenarios

In[∘]:=

```
5.35 Log[2]
```

Out[∘]=

```
3.70834
```

In[∘]:=

```
3.708337415995707`
```

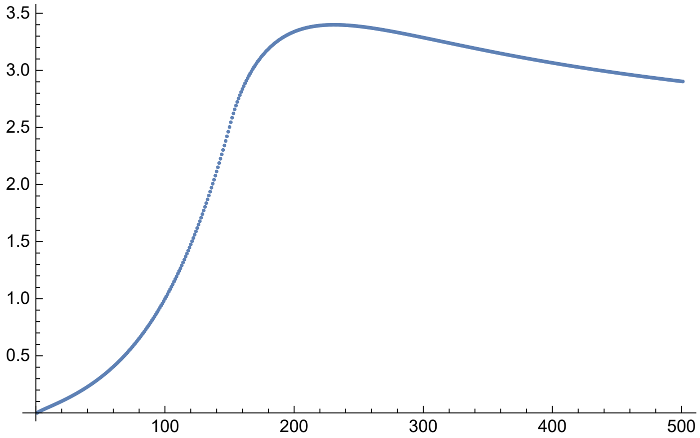
Out[∘]=

```
3.70834
```

There is the climate forcing corresponding to this CO2 scenario:

```
In[•]:= forc = Table[5.35 * Log[carb[[i]] / 280], {i, 1, 501}];
        ListPlot[forc]
```

Out[•]=



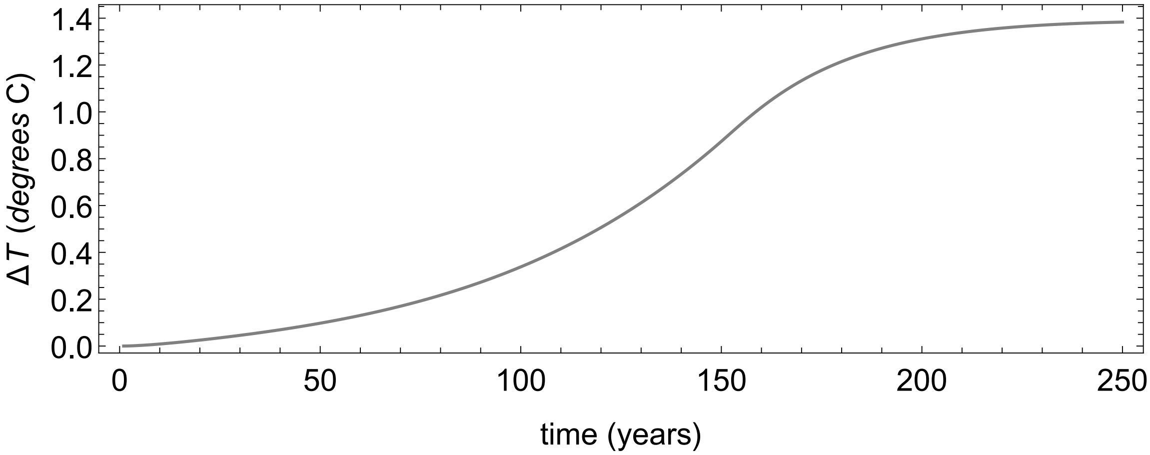
The climate model:

```
In[•]:= g = (1 / c1) * Exp[-# / τ1] + (1 / c2) * (1 - Exp[-# / τ2]) + (1 / c3) * (1 - Exp[-# / τ3]) &;
        matrise = Table[g[i - j] * UnitStep[i - j], {i, 1, 501}, {j, 1, 501}];
```

Doubling CO2 corresponds to 3.7 in W/m^2:

```
In[•]:= temp = matrise.(forc / (2 * 3.7));
        temp = temp[[1 ;; 250]];
        ListPlot[temp, PlotRange → All, Joined → True, Axes → False , Frame → True,
         AspectRatio → 1 / 3, ImageSize → 600, FrameStyle → Directive[Black, 16],
         FrameLabel → {"time (years)", "ΔT (degrees C)"}, PlotStyle → Gray]
```

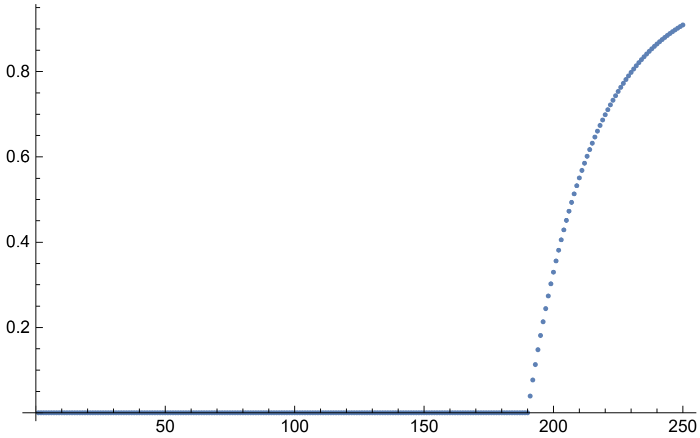
Out[•]=



Reduction rate in emissions:

```
In[∘]:= R = 1 - Table[S[t, 190, 0.8, 0.02, 0.02], {t, 1, 250}] /
            Table[S[t, 500, 0.8, 0.02, 0.02], {t, 1, 250}];
        ListPlot[
         R]
```

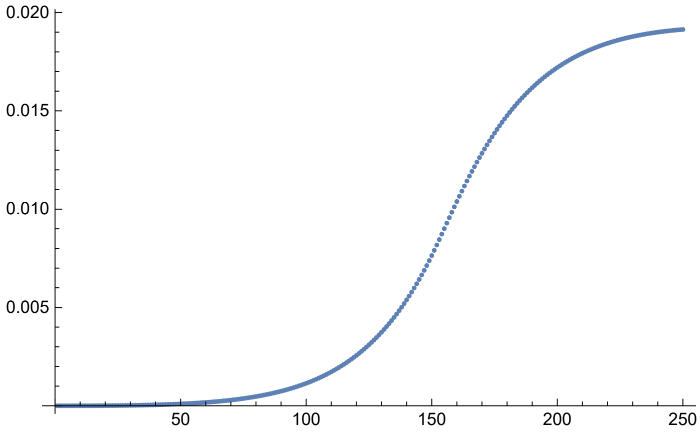
Out[∘]=



The damage function:

```
In[∘]:= ψ1 = 0.;
        ψ2 = 0.01;
        ψ6 = 0.0;
        damage = ψ1 * temp + ψ2 * temp^2 + ψ6 * temp^6;
        QL1 = ListPlot[damage]
```

Out[∘]=



Parameters in the economic model:

```
In[∘]:= s = 0.3;
        α = 0.1;
        δ = 0.2;
        A = 0.5;
        β = 0.1;
        ρ = 0.04;
        τ = 400;
```

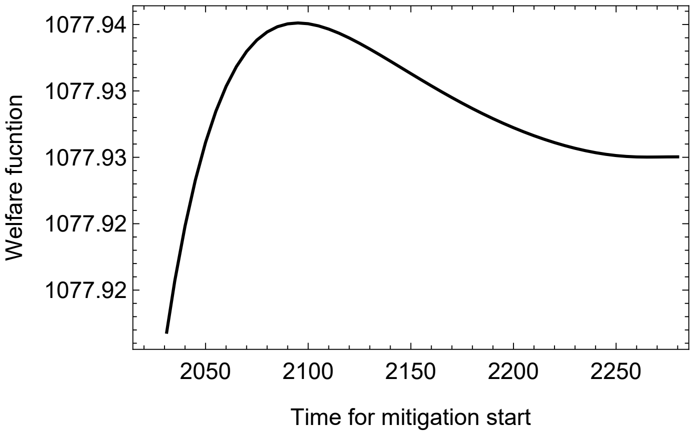
Running the integrated assessment model:

```
In[◦]:= liste = {};
        Monitor[
          Do[
            carb = Table[
              Sum[ (t - x + 0.5)^-0.2 * S[x + 0.5, mittime, 0.8, 0.02, 0.02], {x, 0, t}], {t, 0, 500}];
        carb = 280 + ((390 - 280) / carb[[130]]) * carb;
            forc = Table[5.35 * Log[carb[[i]] / 280], {i, 1, 501}];
            temp = matrise.(forc / (2 * 3.7));
            temp = temp[[1 ;; τ]];
            R = 1 - Table[S[t, mittime, 0.8, 0.02, 0.02], {t, 1, τ}] /
               Table[S[t, 500, 0.8, 0.02, 0.02], {t, 1, τ}];
            damage = ψ1 * temp + ψ2 * temp^2 + ψ6 * temp^6;
            Y = 10;
            K = 1;
            P = 1.;
            results = {};
            Do[
             K = (1 - δ) * K + s Y;
             YG = A * (K^α) * (P)^(1 - α);
             P = 1.015 * P;
             Y = (1 - damage[[t]] - β * R[[t]]^2) * YG;
             results = Append[results, {1880 + t, Y, K, P}];
             , {t, 1, τ}];
            liste = Append[liste,
              {1880 + mittime,
               Plus @@ (results[[All, 4]] * Log[(10^12) * ((1 - s) * results[[All, 2]]) /
                     results[[All, 4]]] * ((1 / (1 + ρ))^Range[1, τ]))}
             ];
            , {mittime, 140, 400, 5}];
          , mittime];
```

Plotting welfare as a function of when mitigation starts.

In[ ]:=
```
PL1 = ListPlot[liste, Axes → False, Frame → True,
  FrameLabel → {"Time for mitigation start", "Welfare fucntion"},
  FrameStyle → Directive[Black, 12], Joined → True, PlotStyle → Black]
```

Out[ ]=



In[ ]:=
```
liste[[Last[Ordering[liste[[All, 2]]]]]]
```

Out[ ]=
```
{2095, 1077.94}
```

For these parameters and this scenario (2% reduction per year after mitigation start), welfare is maximal if mitigation starts in 2095. This conclusion will change if we change the parameters.

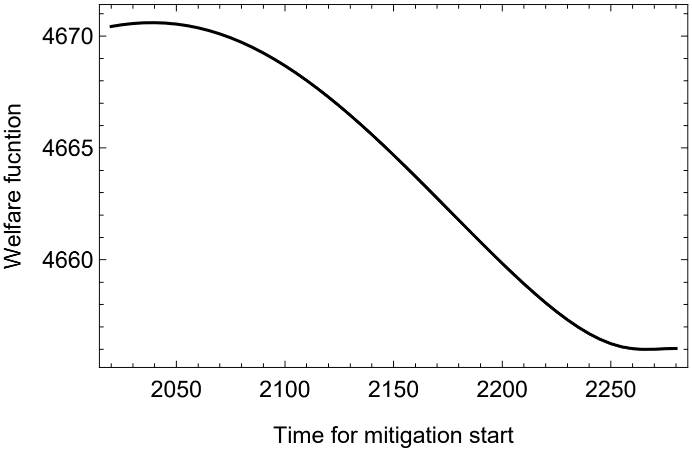
Now, try another parameters.

In[ ]:=
```
s = 0.1;
α = 0.1;
δ = 0.05;
A = 0.5;
β = 0.1;
ρ = 0.02;
τ = 400;
```

```
In[∘]:= liste = {};
        Monitor[
          Do[
            carb = Table[
              Sum[ (t - x + 0.5)^-0.2 * S[x + 0.5, mittime, 0.8, 0.02, 0.02], {x, 0, t}], {t, 0, 500}];
        carb = 280 + ((390 - 280) / carb[[130]]) * carb;
            forc = Table[5.35 * Log[carb[[i]] / 280], {i, 1, 501}];
            temp = matrise.(forc / (2 * 3.7));
            temp = temp[[1 ;; τ]];
            R = 1 - Table[S[t, mittime, 0.8, 0.02, 0.02], {t, 1, τ}] /
                Table[S[t, 500, 0.8, 0.02, 0.02], {t, 1, τ}];
            damage = ψ1 * temp + ψ2 * temp^2 + ψ6 * temp^6;
            Y = 10;
            K = 1;
            P = 1.;
            results = {};
            Do[
             K = (1 - δ) * K + s Y;
             YG = A * (K^α) * (P)^(1 - α);
             P = 1.015 * P;
             Y = (1 - damage[[t]] - β * R[[t]]^2) * YG;
             results = Append[results, {1880 + t, Y, K, P}];
             , {t, 1, τ}];
            liste = Append[liste,
              {1880 + mittime,
               Plus @@ (results[[All, 4]] * Log[(10^12) * ((1 - s) * results[[All, 2]]) /
                      results[[All, 4]]] * ((1 / (1 + ρ))^Range[1, τ]))}
             ];
            , {mittime, 140, 400, 5}];
          , mittime];

        PL1 = ListPlot[liste, Axes -> False, Frame -> True,
          FrameLabel -> {"Time for mitigation start", "Welfare fucntion"},
         FrameStyle -> Directive[Black, 12], Joined -> True, PlotStyle -> Black]
```

Out[∘]=



```
In[•]:= liste[[Last[Ordering[liste[[All, 2]]]]]]
Out[•]= {2040, 4670.6}
```

For these parameters, welfare is maximal, if we start mitigation in 2040, because the welfare is maximized at this point.

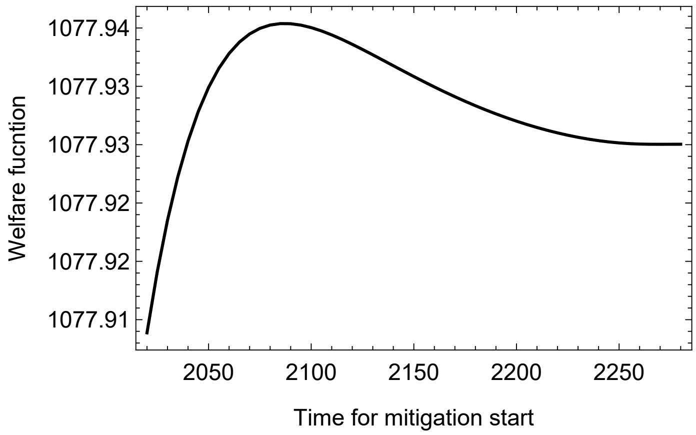
Now, try to change the scenario for the first set of parameters. Now there is a 1% reduction instead of the 2%.

```
In[•]:= s = 0.3;
        α = 0.1;
        δ = 0.2;
        A = 0.5;
        β = 0.1;
        ρ = 0.04;
        τ = 400;
```

```
In[∘]:= liste = {};
        Monitor[
          Do[
            carb = Table[
              Sum[ (t - x + 0.5)^-0.2 * S[x + 0.5, mittime, 0.8, 0.02, 0.01], {x, 0, t}], {t, 0, 500}];
        carb = 280 + ((390 - 280) / carb[[130]]) * carb;
            forc = Table[5.35 * Log[carb[[i]] / 280], {i, 1, 501}];
            temp = matrise.(forc / (2 * 3.7));
            temp = temp[[1 ;; τ]];
            R = 1 - Table[S[t, mittime, 0.8, 0.02, 0.01], {t, 1, τ}] /
               Table[S[t, 500, 0.8, 0.02, 0.01], {t, 1, τ}];
            damage = ψ1 * temp + ψ2 * temp^2 + ψ6 * temp^6;
            Y = 10;
            K = 1;
            P = 1.;
            results = {};
            Do[
             K = (1 - δ) * K + s Y;
             YG = A * (K^α) * (P)^(1 - α);
             P = 1.015 * P;
             Y = (1 - damage[[t]] - β * R[[t]]^2) * YG;
             results = Append[results, {1880 + t, Y, K, P}];
             , {t, 1, τ}];
            liste = Append[liste,
              {1880 + mittime,
               Plus @@ (results[[All, 4]] * Log[(10^12) * ((1 - s) * results[[All, 2]]) /
                     results[[All, 4]]] * ((1 / (1 + ρ))^Range[1, τ]))}
             ];
            , {mittime, 140, 400, 5}];
          , mittime];

In[∘]:= PL1 = ListPlot[liste, Axes -> False, Frame -> True,
          FrameLabel -> {"Time for mitigation start", "Welfare fucntion"},
         FrameStyle -> Directive[Black, 12], Joined -> True, PlotStyle -> Black]
```

Out[∘]=



```
In[•]:= liste[[Last[Ordering[liste[[All, 2]]]]]]

Out[•]= {2085, 1077.94}
```

For this scenario the mitigation should start in 2085.

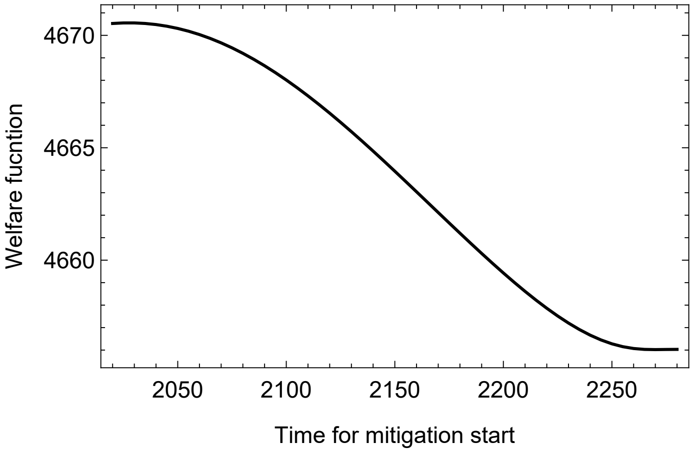
Now, let us try the 1% reduction for the second set of parameters.

```
In[•]:= s = 0.1;
        α = 0.1;
        δ = 0.05;
        A = 0.5;
        β = 0.1;
        ρ = 0.02;
        τ = 400;

In[•]:= liste = {};
        Monitor[
          Do[
            carb = Table[
              Sum[ (t - x + 0.5)^-0.2 * S[x + 0.5, mittime, 0.8, 0.02, 0.01], {x, 0, t}], {t, 0, 500}];
        carb = 280 + ((390 - 280) / carb[[130]]) * carb;
            forc = Table[5.35 * Log[carb[[i]] / 280], {i, 1, 501}];
            temp = matrise.(forc / (2 * 3.7));
            temp = temp[[1 ;; τ]];
            R = 1 - Table[S[t, mittime, 0.8, 0.02, 0.01], {t, 1, τ}] /
               Table[S[t, 500, 0.8, 0.02, 0.01], {t, 1, τ}];
            damage = ψ1 * temp + ψ2 * temp^2 + ψ6 * temp^6;
            Y = 10;
            K = 1;
            P = 1.;
            results = {};
            Do[
             K = (1 - δ) * K + s Y;
             YG = A * (K^α) * (P)^(1 - α);
             P = 1.015 * P;
             Y = (1 - damage[[t]] - β * R[[t]]^2) * YG;
             results = Append[results, {1880 + t, Y, K, P}];
             , {t, 1, τ}];
            liste = Append[liste,
              {1880 + mittime,
               Plus @@ (results[[All, 4]] * Log[(10^12) * ((1 - s) * results[[All, 2]]) /
                     results[[All, 4]]] * ((1 / (1 + ρ))^Range[1, τ]))}
             ];
            , {mittime, 140, 400, 5}];
          , mittime];
```

```
In[∘]:= PL1 = ListPlot[liste, Axes -> False, Frame -> True,
          FrameLabel -> {"Time for mitigation start", "Welfare fucntion"},
         FrameStyle -> Directive[Black, 12], Joined -> True, PlotStyle -> Black]
```

Out[∘]=



```
In[∘]:= liste[[Last[Ordering[liste[[All, 2]]]]]]
Out[∘]= {2030, 4670.55}
```

Now, we see that the mitigation should start in 2030.

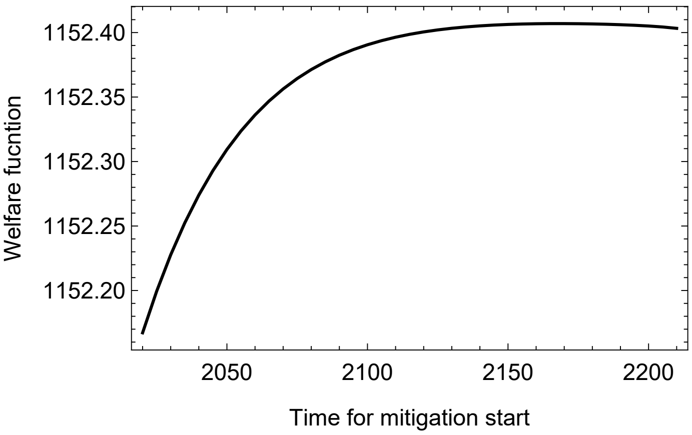
One more set of parameters. Here we again assume 2% reduction.

```
In[∘]:= s = 0.3;
        α = 0.3;
        δ = 0.2;
        A = 2;
        β = 0.3;
        ρ = 0.04;
        τ = 500;
```

```
In[∘]:= liste = {};
        Monitor[
          Do[
            carb = Table[
              Sum[ (t - x + 0.5)^-0.2 * S[x + 0.5, mittime, 0.8, 0.02, 0.02], {x, 0, t}], {t, 0, 500}];
        carb = 280 + ((390 - 280) / carb[[130]]) * carb;
            forc = Table[5.35 * Log[carb[[i]] / 280], {i, 1, 501}];
            temp = matrise.(forc / (2 * 3.7));
            temp = temp[[1 ;; τ]];
            R = 1 - Table[S[t, mittime, 0.8, 0.02, 0.02], {t, 1, τ}] /
               Table[S[t, 500, 0.8, 0.02, 0.02], {t, 1, τ}];
            damage = ψ1 * temp + ψ2 * temp^2 + ψ6 * temp^6;
            Y = 10;
            K = 1;
            P = 1.;
            results = {};
            Do[
             K = (1 - δ) * K + s Y;
             YG = A * (K^α) * (P)^(1 - α);
             P = 1.015 * P;
             Y = (1 - damage[[t]] - β * R[[t]]^2) * YG;
             results = Append[results, {1880 + t, Y, K, P}];
             , {t, 1, τ}];
            liste = Append[liste,
              {1880 + mittime,
               Plus @@ (results[[All, 4]] * Log[(10^12) * ((1 - s) * results[[All, 2]]) /
                     results[[All, 4]]] * ((1 / (1 + ρ))^Range[1, τ]))}
             ];
            , {mittime, 140, 400, 5}];
          , mittime];
```

```
In[∘]:= PL1 = ListPlot[liste, Axes -> False, Frame -> True,
          FrameLabel -> {"Time for mitigation start", "Welfare fucntion"},
         FrameStyle -> Directive[Black, 12], Joined -> True, PlotStyle -> Black]
```

Out[∘]=



```
In[ ]:= liste[[Last[Ordering[liste[[All, 2]]]]]]
Out[ ]= {2170, 1152.41}
```

If we use this set of parameters, the mitigation should start in 2170.