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Constraining Transient Climate Response Using the Historical Temperature Record

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“All models are wrong, but some are usefull”
–George Box

Abstract

Anthropogenic greenhouse gas emissions alter the planet's energy balance and cause heating of the surface and oceans. Estimates of the implications of continued emissions are necessary to assess the severity of the situation. The transient climate response (TCR) is a measure of the climate's response to a continued increase in CO₂ concentration, and indicates how much the temperature will increase in the coming decades if emission continues. However, uncertainty in estimates are still large and projections of TCR vary significantly between climate models in the Coupled Model Intercomparison Project (CMIP5) ensemble.

A new method to constrain the likely range of TCR is developed on the basis of a simple energy balance model (EBM). From recent estimates of historical radiative forcing and the global instrumental temperature record, a probability density function (pdf) for TCR is calculated. A 5 – 95% confidence interval is found, that suggests that models with the highest TCR values are inconsistent with the observed temperature record.

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Acronyms

AOGCM Atmospheric-ocean general circulation model

AR5 Assessment Report 5

EBM Energy balance model

GCM General circulation model

IPCC Intergovernmental Panel on Climate Change

MMMRT Multi-model mean relaxation time

MSRT Model-specific relaxation time

OBM One box model

SST Sea surface temperature

TBM Two box model

TOA Top of the atmosphere

Nomenclature

Symbol	Description	Unit
λ	Climate sensitivity	$\text{m}^2 \text{K/W}$
α	Climate sensitivity parameter	$\text{W/m}^2 \text{K}$
GMST	Global mean surface temperature	K
F	Forcing	W/m^2
RF	Radiative forcing	W/m^2
ERF	Effective radiative forcing	W/m^2
ECS	Equilibrium climate sensitivity	K
TCR	Transient climate response	K
T	Temperature	K
N	Net radiative imbalance	W/m^2
ϵ	Emissivity	Unitless
<i>Two Box Model</i>		
T	Temperature anomaly	K
T_0	Temperature anomaly of the deep ocean	K
T_{eq}	Equilibrium temperature anomaly	K
τ_f	Characteristic time scale (fast)	Years
τ_s	Characteristic time scale (slow)	Years
a_f	Amplitude fast response	Unitless
a_s	Amplitude slow response	Unitless



Introduction

1.1 Motivation

Since 1880 the global mean surface temperature has risen by about 0.85 degrees Celsius [Pachauri et al., 2014]. A rising greenhouse gas concentration is heating the planet, and the overwhelming consensus among climate scientist is that this increase results from human activity [Cook and Skuce, 2013].

Hurricanes like the ones in the Caribbean in 2017 are reinforced by the increasing ocean temperatures and cause massive destruction to cities and economies, as well as loss of human life. Despite the overwhelming evidence that anthropogenic greenhouse gas emissions has the potential to bring great harm to the economy as well as human civilization, experts and politicians still argue about how to best deal with the situation.

Climate science lays the foundation for political discussion on climate change. Given different socio-economic emission scenarios, the fundamental problem is to predict the future climate as accurately as possible. The costs of mitigation, along with the continuing high uncertainty of climate predictions, make politicians hesitant to pass unpopular laws enforcing emission reductions.

Climate models used as part of the Coupled Model Intercomparison Project Phase 5 (CMIP5) estimate a so-called *transient climate response* (TCR), which is a measure of the sensitivity of the climate to an external forcing on a timescale of about a century. This number is highly interesting, scientifically and especially politically, as it indicates how extensive the climate change of

the coming century will be if we continue perturbing the climate as we are doing today. The TCR value of the CMIP5 models vary from 1.1°C increase for a doubling of CO₂, to 2.6°C, and narrowing this range would strengthen the foundation on which mitigation and adaptation policies are to be made [Collins et al., 2013].

Temperature data has been recorded systematically since 1850. How useful is this record? Is it possible to narrow the likely range for TCR? This thesis aims at trying to answer this question.

1.2 Research Question

Given the recorded instrumental temperature record and a set of historical simulations from an ensemble of CMIP5 models, is it possible to constrain the likely values of TCR and thus discard some models as being inconsistent with observation?

1.3 Contribution

A new method for constraining the transient climate response is developed. Historical model simulations, together with the historical temperature record are used to constrain the likely range of TCR. The method is based on the fitted responses of a simple energy balance model (EBM) to adjusted forcings based on the historical simulations of the ensemble of CMIP5 models. The statistical framework is similar to the one used by Cox et al. (2018), however they attempted to constrain equilibrium climate sensitivity using a different approach [Cox et al., 2018].

1.4 Structure of Thesis

The rest of the thesis has the following structure:

Chapter 2 starts by introducing some important underlying concepts that are essential to the understanding of the thesis. The chapter also looks at a simple EBM used to fit the temperature response of the CMIP5 models.

Chapter 3 introduces the methods used to approach the research question. It looks at how to process the data to be able to compare models to the historical record and the statistical framework is explained.

Chapter 4 presents and discusses the results from applying the methods explained in chapter 3.

Chapter 5 summarizes the work, answers the research question and concludes the work of the thesis. It also presents some possible future studies.

/2

Theory and Background

This chapter introduces concepts that are essential to the understand the thesis. The concepts of forcing and climate sensitivity are presented in more technical terms. We look at the concept of feedbacks and how it is distinguished from forcing. Further, it looks at how the concept of effective forcing is needed to generalize forcing to account for different forcing agents. Two methods used to estimate effective forcing from models are explained. Moreover, the *two-box model* (TBM) and its response function are introduced, along with an earlier study on the TBM's ability to reproduce temperature responses of complex models.

2.1 Climate models

Through the years, large amounts of data has been collected about the climates of the past. From geological evidence we can approximately recreate temperatures back millions of years, and from ice cores we can recreate CO₂ concentrations from the past 800 000 years [Masson-Delmotte et al., 2013]. These data, along with known Milankovitch cycles¹, serve as guidelines for estimating climate sensitivity. However, the rate of change in the climate system seen today is unprecedented. This means there is limited use for such low-

1. Milankovitch cycles are cycles in eccentricity, axial tilt and precession of earth's orbit around the sun affecting the solar constant and distribution of energy of the planet.

resolution time series of forcing, CO₂ concentrations and temperature when predicting climate change for the coming century. This is why we need climate models.

Atmospheric-ocean (coupled) general circulation model (AOGCM) is the popular name of a fully coupled climate model that describes the dynamics of all the important climate variables. It is the most comprehensive tool available to study climate change and make climate predictions, built on the best knowledge acquired through the years, in a wide range of scientific fields. A model works by integrating a (large) set of discretized differential equations, based on widely accepted principles of physics, chemistry and fluid motion, along with properly chosen initial conditions. It uses a 3D-grid of the planet and only looks for solutions on these grid points. Important climate variables that are smaller than the resolution of the grid, like clouds, are parameterized by other variables, for instance temperature and humidity [McFarlane, 2011].

2.1.1 Global Mean Surface Temperature And Energy Balance Models

Although climate models solve for a large number of climate variables, it is the global mean surface temperature (GMST) that is of highest interest, because most of the important climate variables directly relate to it. Energy balance models (EBMs) are simple climate models that can be useful to summarize the thermal properties of the AOGCMs. They have the potential to investigate the temperature responses of AOGCMs for a wide range of scenarios, with limited computational costs.

2.2 Forcing and Climate Sensitivity

Radiative forcing (RF) and climate sensitivity are two frequently used concepts in climate change analysis. They are useful when comparing the size of responses between models and the effects of different forcing agents.

A radiative forcing is a change in the planetary radiation balance due to some perturbation, either internal or external. It is most often expressed in Watts per square meter (W/m²) and defined to be positive if the net flux is directed downwards [Myhre et al., 2013]. A nonzero net flux changes the total energy in the climate system and must lead to a modification of the climate system.

Climate sensitivity relates the surface temperature response to the imposed forcing. Equilibrium climate sensitivity (ECS) is the equilibrium temperature

change following a doubling of CO₂ concentration². Our focus of study is not the ECS, but the *transient climate response* (TCR), defined to be the mean surface temperature change in a 20-year period centered at a CO₂ doubling from a 1% per year increase over 70 years with subsequent CO₂ stabilization, see figure 2.1. TCR is always lower than ECS, because heat uptake in the ocean delay temperature increase [Otto et al., 2013].

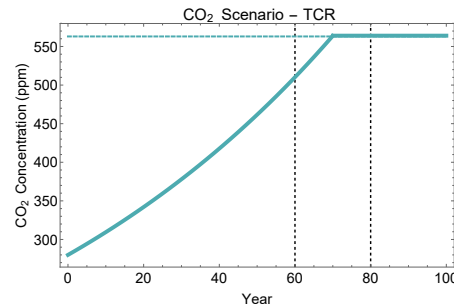


Figure 2.1: CO₂ concentration scenario used for calculating TCR. TCR is defined as the average temperature change between year 60 and 80.

Just like ECS, TCR serves as a tool for comparing responses of different models to the same standardized forcing. However, the two values serve different purposes, and they are both instructive in different ways. TCR measures the response when the system is in a transient state, still in disequilibrium from a partly ongoing increase in forcing.

Climate models respond differently on different time scales, and models giving a high ECS, do not always produce a high TCR. Nevertheless, the correlation is high and models producing a large TCR often produce high ECS as well, as illustrated in figure 2.2. All values of TCR and ECS used in this thesis refers to the values provided in Assessment Report 5 (AR5) from the Intergovernmental Panel on Climate Change (IPCC), published in 2013.

2.3 Feedbacks

Climate feedbacks result from the interconnection of climate variables. Changing one variable leads to a change in some other, which again changes the first. One familiar example is the ice-albedo feedback on temperature: An increase in temperature causes ice to melt, reducing the albedo, accelerating temperature increase further and causing even more ice to melt.

2. The definition of equilibrium does not account for the longest (e.g geological) time scales.

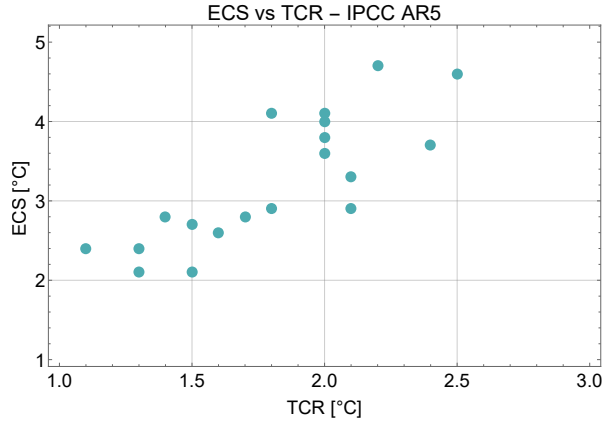


Figure 2.2: Scatterplot of ECS versus TCR given in IPCC AR5 for the 19 models used in this thesis.

If there were no feedbacks, then finding the change in GMST would be straight forward, given that we know the radiative imbalance. A forcing such as a doubling of CO_2 (around 3.7 W/m^2 [Myhre et al., 1998]) would cause a warming easily calculated using Stefan-Boltzmann's law, where the change in outward energy flux is given by

$$\Delta H \approx \frac{dH}{dT} \Delta T = \epsilon(4\sigma T_0^3) \Delta T, \quad (2.1)$$

where $H = \epsilon\sigma T^4$ is Stefan-Boltzmann's law and ϵ is emissivity of the planet (around 0.7). Using $T_0 = 288 \text{ K}$, we find that the temperature increase needed to balance the radiative forcing is

$$\Delta T \approx \frac{3.7 \text{ W/m}^2}{(0.7 \times 4 \times 5.67 \cdot 10^{-8} \times 288^3) \text{ W/m}^2 \text{ K}} \approx 1.0 \text{ K}. \quad (2.2)$$

However, there are a high number of internally interconnected agents, Ψ , in the climate system also influencing the planetary energy budget. The infinitesimal heat loss dH is therefore *not* given by equation (2.1), but more adequately described by

$$dH = dT \left(\frac{\partial H}{\partial T} + \sum_i \frac{\partial H}{\partial \Psi_i} \frac{d\Psi_i}{dT} \right), \quad (2.3)$$

where the first term is the black body radiation from (2.1) and the remaining terms correspond to changes in heat loss due to changes in other climate variables (that change due to temperature change). The magnitude and sign

of the terms in the sum are uncertain quantities, and determining this can be a subtle problem.

The partial derivatives in (2.3) are generally found to be state dependent, meaning that the change in the energy budget per change in the agent Ψ_i (i.e. $\partial H/\partial\Psi_i$) depends on the climate's state at that particular time. Moreover, the partial derivative might also depend on how the change dT occurs [Sherwood et al., 2015].

Despite the inaccuracies, the partial derivatives are often assumed to be constant, and in particular that

$$\frac{dT}{dH} = \left(\frac{\partial H}{\partial T} + \sum_i \frac{\partial H}{\partial \Psi_i} \frac{d\Psi_i}{dT} \right)^{-1} = \lambda, \quad (2.4)$$

where λ is the constant (state independent) climate sensitivity per unit forcing. A feedback is said to be *amplifying* if

$$\frac{\partial H}{\partial \Psi_i} \frac{d\Psi_i}{dT} < 0,$$

and *stabilizing* if not. According to AR5, it is *extremely likely*³ that the total feedback is positive. That is, the sum in (2.3) is negative and the temperature increase from a doubling of CO₂ is higher than the one degree we calculated above [Sherwood et al., 2015, Pachauri et al., 2014].

Traditionally, finding the climate sensitivity has been equivalent to the problem of finding the constant λ in equation (2.4), as the forcing from a doubling of CO₂ has been assumed to be known. However, since the forcing from a doubling of CO₂ is uncertain as well, the term climate sensitivity has been redefined to mean sensitivity to emissions. This depends on the magnitude of the forcing generated from a doubling of CO₂ as well [Stevens and Schwartz, 2012]. This is important, since it means that estimating TCR is not only about finding the feedbacks, but also estimating the forcing from a CO₂ doubling.

2.4 Adjustments and Effective Radiative Forcing

Radiative forcing is, as mentioned, frequently used to compare the size of responses to different forcing agents. Understanding how to define forcing in such a way that it can be used for comparison is thus essential, however, not straightforward.

3. This means that the probability is greater than 95%.

To define the forcing to be the instantaneous radiative imbalance at the top of the atmosphere (TOA) is often an inaccurate measure of the forcing agent [Hansen et al., 1997]. The temperature response in the stratosphere is not analogous to that of the surface temperature. There are processes within the stratosphere, often referred to as adjustments, that are different depending on the forcing agent. An increase in the greenhouse gas concentration actually cools the stratosphere while a solar forcing warms it, affecting the TOA energy budget in opposite directions. This implies that a given instantaneous TOA energy imbalance can correspond to a difference in the heating of the surface and troposphere if the agent were CO₂, compared to a solar forcing. The net radiative forcing at the TOA after stratospheric adjustment is called the *adjusted forcing* (AF)⁴, and is a popular way of defining forcing [Hansen et al., 2005].

Within the troposphere, there are also adjustments in response to the imposed forcing, that alter TOA energy flux balance before any change in surface temperature has taken place. One example of such an adjustment is the so-called “semi-direct effect” of aerosols, which is brought about by an uneven distribution of tropospheric heating due to aerosols. This has a destabilizing effect on the troposphere, driving circulation, affecting cloud cover and precipitation. These changes are not triggered by a change in the surface temperature, and should more accurately be considered an adjustment to the aerosol forcing, again changing the forcing, not as a feedback. This is because the change is triggered by the forcing directly and not by a change in temperature. The subsequent TOA flux imbalance, after adjustments in both the stratosphere and troposphere have taken place, is called the *effective radiative forcing* (ERF) [Sherwood et al., 2015, Gregory and Webb, 2008, Hansen et al., 2005]. For some forcing agents, the difference between the instantaneous TOA imbalance, AF and ERF is substantial, and it is important to distinguish between the different definitions. Forcing data used in subsequent analysis in this thesis will be ERF⁵, found using methods that will be explained in the next section.

4. Some authors, as well as IPCC AR5, refer to this as radiative forcing.

5. At least more similar to ERF than the other definitions.

2.5 Estimating Climate Sensitivity and Effective Radiative Forcing

2.5.1 Gregory Method

This method, proposed by Gregory et al. (2004), is used to estimate the ERF from a doubling of CO_2 (denoted Q) of the CMIP5 models used in AR5. The method uses a simple linear regression in a $N - T$ plane, where N is the net radiative imbalance at TOA and T is the GMST anomaly from a preindustrial baseline.

Let F be the forcing (positive downwards) and H be the outgoing radiative response (positive upwards), both measured at TOA, both initially zero. Intuitively, the net downward heat flux, $N = F - H$, is the rate of increase of energy in the climate system. One major assumption is linearity between temperature rise and outward heat flux, $H = \alpha T$, such that

$$N(t) = F(t) - \alpha T(t), \quad (2.5)$$

where α is the *climate sensitivity parameter*. From our previous discussion, linearity implies that equation (2.4) holds (and $\alpha = 1\lambda$).

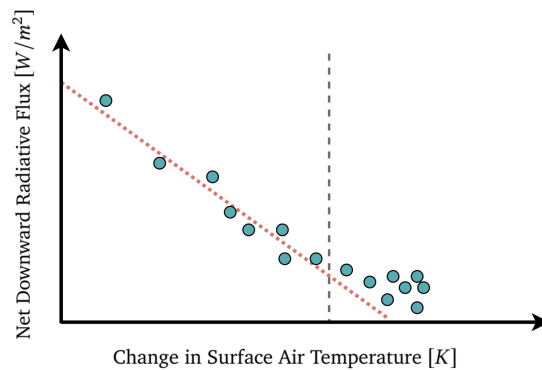


Figure 2.3: Illustration of the method proposed by Gregory et al (2004). The red dotted line is a linear fit of the points to the left of the grey dotted line, whereby the grey dotted line represents the cut-off time. The intersection of the x - and y -axis estimates the equilibrium temperature and the ERF, respectively. The slope estimates the magnitude of the climate sensitivity parameter α .

By plotting $N(t) = F - \alpha T(t)$ against $T(t)$, one should, in the case of linearity, get a straight line with slope $-\alpha$ that intersects the x -axis at $T = F/\alpha$ and y -axis at F . If α is constant, there is no need to run an AOGCM until its steady state to estimate the climate sensitivity, only far enough to perform

regression on the slope of $N(t)$ against $T(t)$. Figure 2.3 illustrates the method whereby the black dotted line represents the cut-off time after which the points (N, T) are no longer part of the regression. Forcing obtained using this method should, in particular, include adjustments and other responses like the semi-direct effect of aerosols that do not change radiative balance at the TOA instantaneously. This is because the regression points are calculated as averages over years, longer than the time scales of the adjustments⁶. This method therefore provides a tool for estimation of the ERF, climate sensitivity and equilibrium temperature change. Also using figure 2.3, we can think of a climate feedback as a mechanism that changes the slope of the curve, while a forcing changes the intercept⁷. Gregory et al. (2004) found that, when running $4\times\text{CO}_2$, α behaved non-linear when the temperature change got high ($> 4^\circ\text{C}$). A newer study from Andrews et al. (2012) also found similar non-linearity, while estimates done on $2\times\text{CO}_2$ showed less non-linearity as the temperature increase was lower [Andrews et al., 2012, Gregory et al., 2004].

2.5.2 Forster Method For Estimating Time Varying Forcing From Models

The Gregory method provides a procedure for computing the climate sensitivity parameter and the ERF from a constant forcing in a climate model. In order to calculate time varying ERF however, some other method is needed.

Forster et al. (2013) proposed a simple two-step method for obtaining a time series for the ERF. Step one uses $4\times\text{CO}_2$ simulations to estimate α using the Gregory method. Step two uses the estimate of α and

$$F(t) = N(t) + \alpha T(t), \quad (2.6)$$

together with the diagnostics of $N(t)$ and $T(t)$, obtained from the climate model, to obtain the time series of the forcing $F(t)$ in the model [Forster et al., 2013]. Hereafter, we refer to forcing calculated this way as *Forster forcing*.

Diagnosing forcing this way produces large year-to-year fluctuation in the forcing due to natural fluctuation in temperature. This is because the linearity does not hold in the case of large interannual variability [Trenberth et al., 2015]. However, over time, the linearity is a satisfactory assumption, which means that the trend in forcing should be approximately captured by equation (2.6) [Gregory et al., 2004].

6. Even though the value of F is calculated at the intersect where $T = 0$, this does not mean that this is the instantaneous forcing. It is the net radiative imbalance before the surface temperature has had time to respond to the forcing.

7. Assuming state independence for the climate sensitivity, as the slope might depend on the magnitude of the forcing otherwise.

2.5.3 Hansen Forcing

The period since 1850 is called the "historical period" in climate science [Miller et al., 2014]. This is the period where we have instrumental data of climate variables, which also starts before significant anthropogenic forcing had begun⁸. Our ability to reproduce the forcing of this period is highly important in order to infer something about the sensitivity of the climate from instrumentally recorded changes climate.

An especially comprehensive study of the historical forcing that has been extensively used in studies of historical climate is presented in Hansen et al. (2011), and has been updated since. As with the Forster forcing, it uses models (six different versions of the GISS-E2 model) to analyze the effect of different forcing agents and simulate trends in surface temperature, atmospheric temperature, sea ice and ocean heat uptake [Miller et al., 2014, Hansen et al., 2011].

This forcing (hereafter referred to as *the Hansen forcing*) does not have large fluctuations like Forster forcing, since it is calculated as a mean of an ensemble of different simulations from different models. The Hansen forcing is displayed in figure 2.4 (left), along with the Forster forcing from one of the CMIP5 models used in this thesis, the MIROC5 model (right). The large fluctuations in the Forster forcing are clearly seen.

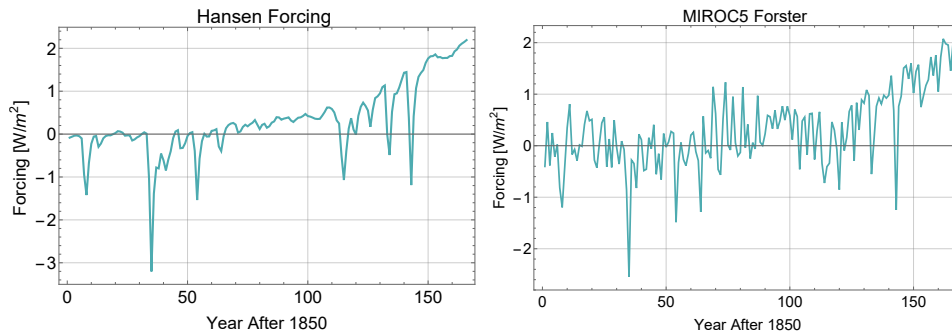


Figure 2.4: Hansen forcing (left) and Forster forcing from MIROC5 show the large fluctuations of the Forster forcing.

When plotting the mean of all the different Forster forcings used in this thesis, the fluctuations evidently cancel, and what we get is indeed very close to the forcing data of Hansen et al. (2011)⁹, see figure 2.5.

8. We can therefore assume some sort of equilibrium prior to the period.

9. The data set has been extended since 2011 to include the last few years.

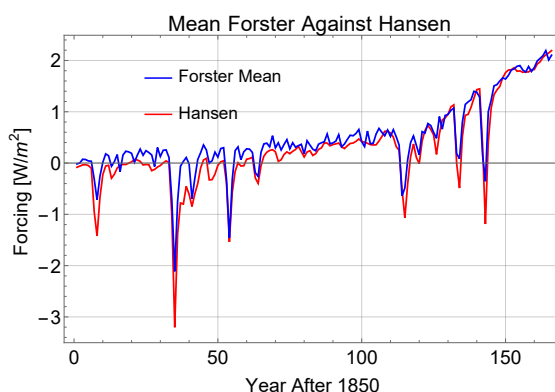


Figure 2.5: Plot of the mean of all 15 forcings from Forster et al. (2013) used in this thesis and that has data back to 1850 (blue), along with the Hansen forcing also used (red).

2.6 The Importance of TCR

In the transient regime, as long as the linearity holds, TCR maps roughly to other forcing scenarios by a factor equal to the forcing of that scenario to a doubling of CO_2 :

$$T = \text{TCR} \cdot \frac{F}{F_{2 \times \text{CO}_2}}. \quad (2.7)$$

This means that the uncertainty in the most likely forcing scenarios¹⁰ maps directly to the uncertainty in the estimates of TCR and vice versa. TCR is therefore not only a non-observable value for comparison of models, but also a guideline for the severity of climate change in the most likely future climate scenarios [Gregory and Forster, 2008, Collins et al., 2013].

High uncertainty in historical forcing, uncertainty in aerosols' effect on forcing, cloud feedbacks and ocean heat uptake¹¹ makes estimates of TCR highly uncertain [Myhre et al., 2015, Otto et al., 2013].

2.7 Two-Box Model

One simple and popular EBM used to model temperature response to forcing is the *two-box model* (TBM). It models the earth as a uniform planet of two distinct thermal layers, one for the atmosphere, biosphere and upper ocean,

10. Forcing is likely to continue to increase throughout this century due to enduring greenhouse gas emissions.

11. The list is longer, but these are the most important.

and one for the deep ocean. Temperatures, one for each layer, are the only climate variables it models.

The TBM is characterized by equations (2.8) and (2.9) and illustrated in figure 2.6.

$$C \frac{dT}{dt} = F - \alpha T - \gamma(T - T_0) \quad (2.8)$$

$$C_0 \frac{dT_0}{dt} = \gamma(T - T_0), \quad (2.9)$$

where:

- T ~ Characteristic temperature perturbations for the atmosphere/upper-ocean layer
- T_0 ~ Characteristic temperature perturbations for the deep-ocean layer
- α ~ Climate sensitivity parameter (free)
- γ ~ Heat exchange coefficient (free)
- F ~ Radiative forcing
- C ~ Effective heat capacity per unit area of the upper-ocean layer, (free)
- C_0 ~ Effective heat capacity per unit area of the deep-ocean layer (free)

The two layers (boxes) are interconnected through heat transfer, with heat going from the upper ocean to the deep ocean at a rate given by $C_H = \gamma(T - T_0)$. The upper layer also gains energy through the radiative forcing F and gives off energy according to the linear assumption $H = \alpha T$. The net heat influx to the climate system (both layers) equals the net flux at TOA, $N = F - \alpha T$.

2.7.1 Two-Box Model Explained

Although the TBM is extremely simple compared to AOGCMs, the equations that describe its dynamics are grounded in the physical and dynamic properties of the planet.

First of all, the ocean's importance to the dynamics of the climate can not be overstated. According to IPCC AR5, there is high confidence that the oceans

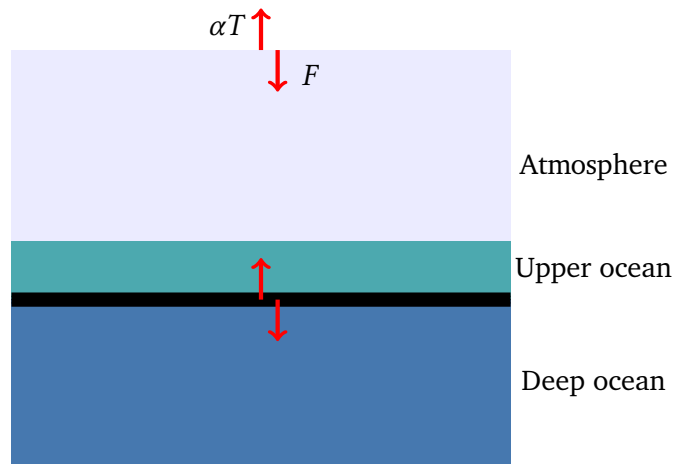


Figure 2.6: Simple illustration of the TBM. Red arrows indicate boundaries where heat is exchanged. The upper layer exchanges heat with the external universe and with the bottom layer at the boundary between the upper and deep ocean.

were responsible for around 90% of the heat uptake in the climate system between 1971 and 2010 [Pachauri et al., 2014]. The ocean has its own dynamics and a thermal profile, which leads to the distinction between the upper and deep ocean in the TBM. The upper part of the ocean is often called the *mixed layer*, which is a layer with a small temperature gradient due to active turbulence and overturning of fluid masses due to wind and waves. The mixed layer interacts directly with the atmosphere and exchanges heat on short time scales (daily to seasonally). Below the mixed layer is the thermocline¹², that separates the mixed layer from the much colder deep ocean below.

It is the structure of the ocean, with the mixed layer and deep ocean separated by a steep thermocline that is behind the reasoning. The atmosphere and upper ocean could in terms of its thermal properties be regarded as one single thermal layer while the deep ocean must be thought of as a separate part of the climate system. The deep ocean exchanges heat with the upper ocean in more indirect ways that are more indirect, and on longer time scales.

It is important to understand that even though there is only one temperature for each layer, this does not mean that it pictures the planet as only having two distinct temperature. There are obviously large temperature gradients within both layers, but this does not inhibit the use of only one temperature, since this temperature should be seen as an average.

12. The thermocline is defined to be a thin layer in a large body of fluid in which the temperature changes more rapidly than layers above and below.

An even simpler climate model is the *one-box model* (OBM), described by only one equation:

$$C \frac{dT}{dt} = F - \alpha T - \gamma T.$$

In this model, the deep ocean acts as an infinite external heat sink that absorbs heat from the climate system at a rate γT . The TBM becomes equivalent to the OBM in the limit $C_0 \rightarrow \infty$.

It is important to note that the temperatures in (2.8) and (2.9) are temperature deviations from equilibrium, and that heat exchange between the upper and deep ocean is always going on, even when the system is in equilibrium. Moreover, this means that although the mechanisms of heat exchange between the upper and deep ocean is highly complicated, it should be possible to describe the additional disequilibrium heat exchange between the two layers as a linear function of the disequilibrium difference in temperature ($T - T_0$), in the limit where the perturbations are small. This is however only if the climate system is not in an equilibrium state within close proximity of a bifurcation point, often referred to as a *tipping point*, where a smooth change in temperature leads to abrupt changes to the mechanisms of ocean circulation.

2.7.2 Response Function

Equations (2.8) and (2.9) form a set of coupled linear ordinary differential equations that can be solved analytically (see Appendix A). The solution is given by

$$T(t) = T^*(t) + \frac{a_f}{\alpha\tau_f} \int_0^t F(s)e^{-(t-s)/\tau_f} + \frac{a_s}{\alpha\tau_s} \int_0^t F(s)e^{-(t-s)/\tau_s} \quad (2.10)$$

$$T_0(t) = T_0^*(t) + \frac{\phi_f a_f}{\alpha\tau_f} \int_0^t F(s)e^{-(t-s)/\tau_f} + \frac{\phi_s a_s}{\alpha\tau_s} \int_0^t F(s)e^{-(t-s)/\tau_s}, \quad (2.11)$$

where $T^*(t)$ and T_0^* are the homogeneous parts of the solutions that depend on the initial state of the system and, in particular, they equal zero if (and only if) the initial temperature anomaly is set to zero.

The particular part of the solution in equation (2.10) is a convolution integral between the forcing function and a *response function*¹³, where the response function (for the upper layer temperature) is given as

$$G(t) = \frac{a_f}{\alpha\tau_f} e^{-t/\tau_f} + \frac{a_s}{\alpha\tau_s} e^{-t/\tau_s} \quad t \geq 0. \quad (2.12)$$

13. The response function is (when multiplied by a unit step function) a Green's function that is the solution to the temperature response for a forcing pulse (Dirac delta function).

The response function explains how the response to some forcing propagates through time and the TBM is characterized by a response function which is the sum of two distinct exponentials, one with a short(er) characteristic time scale (τ_f) and the other with a long(er) (τ_s).

In the case of the step forcing, the convolution integral becomes trivial and the solution (given $T(0) = T_0(0) = 0$) of (2.10) becomes

$$T(t) = \frac{F_0}{\alpha} \left(1 - a_f e^{-t/\tau_f} - a_s e^{-t/\tau_s} \right), \quad (2.13)$$

where F_0 is the magnitude of the step forcing. This temperature response is plotted in figure 2.7.

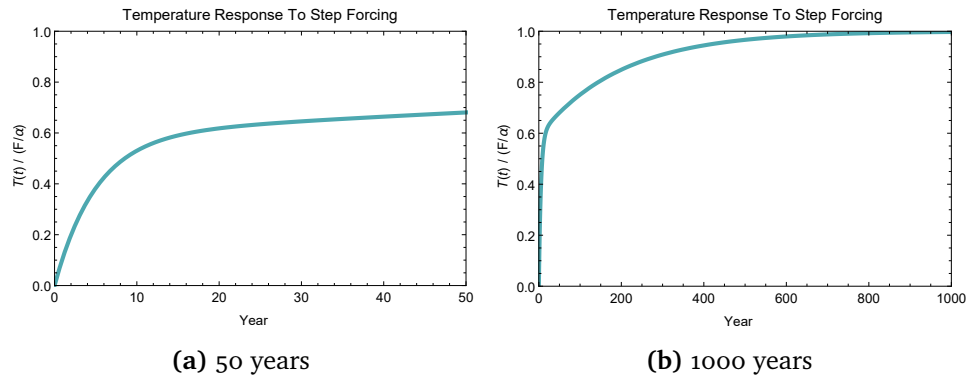


Figure 2.7: Temperature response to an abrupt change in forcing showing the two distinct exponential responses over (a) 50 years and (b) 1000 years. The parameters used are multi-model means given in Geoffroy et al. (2013) [Geoffroy et al., 2013].

2.7.3 Two-Box Model Fitted Using AOGCMs

We now take a look at the TBM's ability to fit the temperature response of the excessively more complicated AOGCMs in the idealized case of a $2\times\text{CO}_2$ step forcing.

Geoffroy et al. (2013) suggested a procedure for tuning the free parameters of TBM to fit different CMIP5 AOGCMs. The fitting is based on two steps, the first for estimating the forcing F_0 and climate sensitivity parameter α , and the second for estimating a_f , a_s , τ_f and τ_s from equation (2.13).

The first step uses the net flux at the top of the climate model (assumed to be equivalent to the TOA) and the temperature response in order to estimate F_0 and α using the Gregory method of section 2.5.1. In this way, the forcing includes both stratospheric and tropospheric adjustments.

The second step uses only the upper layer temperature response and equation (2.13). By assuming $\tau_f \ll 30$ years, equation (2.13) can be approximated by

$$T \approx \frac{F_0}{\alpha} \left(1 - a_s e^{-t/\tau_s}\right) \Rightarrow \ln \left(1 - \frac{\alpha T}{F_0}\right) \approx \ln a_s - \frac{1}{\tau_s} t, \quad (2.14)$$

from which a_s and τ_s is found from linear regression against t over the period 30-150 years. Then $a_f = 1 - a_s$, and τ is found by solving for τ_f in equation (2.13) and averaging over the first 10 years.

Figure 2.8 shows two plots from the Geoffroy paper showing the fitted TBM output against the temperature response of two different AOGCMs. The red curve corresponds to the $2\times\text{CO}_2$ step forcing scenario, to which the parameters are fitted, and gives a quite satisfactory fit. The blue curve corresponds to a simulation of the TCR-scenario, and also shows a quite accurate fit.

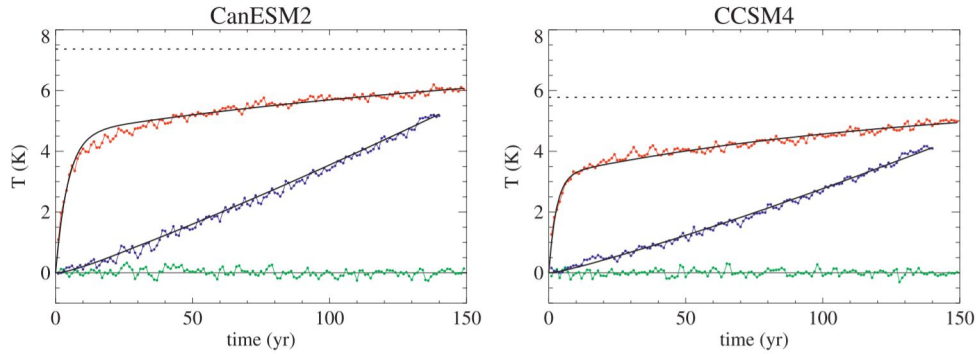


Figure 2.8: Figure from Geoffroy et al. (2013) showing the fit of the TBM tuned to an instant doubling of CO_2 simulation (red). Using the tuned parameters, the TBM is able to reproduce the temperature of a TCR-scenario accurately (blue curve). The green series is a control simulation without external forcing.

Importantly, the TBM plot (black) for the TCR scenario uses the parameters estimated from the $2\times\text{CO}_2$ only. The only thing that differs between the two TBM curves is the forcing, where the forcing used in the TCR scenario is linearly increasing instead of instantaneous. This forcing is based on the approximate logarithmic relationship between CO_2 concentration and forcing gives a linear forcing

$$F(t) = at,$$

where the constant a is found by setting $t = 70$ years and using the estimate of F_0 from the $2\times\text{CO}_2$ simulation.

The important finding from this study is that the TBM is able to reproduce temperature response from AOGCMs accurately, and parameters fitted to one

scenario seems to reproduce temperature response of different scenarios accurately as well, given a forcing that is representative for that scenario. This should mean that parameters found when fitting the TBM to any scenario could be considered parameters characteristic to that particular AOGCM (i.e there is only one set of parameters for each model, not one set for each AOGCM/scenario pair).

Parameters can however vary significantly between different AOGCMs, and the inter-model differences are generally much larger than the residual error of the TBM fit (see figure 2.8). This means that although the fit is imperfect, it generally suffices as a quantitative tool to investigate the responses of different models.

/ 3

Methods and Implementation

This chapter looks at how historical forcings from models are adjusted, how the TBM is fitted to the AOGCMs, and how a metric for comparing models to instrumental data is developed. Finally, the statistical methods used to obtain a probability density for the TCR, based on the model ensemble and the instrumental temperature record, is explained.

3.1 Overview of Method

Imagine there exist some metric, call it ν , and a function Φ , such that the function Φ evaluated at ν gives the TCR corresponding to the metric ν :

$$\Phi(\nu) = \text{TCR}. \quad (3.1)$$

This method uses an informed metric ν , which can be computed both from AOGCM simulation data and from historical data for comparison. Further, it uses the climate models to estimate the function Φ . As the function must be estimated from data, the function is transformed to a probability density function $P(\text{TCR} | \nu)$. Furthermore, there is uncertainty associated with ν as well, so ν is also transformed to a pdf $P(\nu)$. The estimates culminate in a

probability density function (pdf) for TCR, constrained by observation:

$$P(\text{TCR}) = \int P(\text{TCR} | \boldsymbol{\nu}) P(\boldsymbol{\nu}) d\boldsymbol{\nu} \quad (3.2)$$

3.2 Independence of Models

Available for this study, is data from 19 different IPCM5 AOGCMs. These models are built by independent research teams and we will throughout this thesis assume independence between time series produced by the different models. However, these models are not fully independent, often using the same real world data for calibration as well as similar or identical software to describe parts of the model. For instance, here is a quote from the European Network for Earth System Modelling (ENES) about the NorESM model (one of the 19 models used in this thesis):

The NorESM-family of models are based on the Community Climate System Model version 4 (CCSM4) of the University Corporation for Atmospheric Research, but differs from the latter by, in particular, an isopycnic coordinate ocean model and advanced chemistry-aerosol-cloud-radiation interaction schemes [ENES, 2018].

The CCSM4 model mentioned is also one of the models used in this thesis, so there are obviously some interconnection and the assumption of independence is therefore at best a good approximation.

3.3 Method For Comparing AOGCMs To Observation

3.3.1 Adjusting Forcing

With the goal of fitting response functions to the temperature response of each particular AOGCM model, we must make sure that the forcing applied when fitting the response function mimics the forcing experienced by that particular model.

Different models respond differently and produce different historical forcings, and the Forster method provides a tool for investigation of the forcing experienced by individual models. However, as mentioned, the forcing series obtained using this method inherits the same fluctuations as the temperature of the

internally fluctuating climate system. It is important to separate the *forced response* from the internal variability of the climate, as the realized temperature response will not in general follow the forced response.

Furthermore, since we are estimating the response of the model, there will be a problem with circular reasoning when the forcing depends directly on the temperature response. The forcing might end up explaining the natural fluctuation as well.

Despite the shortcomings of the Forster method as a tool for investigating the response, the Forster forcing should, over time (in the regime where linearity holds) depict the forcings experienced by each model adequately. By smoothing the time series, we can obtain a forcing series that characterizes the actual forcing better. At the same time, we avoid the problem of an overfitted forcing leading to an explanation for the internal variability. However, the actual historical forcing (though unknown) is not particularly smooth. Especially large volcanic eruptions are important events that greatly effect the forcing, and it is important to keep these natural short time scale forcing events as part of our adjusted forcing.

In order to (try to) bypass all these problems, we will use the Hansen forcing as our starting point, and adjust this forcing by a time series that is the difference between a smoothed Forster forcing and a smoothed Hansen forcing. In this way, we will account for the differences between models and at the same time make sure that the forcing accounts for important natural changes to forcings. A simple non-technical overview of the method is illustrated in figure 3.1.

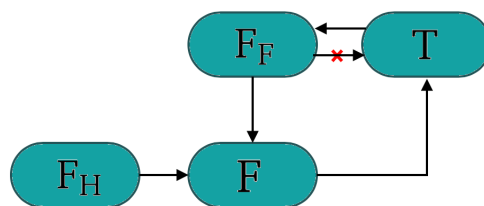


Figure 3.1: Illustration of the process for obtaining the forcing used to fit the response function to the temperature data. F_F , F_H , and F denotes the Forster forcing, Hansen forcing and final forcing, respectively. The red cross illustrates the problem of circular reasoning when using the Forster forcing.

By introducing this correction of the Hansen forcing, we have in a way indirectly connected each AOGCM model with the TBM. The forcing surface temperature to change now depends on each AOGCM. The response function obtained when fitting the forcing to the temperature response therefore not only depends on the temperature response, but also on each model's different imposed forcing.

This is essential to the method.

One problem with this method might be that if models respond very differently to the important natural forcings, then using the Hansen forcing and just adding a smoothed trend difference would not allow for such differences. This is problematic, however even if there are some differences in the forcing from large natural forcing agents, these differences will only be problematic over short periods of time, and one can therefore argue that this method is reasonable.

The appropriate window size for smoothing is not clear. We want a sufficiently large window as to effectively smooth the internal fluctuation, however short enough to capture possible trend differences between the two forcing series.

Figure 3.2 shows the procedure for obtaining the adjusted forcing for the MIROC5 model using a smoothing window of 16 years. The plot in the upper left corner is the smoothed forcings of the MIROC5 and Hansen forcing from figure 2.4. The difference between the two (shown in the upper right corner) is added to the Hansen forcing to give the adjusted MIROC5 forcing. The final adjusted forcing is shown in the bottom plot, together with the Hansen forcing.

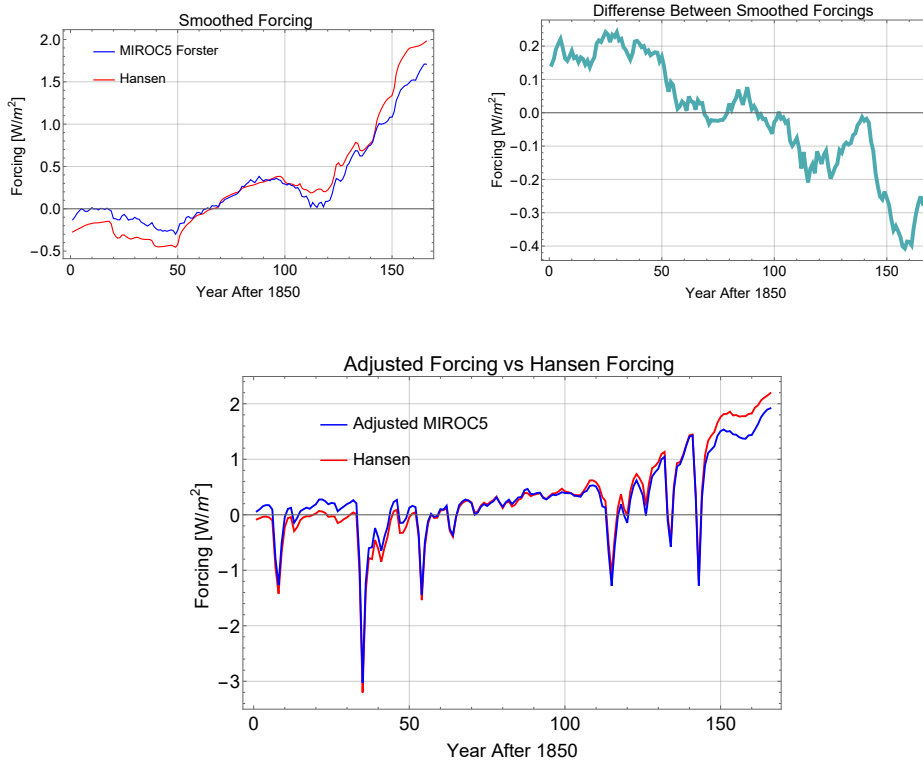


Figure 3.2: Procedure for obtaining the adjusted forcing for the MIROC5 model. Upper left shows the smoothed version of the two time series in figure 2.4, using a 16-year window. The upper right shows the difference (correction) between the two smoothed forcings. The lower plot shows the adjusted forcing together with the original Hansen forcing

3.3.2 Fitting the Response Function

Having established which forcing to use for each model, we can turn our attention to the fitting of the response functions. Again, the TBM provides a response function

$$G(t) = c_1 e^{-t/\tau_f} + c_2 e^{-t/\tau_s}, \quad t \geq 0. \quad (3.3)$$

We want to minimize the sum of squared errors

$$\sum_n (\hat{T}_n - T_n)^2, \quad (3.4)$$

where T_n is the n th element of the time series \mathbf{T} from the AOGCM or from the instrumental record, and

$$\hat{T}_n = c_0 + \sum_{s=0}^n F(s) \left(c_1 e^{-(n-s)/\tau_f} + c_2 e^{-(n-s)/\tau_s} \right) \quad (3.5)$$

is the fitted TBM temperature in year n of the time series¹, where F is the forcing. Note that the fitting of the TBM using the regression fit of Geoffroy et al. (2013) is not possible, as it requires both a constant forcing and data of the TOA energy flux. Our method of fitting is more direct, and can be applied to any forcing scenario.

One problematic deficiency of the data is the short length of the time series. This means overfitting is a concern. Geoffroy et al. (2013) gave a multi-model mean of 219 ± 69 years for the long time scale, which is longer than the time series itself, and indicates that overfitting and hence bad generalization properties is likely [Geoffroy et al., 2013]. Fitting all four (five counting c_0) parameters might therefore lead to non-physical solutions due to overfitting.

To overcome this, we could fix the time scales to the multi-model mean relaxation times (MMMRT) given in Geoffroy et al. (2013), based on the instantaneous doubling of CO_2 . For models that were part of the study, we can use model-specific relaxation times (MSRT), assuming forcing-independent model parameters following section 2.7.3.

Based on the discussion above, we fix τ_f and τ_s such that equation (3.5) reduces to

$$\begin{aligned} \hat{T}_n &= c_0 + c_1 \sum_{s=0}^n F(s) e^{-(n-s)/\tau_f} + c_2 \sum_{s=0}^n F(s) e^{-(n-s)/\tau_s} ds \quad (3.6) \\ &= c_0 + c_1 T_{1n} + c_2 T_{2n}, \end{aligned}$$

where T_{1n} and T_{2n} are the n th elements of the new time series \mathbf{T}_1 and \mathbf{T}_2 , that depend only on the forcing. Finding the constants c_1 and c_2 for each model is therefore reduced to solving the normal equations to the inconsistent linear system

$$\mathbf{A}\mathbf{c} = \mathbf{T}, \quad (3.7)$$

where $\mathbf{A} = [\mathbf{1}, \mathbf{T}_1, \mathbf{T}_2]$ and $\mathbf{c} = [c_0, c_1, c_2]^T$.

Figure 3.3 shows the fitted temperature response of another CMIP5 model, the NorESM1-M model, using two different response functions. These are the TBM response function and the OBM response function, which only has one

1. The series starts in 1850, 1860 or 1861 depending on the model.

single exponential. However, the two fits are not very different, and the need for a long time scale is not easily visible, compared to figure 2.8, where it is. In fact, the fit using only the short relaxation time is almost just as good, with the standard deviation being only 11% higher. The two bottom plots show the time series T_1 and T_2 . The OBM fit is simply a shifted multiple of T_1 .

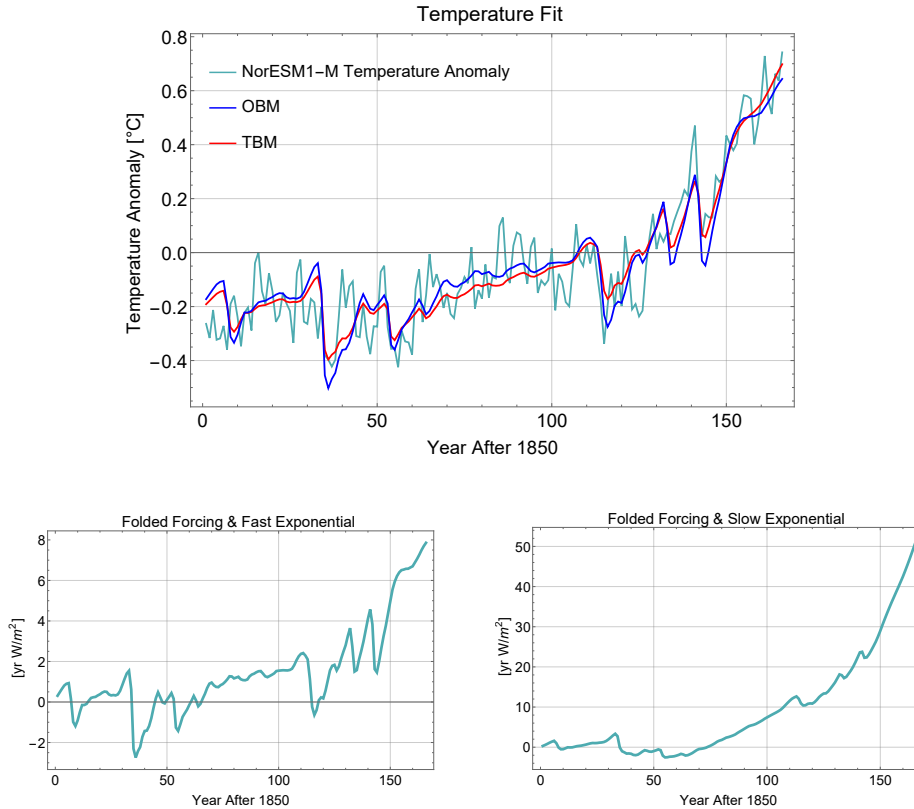


Figure 3.3: Temperature fit of the NorESM1-M model (top) using both the OBM and TBM. The standard deviation of the fits are 0.118 and 0.106°C for the OBM and TBM, respectively. The bottom left plot shows the folded forcing and exponential with short relaxation (T_1), while the bottom right plot shows the folded forcing and exponential with long relaxation time (T_2)

There is a straightforward theoretical explanation for this good fit using only one short relaxation time. Although the time series dates back 166 years (and therefore an exponential with relaxation time of around 200 years could be important), the forcing averages at around zero until the last few decades, where the increase is significant. This means that the slow response does not gain much net momentum in this period. That is, there is not much net heat flux to the deep ocean, and as long as the temperature of the deep ocean is unchanged (i.e. $(T - T_0) \approx T$), the TBM is dynamically no different from the

OBM where the deep ocean is an external heat sink. In that case, the short time scale is approximately given by

$$\tau_f \approx \frac{C}{\alpha + \gamma}, \quad (3.8)$$

which is the characteristic time scale of the OBM.

As the deep ocean starts to heat up during the latter part of the 20th century, the TBM begins to deviate from the OBM. There is a tendency that the OBM fit slightly overshoots the temperature response in the first part of the response in order to be able to fit the sharp rise in temperature of the last few decades.

From the TBM equations, we have an important equality given by,

$$c_1 \tau_f + c_2 \tau_s = \frac{1}{\alpha}. \quad (3.9)$$

This quantity, $1/\alpha$, is the climate sensitivity per unit forcing². Using the fit from figure 3.3 we find the response functions

$$G_{\text{OBM}}(t) = 0.104e^{-t/4.1} \quad (3.10)$$

$$G_{\text{TBM}}(t) = 0.051e^{-t/4.1} + 9.62 \times 10^{-3}e^{-t/219}, \quad (3.11)$$

which gives climate sensitivities per unit forcing of $2.32 \text{ K m}^2/\text{W}$ for the TBM³. The climate sensitivity of the CMIP5 models used in Geoffroy et al. (2013) have all a climate sensitivity per unit forcing (ECS/ Q) between 0.55 and $1.59 \text{ K m}^2/\text{W}$, and we should expect the TBM climate sensitivity parameter α to be within an a priori range guided by these values. The very high value obtained using the TBM fit demonstrates the problem of having such a short temperature record (or rather, such a short period with significant forcing). The value of c_2 can be shifted substantially by different internal fluctuations, but physically the value of c_2 should not depend on these fluctuations. It is however not merely due to the the late heating of the oceans that we can expect large variations in the fitting constants, but it is also in combination with the complexity of the climate system. The TBM should on the aggregate depict the temperature of the land/sea surface quite well, but the complexity of the climate system makes possible large deviations from the TBM behavior

2. It is more precisely known as the feedback parameter, since the climate sensitivity often is used in the context of forcing from CO_2 .
3. One cannot say the OBM has an equilibrium climate sensitivity, only what Held et al. (2010) calls the *transient sensitivity*, which is equal to $1/(\alpha + \gamma)$.

over substantial periods⁴.

From the discussion above it is clear that further constraints should be placed on the problem. We allow a climate sensitivity between 0.2 and 2.0 m²/W, corresponding to an ECS of about 0.8 to 7°C⁵. This is a highly vague constraint that should not narrow the possible outcomes of the study, since we assume that indeed the climate sensitivity is within this range. It is only a needed guideline to narrow the solution space in order to obtain reasonable results. The non-linearity of the problem means that local or even global minima likely exist within the domain (i.e. not at the boundary), and reducing the domain might give completely different solutions. As an additional constraint, we also constrain the values c_1 and c_2 individually, again using the values in Geoffroy et al. (2013) as guidelines, and allow for some extra deviation in both directions.

3.3.3 The Fourier Transform Amplitude As a Metric

In order to accentuate nuances in the response function, it is favorable to change domain to the frequency domain, by taking the Fourier transform. The Fourier transform takes the function $G(t)$ and decomposes it into its modes of frequency f . The Fourier transform of $G(t)$, $\widehat{G}(f)$, is a complex function of the frequency f whose amplitude represent the amount of frequency f that is present in $G(t)$.

By taking the amplitude of the Fourier transform of the response function and multiplying it with the factor Q (the forcing from a doubling of CO₂) we get a function we call $R(f)$. When correlating values of $R(f)$ to the TCR of the models for different frequencies, we can find a frequency f^* which gives the best (or one of the best) correlation(s), and use $R(f^*)$ as the informed metric. It contains information both about the AOGCMs response to forcing and the estimate forcing magnitude from a doubling of CO₂ in that AOGCM, the two components that in theory should determine TCR.

Using the linearity of the Fourier transform we have

$$\mathcal{F}_t[G(t)] = \mathcal{F}_t \left[c_1 e^{-t/\tau_f} + c_2 e^{-t/\tau_s} \right] = c_1 \mathcal{F}_t \left[e^{-t/\tau_f} \right] + c_2 \mathcal{F}_t \left[e^{-t/\tau_s} \right], \quad (3.12)$$

4. Although Geoffroy et al. (2013) only uses a time series of 150 years to estimate parameters, roughly the length of our historical time series, it is much better suited for estimating the longest response time because there is a much more significant heat flux in to the deep ocean at an early stage of the time series, and therefore the longest time scale is much more detectable.

5. Using a value for Q of 3.5 W/m².

and since the response function requires $t \geq 0$, the function is square integrable and the existence of a Fourier transform is guaranteed. The transform for this negative exponential is given as

$$\mathcal{F}_t \left[e^{-t/\tau_f} \right] (f) = \frac{\tau_f}{1 + i2\pi\tau_f f}, \quad (3.13)$$

where $i = \sqrt{-1}$ is the imaginary unit. From this we have

$$\widehat{G}(f) = \frac{c_1\tau_f}{1 + i2\pi\tau_f f} + \frac{c_2\tau_s}{1 + i2\pi\tau_s f}. \quad (3.14)$$

Taking the amplitude and multiplying with Q gives:

$$\begin{aligned} R(f) &= Q|\widehat{G}(f)| \\ &= Q \left[\left(\frac{c_1\tau_f}{(2\pi\tau_f f)^2 + 1} + \frac{c_2\tau_s}{(2\pi\tau_s f)^2 + 1} \right)^2 + \left(\frac{2\pi c_1\tau_f^2 f}{(2\pi\tau_f f)^2 + 1} + \frac{2\pi c_2\tau_s^2 f}{(2\pi\tau_s f)^2 + 1} \right)^2 \right]^{\frac{1}{2}}. \end{aligned}$$

So far, what has been done is to establish a method such that, given a time series of temperature and a corresponding time series of forcing, we can produce a function $R(f)$. If this function (when calculated for each of the available AOGCMs), for some range of frequencies, correlates well with the TCR of the AOGCMs, then the value of the function at those frequencies can predict the value of TCR, with some statistical uncertainty. We have found our metric ν (the value of R at some frequency), and in the next section we establish the framework for this statistical analysis.

3.4 Method For Constraining TCR From Obsevation

3.4.1 Conditional Distribution of TCR

Correlation is a measure of linear dependence (see Appendix A.2), so the natural choice for Φ is a linear function, such that

$$\hat{y} = ax + b, \quad (3.15)$$

where \hat{y} is the estimated value of TCR when x corresponds to a value of R^6 .

6. By the value of R we mean the value of the function $R(f)$ at some frequency.

Here, we seek to minimize

$$s^2 = \frac{1}{N-2} \sum_{n=1}^N (y_n - \hat{y}_n)^2 = \frac{1}{N-2} \sum_{n=1}^N (y_n - ax_n - b)^2, \quad (3.16)$$

where y_n is the TCR for model $n \in \{1, 2, \dots, N\}$, and x_n is the corresponding value of R . The best estimates for a and b are found by differentiating (3.16) with respect to a and b and equating it to zero.

For our purpose, the goal is to predict the value of the next observation y_{N+1} (corresponding to the correct TCR of the climate system), given a value $x = x_0$ ($R = R_0$). The distribution of y_{N+1} given $x = x_0$ can be proven to be a normal distribution centered at the regression line [Walpole et al., 2014]. The prediction error (or standard deviation) is given by

$$\sigma_f(x) = s \sqrt{1 + \frac{1}{N} + \frac{(x - \bar{x})^2}{\sigma_x^2 N}}. \quad (3.17)$$

The prediction error increases slightly away from the sample mean, as it depends on $(x - \bar{x})^2 / \sigma_x^2$, where σ_x^2 is given by

$$\sigma_x^2 = \frac{1}{N} \sum_{n=1}^N (x_n - \bar{x})^2. \quad (3.18)$$

This is because the uncertainty in the slope. It always goes true the sample mean, causing higher uncertainty away from the sample mean [Walpole et al., 2014].

3.4.2 Distribution of the Observed Metric

There exist only one observable realization of the historical climate, corresponding to only the one forcing scenario - the historical forcing - which is highly uncertain. This inevitable fact is one of the factors that constrain our ability to predict future climates, and one of the reasons why we are so dependent upon models. The uncertainty regarding historical forcing, together with large unknown internal variability of the climate system, make predictions statistically problematic. The value of R is therefore also highly uncertain, and this must be accounted for in some way.

In the case where we use MMMRT, the uncertainty in forcing is accounted for by varying the forcing across the 19 different forcings produced by our adjustment method of section 3.3.1. This gives us 19 different best fits and 19

different values for R . These give us a guidance to the "true"⁷ value of R , as well as a measure of how sensitive the value is to varying forcing. In the MSRT case, we combine the forcing from each model with the MSRTs of each model to produce a set of forcing and relaxation times.

To estimate uncertainty due to internal variability, we again rely on the models. Available to this study are also ten different historical simulations of the CNRM-CM5 model, which uses equal initial conditions and forcing scenarios. These simulations can be used to estimate the variance in the value of R due to internal variability. We use the adjusted forcing of the CNRM-CM5 model, fit the response function to all the ten temperatures, and subsequently get ten values of R from which we can estimate the variance. Figure 3.4 shows two of these simulations, depicting the large variations.

The temperature record used is the HadCRUT4 data, developed by the Climatic Research Unit (University of East Anglia) in conjunction with the Hadley Centre (UK Met Office). They provide monthly land and sea surface temperatures (SST) dating back to 1850 [Climatic Research Unit, 2018]. As with anything, there are uncertainties regarding these data as well, especially before 1880. However we will not account for these uncertainties directly, as the uncertainty in a way is incorporated into the uncertainty due to internal variability.

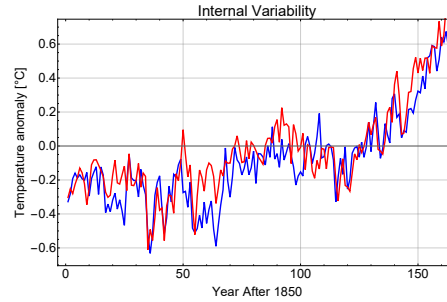


Figure 3.4: Two different simulations from the same model using identical perturbation scenarios and initial conditions, depicting the large variations.

Mathematically, the variance of R due to forcing is estimated as

$$s_F^2 = \frac{1}{N_F - 1} \sum_{n=1}^{N_F} (R_n - \overline{R_F})^2, \quad (3.19)$$

where N_F corresponds to the number of different forcings available, and $\overline{R_F}$ is the mean of all $R_n \in (R_1, R_2, \dots, R_{N_F})$. Similarly, the variance of R due to the

7. By true we mean the value of R we would get if we knew the historical forcing and fitted it to the temperature record. However, there is no correct value of R , due to internal variability of the climate.

internal variability of the climate system is estimated as

$$s_I^2 = \frac{1}{N_I - 1} \sum_{k=1}^{N_I} \left(R_k - \overline{R_I} \right)^2, \quad (3.20)$$

where N_I is the number of CNRM-CM5 simulation available (equal to 10), and $\overline{R_I}$ is the mean of all $R_k \in (R_1, R_2, \dots, R_{N_I})$. The total variance in R from the two sources of uncertainty is found using the Pythagorean Theorem of Statistics⁸,

$$s_{\text{tot}}^2 = s_F^2 + s_I^2. \quad (3.21)$$

By combining the variability due to forcing uncertainty and internal variability, we have accounted for the two main sources of uncertainty regarding the historical value of R . We have therefore estimated the variance of R . However, we have no prior knowledge of the shape of the distribution of R , and it is unlikely to expect R having any theoretical distribution, given the complexity of the climate system. A normal distribution is of course a natural choice and might be suitable, however the negative support makes it inconsistent with the positivity of $R(f)$ and this might be problematic. The positive support of the gamma distribution makes it a natural alternative to the normal distribution. A third possibility is to use an empirical kernel distribution based on the finite set of data points, however only the two first will be examined.

3.5 An Idealized Case

As an introduction to the method, as well as a reference for later comparison, we now examine the first part of the method using the response functions estimated by Geoffroy et al. (2013). The upper left plot in figure 3.5 shows the the correlation between $R(f)$, calculated using these TBM values. It shows a correlation above 0.9 for frequencies the order of 10^{-2} yr^{-1} , and just under 0.8 for frequencies lower than 10^{-3} yr^{-1} .

In the upper right figure, we repeat the plot for ECS. While the correlation is around 0.8 for frequencies which gave the best correlation to TCR, the correlation is now above 0.95 as the frequency goes to zero. This highlights the differences in the time scale at which these two values are important. TCR is most informative on a 100-year scale or so, while ECS is important on time scales of millennia. The high correlation of TCR on very low frequencies is a

8. We have assumed that the internal variability and the forcing are independent.

direct consequence of the high correlation between TCR and ECS. A scatterplot of the value of R when $f = 0.01 \text{ yr}^{-1}$ is shown in the bottom figure, together with the least squares regression fit. As the correlation is so high we can predict the TCR of a model based on the value of $R(f = 0.01)$, with low uncertainty.

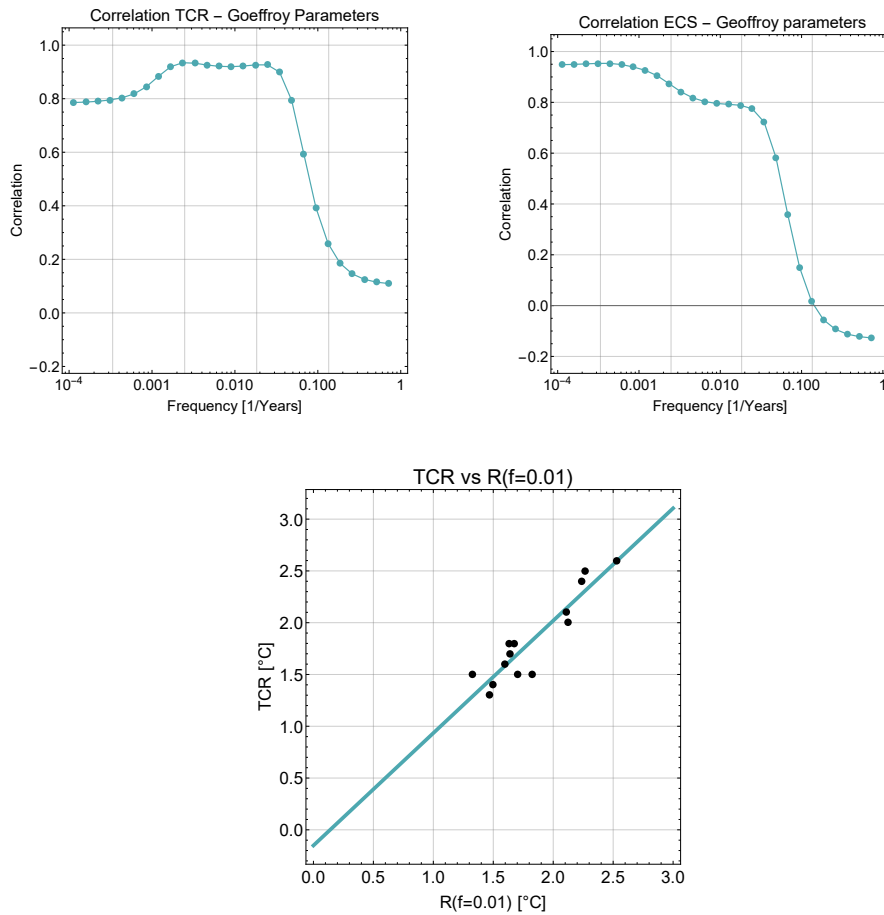


Figure 3.5: Correlation between $R(f)$ and TCR (upper left), $R(f)$ and ECS (upper right), using TBM parameters from Geoffroy et al. (2013). The scatterplot of the value of $R(f = 0.01)$ against TCR is shown in the bottom, together with a least squared regression fit.

Figure 3.6 shows the function $R(f)$ for three different models. The numbers in the upper right corner indicate the TCR (left) and ECS of the models. BCC-CSM1-1 and NorESM1-M (names not shown) both have an ECS of 2.8°C , but their TCR differ, 1.7 and 1.4°C , respectively. The CNRM-CM5 model has both a high TCR and ECS, at 2.1 and 3.3° , respectively. Since the correlation is so high, these similarities and differences are clearly seen in the plot as the TCR correlates best with frequencies between 0.001 yr^{-1} and 0.01 yr^{-1} , while frequencies lower than 0.001 yr^{-1} correlates almost perfectly with ECS.

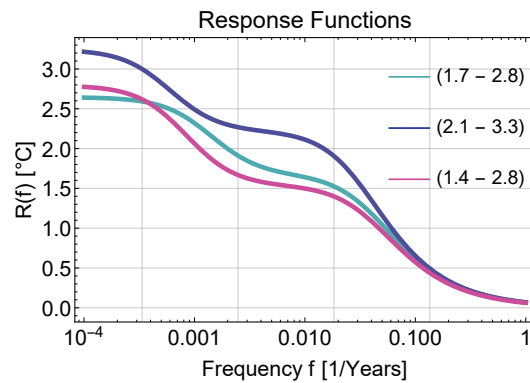


Figure 3.6: $R(f)$ for three different models. The models' respective TCR (left) and ECS is shown for comparison.

/4

Results and Discussion

This chapter presents and discusses the results obtained when applying the methods presented the previous chapter to the data. The resulting forcings from the adjustments and the the correlation between $R(f)$ and TCR is presented. Subsequently, an informative figure that visualizes the most important results is discussed. Then the final pdf for TCR is presented. Finally, we discuss the significance of the result, examine its robustness and discuss possible issues with our method.

We experiment both with the case where we use 13 models with MSRT and the case where we use our full ensemble of 19 models with MMMRT.

4.1 Adjusted Forcings

Figure 4.1 (upper) shows the corrected series for the 15 models having data back to 1850, using a constant smoothing window of 16 years, while the bottom figure shows the subsequent adjusted forcings. There are considerable differences between the models, with an ERF of $1.7 \pm 0.9 \text{ W/m}^2$ in 2003 (year 153). This spread is mainly driven by differences in forcing from greenhouse gases and aerosols, and little by natural forcing [Forster et al., 2013].

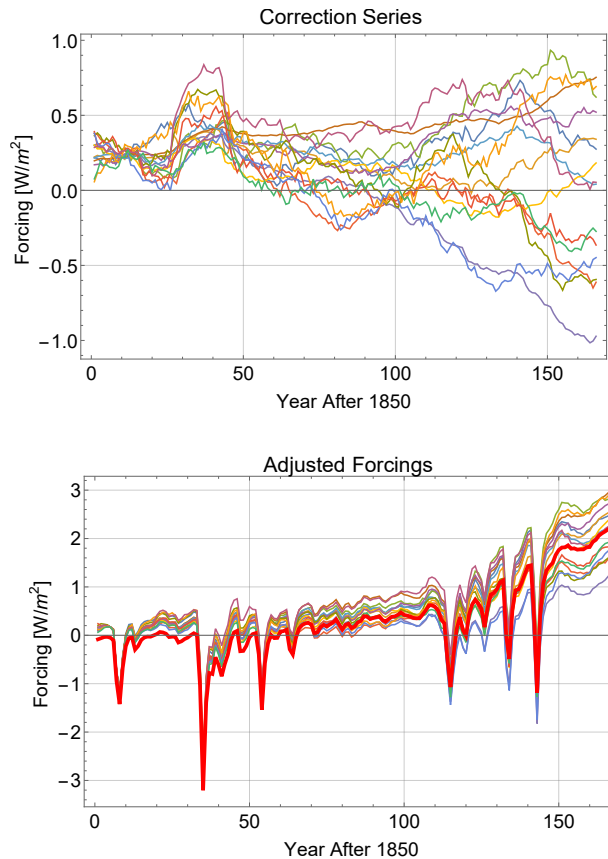


Figure 4.1: Results from adjustment of the 15 models having data back to 1850. A smoothing window of 16 years is used. The upper plot shows the adjusted series, similar to figure 3.2 (upper right), while the adjusted forcings are plotted below, along with the Hansen forcing (thick red), for comparison.

4.2 Correlation Plots

Figure 4.2 shows the obtained plots of correlation between $R(f)$ and TCR in six different cases. The left column shows the correlation when we use MMMRT (4.1 and 219 years) for all 19 models, while the right column shows the correlation when the 13 models used in Geoffroy et al. (2013).

The upper row shows the resulting correlation when the method of the previous chapter is used. It is the correlation between $R(f)$ and TCR obtained by fitting the response function to each model's historic temperature simulation, using that particular model's adjusted forcing. The correlation is high, however, not as high as when we used fitted parameters from the $2\times\text{CO}_2$ simulation in Geoffroy et al. (2013). We again find the best correlation for frequencies around 0.01 yr^{-1} , as expected. MSRT perform slightly better (around 0.9), compared to

using MMMRT (around 0.86). The shape of the MSRT-graph also inherits some of the characteristics as figure 3.5 because it uses the exact same relaxation times.

The plots in the middle row illustrate the importance of using model-specific forcings. It shows that when we fit the response function to model data using the same forcing (the Hansen forcing), the correlation drops drastically to a value that is much less informative and practically useless. This is because in that case, the forcing that is used is not the forcing that causes the temperature response. Because of this, the resulting response function does not depict the true transient response of the model and therefore the correlation is somewhat lost. There is still some positive correlation, probably due to a combination of the factor Q (that correlates with TCR), together with the fact that models with high TCR often produce higher than average historical temperatures. Since the forcing is constant in this case, the models with higher temperatures obtain a correspondingly larger response function, which also correlate with TCR. Furthermore, this plot also demonstrates that using model-specific relaxation times does not necessarily produce a higher correlation.

The bottom row presents the correlation using only the Forster forcing without any smoothing or adjustments. Interestingly, the correlation peaks at roughly the same value as the adjusted forcing. However, the main reason for this is rather straightforward. The high fluctuations in the Forster forcing are mainly interannual fluctuations, shorter than the relaxation time of the fast response. Therefore, the data is indirectly smoothed when fitted to the response function. The difference between the two is therefore not well shown in the correlation plot.

In general, we can expect comparable results using only the Forster forcing, due to the characteristics of the fluctuations and the use of a TBM. However, to be precise, we need to make sure that the fitting of the historical temperature has the same starting point as the fitting of the models. By this, we mean that, in principle, the definition of forcing could have been $F = N + \alpha T$. However, when using historical data this definition is a dead end, since we do not know the historical N ¹. If we possessed satellite data of N dating back to 1850 we could have compared models with historical climate directly using the this definition, and there would not be a need for any adjustment to make sure fitting of model data and fitting of historical data had the same "type" of forcing. Until we have more satellite data available, using the Forster forcing directly is inaccurate.

1. There would still be large uncertainty in the forcing due to the unknown value of α .

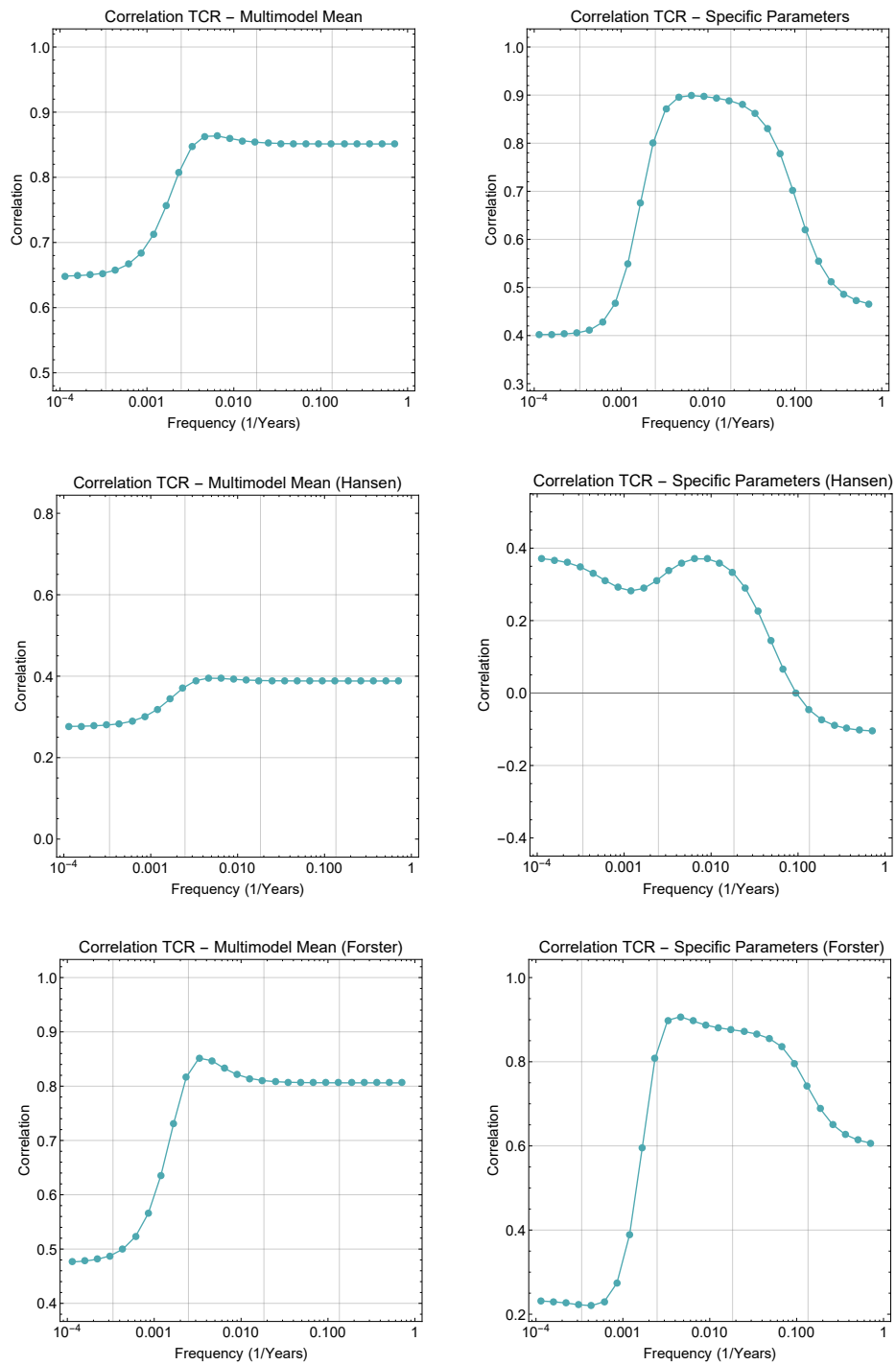


Figure 4.2: Correlation plots using MMMRT (left column) and MSRT (right column) for the adjusted forcing (upper row), Hansen forcing (middle row) and Forster forcing (bottom row).

4.3 Resulting Response Functions

As stated in section 3.4.2, we account for uncertainty in historical forcing by varying the forcing applied to the TBM across the adjusted forcings to get 19 (13) different response functions and 19 (13) different values of R .

If we look in more detail at the specific numbers, we find that there is a statistically significant difference between the average values of c_1 and c_2 in the response functions fitted to models and the response functions fitted to historical temperature. For the MMMRT case with 19 models, the average value of c_1 when fitted to models is about $0.092 \text{ K m}^2/\text{W}$, while it is 25% lower when fitted to historical data (0.069). Furthermore, the value of c_2 averages at $4.77 \cdot 10^{-3} \text{ K m}^2/\text{W}$ when fitted to models, and is 18% higher when fitted to historical temperature ($5.61 \cdot 10^{-3}$), see table 4.1.

Table 4.1: Average values of the constants c_1 , c_2 and $1/\alpha$ from the fitting of model simulations and instrumental temperature record

	Average c_1	Average c_2	Average $1/\alpha$
Model	$0.092 \text{ K m}^2/\text{W yr}$	$4.77 \cdot 10^{-3} \text{ K m}^2/\text{W yr}$	$1.42 \text{ K m}^2/\text{W yr}$
Historical	$0.069 \text{ K m}^2/\text{W yr}$	$5.61 \cdot 10^{-3} \text{ K m}^2/\text{W yr}$	$1.51 \text{ K m}^2/\text{W yr}$
Difference	$-0.023 \text{ K m}^2/\text{W yr}$	$0.84 \cdot 10^{-3} \text{ K m}^2/\text{W yr}$	$0.09 \text{ K m}^2/\text{W yr}$

In terms of the climate sensitivity of the TBM ($1/\alpha$), these differences average out to give approximately the same value, which means that there seems to be no significant evidence of a higher/lower ECS from these results. From equation (2.12) we have

$$c_1 = \frac{a_f}{\alpha \tau_f} \quad c_2 = \frac{a_s}{\alpha \tau_s}.$$

This suggests that, since the climate sensitivity parameter is approximately equal and the characteristic time scales are equal, the differences between the fitting of the model simulation and the historical data lies in the amplitudes of the fast and slow response, a_f and a_s . On average, the slow component is larger when the TBM is fitted to historical data, which means that the temperature response to forcing is more delayed, leading to a lower average TCR^2 .

It is however too early to conclude that the TCR is likely significantly lower than some models predict. There is still uncertainty regarding internal variability and how well the models' TCR is captured by the response function. Statistics are needed for this purpose.

2. TCR is not only determined by the response of a model, but rather the product of forcing from a doubling of CO_2 and response. However, we know that the forcings used were the same in both sets.

4.4 An Illustrative Plot

Figure 4.3 packs much of the statistical data used to obtain our resulting pdf of TCR into one plot: One for the MSRT case (top) and one for the MMMRT case (bottom). For each of the plots, the result was found using a smoothing of 16 years.

The black points are the values of R (x -axis), with $f = 0.01 \text{ yr}^{-1}$, corresponding to fits of the temperature data from model simulations, with each model's TCR on the y -axis (as in figure 3.5). The numbers indicate which model each point represents, and is found in table B.1 in appendix B. The best linear fit to the data is also shown, which at every value of R corresponds to the center of the conditional distribution of TCR. The dashed lines represent one standard deviation of the conditional distribution, using the method of section 3.4.1.

The red points represent the values of R corresponding to the historical temperature fits, and here the numbers indicate which model's adjusted forcing is used. A normal distribution and a gamma distribution for $P(R)$ is also shown, where the mean of the distributions equals the mean of the red points. The variance is the sum of the sample variance of the red points and the sample variance from the ten runs using the CNRM-CM5 simulations³.

In the MSRT plot, we see that the correlation between R and TCR is high (0.9). It is noticeably better than in the MMMRT plot, which has a correlation around 0.85. This narrows the conditional pdf in the MSRT case compared to using MMMRT case. Furthermore, looking at the MMMRT plot in particular, we find that the average value of R is significantly lower for the historical data fits (red points), compared to the model simulation fits (black points). The mean is 1.06°C for the red points and 1.38°C for the black points, which is consistent with what we found by investigating the response function in the previous section.

The standard deviation due to internal variability in the same plot (bottom) is 0.264°C . The standard deviation due to variable forcing is 0.288°C . This gives a total standard deviation of 0.391°C . This also means that the estimates of uncertainty in forcing and internal variability are approximately equally important to the total variance.

Looking at the normal distribution, it seems that indeed there is a tendency that it gives nonphysically high probability for the extreme low (and even negative)

3. The additional variance due to internal variability means the pdfs are wider than if they were fitted to the points only.

values of R . A gamma distribution looks like the most reasonable choice. We do however examine the both in table 4.2 of the next section.

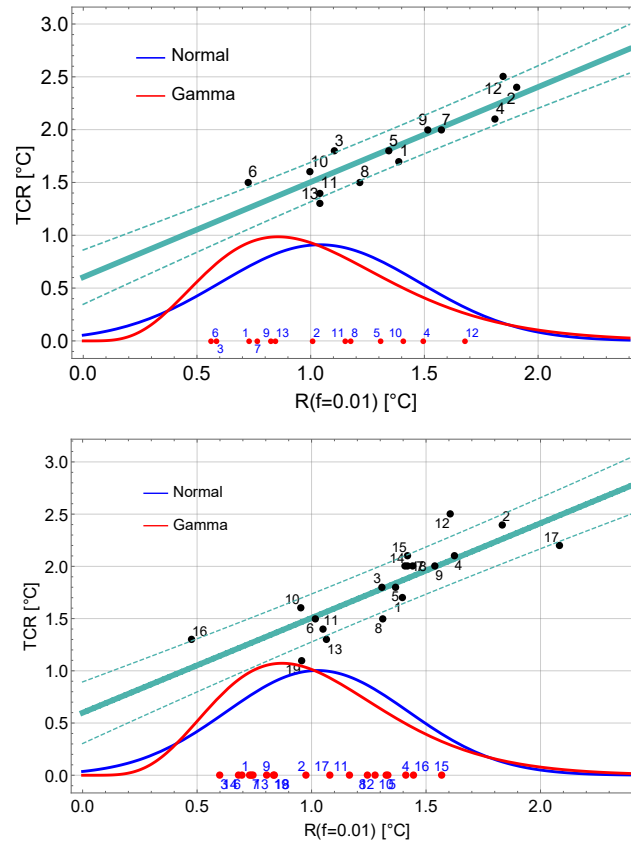


Figure 4.3: Scatterplot of the values of $R(f = 0.01)$ against TCR (black points), with numbers corresponding to each model in table B.1 in Appendix B, together with a least square fit of the points and one standard deviation. Red points represent historical fits, using the forcing corresponding to the numbers. A normal and gamma distribution for $P(R)$ is also shown by the blue and red curve, respectively. A 16-year smoothing window is used. The upper plot is for the MSRT case, while the bottom is for the MMMRT case.

4.4.1 Remarks

One important point to keep in mind is that the values of R found when fitting the response function to the historical data uses the exact same forcing *and* relaxation times as the corresponding black point. This is important to the statistics of the problem. Each number in the MSRT plot corresponds to a set of both forcing and relaxation times, not only forcing. This is essential to make sure the fitting of the two data sets is without bias. For the same reason we must

also adjust the Forster forcing to validate our study, as discussed earlier.

As mentioned, the spread of the red points is due to forcings, since the temperature is always the same. In the MSRT case, the spread also depends on the relaxation times, since they are different. However, even in the MMMRT case, where forcing is the only difference between the points, it is not straightforward to quantify the difference directly in terms of the forcing applied. When a large forcing is applied, the fitted response function in general gets smaller, again since the temperature response is always the same. However, it is not only the Fourier transform of the response function that determines the value of R , it is also the factor Q . On average, models with higher historical forcing also have a higher value of Q , and this works in the opposite direction. This means the value of R could be larger for a model with larger forcing, even though the response function is smaller.

TCR is measure of temperature change to forcing from CO_2 , and the factor Q is a measure of forcing from CO_2 ⁴. However, the historical forcing is not only from CO_2 , and therefore the value of Q does not have a direct link to the historical forcing of the models. Forcing from other agents is also causing large inter-model spread in historical forcing [Forster et al., 2013]. Even if the historical forcing was only due to CO_2 , a simple factor Q could not compensate for differences in forcing as there are several non-linear mathematical operations between forcing and a value of R .

However, we know that the response function measures (with somewhat high accuracy) the response to of the models to (any) forcing, and the factor Q is a quantity about the magnitude of the forcing from a doubling of CO_2 . In combination, they should give some indication to the value of TCR in models.

Figure 4.4 shows the forcing and response functions of the model 6 (GISS-E2-R) and model 12 (HasGEM2-ES), the two models that that give the lowest and highest value of R in the MSRT plot. The much larger historical forcing of the GISS-E2-R gives a much smaller response function. From the resulting $R(f)$, it is clear that the higher value of Q for the GISS-E2-R (3.8 W/m^2 to 2.9 W/m^2) does not cancel this difference in this case.

Another important point to add is that we do of course not assume that there only is internal variability in the real climate system. After all, we used a model to estimated the internal variability. However, this uncertainty is part of the uncertainty in the linear regression prediction interval. The points spread out not only because the TBM is imperfect, but also due to the internal variability,

4. This is why we multiply with Q for the value of R , to make it comparable to TCR.

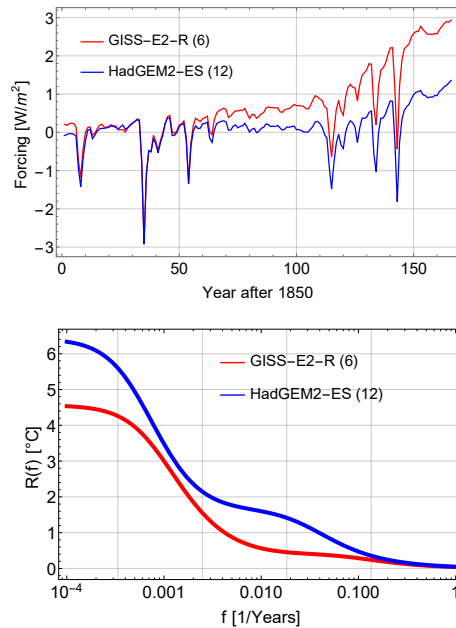


Figure 4.4: Adjusted forcing (upper plot) and the function $R(f)$ to historical fits using two different models in the MSRT case. A smoothing window of 16 years is used

resulting in a larger prediction error.

4.5 Probability Distribution of TCR

From the conditional pdf, estimated from the scatterplot (black points) in figure 4.3, and distribution of R , shown in the same figure, we can calculate the pdf of TCR. Figure 4.5 shows the resulting pdfs using a 16-year smoothing window and a gamma distribution. In the same plots, histograms showing the distribution of values of TCR from the models used to calculate the pdfs are plotted. The top plot shows the MMMRT case (19 models in the histogram), while the MSRT case is shown below. The pdfs using a normal distribution are presented in Appendix C, so is the pdf using only Forster forcing.

There is a significant shift towards lower values for TCR in both cases. Table 4.2 gives the results using both 19 and 13 models, for different smoothing window sizes and distributions of R (gamma or normal). We get the most constraining result using 19 models and a gamma distribution. Varying the smoothing window size only changes the results slightly. If we use the gamma distribution, we get an interval that encompasses all the different 5 – 95% intervals from

both cases of $1.0 - 2.3^{\circ}\text{C}$, and the most narrow interval is $1.05 - 2.19^{\circ}\text{C}$, using MMMRT. We expand the best interval to the nearest decimal, and obtain our final result, which is $1.0 - 2.2^{\circ}\text{C}$ ⁵.

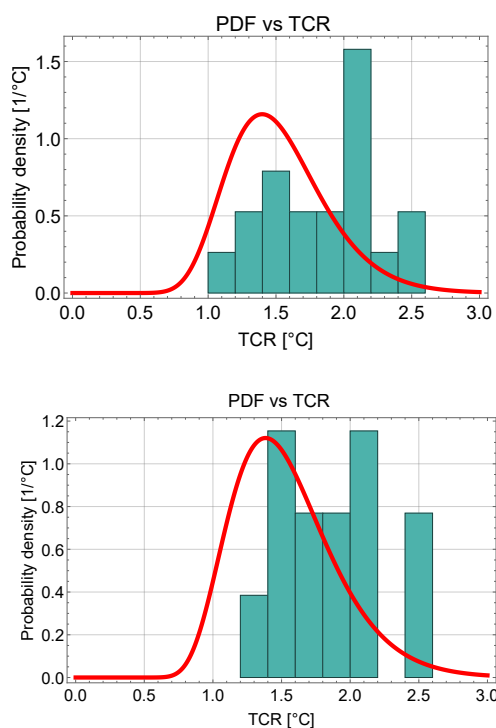


Figure 4.5: Final pdfs for the MSRT case (top) and MMMRT case (bottom) using a 16-year smoothing and a gamma distribution for $P(R)$. Histograms are also plotted, and they show the distribution of values for TCR for the models used to calculate the pdfs.

Table 4.2: Results for the two methods using different smoothing window and distribution for R , in both the MMMRT and MSRT case. For the columns, the number indicates the smoothing window size and letter indicates distribution - G for gamma and N for normal. The number indicates the mean, and the numbers in the parentheses gives the 5 – 95% confidence interval.

	10 - G	10 - N	20 - G	20 - N
MMMRT	1.51 (1.05-2.19)	1.55 (0.97-2.12)	1.51 (1.03-2.22)	1.56 (0.96-2.17)
MSRT	1.48 (1.00-2.26)	1.55 (0.93-2.22)	1.51 (1.02-2.30)	1.57 (0.94-2.25)

Figure 4.6 shows the mean and 5 – 95% confidence level for the TCR, as function of the period $1/f$. The intervals are obtained using the MSRT case

5. We could use the most narrow, but it common to use only one decimal.

and a gamma function for $P(R)$. Our interval for $f = 0.01 \text{ yr}^{-1}$ is almost encompassed by all the other 5 – 95% intervals. However, the lowest frequency (where the correlation is bad) has a slightly higher lower bound. This means that the lowest values in the case of $f = 0.01 \text{ yr}^{-1}$ are not consistent with all the intervals. This is slightly problematic, but only concerns the values lower than 1.1°C .

The constraint is not visible at all using a period of 1000 years. Furthermore, this shows that the result is not very sensible to choice in f , as long as we choose a frequency that correlates well with TCR, adding to our confidence in the result (apart from the slight inconsistency of the lowest vales).

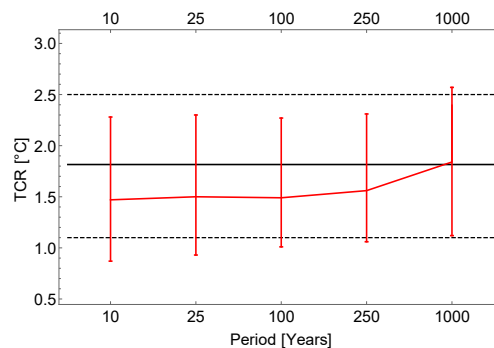


Figure 4.6: Mean and 5 – 95% confidence level as function of the period ($1/f$) using the MSRT case and a gamma distribution. Black solid line is mean of model TCR and dashed lines indicate maximum and minimum value for TCR for the 19 models (1.1 and 2.5°C).

4.5.1 Implications

If we compare our interval to the TCR of the models used to estimate the conditional pdf, we find that two of the models are outside this interval, namely CanESM2 (2.4°C) and HadGEM2-ES (2.5°C)⁶. These models therefore seem to be inconsistent with the observed temperature record.

It is rather interesting how a result like this is at all possible, using such simple tools and the historical temperature record. Sanderson and Knutti et al. (2012) argue that most of the large scale observations is already used to develop the models, and is therefore of little use for further constraining models. [Sanderson and Knutti, 2012]. In AR5, IPCC concluded that the result from different studies based on surface temperature, ocean heat uptake and RF agreed well with their *likely*⁷ range of 1.0°C to 2.5°C .

6. This is independent of what interval is used.

7. The term *likely* means a greater than 66% chance in the IPCC reports.

However, the result is consistent with some recent studies which use historical data to constrain TCR [Lewis and Curry, 2018]. Recent revisions of greenhouse gas forcing and post-1990 aerosol and ozone forcing is one possible explanation [Lewis and Curry, 2018]. Regarding this, it should also be mentioned that IPCC use many other studies that do not use the temperature record to constrain the outcome, and IPCC tend to weigh such studies significantly as well [Collins et al., 2013].

One comparable study to our study is the study of Otto et al. (2013). They estimate TCR as the slope of temperature change to forcing from equation (2.7), and use the mean Forster forcing. They conclude with a 5 – 95% confidence interval of 0.9°C to 2.0°C [Otto et al., 2013]. In their study, they added additional 0.08°C standard deviation (Gaussian white noise) to the HadCRUT4 data series to account for internal variability. In addition, they used recent estimates of aerosol forcing to scale the forcing series in order to add 0.3 W/m^2 in 2010. This scaling lowers the estimated TCR, which might explain the even lower estimate for TCR, compared to our result.

As the "historical period" expands into the future, our ability to predict TCR should increase drastically, as the total forcing compared to pre-industrial levels increase towards higher and higher values and the temperature record is constantly updated. ECS remains a much harder problem, and using our method to estimate ECS is likely to be very inaccurate.

It is however important to stress that our result should be seen in context with the histogram, as it is the models pictured in the histogram that are used to estimate the final distribution of TCR. The estimate is very much dependent upon the models used.

We leave it for future studies work to further investigate the reasons for this result.

4.6 Robustness of Method - OBM

As an additional way to test the robustness of the results, we will use the OBM as well, although only for the MMMRT case. We have seen how the OBM only had a slightly less satisfactory fit to historic temperature, as the longer time scale only really becomes significant the last couple of decades. Using the fast

relaxation time (4.1 years),⁸ we find a correlation of 0.815⁹, as seen in figure 4.7.

Despite the lower correlation, the resulting pdf is nearly identical to the one obtained using the TBM, as seen in figure 4.7 (bottom). It gives a slightly higher mean, at 1.56°C with a 5 – 95% confidence interval between 1.05°C and 2.26°C.

4.6.1 Important Note

Our method has now been shown to give consistent results, with only small variations, using MMMRT, MSRT and even the simpler OBM. It is however important to note that the methods give approximately the same results because they are physically grounded in approximately the same way. We cannot use a model that is not physically meaningful. For instance, if we use the OBM, but choose the slow response time (219 years), the correlation drops to 0.18 and we obtain no constraining results whatsoever.

Furthermore, it is possible to develop a method that constrains the values even more, for instance using machine learning. There are certainly features in the data that separates the model simulations from historical data, and some method, say a neural network, could give practically any result if necessary.

Results obtained using our TBM approach are however grounded in a physically sound model and the results obtained are therefore legitimized by the evidence that the TBM is able to capture temperature responses to forcing accurately [Geoffroy et al., 2013].

The result using the OBM is gratifying, as it shows that as long as we use a physically grounded model, the method is robust and consistent.

4.7 Some Possible Issues Related to the Method

The robustness of the method used has been shown to be high. However, there are some questions to be asked regarding some aspects of the method.

8. As we saw in section 3.3.2 the theoretical value of the OBM relaxation time is slightly shorter due to the more efficient heat uptake of the infinite deep ocean, but we stick with the same for simplicity.
9. The only thing that separates the response functions when using a constant relaxation time is the product $Q \cdot c_1$, and therefore the correlation is independent of f .

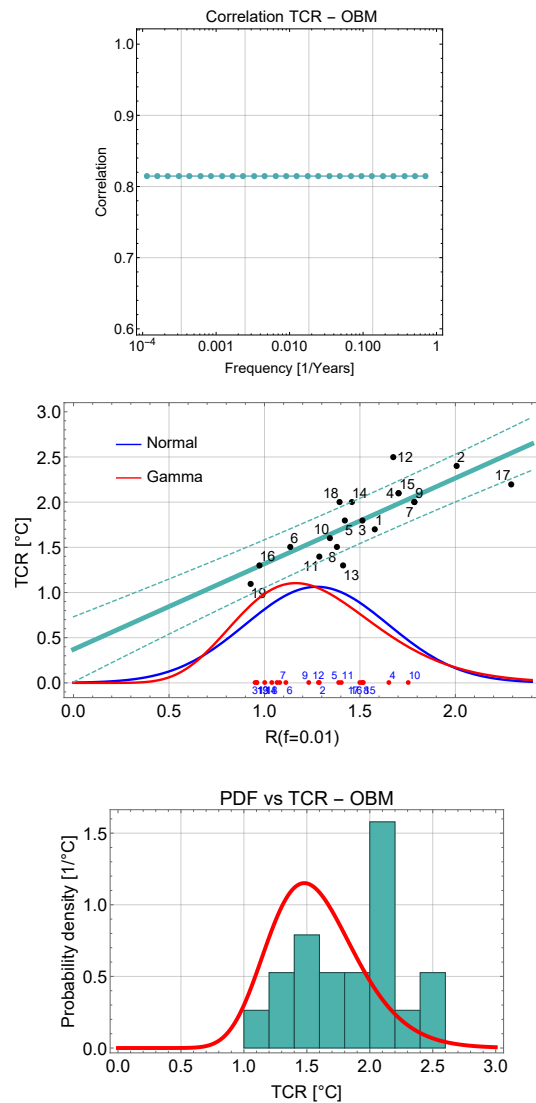


Figure 4.7: Results using the OBM model. From the top is the correlation between $R(f)$ and TCR, the scatterplot of the fits, and finally the pdf, together with the histogram.

4.7.1 Forcing

One obvious possible inaccuracy is the way we use the forcing data. Using model-specific forcings is necessary to adequately describe the response of the models. We have attempted to preserve the forcing as model-specific, while at the same time could be used to compare models to observation without bias. However, this method is novel, and its implications are not well tested.

4.7.2 Internal Variability

Estimating internal variability from a single model could be a source of bias. We have also accounted for internal variability by widening $P(R)$ according to the Pythagorean theorem of statistics. This assumes independence between the forcing and the internal variability. Some co-variance is however likely to exist.

4.7.3 Two-Box Model

The TBM has proven effective. However, some questions might be asked about the TBM's applicability.

In section 2.7.3 we saw how Geoffroy et al. (2013) found that the TBM managed to reproduce the 1% per year increase in CO₂ concentration scenario, using parameters fitted to the instant CO₂ doubling scenario. They therefore concluded that the TBM is able to adequately reproduce the temperature response of different scenarios. As mentioned, this should mean that the parameters found instant CO₂ doubling should give a good fit to any scenario¹⁰.

When we use the parameters they found for the 2×CO₂ scenario, we find that the parameters differ from the ones we find, some significantly. Figure 4.8 compare the fit for two different models. It is obvious that for the CCSM4 their parameters do not give a satisfactory fit. If we compare the numbers we find that c_1 for our fit is 0.10 K m²/W yr, while their value is 0.16 K m²/W yr. The value of c_2 is 0.0054 K m²/W yr and 0.0027 K m²/W yr for our and their fit, respectively.

There might be several explanations to this difference between the parameters. The most obvious is clearly the simplicity of the TBM. An instant doubling of CO₂ is a very different scenario than the historical forcing scenario, which means that the climate system can react differently. This does not necessarily imply that the TBM is not an adequate model for our purposes, but perhaps indicates that the TBM is only applicable within a range of scenarios somewhat similar to the tuning scenario. After all, we do get a good fit from using the response function of the TBM, even when the two time scales are set prior to the fit (which somewhat inhibits the fitting ability). The linearity assumption of the TBM should hold especially well in the historical case, since the size of

10. This is of course a necessary assumption to begin with, since there is no point in having a model if you need to tune the model to another model for every forcing scenario. However, a model can still be useful even though it does not perform well for every scenario, but perhaps some range of scenarios.

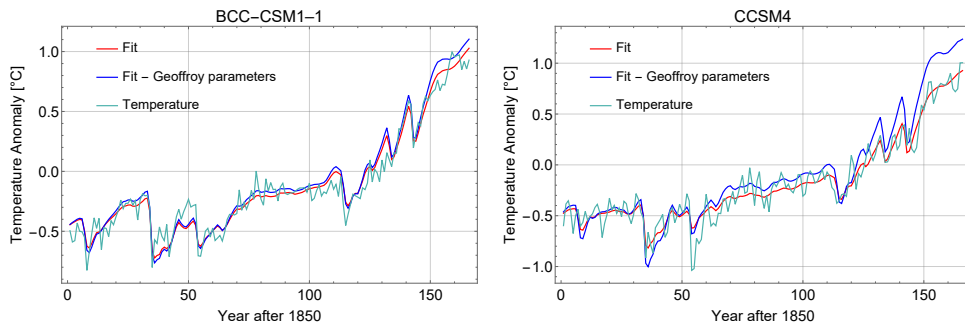


Figure 4.8: Fitted temperature response using both the fit to this particular forcing scenario, and a fit using the parameters from Geoffroy et al. (2013).

the perturbation is smaller.

Another possible explanation for the deviation from the instant CO_2 doubling parameters is the fact that the forcings used are different. Not just the magnitude, but also in the way they are estimated. The forcing used in Gregory et al. (2013) was calculated using the Gregory method and should be a good estimate of the true ERF. The forcing they used in the testing scenario (TCR scenario) was directly linked to the forcing of the tuning scenario through the logarithmic depends of forcing to CO_2 concentration, and therefore no new estimation method of the forcing was needed. Therefore, the forcing was guaranteed to be the same kind of forcing in both scenarios.

The Forster forcing is based on the same idea, but on a much shorter time scale, and with a constantly varying forcing. Our smoothing aims to restore the linearity relation and therefore correct the forcing to represent the trend in ERF in each time series. It is however not clear how well our forcing mimics the Gregory ERF. If the the different, the response of the TBM should also be different, which might explain some of the deviation. This does not, however, imply that the TBM is less suited to our problem. It just means that the calibration of the model is different. On the other hand, the MSRT gave the best correlation, which indicates that there is some applicability of the $2\times\text{CO}_2$ fit to our purpose as well

There are most certainly some questions to be asked about how well suited the TBM is for our particular method, and whether we could have used a better, perhaps more complicated model. The important point is that the correlation between R is still high using our fitting method to historical forcing, so the model seems to a nearly equally good job at extract information import for TCR. The fitting in Geoffroy et al. (2013) also had a much better staring point than for fitting given the instant high forcing, and therefore there is less noise in the fitting.

4.8 Underlying Issues

Finally, there are some questions to be asked about the validity of this study as a whole.

First, the problem is somewhat circular, as we assume that the models tell an accurate story about the nature of TCR when we build our statistical framework, while the concluding interval suggests that some models are unlikely to have an accurate estimate of TCR.

Second, doing statistics based almost entirely on models to disprove the models is perhaps superficial, even though there is this important extra data from the historical instrumental record upon which we lay our confidence in the result.

/5

Concluding Remarks

5.1 Summary

In this thesis, a method has been developed to constrain the likely values of TCR produced by an ensemble of CMIP5 models, based on the historic temperature record from 1850. The method was based on the simple but illustrative and physically sound TBM, which is able to reproduce the temperature of the complex AOGCMs accurately. In the fitting of the response function of the TBM to the data, the data was compressed into a scalar (R), that could be compared between model simulations and historical data.

In order to best compare historical data to model data, we smoothed and adjusted forcing data from each model. This was done to ensure that the forcing used, when fitting both historical and model data, was not directly linked to the temperature fluctuations of the model. When models were fitted to the TBM, the subsequent high correlation between the value of R and TCR promised high predictive ability of TCR just from this scalar.

To account for the uncertainty in historical forcing, we applied the different adjusted forcings of every model to the TBM, and fitted to historical data. Additional uncertainty in the value of R was included to account for the variability in the climate system, and we did so by using data from different simulations of the same model.

Based on the linear regression between R and TCR from the model simulation,

and the distribution of R from historical fits based on different forcings (as well as the estimate of internal variability), a pdf for TCR was estimated.

5.2 Conclusion

Reducing uncertainty in the TCR is important as it maps directly to uncertainty in climate change of the coming century. This study set out to investigate the following question:

Given the recorded instrumental temperature record and a set of historical simulations from an ensemble of CMIP5 models, is it possible to constrain the likely values of TCR and thus discard some models as being inconsistent with observation?

Based on the results of this study, the answer is found to be yes.

Our resulting 5 – 95% confidence interval predicts a TCR of 1.0 – 2.2°C. Two of the AOGCMs used in this study have a TCR which is outside this range, CanESM2 (2.4°C) and HadGEM2-ES (2.5°C). These models therefore seem to be inconsistent with observations.

This result has been shown to be robust to changes in parameters, and provides further evidence to recent studies suggesting that the likely range of TCR is lower than what some climate models predict.

However, questions remain regarding the validity of using the forcing data in such a way, and whether the uncertainties are properly included.

5.3 Future Work

Simply acquiring more data as time passes would improve the ability to constrain TCR from observation, because the rapid increase in greenhouse gases of the past decades means the response of the climate to a rapidly increasing CO₂ concentration is unraveling. CMIP6 is on the doorsteps as well, and repeating this study using this updated ensemble of models along with even more recent temperature records would likely give more accurate results.

Another possibility is to use the most recent Paleoclimate Modeling Intercomparison Project (PMIP₃) to repeat the study with the goal of constraining ECS. Furthermore, one can have a different measure of climate change and use a

similar method (possibly with some other metric). One possibility is to use the temperature in year 2100 in a given RCP scenario as the target variable.



Technical Theory

A.1 Solving The Two Box Model

The two-box model equations are given by

$$C \frac{dT}{dt} = F - \lambda T - \gamma(T - T_0) \quad (\text{A.1})$$

$$C_0 \frac{dT_0}{dt} = \gamma(T - T_0) \quad (\text{A.2})$$

By rewriting the set of equations in matrix form we get the following matrix equation

$$\frac{d\mathbf{x}}{dt} = A\mathbf{x} + \mathbf{b} \quad (\text{A.3})$$

Where

$$\mathbf{x}(t) = \begin{bmatrix} T \\ T_0 \end{bmatrix}, \quad A = \begin{bmatrix} -(\lambda + \gamma)/C & \gamma/C \\ \gamma/C_0 & -\gamma/C_0 \end{bmatrix}, \quad \mathbf{b}(t) = \begin{bmatrix} F/C \\ 0 \end{bmatrix}.$$

The solution, \mathbf{x}^* , to the homogeneous system ($\mathbf{b} = 0$) is given by

$$\mathbf{x}^*(t) = e^{tA}\mathbf{x}(0),$$

where e^{tA} is the matrix exponential [Anton, 2010, Edwards et al., 2004].

In order to express e^{tA} directly, not as an infinite series, one can diagonalize A so that $A = \Phi D \Phi^{-1}$, where D is the diagonal matrix whose diagonal elements are the eigenvalues of A and Φ is the matrix with the corresponding eigenvectors as columns. It can be shown (with some algebra) that

$$D = \begin{bmatrix} -1/\tau_f & 0 \\ 0 & -1/\tau_s \end{bmatrix}, \text{ and } \Phi = \begin{bmatrix} 1 & 1 \\ \phi_f & \phi_s \end{bmatrix}$$

Where

$$\tau_f = \frac{CC_0}{2\lambda\gamma}(b - \sqrt{\delta}), \quad \tau_s = \frac{CC_0}{2\lambda\gamma}(b + \sqrt{\delta}), \quad \phi_f = \frac{C}{2\gamma}(b^* - \sqrt{\delta}), \quad \phi_s = \frac{C}{2\gamma}(b^* + \sqrt{\delta})$$

and

$$b = \frac{\lambda + \gamma}{C} + \frac{\gamma}{C_0}, \quad b^* = \frac{\lambda + \gamma}{C} - \frac{\gamma}{C_0}, \quad \delta = b^2 - 4\frac{\lambda\gamma}{CC_0}$$

It can be proved that by using the diagonalization of A , that the exponential can be expressed

$$e^{tA} = e^{\Phi D \Phi^{-1}} = \Phi e^{tD} \Phi^{-1} = \Phi \begin{bmatrix} e^{-t/\tau_f} & 0 \\ 0 & e^{-t/\tau_s} \end{bmatrix} \Phi^{-1} \quad (\text{A.4})$$

Which gives us the general solution of the homogeneous system as follows

$$T^*(t) = \frac{1}{\phi_s - \phi_f} \left(T_1 e^{-t/\tau_f} + T_2 e^{-t/\tau_s} \right) \text{ and}$$

$$T_0^*(t) = \frac{1}{\phi_s - \phi_f} \left(\phi_f T_1 e^{-t/\tau_f} + \phi_s T_2 e^{-t/\tau_s} \right)$$

Where $T_1 = \phi_s T(0) - T_0(0)$ and $T_2 = -\phi_f T(0) + T_0(0)$

We can find the particular solution by using *variation of parameters*. By assuming that a particular solution has the form $\mathbf{x}(t) = e^{tA} \mathbf{U}(t)$, we can insert this into A.3 to get $\mathbf{U}'(t) = (e^{tA})^{-1} \mathbf{b}(t)$. Using equation A.4 it is possible integrate this expression to obtain the vector \mathbf{U} . By combining the homogeneous solution with the particular solution we get the following general solutions

$$T(t) = T^*(t) + \frac{1}{C(\phi_s - \phi_f)} \left[\phi_s \int_0^t ds F(s) e^{-(t-s)/\tau_f} - \phi_f \int_0^t ds F(s) e^{-(t-s)/\tau_s} \right]$$

and

$$T_0(t) = T_0^*(t) + \frac{\phi_f \phi_s}{C(\phi_s - \phi_f)} \left[\int_0^t ds F(s) e^{-(t-s)/\tau_f} - \int_0^t ds F(s) e^{-(t-s)/\tau_s} \right]$$

In the case of a step forcing the analytic solution is straightforward and can be written as

$$T(t) = T_{eq} - a_f T_{eq} e^{-t/\tau_f} - a_s T_{eq} e^{-t/\tau_s} \quad (\text{A.5})$$

$$T_0(t) = T_{eq} - \phi_f a_f T_{eq} e^{-t/\tau_f} - \phi_f a_s T_{eq} e^{-t/\tau_s} \quad (\text{A.6})$$

where

$$a_f = \frac{\phi_s \tau_f}{C(\phi_s - \phi_f)} \lambda, \quad a_s = -\frac{\phi_f \tau_s}{C(\phi_s - \phi_f)} \lambda$$

are the amplitudes of the fast and slow temperature responses and $T_{eq} = F/\lambda$. Importantly, $a_f + a_s = 1$ and $\phi_f a_f + \phi_s a_s = 1$. [Geoffroy et al., 2013]

A.2 Correlation

The correlation measures the linear relationship between two variables. The most common measure is the "Pearson correlation coefficient", often just referred to as the correlation coefficient [Walpole et al., 2014]. The coefficient is an estimate of the predictive relationship between the two variables, and a high (absolute) suggest a strong predictive relationship where knowing one variable is enough to predict the other with low uncertainty.

Mathematically, the coefficient is given by

$$r = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2 \sum_{i=1}^n (y_i - \bar{y})^2}}, \quad (\text{A.7})$$

and this expression is the one used in the Correlation[] function in Mathematica.



Tables

Table B.1: Table of CMIP5 models used in this thesis, with their corresponding TCR, ECS and Q . For models that were part of the study of Geoffroy et al. (2013), the respective relaxation times are also shown.

#	Name	TCR [°C]	ECS [°C]	Q [W/m ²]	τ_f [yr]	τ_s [yr]
1	BCC-CSM1-1	1.7	2.8	3.2	4.0	126
2	CanESM2	2.4	3.7	3.8	4.5	193
3	CCSM4	1.8	2.9	3.6	2.8	132
4	CNRM-CM5	2.1	3.3	3.6	5.2	289
5	CSIRO-Mk3-6-0	1.8	4.1	2.6	3.9	200
6	GISS-E2-R	1.5	2.1	3.8	1.6	184
7	IPSL-CM5A-LR	2.0	4.1	3.1	5.5	286
8	MIROC5	1.5	2.7	4.1	3.5	285
9	MPI-ESM-LR	2.0	3.6	4.1	3.9	164
10	MRI-CGCM3	1.6	2.6	3.2	4.3	150
11	NorESM1-M	1.4	2.8	3.1	4.0	218
12	HadGEM2-ES	2.5	4.6	2.9	5.3	280
13	GFDL-ESM2M	1.3	2.4	3.4	3.6	197
14	ACCESS10	2.0	3.8	3.0	-	-
15	BCC-CSM1-1-M	2.1	2.9	3.6	-	-
16	INM-CM4	1.3	2.1	3.0	-	-
17	MIROC-ESM	2.2	4.7	4.3	-	-
18	GFDL-CM3	2.0	4	3.0	-	-
19	GFDL-ESM2G	1.1	2.4	3.1	-	-



Additional Results

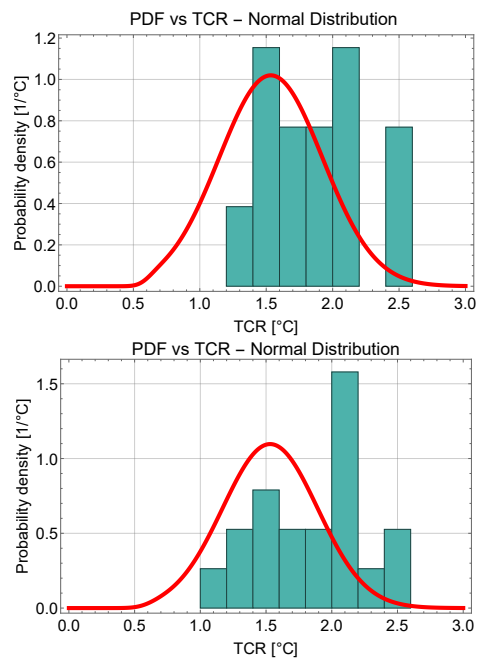


Figure C.1: Pdfs for the MSRT and MMMRT cases using a normal distribution for $P(R)$. All parameters are the same as in figure 4.5.

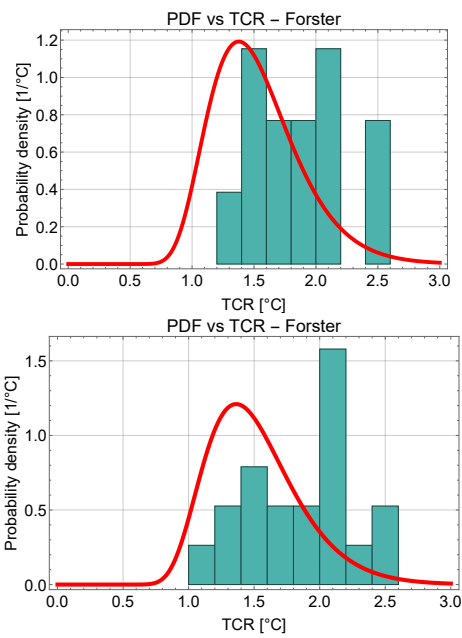


Figure C.2: Pdfs for the MSRT and MMMRT cases using the Forster forcing directly without adjustment. A gamma distribution is used for comparison with figure 4.5.



Mathematica Code

```
SetDirectory["C:\\Users\\Olaf\\Dropbox\\Master\\Data"];
```

```
Gofmat =
```

```
{{"BCC-CSM1-1", 7.6, 53, 0.67, 4., 126}, {"BNU-ESM", 7.4, 90, 0.53, 5.0, 267},  
, {"CanESM2", 7.3, 71, 0.59, 4.5, 193}  
, {"CCSM4 ", 6.1, 69, 0.93, 2.8, 132}  
, {"CNRM-CM5", 8.4, 99, 0.50, 5.2, 289}, {"CSIRO-Mk3.6.0",  
6.0, 69, 0.88, 3.9, 200}, {"GFDL-ESM2M", 8.1, 105, 0.90, 3.6, 197},  
{"GISS-E2-R", 4.7, 126, 1.16, 1.6, 184}, {"HadGEM2-ES", 6.5, 82, 0.55, 5.3, 280},  
{"IPSL-CM5A-LR", 7.7, 95, 0.59, 5.5, 286}, {"MIROC5", 8.3, 145, 0.76, 3.5, 285},  
{"MPI-ESM-LR", 7.3, 71, 0.72, 3.9, 164}, {"MRI-CGCM3", 8.5, 64, 0.66, 4.3, 150},  
{"NorESM1-M", 8.0, 105, 0.88, 4.0, 218}, {"FGOALS-s2", 7.0, 127, 0.76, 4.2, 317},  
{"multimodel mean without INM-CM4", 7.3, 91, 0.74, 4.1, 219}};
```

```
b = Table[(Lambda2box[[i]] + Gofmat[[i, 4]]) / Gofmat[[i, 2]] +  
Gofmat[[i, 4]] / Gofmat[[i, 3]], {i, 1, 15}];  
bstar = Table[(Lambda2box[[i]] + Gofmat[[i, 4]]) / Gofmat[[i, 2]] -  
Gofmat[[i, 4]] / Gofmat[[i, 3]], {i, 1, 15}];  
delta = Table[b[[i]]^2 - ((4 * Lambda2box[[i]] * Gofmat[[i, 4]]) /  
(Gofmat[[i, 2]] * Gofmat[[i, 3]])), {i, 1, 15}];  
phif = Table[(Gofmat[[i, 2]] / (2 * Lambda2box[[i]])) *  
(bstar[[i]] - Sqrt[delta[[i]]]), {i, 1, 15}];  
phis = Table[(Gofmat[[i, 2]] / (2 * Lambda2box[[i]])) *  
(bstar[[i]] + Sqrt[delta[[i]]]), {i, 1, 15}];  
tf = Table[(Gofmat[[i, 2]] * Gofmat[[i, 3]] / (2 * Lambda2box[[i]] * Gofmat[[i, 4]])) *  
(b[[i]] - Sqrt[delta[[i]]]), {i, 1, 15}];  
ts = Table[(Gofmat[[i, 2]] * Gofmat[[i, 3]] / (2 * Lambda2box[[i]] * Gofmat[[i, 4]])) *  
(b[[i]] + Sqrt[delta[[i]]]), {i, 1, 15}];  
af = Table[(phis[[i]] * tf[[i]] * Lambda2box[[i]]) /  
(Gofmat[[i, 2]] * (phis[[i]] - phif[[i]])), {i, 1, 15}];  
c1gof = Table[af[[i]] / (Lambda2box[[i]] * tf[[i]]), {i, 1, 15}];  
c2gof = Table[(1 - af[[i]]) / (Lambda2box[[i]] * ts[[i]]), {i, 1, 15}];  
  
c1gof = Drop[c1gof, {2}]  
c2gof = Drop[c2gof, {2}]  
tfgof = Drop[tf, {2}]  
tsgof = Drop[ts, {2}]  
tn1 = {tfgof[[1]], tfgof[[2]], tfgof[[3]], tfgof[[4]], tfgof[[5]],  
tfgof[[7]], tfgof[[9]], tfgof[[10]], tfgof[[11]], tfgof[[12]],  
tfgof[[13]], 4.1, 4.1, 4.1, 4.1, tfgof[[8]], 4.1, tfgof[[6]], 4.1};  
tn2 = {tsgof[[1]], tsgof[[2]], tsgof[[3]], tsgof[[4]], tsgof[[5]],  
tsgof[[7]], tsgof[[9]], tsgof[[10]], tsgof[[11]], tsgof[[12]],  
tsgof[[13]], 219, 219, 219, 219, tsgof[[8]], 219, tsgof[[6]], 219};  
tn11 = Table[tn1[[i]], {i, {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 16, 18, 19}}];  
tn22 = Table[tn2[[i]], {i, {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 16, 18, 19}}];  
imppar = Table[{Lambda2box[[i]], af[[i]], 1 - af[[i]], tf[[i]], ts[[i]]}, {i, 1, 14}];  
Lambda2box = {1.21, 0.93, 1.03, 1.24, 1.11,  
0.61, 1.34, 1.70, 0.65, 0.79, 1.58, 1.14, 1.26, 1.11, 0.88};  
Seq = 1 / Lambda2box;
```

```

AR5mat =
  {"BCC-CSM1-1", 3.2, 2.8, 1.7, 0.9, 1.1}, {"BNU-ESM", 3.9, 4.1, 2.6, 1.1, 1.0}
, {"CanESM2", 3.8, 3.7, 2.4, 1, 1}
, {"CCSM4 ", 3.6, 2.9, 1.8, 0.8, 1.2}
, {"CNRM-CM5", 3.6, 3.3, 2.1, 0.9, 1.1}, {"CSIRO-M k3.6.0",
  2.6, 4.1, 1.8, 1.6, 0.6}, {"GFDL-ESM2M", 3.4, 2.4, 1.3, 0.7, 1.4},
  {"GISS-E2-R", 3.8, 2.1, 1.5, 0.6, 1.8}, {"HadGEM2-ES", 2.9, 4.6, 2.5, 1.6, 0.6},
  {"IPSL-CM5A-LR", 2.7, 2.6, 1.5, 1, 1}, {"MIROC5", 4.1, 2.7, 1.5, 0.7, 1.5},
  {"MPI-ESM-LR", 4.1, 3.6, 2, 0.9, 1.1}, {"MRI-CGCM3", 3.2, 2.6, 1.6, 0.8, 1.2},
{"NorESM1-M", 3.1, 2.8, 1.4, 0.9, 1.1}, {"FGOALS-g2-NOT WORKING", 1, 1, 1, 1, 1},
  {"multimodel mean without INM-CM4", 7.3, 91, 0.74, 4.1, 219}};
AR5matnew =
  {"BCC-CSM1-1", 3.2, 2.8, 1.7, 0.9, 1.1}, {"BNU-ESM", 3.9, 4.1, 2.6, 1.1, 1.0}
, {"CanESM2", 3.8, 3.7, 2.4, 1, 1}
, {"CCSM4 ", 3.6, 2.9, 1.8, 0.8, 1.2}
, {"CNRM-CM5", 3.6, 3.3, 2.1, 0.9, 1.1}, {"CSIRO-M k3.6.0",
  2.6, 4.1, 1.8, 1.6, 0.6}, {"GFDL-ESM2M", 3.4, 2.4, 1.3, 0.7, 1.4},
  {"GISS-E2-R", 3.8, 2.1, 1.5, 0.6, 1.8}, {"HadGEM2-ES", 2.9, 4.6, 2.5, 1.6, 0.6},
  {"IPSL-CM5A-LR", 3.1, 4.1, 2, 1, 1}, {"MIROC5", 4.1, 2.7, 1.5, 0.7, 1.5},
  {"MPI-ESM-LR", 4.1, 3.6, 2, 0.9, 1.1}, {"MRI-CGCM3", 3.2, 2.6, 1.6, 0.8, 1.2},
{"NorESM1-M", 3.1, 2.8, 1.4, 0.9, 1.1},
  {"Acess10", 3, 3.8, 2.0, 4.1, 219}, {"bcc-csm1-1m", 3.6, 2.9, 2.1, 4.1, 219},
  {"gfdl-cm3", 3, 4, 2.0, 4.1, 219}, {"gfdl-esm2g", 3.1, 2.4, 1.1, 4.1, 219},
  {"inm-cm4", 3, 2.1, 1.3, 4.1, 219}, {"miroc-esm", 4.3, 4.7, 2.2, 4.1, 219}};

Clisen =
  Table[{AR5mat[[i, 3]]/AR5mat[[i, 2]], AR5mat[[i, 4]]/AR5mat[[i, 2]]}, {i, 1, 14}];
Clisennew = Table[{AR5matnew[[i, 3]]/AR5matnew[[i, 2]],
  AR5matnew[[i, 4]]/AR5matnew[[i, 2]]}, {i, 1, 20}];

AR5 = Join[AR5mat, Clisen, 2];
AR5new = Join[AR5matnew, Clisennew, 2];
TCR14 = Table[AR5[[i, 4]], {i, 1, 14}];
TCR = Drop[TCR14, {2}];
TCRn = Table[AR5new[[i, 4]], {i, 1, 20}];
TCRn = Drop[TCRn, {2}];
ECSn = Table[AR5new[[i, 3]], {i, 1, 20}];
ECSn = Drop[ECSn, {2}];
ECSn =
  Table[ECSn[[i]], {i, {1, 2, 3, 4, 5, 7, 9, 10, 11, 12, 13, 14, 15, 18, 19, 8, 16, 6, 17}}];
TCRn = Table[TCRn[[i]],
  {i, {1, 2, 3, 4, 5, 7, 9, 10, 11, 12, 13, 14, 15, 18, 19, 8, 16, 6, 17}}];
TCRname = {"BCC-CSM1-1", "CanESM2", "CCSM4", "CNRM-CM5", "CSIRO-M k3.6.0",
  "GISS-E2-R", "IPSL-CM5A-LR", "MIROC5", "MPI-ESM-LR", "MRI-CGCM3",
  "NorESM1-M", "Acess10", "bcc-csm1-1m", "inm-cm4", "miroc-esm",
  "HadGEM2-ES", "gfdl-cm3", "GFDL-ESM2M", "gfdl-esm2g"};
scatsens = Table[{TCRn[[i]], ECSn[[i]]}, {i, 1, 19}];
plsen = ListPlot[scatsens, LabelStyle -> {15, RGBColor[184/256, 101/256, 71/256]},
  PlotStyle -> {PointSize[0.02], RGBColor[0.3, 0.67, 0.69]}];
Show[plsen, PlotLabel -> Style["ECS vs TCR - IPCC AR5", 22, Black],
  Frame -> True, FrameLabel -> {"TCR [°C]", "ECS [°C]"},
  LabelStyle -> {20, Black}, PlotRange -> {{1, 3}, {1, 5}}, GridLines -> Automatic]

```

```

ECS14 = Table[AR5[[i, 3]], {i, 1, 14}]
ECS = Drop[ECS14, {2}];
Q14 = Table[AR5[[i, 2]], {i, 1, 14}];
Q = Drop[Q14, {2}];
Qn = Table[AR5new[[i, 2]], {i, 1, 20}];
Qn = Drop[Qn, {2}];
Qn =
  Table[Qn[[i]], {i, {1, 2, 3, 4, 5, 7, 9, 10, 11, 12, 13, 14, 15, 18, 19, 8, 16, 6, 17}}]
sjekkecsper = Table[ECSn[[i]]/Qn[[i]], {i, 1, 19}];
Min[sjekkecsper]
Max[sjekkecsper]

data = Drop[ReadList["effektivforcing.csv", String], 0];
data[[150]] =
  "1999;1,0296;2,437;1,4666;1,7507;2,3697;1,5102;1,1762;2,6721;1,0;1,7391;2,4549;-99,
    99;2,2879;0,7953;2,4307;2,0774;-99,99;1,0229;1,4754;1,6457;-99,99;0,7256;1,1693
  ";
data = Table[ToExpression[Map[StringJoin[{{#[[1]], "."}, #[[2]]}] &, Map[
  StringSplit[#, ","] &, Drop[StringSplit[data[[i]], ";"], 1]]], {i, 1, 166}];
modeller = {"ACCESS1-0", "bcc-csm1-1-m", "bcc-csm1-1", "CanESM2", "CCSM4",
  "CNRM-CM5", "CSIRO-Mk3-6-0", "FGOALS-s2", "GFDL-CM3", "GFDL-ESM2G", "GFDL-ESM2M",
  "GISS-E2-H", "GISS-E2-R", "HadGEM2-ES", "inmcm4", "IPSL-CM5A-LR", "IPSL-CM5B-LR",
  "MIROC-ESM", "MIROC5", "MPI-ESM-LR", "MPI-ESM-P", "MRI-CGCM3", "NorESM1-M"};
dataall = Table[{data[[All, i]]}, {i, {3, 4, 5, 6, 7, 11, 13, 14, 16, 19, 20, 22, 23}}];
dataallall = Table[{data[[All, i]]},
  {i, {3, 4, 5, 6, 7, 11, 13, 14, 16, 19, 20, 22, 23, 1, 2, 9, 10, 15, 18}}];

Tmd1 = {"BCC-CSM1-1",
  Import["bcc-csm1-1_glanom_1850-2100_historicalandrcp85.txt", "Table"]};
Tmd2 = {"BNU-ESM", Import["BNU-ESM_glanom_1850-2100_historicalandrcp85.txt",
  "Table"]};
Tmd3 = {"CanESM2", Import["canesm2_glanom_1850-2100_historicalandrcp85.txt",
  "Table"]};
Tmd4 = {"CCSM4", Import["ccsm4_glanom_1850-2100_historicalandrcp85.txt", "Table"]};
Tmd5 =
  {"CNRM-CM5", Import["cnrmcm5_glanom_1850-2055_historicalandrcp85.txt", "Table"]};
Tmd6 = {"CSIRO-Mk3.6.0",
  Import["csiro-mk3-6-0_glanom_1850-2100_historicalandrcp85.txt", "Table"]};
Tmd7 = {"GFDL-ESM2M", Import[
  "gfdl-esm2m_glanom_1861-2020_historicalandrcp85.txt", "Table"]};
Tmd8 = {"GISS-E2-R", Import["GISS-E2-R_glanom_1850-2100_historicalandrcp85.txt",
  "Table"]};
Tmd9 = {"HadGEM2-ES", Import["hadgem2_glanom_1860-2054_historicalandrcp85.txt",
  "Table"]};
Tmd10 = {"IPSL-CM5A-LR", Import[
  "IPSL-CM5A-LR_glanom_1850-2100_historicalandrcp85.txt", "Table"]};
Tmd11 = {"MIROC5", Import["MIROC5_glanom_1850-2100_historicalandrcp85.txt",
  "Table"]};
Tmd12 = {"MPI-ESM-LR", Import["MPI-ESM_glanom_1850-2100_historicalandrcp85.txt",
  "Table"]};
Tmd13 = {"MRI-CGCM3", Import["MRI-CGCM3_glanom_1850-2100_historicalandrcp85.txt",
  "Table"]};

```

```

Tmd14 = {"NorESM1-M", Import["NorESM1-M_glanom_1850-2100_historicalandrcp85.txt",
  "Table"]};
Tm18 = {"MIROC-ESM", Import["MIROC-ESM_glanom_1850-2100_historicalandrcp85.txt",
  "Table"]};
Tm1 = {"Access10", Import["access10_glanom_1850-2100_historicalandrcp85.txt",
  "Table"]};
Tm2 = {"bcc-csm1-1m", Import["bcc-csm1-1-m_glanom_1850-2100_historicalandrcp85.txt",
  "Table"]};
Tm15 = {"inm-cm4", Import["inm-cm4_glanom_1850-2100_historicalandrcp85.txt",
  "Table"]};
Tm9 = {"gfdl-cm3", Import[
  "gfdl-cm3_glhistoricalanom_1860-2020_historicalandrcp85.txt", "Table"]];
Tm10 = {"gfdl-esm2g", Import[
  "gfdl-esm2g_glhistoricalanom_1861-2020_historicalandrcp85.txt", "Table"]];
Tmdmat = {Tmd1, Tmd2, Tmd3, Tmd4, Tmd5, Tmd6, Tmd7, Tmd8,
  Tmd9, Tmd10, Tmd11, Tmd12, Tmd13, Tmd14};
Tmdmatnew = {Tmd1, Tmd2, Tmd3, Tmd4, Tmd5, Tmd6, Tmd7, Tmd8, Tmd9, Tmd10,
  Tmd11, Tmd12, Tmd13, Tmd14, Tm1, Tm2, Tm9, Tm10, Tm15, Tm18};
Tmdmat2 = Drop[Tmdmat, {2}];
Tmdmat2 = Drop[Tmdmat, {6}];
Tmdmat2 = Drop[Tmdmat, {7}];
Tmdmatnew2 = Drop[Tmdmatnew, {2}];
Tmdmatn1 = {Tmd1, Tmd3, Tmd4, Tmd5, Tmd6,
  Tmd8, Tmd10, Tmd11, Tmd12, Tmd13, Tmd14, Tm1, Tm2, Tm15, Tm18};
Tmdmatn2 = {Tmd9, Tm9};
Tmdmatn3 = {Tmd7, Tm10};

```

In[3372]:=

```

imppargof =
  Table[{Lambda2box[[i]], af[[i]], 1 - af[[i]],  $\tau f[[i]]$ ,  $\tau s[[i]]$ , Q14[[i]]}, {i, 1, 14}]

```

In[3373]:= Clear[f]

```

R =
  Map[#[[6]] * (((#[[2]]/#[[1]]) / ((2 * Pi * #[[4]] * f)^2 + 1) + (#[[3]]/#[[1]]) /
    ((2 * Pi * #[[5]] * f)^2 + 1))^2 + ((2 * Pi * #[[2]] * #[[4]] * f/#[[1]]) /
    ((2 * Pi * #[[4]] * f)^2 + 1) + (2 * Pi * #[[3]] * #[[5]] * f/#[[1]]) /
    ((2 * Pi * #[[5]] * f)^2 + 1))^2)^(0.5) &, imppargof]

```

In[3375]:=

```

korECS = Table[{(1.4)^(-i), Correlation[ECS14, R /. f -> (1.4)^(-i)]}, {i, 1, 27}]

```

```

In[3518]= PL111 = ListLogLinearPlot[korECS, Frame → True, FrameStyle → Directive[Black, 20],
  PlotStyle → {PointSize[0.02], RGBColor[0.3, 0.66, 0.69]};
PL222 = ListLogLinearPlot[korECS, Frame → True, FrameStyle → Directive[Black, 20],
  PlotStyle → RGBColor[0.3, 0.66, 0.69], Joined → True];
G222 = Show[{PL111, PL222}, FrameLabel → {"Frequency [1/Years]", "Correlation"},
  AspectRatio → 1, Background → None, PlotRange → {{Log[0.0001], Log[1]}, {-0.2, 1}},
  PlotLabel → Style["Correlation ECS - Geoffroy parameters", 22, Black],
  GridLines → Automatic]

In[3438]= aa = LineLegend[
  {RGBColor[0.3, 0.67, 0.69], RGBColor[0.1, 0.2, 0.6], RGBColor[0.8, 0.3, 0.6]},
  {Style["(1.7 - 2.8)", 18], Style["(2.1 - 3.3)", 18], Style["(1.4 - 2.8)", 18]},
  LegendMarkerSize → 40, LegendMarkers → Graphics[Thickness[3]]]
Clear[f]
R =
Map[#[[6]] * (((#[[2]]/#[[1]]) / ((2 * Pi * #[[4]] * f)^2 + 1) + (#[[3]]/#[[1]]) /
  ((2 * Pi * #[[5]] * f)^2 + 1))^2 + ((2 * Pi * #[[2]] * #[[4]] * f/#[[1]]) /
  ((2 * Pi * #[[4]] * f)^2 + 1) + (2 * Pi * #[[3]] * #[[5]] * f/#[[1]]) /
  ((2 * Pi * #[[5]] * f)^2 + 1))^2)^(0.5) &, imppargof]

In[8892]=
imp1 =
Table[{Lambda2box[[i]], af[[i]], 1 - af[[i]], 4.1, ts[[i]], Q14[[i]]}, {i, 1, 14}];
Correlation[Q14 * (Drop[af, {15}] / Drop[Lambda2box, {15}]), TCR14]
PR1 = LogLinearPlot[{R[[1]]},
  {f, 0.0001, 1}, Frame → True, FrameStyle → Directive[Black, 20],
  PlotStyle → {{Thickness[0.01], RGBColor[0.3, 0.67, 0.69]}},
  FrameLabel → {"Frequency f [1/Years]", "R(f) [°C]"}];
PR2 = LogLinearPlot[{R[[5]]}, {f, 0.0001, 1}, Frame → True, FrameStyle →
  Directive[Black, 20], PlotStyle → {{Thickness[0.01], RGBColor[0.3, 0.3, 0.6]}},
  FrameLabel → {"Frequency f (1/Years)", "R(f)"}];
PR3 = LogLinearPlot[{R[[14]]}, {f, 0.0001, 1}, Frame → True, FrameStyle →
  Directive[Black, 20], PlotStyle → {{Thickness[0.01], RGBColor[0.8, 0.3, 0.6]}},
  FrameLabel → {"Frequency [1/Years]", "R(f) [°C]"}];
Show[PR1, PR2, PR3, PlotRange → {0, 3.5},
  PlotLabel → Style["Response Functions", 22, Black],
  Epilog → {Inset[aa, Scaled[{0.83, 0.68}]]}, GridLines → Automatic]
Table[{TCR14[[i]], ECS14[[i]]}, {i, {1, 5, 14}}]

R1 = Map[#[[6]] * ((#[[2]]/#[[1]]) / ((2 * Pi * #[[4]] * f)^2 + 1)) &, imppargof]

korTCS = Table[{(1.4)^(-i), Correlation[TCR14, R /. f → (1.4)^(-i)]}, {i, 1, 27}]
PLTCS1 = ListLogLinearPlot[korTCS, Frame → True, FrameStyle → Directive[Black, 20],
  PlotStyle → {PointSize[0.02], RGBColor[0.3, 0.66, 0.69]};
PLTCS2 = ListLogLinearPlot[korTCS, Frame → True, FrameStyle → Directive[Black, 20],
  PlotStyle → RGBColor[0.3, 0.66, 0.69], Joined → True];
GTCS = Show[{PLTCS1, PLTCS2}, FrameLabel → {"Frequency [1/Years]", "Correlation"},
  AspectRatio → 1, Background → None, PlotRange → {{Log[0.0001], Log[1]}, {-0.2, 1}},
  PlotLabel → Style["Correlation TCR - Goeffroy Parameters", 22, Black],
  Axes → None, GridLines → Automatic]

```

In[3511]=

```

f = 0.01;
corR = MapThread[{-#1, #2} &, {R, TCR14}]
Correlation[corR]
lincorR = LinearModelFit[corR, x, x]
P3 = Plot[lincorR[t], {t, 0, 3}, Frame → True, FrameStyle → Directive[Black, 20],
  PlotStyle → {Thickness[0.01], RGBColor[0.3, 0.66, 0.69]}}];
P4 = ListPlot[corR, Frame → True, FrameStyle → Directive[Black, 20],
  PlotStyle → {PointSize[0.02], Black}];
Show[P3, P4, PlotRange → All, FrameLabel → {"R(f=0.01) [°C]", "TCR [°C]"},
  AspectRatio → 1, Background → None, PlotRange → {{Log[0.0001], Log[1]}, {-0.2, 1}},
  PlotLabel → Style["TCR vs R(f=0.01)", 24, Black], GridLines → Automatic]

```

```

FF[x_] = 1 - 0.59 * Exp[-x / 4] + 0.41 * Exp[-x / 220];

```

```

plo1 = Plot[1 - 0.59 * Exp[-x / 5] - 0.41 * Exp[-x / 200], {x, 0, 2000},
  PlotRange → {{0, 50}, {0, 1}}, Frame → True, FrameStyle → Directive[Black, 14],
  PlotStyle → {Thickness[0.01], RGBColor[0.3, 0.66, 0.69]}, FrameLabel → {"Year \!\!\(*
StyleBox[\\"", \nFontSlant->\\"Italic\"]) \!\!\(*
StyleBox[\\" \", \nFontSlant->\\"Italic\"]) \!\!\(*
StyleBox[\\"T\", \nFontSlant->\\"Italic\"]) \!\!\(*
StyleBox[\\"(\", \nFontSlant->\\"Italic\"]) \!\!\(*
StyleBox[\\"t\", \nFontSlant->\\"Italic\"]) \!\!\(*
StyleBox[\"]\", \nFontSlant->\\"Italic\"]) / (F/α)",
  PlotLabel → Style["Temperature Response To Step Forcing", 16, Black]
plo2 = Plot[1 - 0.59 * Exp[-x / 5] - 0.41 * Exp[-x / 200], {x, 0, 2000},
  PlotRange → {{0, 1000}, {0, 1}}, Frame → True, FrameStyle → Directive[Black, 14],
  PlotStyle → {Thickness[0.01], RGBColor[0.3, 0.66, 0.69]}, FrameLabel → {"Year \!\!\(*
StyleBox[\\"", \nFontSlant->\\"Italic\"]) \!\!\(*
StyleBox[\\" \", \nFontSlant->\\"Italic\"]) \!\!\(*
StyleBox[\\"T\", \nFontSlant->\\"Italic\"]) \!\!\(*
StyleBox[\\"(\", \nFontSlant->\\"Italic\"]) \!\!\(*
StyleBox[\\"t\", \nFontSlant->\\"Italic\"]) \!\!\(*
StyleBox[\"]\", \nFontSlant->\\"Italic\"]) / (F/α)",
  PlotLabel → Style["Temperature Response To Step Forcing", 16, Black]

```

In[91]:=

```

kj1 = Import["csiro_r1i1p1_glanom_1850-2012_historicalNat.txt", "Data"];
kj2 = Import["csiro_r2i1p1_glanom_1850-2012_historicalNat.txt", "Data"];
kj3 = Import["csiro_r3i1p1_glanom_1850-2012_historicalNat.txt", "Data"];
kj4 = Import["csiro_r4i1p1_glanom_1850-2012_historicalNat.txt", "Data"];
kj5 = Import["csiro_r5i1p1_glanom_1850-2012_historicalNat.txt", "Data"];
kj6 = Import["csiro_r6i1p1_glanom_1850-2012_historicalNat.txt", "Data"];
kj7 = Import["csiro_r7i1p1_glanom_1850-2012_historicalNat.txt", "Data"];
kj8 = Import["csiro_r8i1p1_glanom_1850-2012_historicalNat.txt", "Data"];
kj9 = Import["csiro_r9i1p1_glanom_1850-2012_historicalNat.txt", "Data"];
kj10 = Import["csiro_r10i1p1_glanom_1850-2012_historicalNat.txt", "Data"];
ki1 = Import["cnrmcm5_r1i1p1_glanom_1850-2012_historicalandext.txt", "Data"];
ki2 = Import["cnrmcm5_r2i1p1_glanom_1850-2012_historicalandext.txt", "Data"];
ki3 = Import["cnrmcm5_r3i1p1_glanom_1850-2012_historicalandext.txt", "Data"];
ki4 = Import["cnrmcm5_r4i1p1_glanom_1850-2012_historicalandext.txt", "Data"];
ki5 = Import["cnrmcm5_r5i1p1_glanom_1850-2012_historicalandext.txt", "Data"];
ki6 = Import["cnrmcm5_r6i1p1_glanom_1850-2012_historicalandext.txt", "Data"];
ki7 = Import["cnrmcm5_r7i1p1_glanom_1850-2012_historicalandext.txt", "Data"];
ki8 = Import["cnrmcm5_r8i1p1_glanom_1850-2012_historicalandext.txt", "Data"];
ki9 = Import["cnrmcm5_r9i1p1_glanom_1850-2012_historicalandext.txt", "Data"];
ki10 = Import["cnrmcm5_r10i1p1_glanom_1850-2012_historicalandext.txt", "Data"];

kj = {kj1, kj2, kj3, kj4, kj5, kj6, kj7, kj8, kj9, kj10};
ki = {ki1, ki2, ki3, ki4, ki5, ki6, ki7, ki8, ki9, ki10};

```

In[297]:=

```

forcR = ReadList["F.txt", String];
forcR = Partition[
  Flatten[Map[StringSplit[#, ","] &, Drop[Drop[StringSplit[forcR], 5], -5]]], 10];
forcR = ToExpression[forcR[[All, 10]]];
T11[t_] := Sum[fac1[[4, s]] * Exp[-(t - s) / τ1], {s, 0, t}];
T222[t_] := Sum[fac1[[4, s]] * Exp[-(t - s) / τ2], {s, 0, t}];
T11 = Table[T11[t][[1]], {t, 1, 163}];
T22 = Table[T222[t][[1]], {t, 1, 163}];

constski = {};
constski2 = {};
Do[

  cc = ki[[n]][[All, 2]];
  dd = T11;
  ee = T22;
  ff = ConstantArray[1, 163];
  mm = Transpose[{dd, ee, ff}];
  mm2 = Transpose[{dd, ff}];
  ss = LeastSquares[mm, cc];
  ss2 = LeastSquares[mm2, cc];
  constski = Append[constski, ss];
  constski2 = Append[constski2, ss2];

, {n, 1, 10}]

```



```

intvar = Show[ll1, ll2, Frame → True, FrameStyle → Directive[Black, 24],
  FrameLabel → {"Year After 1850", "Temperature anomaly [°C]"},
  PlotLabel → Style["Internal Variability", 28, Black],
  GridLines → Automatic, ImageResolution → 100000000]
Export["intvar2.jpg", intvar]

```

In[8132]=

```

bb = LineLegend[{Blue, Red},
  {Style["Adjusted MIROC5", 18], Style["Hansen", 18]}, LegendMarkerSize → 30]

```

In[8164]=

```

globaltemp = Import["Hadcrut4_monthly_to2017.txt", "Table"][[1 ;; ;; 2]];
st = 30;
smt = 8;
excut = 0;
cut = 0;
smt2 = 8;
cend = 60;
forcalln = {};
forcalln2 = {};
forcallsame = {};
forcallsameF = {};
forcallsameF2 = {};
forcallsameF3 = {};
korsave = {};
fac1 = {};
fac2 = {};
fac3 = {};
Do[
  F = dataallall[[n, 1]];
  FSF = F;
  MM = MeanFilter[F, smt];
  MMp = Drop[MM, -cend];
  ML = MeanFilter[F, smt2];
  ML2 = Drop[ML, 166 - cend];
  MMP = Join[MMp, ML2];
forc = ReadList["F.txt", String];
forc = Partition[
  Flatten[Map[StringSplit[#, ", "] &, Drop[Drop[StringSplit[forc], 5], -5]]], 10];
F = ToExpression[forc[[All, 10]]];
FS = F;
MMM = MeanFilter[F, smt];
MML = MeanFilter[F, smt2];
MML2 = Drop[MML, 166 - cend];
MMMp = Drop[MMM, -cend];
MMMP = Join[MMMp, MML2];
korreksjonM = (MMM - MM);
korreksjonM = Drop[korreksjonM, cut];
MJ = ConstantArray[0, cut];
MA = MML - ML;
MA = Drop[MA, 166 - cend];
korreksjonM = Join[MJ, korreksjonM];
korreksjonM = Drop[korreksjonM, -cend];
korreksjonM = Join[korreksjonM, MA];
forc = ReadList["F.txt", String];

```

```

forc = Partition[
  Flatten[Map[StringSplit[#, ", "] &, Drop[Drop[StringSplit[forc], 5], -5]]], 10];
F = ToExpression[forc[[All, 10]]];
F2 = F - korreksjonM;
forcalln = Append[forcalln, F2];
forallsame = Append[forallsame, FS];
forallsameF = Append[forallsameF, FSF];
fac1 = Append[fac1, F2];
korsave = Append[korsave, korreksjonM];
, {n, {1, 2, 3, 4, 5, 7, 9, 10, 11, 12, 13, 14, 15, 18, 19}}]
ListPlot[FSF, Joined → True,
  PlotStyle → {Thickness[0.004], RGBColor[0.3, 0.67, 0.69]},
  PlotLabel → Style["MIROC5 Forster ", 22, Black], Frame → True,
  FrameLabel → {"Year After 1850", "Forcing [W/m2"]},
  LabelStyle → {20, Black}, PlotRange → All, GridLines → Automatic]
ListPlot[{MMMP, MMP}, Joined → True,
  PlotLabel → Style["Smoothed Forcing ", 22, Black], Frame → True,
  FrameLabel → {"Year After 1850", "Forcing [W/m2 "]},
  LabelStyle → {20, Black}, PlotRange → All, GridLines → Automatic,
  PlotStyle → {{Thickness[0.003], Red}, {Thickness[0.003], Blue}},
  Epilog → {Inset[bb, Scaled[{0.25, 0.84}]]}]

Do[
  F = dataallall[[n, 1]];
  F = Drop[F, 10];
  FSF = F;
  fj = ConstantArray[0, 10];
  FSF = Join[fj, FSF];
  forallsameF2 = Append[forallsameF2, FSF];
  MM = MeanFilter[F, smt];
  ML = MeanFilter[F, smt2];
forc = ReadList["F.txt", String];
forc = Partition[
  Flatten[Map[StringSplit[#, ", "] &, Drop[Drop[StringSplit[forc], 5], -5]]], 10];
F = ToExpression[forc[[All, 10]]];
F = Drop[F, 10];
FS = F;
MMM = MeanFilter[F, smt];
MML = MeanFilter[F, smt2];
korreksjonM = (MMM - MM);
korreksjonM = Drop[korreksjonM, cut];
MJ = ConstantArray[0, cut];
MA = MML - ML;
MA = Drop[MA, 156 - cend];
korreksjonM = Join[MJ, korreksjonM];
korreksjonM = Drop[korreksjonM, -cend];
korreksjonM = Join[korreksjonM, MA];
koradd10 = ConstantArray[0, 10];
korreksjonM = Join[koradd10, korreksjonM];
forc = ReadList["F.txt", String];
forc = Partition[
  Flatten[Map[StringSplit[#, ", "] &, Drop[Drop[StringSplit[forc], 5], -5]]], 10];
F = ToExpression[forc[[All, 10]]];
FF = Thread[{#1, #2} &[Range[Length[F]], F]];
F2 = F - korreksjonM;

```

```

forcalln = Append[forcalln, F2];
fac2 = Append[fac2, F2];

, {n, {8, 16}}]

Do[
  F = dataallall[[n, 1]];
  F = Drop[F, 11];
  FSF = F;
  fj = ConstantArray[0, 11];
  FSF = Join[fj, FSF];
  forcallsameF3 = Append[forcallsameF3, FSF];
  MM = MeanFilter[F, smt];
  ML = MeanFilter[F, smt2];
  forc = ReadList["F.txt", String];
  forc = Partition[
    Flatten[Map[StringSplit[#, ", "] &, Drop[Drop[StringSplit[forc], 5], -5]]], 10];
  F = ToExpression[forc[[All, 10]]];
  F = Drop[F, 11];
  FS = F;
  MMM = MeanFilter[F, smt];
  MML = MeanFilter[F, smt2];
  korreksjonM = (MMM - MM);
  korreksjonM = Drop[korreksjonM, cut];
  MJ = ConstantArray[0, cut];
  MA = MML - ML;
  MA = Drop[MA, 155 - cend];
  korreksjonM = Join[MJ, korreksjonM];
  korreksjonM = Drop[korreksjonM, -cend];
  korreksjonM = Join[korreksjonM, MA];
  koradd10 = ConstantArray[0, 11];
  korreksjonM = Join[koradd10, korreksjonM];
  forc = ReadList["F.txt", String];
  forc = Partition[
    Flatten[Map[StringSplit[#, ", "] &, Drop[Drop[StringSplit[forc], 5], -5]]], 10];
  F = ToExpression[forc[[All, 10]]];
  FF = Thread[{#1, #2} &[Range[Length[F]], F]];
  F2 = F - korreksjonM;
  forcalln = Append[forcalln, F2];
  fac3 = Append[fac3, F2];

, {n, {6, 17}}]

fko = ListPlot[Mean[forcallsameF], Joined → True];
feo = ListPlot[forcallsame[[1]], Joined → True, PlotStyle → Red];
ListPlot[{forcallsame[[1]], fac1[[15]]}, Joined → True,
  PlotLabel → Style["Adjusted Forcing vs Hansen Forcing ", 22, Black],
  Frame → True, FrameLabel → {"Year After 1850", "Forcing [W/m2 "},
  LabelStyle → {20, Black}, PlotRange → All, GridLines → Automatic,
  PlotStyle → {{Thickness[0.0035], Red}, {Thickness[0.0035], Blue}},
  Epilog → {Inset[bb, Scaled[{0.3, 0.8}]]}]
ListPlot[-korsave, Joined → True, PlotLabel → Style["Correction Series ", 22, Black],
  Frame → True, FrameLabel → {"Year After 1850", "Forcing [W/m2 "},

```

```

LabelStyle → {20, Black}, PlotRange → All, GridLines → Automatic]
ListPlot[forcallsame[[1]], PlotStyle → {{Thickness[0.005], RGBColor[0.3, 0.67, 0.69]}},
Joined → True, PlotLabel → Style["Hansen Forcing ", 22, Black],
Frame → True, FrameLabel → {"Year After 1850", "Forcing [W/m2 "}],
LabelStyle → {20, Black}, PlotRange → All, GridLines → Automatic]
ListPlot[fac1[[15]], Joined → True,
PlotStyle → {{Thickness[0.004], RGBColor[0.3, 0.67, 0.69]}},
PlotLabel → Style["NorESM1-M Forster", 22, Black], Frame → True,
FrameLabel → {"Year After 1850", "Forcing [W/m2 "}],
LabelStyle → {20, Black}, PlotRange → All, GridLines → Automatic]
P0 = ListPlot[fac1, Joined → True,
PlotLabel → Style["Smoothed Forcing data ", 22, Black],
Frame → True, FrameLabel → {"Year After 1850", "Forcing [W/m2 "}],
LabelStyle → {20, Black}, PlotRange → All, GridLines → Automatic];
P00 = ListPlot[forcallsame[[1]], Joined → True,
PlotStyle → {{Thickness[0.007], Red}},
PlotLabel → Style["Smoothed Forcing data ", 22, Black],
Frame → True, FrameLabel → {"Year After 1850", "Forcing [W/m2 "}],
LabelStyle → {20, Black}, PlotRange → All, GridLines → Automatic];
Show[P0, P00, PlotLabel → Style["Adjusted Forcings ", 22, Black],
Frame → True, FrameLabel → {"Year After 1850", "Forcing [W/m2 "}],
LabelStyle → {20, Black}, PlotRange → All, GridLines → Automatic]

```

In[8195]=

```

τ1 = 4.1;
τ2 = 219;
TST01n1[t_] := Table[Sum[fac1[[m, s]] * Exp[-(t - s) / τ1], {s, 0, t}], {m, 1, 15}];
TST02n1[t_] := Table[Sum[fac1[[m, s]] * Exp[-(t - s) / τ2], {s, 0, t}], {m, 1, 15}];
TST1n1 = Table[TST01n1[m][[i]][[1]], {m, 1, 166}, {i, 1, 15}];
TST2n1 = Table[TST02n1[m][[i]][[1]], {m, 1, 166}, {i, 1, 15}];
TST01n2[t_] :=
Table[Sum[fac2[[m, s + 10]] * Exp[-(t - s) / τ1], {s, 0, t}], {m, 1, 2}];
TST02n2[t_] := Table[Sum[fac2[[m, s + 10]] * Exp[-(t - s) / τ2], {s, 0, t}], {m, 1, 2}];
TST1n2 = Table[TST01n2[m][[i]], {m, 1, 156}, {i, 1, 2}];
TST2n2 = Table[TST02n2[m][[i]], {m, 1, 156}, {i, 1, 2}];
TST01n3[t_] :=
Table[Sum[fac3[[m, s + 11]] * Exp[-(t - s) / τ1], {s, 0, t}], {m, 1, 2}];
TST02n3[t_] := Table[Sum[fac3[[m, s + 11]] * Exp[-(t - s) / τ2], {s, 0, t}], {m, 1, 2}];
TST1n3 = Table[TST01n3[m][[i]], {m, 1, 155}, {i, 1, 2}];
TST2n3 = Table[TST02n3[m][[i]], {m, 1, 155}, {i, 1, 2}];
TSTM1 = Table[{Tmdmatn1[[i, 2, m, 2]]}, {m, 1, 166}, {i, 1, 15}];
TSTM2 = Table[{Tmdmatn2[[i, 2, m, 2]]}, {m, 1, 156}, {i, 1, 2}];
TSTM3 = Table[{Tmdmatn3[[i, 2, m, 2]]}, {m, 1, 155}, {i, 1, 2}];
TSTMFITn1 =
Table[{TSTM1[[m, i]], {TST1n1[[m, i]], TST2n1[[m, i]]}}, {m, 1, 166}, {i, 1, 15}];
TSTMFITn2 = Table[{TSTM2[[m, i]], {TST1n2[[m, i]], TST2n2[[m, i]]}},
{m, 1, 156}, {i, 1, 2}];
TSTMFITn3 = Table[{TSTM3[[m, i]], {TST1n3[[m, i]], TST2n3[[m, i]]}},
{m, 1, 155}, {i, 1, 2}];
TCRn = Table[AR5new[[i, 4]], {i, 1, 20}];
TCRn = Drop[TCRn, {2}]

```

```

TCRn =
  Table[TCRn[[i]], {i, {1, 2, 3, 4, 5, 7, 9, 10, 11, 12, 13, 14, 15, 18, 19, 8, 16, 6, 17}}]
TCRname = {"BCC-CSM1-1", "CanESM2", "CCSM4", "CNRM-CM5",
  "CSIRO-M k3.6.0", "GISS-E2-R", "IPSL-CM5A-LR", "MIROC5", "MPI-ESM-LR",
  "MRI-CGCM3", "NorESM1-M", "Acess10", "bcc-csm1-1m", "inm-cm4",
  "miroc-esm", "HadGEM2-ES", "gfdl-cm3", "GFDL-ESM2M", "gfdl-esm2g"}
Qn = Table[AR5new[[i, 2]], {i, 1, 20}];
Qn = Drop[Qn, {2}];
Qn =
  Table[Qn[[i]], {i, {1, 2, 3, 4, 5, 7, 9, 10, 11, 12, 13, 14, 15, 18, 19, 8, 16, 6, 17}}]
lbc = 0.2;
ubc = 2;
constsn1 = {};
constsn2 = {};
constsn3 = {};
constsn = {};
constsn1 = {};
constsn2 = {};
constsn3 = {};

```

```

Do[
  Clear[c1, c2, c3];
  cc = TSTMFITn1[[All, n1, 1]];
  dd = TSTMFITn1[[All, n1, 2, 1]];
  ee = TSTMFITn1[[All, n1, 2, 2]];
  ff = ConstantArray[1, 166];
  mm = Transpose[{dd, ee, ff}];
  ss = LeastSquares[mm, cc[[All, 1]]];
  constsn1 = Append[constsn1, ss];
  Clear[c1, c2, c3];
  funcmin[c1_, c2_, c3_] := Total[(mm.{c1, c2, c3} - cc[[All, 1]])^2];
  cval = NMinimize[
    {Total[(mm.{c1, c2, c3} - cc[[All, 1]])^2], {{lbc < c1 * 4.1 + c2 * 219 < ubc},
      {0.02 < c1 < 0.13}, {0 < c2 < 0.006}}}, {c1, c2, c3}];
  cval2 = {cval[[2, 1]][[2]], cval[[2, 2]][[2]]};
  constsn1 = Append[constsn1, cval2];

```

```
, {n1, 1, 15}]
```

```

Do[
  cc = TSTMFITn2[[All, n1, 1]];
  dd = TSTMFITn2[[All, n1, 2, 1]];
  ee = TSTMFITn2[[All, n1, 2, 2]];
  ff = ConstantArray[1, 156];
  mm = Transpose[{dd, ee, ff}];
  ss = LeastSquares[mm, cc[[All, 1]]];
  constsn2 = Append[constsn2, ss];
  Clear[c1, c2, c3];
  funcmin[c1_, c2_, c3_] := Total[(mm.{c1, c2, c3} - cc[[All, 1]])^2];
  cval = NMinimize[
    {Total[(mm.{c1, c2, c3} - cc[[All, 1]])^2], {{lbc < c1 * 4.1 + c2 * 219 < ubc},

```

```

    {0.02 < c1 < 0.13}, {0 < c2 < 0.006}}}, {c1, c2, c3}];
cval2 = {cval[[2, 1]][[2]], cval[[2, 2]][[2]]};
constsn2 = Append[constsn2, cval2];

, {n1, 1, 2}]

Do[

cc = TSTMFITn3[[All, n1, 1]];
dd = TSTMFITn3[[All, n1, 2, 1]];
ee = TSTMFITn3[[All, n1, 2, 2]];
ff = ConstantArray[1, 155];
mm = Transpose[{dd, ee, ff}];
ss = LeastSquares[mm, cc[[All, 1]]];
constsn3 = Append[constsn3, ss];
Clear[c1, c2, c3];
funcmin[c1_, c2_, c3_] := Total[(mm.{c1, c2, c3} - cc[[All, 1]])^2];
cval = NMinimize[
  {Total[(mm.{c1, c2, c3} - cc[[All, 1]])^2], {{lbc < c1 * 4.1 + c2 * 219 < ubc},
  {0.02 < c1 < 0.13}, {0 < c2 < 0.006}}}, {c1, c2, c3}];
cval2 = {cval[[2, 1]][[2]], cval[[2, 2]][[2]]};
constsn3 = Append[constsn3, cval2];

, {n1, 1, 2}]

constsn1 = Append[constsn1, constsn2[[1]]];
constsn2 = Append[constsn2, constsn2[[2]]];
constsn3 = Append[constsn3, constsn3[[1]]];
constsn4 = Append[constsn4, constsn3[[2]]];
constsn = constsn;
constsn1 = Append[constsn1, constsn2[[1]]];
constsn2 = Append[constsn2, constsn2[[2]]];
constsn3 = Append[constsn3, constsn3[[1]]];
constsn4 = Append[constsn4, constsn3[[2]]];
constsn = constsn;
Clear[f]
impparn = Table[{constsn[[i, 1]], constsn[[i, 2]],  $\tau_1$ ,  $\tau_2$ , Qn[[i]]}, {i, 1, 19}];
Rn =
  Map[#[[5]] * (((#[[1]] * #[[3]]) / ((2 * Pi * #[[3]] * f)^2 + 1) + (#[[4]] * #[[2]]) / ((2 *
    Pi * #[[4]] * f)^2 + 1))^2 + ((2 * Pi * #[[1]] * #[[3]] * #[[3]] * f) /
    ((2 * Pi * #[[3]] * f)^2 + 1) + (2 * Pi * #[[2]] * #[[4]] * #[[4]] * f) /
    ((2 * Pi * #[[4]] * f)^2 + 1))^2)^(0.5) &, impparn];
(*TCRn = Table[TCRn[[i]], {i, {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 16, 18}}];
TCRname = Table[TCRname[[i]], {i, {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 16, 18}}];
Rn = Table[Rn[[i]], {i, {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 16, 18}}];*)
TCRn =
  Table[TCRn[[i]], {i, {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 16, 18, 12, 13, 14, 15, 17, 19}}];
TCRname = Table[TCRname[[i]],
  {i, {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 16, 18, 12, 13, 14, 15, 17, 19}}];
Rn = Table[Rn[[i]], {i, {1, 2, 3, 4, 5, 6, 7, 8, 9, 10,
  11, 16, 18, 12, 13, 14, 15, 17, 19}}];
korTCSn = Table[{(1.4)^(-i), Correlation[TCRn, Rn /. f -> (1.4)^(-i)]}, {i, 1, 27}]
PLTCSn = ListLogLinearPlot[korTCSn, Frame -> True, FrameStyle -> Directive[Black, 18],
  PlotStyle -> {PointSize[0.02], RGBColor[0.3, 0.66, 0.69]};

```

```

PLTCS2n = ListLogLinearPlot[korTCSn, Frame → True, FrameStyle → Directive[Black, 18],
  PlotStyle → RGBColor[0.3, 0.66, 0.69], Joined → True];
Show[{PLTCS1n, PLTCS2n}, FrameLabel → {"Frequency (1/Years)", "Correlation"},
  AspectRatio → 1, Background → None, PlotRange → {{Log[0.0001], Log[1]}, {0.4, 1}},
  PlotLabel → Style["Correlation TCR - Multimodel Mean (Forster)", 20, Black],
  Axes → None, GridLines → Automatic]

```

```
erro = {};
```

```

In[8255]:= f = 0.01;
corRn = MapThread[{{#1, #2} &, {Rn, TCRn}]
Correlation[corRn]
corRn1 = Table[corRn[[i]] → i, {i, 1, 19}];
lincorRn = LinearModelFit[corRn, x, x]
P3n = Plot[lincorRn[t], {t, 0, 2.4},
  PlotStyle → {RGBColor[0.3, 0.7, 0.67], Thickness[0.01], Dashed}];
P4n = ListPlot[corRn1, PlotStyle → Black, LabelStyle → {16, Black}];
impparki =
  Table[{constski[[i, 1]], constski[[i, 2]], τ1, τ2, Mean[Q]}, {i, 1, 10}];
kistat =
  Map[#[[5]] * (((#[[1]] * #[[3]] / ((2 * Pi * #[[3]] * f)^2 + 1) + (#[[4]] * #[[2]]) /
    ((2 * Pi * #[[4]] * f)^2 + 1))^2 + ((2 * Pi * #[[1]] * #[[3]] * #[[3]] * f) /
    ((2 * Pi * #[[3]] * f)^2 + 1) + (2 * Pi * #[[2]] * #[[4]] * #[[4]] * f) /
    ((2 * Pi * #[[4]] * f)^2 + 1))^2)^(0.5)) &, impparki];

s2ki = Variance[kistat];
sigmaI = Sqrt[s2ki]

```

```

In[8266]:= Nn = 19;
s2n = (1 / (Nn - 2)) * Sum[(TCRn[[i]] - lincorRn[Rn[[i]])]^2, {i, 1, 19}]
sx2n = (1 / Nn) * Sum[(Rn[[i]] - Mean[Rn])^2, {i, 1, 19}]

Fstatn = {};
Fstatnc = {};
Do[
  T111[t_] := Sum[fac1[[n1, s]] * Exp[-(t - s) / τ1], {s, 0, t}];
  T222[t_] := Sum[fac1[[n1, s]] * Exp[-(t - s) / τ2], {s, 0, t}];

  annualtemp = globaltemp[[]];
  years = globaltemp[[]];
  T11R = Table[T111[t][[1]], {t, 1, 166}];
  T22R = Table[T222[t][[1]], {t, 1, 166}];
  ccr = Drop[annualtemp, -2];
  ddr = T11R;
  eer = T22R;
  ffr = ConstantArray[1, 166];
  mmm = Transpose[{ddr, eer, ffr}];
  sss = LeastSquares[mmm, ccr];
  Fstatn = Append[Fstatn, sss];
  Clear[c1, c2, c3];
  cval = NMinimize[
    {Total[(mm.{c1, c2, c3} - cc[[All, 1]])^2], {{lbc < c1 * 4.1 + c2 * 219 < ubc},
    {0.02 < c1 < 0.13}, {0 < c2 < 0.006}}}, {c1, c2, c3}];

```

```

cval2 = {cval[[2, 1]][[2]], cval[[2, 2]][[2]]};
Fstatnc = Append[Fstatnc, cval2];

, {n1, 1, 15}]
Do[
  T111[t_] := Sum[fac2[[n1, s + 10]] * Exp[-(t - s) / τ1], {s, 0, t}];
  T222[t_] := Sum[fac2[[n1, s + 10]] * Exp[-(t - s) / τ2], {s, 0, t}];
  annualtemp = globaltemp[[;;, 14]];
  years = globaltemp[[;;, 1]];
  T11R = Table[T111[t], {t, 1, 156}];
  T22R = Table[T222[t], {t, 1, 156}];
  ccr = Drop[annualtemp, -2];
  ccr = Drop[ccr, 10];
  ddr = T11R;
  eer = T22R;
  ffr = ConstantArray[1, 156];
  mmr = Transpose[{ddr, eer, ffr}];
  ssr = LeastSquares[mmr, ccr];
  Fstatn = Append[Fstatn, ssr];
  Clear[c1, c2, c3];
  cval = NMinimize[
    {Total[(mmr.{c1, c2, c3} - cc[[All, 1]])^2], {{lbc < c1 * 4.1 + c2 * 219 < ubc},
      {0.02 < c1 < 0.13}, {0 < c2 < 0.006}}}, {c1, c2, c3}];
  cval2 = {cval[[2, 1]][[2]], cval[[2, 2]][[2]]};
  Fstatnc = Append[Fstatnc, cval2];
, {n1, 1, 2}]

Do[
  T111[t_] := Sum[fac3[[n1, s + 11]] * Exp[-(t - s) / τ1], {s, 0, t}];
  T222[t_] := Sum[fac3[[n1, s + 11]] * Exp[-(t - s) / τ2], {s, 0, t}];
  annualtemp = globaltemp[[;;, 14]];
  years = globaltemp[[;;, 1]];
  T11R = Table[T111[t], {t, 1, 155}];
  T22R = Table[T222[t], {t, 1, 155}];
  ccr = Drop[annualtemp, -2];
  ccr = Drop[ccr, 11];
  ddr = T11R;
  eer = T22R;
  ffr = ConstantArray[1, 155];
  mmr = Transpose[{ddr, eer, ffr}];
  ssr = LeastSquares[mmr, ccr];
  Fstatn = Append[Fstatn, ssr];
  Clear[c1, c2, c3];
  cval = NMinimize[
    {Total[(mmr.{c1, c2, c3} - cc[[All, 1]])^2], {{lbc < c1 * 4.1 + c2 * 219 < ubc},
      {0.02 < c1 < 0.13}, {0 < c2 < 0.006}}}, {c1, c2, c3}];
  cval2 = {cval[[2, 1]][[2]], cval[[2, 2]][[2]]};
  Fstatnc = Append[Fstatnc, cval2];
, {n1, 1, 2}]

impparFn = Table[{Fstatnc[[i, 1]], Fstatnc[[i, 2]], τ1, τ2, Qn[[i]]}, {i, 1, 19}];

```



```

RFn =
  Map[#[[5]] * (((#[[1]] * #[[3]] / ((2 * Pi * #[[3]] * f)^2 + 1) + (#[[4]] * #[[2]]) / ((2 *
    Pi * #[[4]] * f)^2 + 1))^2 + ((2 * Pi * #[[1]] * #[[3]] * #[[3]] * f) /
    ((2 * Pi * #[[3]] * f)^2 + 1) + (2 * Pi * #[[2]] * #[[4]] * #[[4]] * f) /
    ((2 * Pi * #[[4]] * f)^2 + 1))^2)^(0.5)) &, impparFn]

```

```

In[8318]:= muFn = Mean[RFn];
varRFn = Variance[RFn]
sigmatotn = Sqrt[sigmaI^2 + varRFn]
sigmaTCR[xxR_] := s2n * (1 + (1/Nn) + ((xxR - Mean[Rn])^2 / (sx2n * Nn)));
sigmaTCRval = Table[{i, sigmaTCR[i]}, {i, 0, 2, 0.05}];
beta_n = muFn / sigmatotn^2
alpha_n = muFn^2 / sigmatotn^2
RFnstatpl = Table[{RFn[[i]], 0} -> i, {i, 1, 19}];
PRGn = PDF[GammaDistribution[alpha_n, 1/beta_n], y]
eplo = LineLegend[{Blue, Red},
  {Style["Normal", 18], Style["Gamma", 18]}, LegendMarkerSize -> 30];
P8n = ListPlot[RFnstatpl, PlotStyle -> {Red, Thick}, LabelStyle -> {Blue, 16}];
P7n = Plot[PDF[SmoothKernelDistribution[RFn], xy],
  {xy, 0, 2.4}, PlotStyle -> {Red, Thickness[0.005]}];
P6n = Plot[PDF[GammaDistribution[alpha_n, 1/beta_n], y], {y, 0, 2.4},
  PlotStyle -> {Red, Thickness[0.005]}];
P5n = Plot[PDF[NormalDistribution[muFn, sigmatotn], y],
  {y, 0, 2.4}, PlotStyle -> {Blue, Thickness[0.005]}];
Pshu = Plot[{lincorRn[y] - Sqrt[sigmaTCR[y]]}, {y, 0, 2.4},
  PlotStyle -> {RGBColor[0.3, 0.7, 0.68], Dashed}];
Pshd = Plot[{lincorRn[y] + Sqrt[sigmaTCR[y]]}, {y, 0, 2.4},
  PlotStyle -> {RGBColor[0.3, 0.7, 0.68], Dashed}];
Show[P3n, P4n, P5n, P6n, P8n, Pshu, Pshd, PlotLabel -> Style["", 22, Black],
  Frame -> True, FrameLabel -> {"R(f=0.01) [°C]", "TCR [°C]"},
  LabelStyle -> {20, Black}, PlotRange -> All, GridLines -> Automatic,
  Epilog -> {Inset[eplo, Scaled[{0.2, 0.8}]]}, Axes -> None]
PTCRgn = PDF[NormalDistribution[lincorRn[y], sigmaTCR[y]], sl]
PRn = PDF[NormalDistribution[muFn, sigmatotn], y]

```

```
PTCRn[s1_] := NIntegrate[
$$\left( \left( 8.31122834323857 \cdot e^{-\frac{217.01027700273085 \cdot (-0.598712477045074 \cdot s1 - 0.9072037451373964 \cdot y)^2}{\left(\frac{20}{19} + 0.43011908801045257 \cdot (-1.3299666004651804 \cdot y)^2\right)^2}} \right) / \right. \\ \left. \left( \frac{20}{19} + 0.43011908801045257 \cdot (-1.3299666004651804 \cdot y)^2 \right) \right) * \\ (660.2135944213335 \cdot e^{-6.479179215630901 \cdot y} y^{5.645497406023648}), \{y, 0, \text{Infinity}\}]$$

```

```
PTCRvaln = Table[{i, PTCRn[i]}, {i, 0, 3, 0.01}];
CDF[PTCRvaln, xyz];
lpl = Table[{TCRn[[i]], 0}, {i, 1, Length[TCRn]}]
P1 = ListPlot[PTCRvaln, Joined → True, PlotStyle → {Red, Thickness[0.01]}];
P2 =
  Plot[PDF[NormalDistribution[Mean[TCRn], Sqrt[Variance[TCRn]]], xtcs], {xtcs, 0, 3}];
P3 = ListPlot[lpl, PlotStyle → {PointSize[0.02], Red}];
P4 = Histogram[TCRn, {0.2}, "PDF", ChartStyle → RGBColor[0.3, 0.7, 0.67]];
Show[P4, P1, PlotLabel → Style["PDF vs TCR - Normal Distribution", 22, Black],
  Frame → True, FrameLabel → {"TCR [°C]", "Probability density [1/°C]"},
  LabelStyle → {20, Black}, PlotRange → All, GridLines → Automatic, Axes → None]
Folist = FoldList[Plus, 0, PTCRvaln[[All, 2]]] * 0.01;
Fotabl = Table[{0.01 * i, Folist[[i]]}, {i, 0, 300}];
len = Length[Folist];
alp = 0.90;
num1 = (1 - alp) / 2;
num2 = (1 + alp) / 2;
num3 = 0.5;
CIL1 = Pick[Folist, UnitStep[Folist - num1], 1];
CIH1 = Pick[Folist, UnitStep[Folist - num2], 1];
CIM1 = Pick[Folist, UnitStep[Folist - num3], 1];

low = Fotabl[[len - Length[CIL1] + 2, 1]];
high = Fotabl[[len - Length[CIH1] + 2, 1]];
min = Fotabl[[len - Length[CIM1] + 2, 1]];
aperro = {f, min, high, low}

erro = Append[erro, aperro]
```

```
In[7503]:= Needs["ErrorBarPlots`"]
```

```

In[6879]:= stde = Sqrt[Variance[TCRn]];
pe1 = {{0.5, Mean[TCRn]}, {5.5, Mean[TCRn]}};
pe2 = {{0.5, 2.5}, {5.5, 2.5}};
pe3 = {{0.5, 1.1}, {5.5, 1.1}};
anotcr = Table[{0.5, TCRn[[i]]}, {i, 1, 13}]
PE5 = ListPlot[anotcr, PlotStyle -> {Black, Thick}];
erropl = Table[{erro[[i, 2]], (erro[[i, 3]] - erro[[i, 4]])/2}, {i, 1, 5}]
erropl2 = Table[{erro[[1]], erro[[i, 2]], (erro[[i, 3]] - erro[[i, 4]])}, {i, 1, 5}]
PE1 =
  ErrorListPlot[erropl, PlotRange -> {0, 3}, Joined -> True, PlotStyle -> {Thick, Red}];
Perro = ErrorListPlot[{{1/erro[[1, 1]], erro[[1, 2]]},
  ErrorBar[(erro[[1, 3]] - erro[[1, 4]])],
  {{1/erro[[2, 1]], erro[[2, 2]]}, ErrorBar[(erro[[2, 3]] - erro[[2, 4]])}]]];
PE2 = ListPlot[pe1, Joined -> True, PlotStyle -> {Black, Thick}];
PE3 = ListPlot[pe2, Joined -> True, PlotStyle -> {Black, Dashed}];
PE4 = ListPlot[pe3, Joined -> True, PlotStyle -> {Black, Dashed}];
Show[PE1, PE2, PE3, PE4,
  FrameTicks -> {{All, {1, "10"}, {2, "25"}, {3, 100}, {4, 250}, {5, 1000}}, Automatic},
  Axes -> False, Frame -> True, PlotRange -> {All, {0.5, 3}},
  FrameLabel -> {"Period [Years]", "TCR [°C] "}, LabelStyle -> {20, Black}]

func[x_] := Piecewise[{{280 * Exp[0.01 * x], x ≤ 70}, {564, x > 70}}]
Pq1 = Plot[{func[x]}, {x, 0, 100},
  PlotStyle -> {Thickness[0.01], RGBColor[0.3, 0.67, 0.69]},
  GridLines -> {{60, 80}, {0, 0}}, GridLinesStyle -> Directive[{Thick, Dashed}]];
Pq2 = Plot[563, {x, 0, 100}, PlotStyle ->
  {Thickness[0.005], Dashed, RGBColor[0.3, 0.67, 0.69]},
  GridLines -> {{80}, {0}}, GridLinesStyle -> {Thick, Dashed}];
Show[Pq1, Pq2, PlotLabel -> Style["CO2 Scenario - TCR", 22, Black],
  Frame -> True, FrameLabel -> {"Year", "CO2 Concentration (ppm)"},
  LabelStyle -> {20, Black}, PlotRange -> All]

scatsens = Table[{TCRn[[i]], ECSn[[i]]}, {i, 1, 18}]
Corre
ListPlot[scatsens, PlotRange -> {{0.8, 3}, {1.8, 5}},
  PlotStyle -> {PointSize[0.03], RGBColor[0.3, 0.67, 0.69]},
  PlotLabel -> Style["ECS & TCR - IPCC AR5", 22, Black],
  Frame -> True, FrameLabel -> {"TCR [°C]", "ECS [°C] "},
  LabelStyle -> {20, Black}, PlotRange -> All, GridLines -> Automatic]

In[5565]:= gg = LineLegend[{RGBColor[0.3, 0.67, 0.69], Blue, Red},
  {Style["NorESM1-M Temperature Anomaly", 18], Style["OBM", 18], Style["TBM", 18]},
  LegendMarkerSize -> 30]
ListPlot[{ccp, T1p, T2p}, Joined -> True,
  PlotLabel -> Style["Temperature Fit ", 22, Black], Frame -> True,
  FrameLabel -> {"Year After 1850", "Temperature Anomaly [°C] "},
  LabelStyle -> {20, Black}, PlotRange -> All, GridLines -> Automatic,
  PlotStyle -> {{Thickness[0.003], RGBColor[0.3, 0.67, 0.69]}, {Thickness[0.003], Red},
  {Thickness[0.003], Blue}}, Epilog -> {Inset[gg, Scaled[{0.28, 0.8}]]}]

```

```

In[5570]:= ListPlot[dd, Joined → True, PlotStyle → {Thickness[0.007], RGBColor[0.3, 0.67, 0.69]},
  PlotLabel → Style["Folded Forcing & Fast Exponential ", 22, Black],
  Frame → True, FrameLabel → {"Year After 1850", "[yr W/m²]"},
  LabelStyle → {20, Black}, PlotRange → All, GridLines → Automatic]
ListPlot[ee, Joined → True, PlotStyle → {Thickness[0.007], RGBColor[0.3, 0.67, 0.69]},
  PlotLabel → Style["Folded Forcing & Slow Exponential ", 22, Black],
  Frame → True, FrameLabel → {"Year After 1850", "[yr W/m²]"},
  LabelStyle → {20, Black}, PlotRange → All, GridLines → Automatic]
ListPlot[fac1[[11]], Joined → True,
  PlotStyle → {Thickness[0.007], RGBColor[0.3, 0.67, 0.69]},
  PlotLabel → Style["NorESM1-M Adjusted Forcing ", 22, Black],
  Frame → True, FrameLabel → {"Year After 1850", "Forcing [yr W/m²]"},
  LabelStyle → {20, Black}, PlotRange → All, GridLines → Automatic]

TCRn = Table[AR5new[[i, 4]], {i, 1, 20}];
TCRn = Drop[TCRn, {2}]
TCRn =
  Table[TCRn[[i]], {i, {1, 2, 3, 4, 5, 7, 9, 10, 11, 12, 13, 14, 15, 18, 19, 8, 16, 6, 17}}]
TCRname = {"BCC-CSM1-1", "CanESM2", "CCSM4", "CNRM-CM5",
  "CSIRO-M k3.6.0", "GISS-E2-R", "IPSL-CM5A-LR", "MIROC5", "MPI-ESM-LR",
  "MRI-CGCM3", "NorESM1-M", "Acess10", "bcc-csm1-1m", "inm-cm4",
  "miroc-esm", "HadGEM2-ES", "gfdl-cm3", "GFDL-ESM2M", "gfdl-esm2g"}
Qn = Table[AR5new[[i, 2]], {i, 1, 20}];
Qn = Drop[Qn, {2}];
Qn =
  Table[Qn[[i]], {i, {1, 2, 3, 4, 5, 7, 9, 10, 11, 12, 13, 14, 15, 18, 19, 8, 16, 6, 17}}]
lbc = 0.2;
ubc = 2;
constsn1 = {};
constsn2 = {};
constsn3 = {};
constsn = {};
constsn1 = Append[constsn1, ss];
Clear[c1, c2, c3];
funcmin[c1_, c2_] := Total[(mm.{c1, c2} - cc[[All, 1]])^2];
cval = NMinimize[{Total[(mm.{c1, c2} - cc[[All, 1]])^2],
  {{lbc < c1 * 4.1 < ubc}, {c1 > 0}}, {c1, c2}];
cval2 = {cval[[2, 1]][[2]], cval[[2, 2]][[2]]};
constsn1 = Append[constsn1, cval2];

, {n, 1, 15}]

```

```

Do[

cc = TSTMFITn2[[All, n, 1]];
dd = TSTMFITn2[[All, n, 2, 1]];
ee = TSTMFITn2[[All, n, 2, 2]];
ff = ConstantArray[1, 156];
mm = Transpose[{dd, ff}];
ss = LeastSquares[mm, cc[[All, 1]]];
constsn2 = Append[constsn2, ss];
Clear[c1, c2, c3];
funcmin[c1_, c2_, c3_] := Total[(mm.{c1, c2, c3} - cc[[All, 1]])^2];
cval = NMinimize[{Total[(mm.{c1, c2} - cc[[All, 1]])^2],
  {{lbc < c1 * 4.1 < ubc}, {c1 > 0}}, {c1, c2}];
cval2 = {cval[[2, 1]][[2]], cval[[2, 2]][[2]]};
constsn2 = Append[constsn2, cval2];

, {n, 1, 2}]

```

```

Do[

cc = TSTMFITn3[[All, n, 1]];
dd = TSTMFITn3[[All, n, 2, 1]];
ee = TSTMFITn3[[All, n, 2, 2]];
ff = ConstantArray[1, 155];
mm = Transpose[{dd, ff}];
ss = LeastSquares[mm, cc[[All, 1]]];
constsn3 = Append[constsn3, ss];
Clear[c1, c2, c3];
funcmin[c1_, c2_, c3_] := Total[(mm.{c1, c2, c3} - cc[[All, 1]])^2];
cval = NMinimize[{Total[(mm.{c1, c2} - cc[[All, 1]])^2],
  {{lbc < c1 * 4.1 < ubc}, {c1 > 0}}, {c1, c2}];
cval2 = {cval[[2, 1]][[2]], cval[[2, 2]][[2]]};
constsn3 = Append[constsn3, cval2]

, {n, 1, 2}]

```

In[6721]:=

```

constsns = Append[constsn1, constsn2[[1]]];
constsns = Append[constsns, constsn2[[2]]];
constsns = Append[constsns, constsn3[[1]]];
constsns = Append[constsns, constsn3[[2]]];
constsn = constsns

```

```

In[6726]:= Clear[f]
impparn = Table[{constsn[[i, 1]], constsn[[i, 2]],  $\tau_1$ ,  $\tau_2$ , Qn[[i]]}, {i, 1, 19}];
Rn = Map[#[[5]] * ((#[[1]] * #[[3]]) / ((2 * Pi * #[[3]] * f)^2 + 1)) &, impparn];
TCRn =
  Table[TCRn[[i]], {i, {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 16, 18, 12, 13, 14, 15, 17, 19}}];
TCRname = Table[TCRname[[i]],
  {i, {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 16, 18, 12, 13, 14, 15, 17, 19}}];
Qn = Table[Qn[[i]], {i, {1, 2, 3, 4, 5, 6, 7, 8, 9, 10,
  11, 16, 18, 12, 13, 14, 15, 17, 19}}];
Rn = Table[Rn[[i]], {i, {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11,
  16, 18, 12, 13, 14, 15, 17, 19}}];
korTCSn = Table[{(1.4)^(-i), Correlation[TCRn, Rn /. f -> (1.4)^(-i)]}, {i, 1, 27}]
PLTCS1n = ListLogLinearPlot[korTCSn, Frame -> True, FrameStyle -> Directive[Black, 18],
  PlotStyle -> {PointSize[0.02], RGBColor[0.3, 0.66, 0.69]};
PLTCS2n = ListLogLinearPlot[korTCSn, Frame -> True, FrameStyle -> Directive[Black, 18],
  PlotStyle -> RGBColor[0.3, 0.66, 0.69], Joined -> True];
Show[{PLTCS1n, PLTCS2n}, FrameLabel -> {"Frequency [1/Years]", "Correlation"},
  AspectRatio -> 1, Background -> None, PlotRange -> {{Log[0.0001], Log[1]}, {0.6, 1}},
  PlotLabel -> Style[" Correlation TCR - OBM", 20, Black],
  GridLines -> Automatic, Axes -> None]

In[6737]:= f = 0.01;
TCRn1 = Table[TCRn[[i]] -> i, {i, 1, 19}];
corRn = MapThread[{#1, #2} &, {Rn, TCRn}];
Correlation[corRn]
corRn1 = Table[corRn[[i]] -> i, {i, 1, 19}];
lincorRn = LinearModelFit[corRn, x, x]
P3n = Plot[lincorRn[t], {t, 0, 2.4},
  PlotStyle -> {RGBColor[0.3, 0.7, 0.683], Thickness[0.01]};
P4n = ListPlot[corRn1, PlotStyle -> Black, LabelStyle -> {16, Black}];
impparki =
  Table[{constski[[i, 1]], constski[[i, 2]],  $\tau_1$ ,  $\tau_2$ , Mean[Qn]}, {i, 1, 10}];
kistat =
  Map[#[[5]] * (((#[[1]] * #[[3]]) / ((2 * Pi * #[[3]] * f)^2 + 1) + (#[[4]] * #[[2]]) /
    ((2 * Pi * #[[4]] * f)^2 + 1))^2 + ((2 * Pi * #[[1]] * #[[3]] * #[[3]] * f) /
    ((2 * Pi * #[[3]] * f)^2 + 1) + (2 * Pi * #[[2]] * #[[4]] * #[[4]] * f) /
    ((2 * Pi * #[[4]] * f)^2 + 1))^2)^(0.5)) &, impparki];

s2ki = Variance[kistat];
sigmaI = Sqrt[s2ki];
Nn = 19
s2n = (1 / (Nn - 2)) * Sum[(TCRn[[i]] - lincorRn[Rn[[i]])]^2, {i, 1, 19}]
sx2n = (1 / Nn) * Sum[(Rn[[i]] - Mean[Rn])^2, {i, 1, 19}]

Fstatn = {};
Fstatnc = {};
Do[
  T111[t_] := Sum[fac1[[n1, s]] * Exp[-(t - s) /  $\tau_1$ ], {s, 0, t}];
  T222[t_] := Sum[fac1[[n1, s]] * Exp[-(t - s) /  $\tau_2$ ], {s, 0, t}];

  annualtemp = globaltemp[[ ; ; , 14]];
  years = globaltemp[[ ; ; , 1]];
  T11R = Table[T111[t][[1]], {t, 1, 166}];
  T22R = Table[T222[t][[1]], {t, 1, 166}];
  ccr = Drop[annualtemp, -2];

```

```

ddr = T11R;
eer = T22R;
ffr = ConstantArray[1, 166];
mmr = Transpose[{ddr, ffr}];
ssr = LeastSquares[mmr, ccr];
Fstatn = Append[Fstatn, ssr];
Clear[c1, c2, c3];
cval = NMinimize[{Total[(mm.{c1, c2} - cc[[All, 1]])^2],
  {{lbc < c1 * 4.1 < ubc}, {c1 > 0}}}, {c1, c2}];
cval2 = {cval[[2, 1]][[2]], cval[[2, 2]][[2]]};
Fstatnc = Append[Fstatnc, cval2];

, {n1, 1, 15}]
Do[
T111[t_] := Sum[fac2[[n1, s + 10]] * Exp[-(t - s) / τ1], {s, 0, t}];
T222[t_] := Sum[fac2[[n1, s + 10]] * Exp[-(t - s) / τ2], {s, 0, t}];
annualtemp = globaltemp[[ ; ; , 14]];
years = globaltemp[[ ; ; , 1]];
T11R = Table[T111[t], {t, 1, 156}];
T22R = Table[T222[t], {t, 1, 156}];
ccr = Drop[annualtemp, -2];
ccr = Drop[ccr, 10];
ddr = T11R;
eer = T22R;
ffr = ConstantArray[1, 156];
mmr = Transpose[{ddr, ffr}];
ssr = LeastSquares[mmr, ccr];
Fstatn = Append[Fstatn, ssr];
Clear[c1, c2, c3];
cval = NMinimize[{Total[(mm.{c1, c2} - cc[[All, 1]])^2],
  {{lbc < c1 * 4.1 < ubc}, {c1 > 0}}}, {c1, c2}];
cval2 = {cval[[2, 1]][[2]], cval[[2, 2]][[2]]};
Fstatnc = Append[Fstatnc, cval2];
, {n1, 1, 2}]

Do[
T111[t_] := Sum[fac3[[n1, s + 11]] * Exp[-(t - s) / τ1], {s, 0, t}];
T222[t_] := Sum[fac3[[n1, s + 11]] * Exp[-(t - s) / τ2], {s, 0, t}];
annualtemp = globaltemp[[ ; ; , 14]];
years = globaltemp[[ ; ; , 1]];
T11R = Table[T111[t], {t, 1, 155}];
T22R = Table[T222[t], {t, 1, 155}];
ccr = Drop[annualtemp, -2];
ccr = Drop[ccr, 11];
ddr = T11R;
eer = T22R;
ffr = ConstantArray[1, 155];
mmr = Transpose[{ddr, ffr}];
ssr = LeastSquares[mmr, ccr];
Fstatn = Append[Fstatn, ssr];
Clear[c1, c2, c3];
cval = NMinimize[{Total[(mm.{c1, c2} - cc[[All, 1]])^2],
  {{lbc < c1 * 4.1 < ubc}, {c1 > 0}}}, {c1, c2}];

```

```

cval2 = {cval[[2, 1]][[2]], cval[[2, 2]][[2]]};
Fstatnc = Append[Fstatnc, cval2];
, {n1, 1, 2}]

impparFn = Table[{Fstatnc[[i, 1]], Fstatnc[[i, 2]],  $\tau_1$ ,  $\tau_2$ , Qn[[i]]}, {i, 1, 19}];
RFn = Map[#[[5]] * ((#[[1]] * #[[3]]) / ((2 * Pi * #[[3]] * f)^2 + 1)) &, impparFn]
muFn = Mean[RFn];
varRFn = Variance[RFn]
sigmatotn = Sqrt[sigmaI^2 + varRFn]
sigmaTCR[xxR_] := s2n * (1 + (1/Nn) + ((xxR - Mean[Rn])^2 / (sx2n * Nn)));
sigmaTCRval = Table[{i, sigmaTCR[i]}, {i, 0, 2, 0.05}];
 $\beta_n$  = muFn / sigmatotn^2
 $\alpha_n$  = muFn^2 / sigmatotn^2
RFnstatpl = Table[{RFn[[i]], 0} → i, {i, 1, 19}];
PRGn = PDF[GammaDistribution[ $\alpha_n$ , 1/ $\beta_n$ ], y]
eplo = LineLegend[{Blue, Red},
  {Style["Normal", 18], Style["Gamma", 18]}, LegendMarkerSize → 30];
P8n = ListPlot[RFnstatpl, PlotStyle → {Red, PointSize[0.01]},
  LabelStyle → {Blue, 12}];
P7n = Plot[PDF[SmoothKernelDistribution[RFn], xy], {xy, 0, 2.4},
  PlotStyle → {Red, Thickness[0.005]}];
P6n = Plot[PDF[GammaDistribution[ $\alpha_n$ , 1/ $\beta_n$ ], y], {y, 0, 2.4},
  PlotStyle → {Red, Thickness[0.005]}];
P5n = Plot[PDF[NormalDistribution[muFn, sigmatotn], y],
  {y, 0, 2.4}, PlotStyle → {Blue, Thickness[0.005]}];
Pshu = Plot[{lincorRn[y] - Sqrt[sigmaTCR[y]]}, {y, 0, 2.4},
  PlotStyle → {RGBColor[0.3, 0.7, 0.68], Dashed}];
Pshd = Plot[{lincorRn[y] + Sqrt[sigmaTCR[y]]}, {y, 0, 2.4},
  PlotStyle → {RGBColor[0.3, 0.7, 0.68], Dashed}];
Show[P3n, P4n, P5n, P6n, P8n, Pshu, Pshd, PlotLabel → Style["", 22, Black],
  Frame → True, FrameLabel → {"R(f=0.01)", "TCR [°C]"},
  LabelStyle → {20, Black}, PlotRange → All, GridLines → Automatic,
  Axes → None, Epilog → {Inset[eplo, Scaled[{0.2, 0.8}]]}]
PTCRgn = PDF[NormalDistribution[lincorRn[y], sigmaTCR[y]], s1]
PRn = PDF[NormalDistribution[muFn, sigmatotn], y]

```



```

In[8383]:= TST01n1[t_] :=
  Table[Sum[fac1[[m, s]] * Exp[-(t - s) / (tau1[[m]])], {s, 0, t}], {m, 1, 15}];
TST02n1[t_] := Table[Sum[fac1[[m, s]] * Exp[-(t - s) / (tau2[[m]])], {s, 0, t}],
  {m, 1, 15}];
TST1n1 = Table[TST01n1[m][[i]][[1]], {m, 1, 166}, {i, 1, 15}];
TST2n1 = Table[TST02n1[m][[i]][[1]], {m, 1, 166}, {i, 1, 15}];
TST01n2[t_] :=
  Table[Sum[fac2[[m, s + 10]] * Exp[-(t - s) / (tau1[[15 + m]])], {s, 0, t}], {m, 1, 2}];
TST02n2[t_] := Table[Sum[fac2[[m, s + 10]] * Exp[-(t - s) / (tau2[[15 + m]])], {s, 0, t}],
  {m, 1, 2}];
TST1n2 = Table[TST01n2[m][[i]], {m, 1, 156}, {i, 1, 2}];
TST2n2 = Table[TST02n2[m][[i]], {m, 1, 156}, {i, 1, 2}];
TST01n3[t_] :=
  Table[Sum[fac3[[m, s + 11]] * Exp[-(t - s) / (tau1[[17 + m]])], {s, 0, t}], {m, 1, 2}];
TST02n3[t_] := Table[Sum[fac3[[m, s + 11]] * Exp[-(t - s) / (tau2[[17 + m]])], {s, 0, t}],
  {m, 1, 2}];
TST1n3 = Table[TST01n3[m][[i]], {m, 1, 155}, {i, 1, 2}];
TST2n3 = Table[TST02n3[m][[i]], {m, 1, 155}, {i, 1, 2}];
TSTM1 = Table[{Tmdmatn1[[i, 2, m, 2]]}, {m, 1, 166}, {i, 1, 15}];
TSTM2 = Table[{Tmdmatn2[[i, 2, m, 2]]}, {m, 1, 156}, {i, 1, 2}];
TSTM3 = Table[{Tmdmatn3[[i, 2, m, 2]]}, {m, 1, 155}, {i, 1, 2}];
TSTMFITn1 =
  Table[{TSTM1[[m, i]], {TST1n1[[m, i]], TST2n1[[m, i]]}}, {m, 1, 166}, {i, 1, 15}];
TSTMFITn2 = Table[{TSTM2[[m, i]], {TST1n2[[m, i]], TST2n2[[m, i]]}},
  {m, 1, 156}, {i, 1, 2}];
TSTMFITn3 = Table[{TSTM3[[m, i]], {TST1n3[[m, i]], TST2n3[[m, i]]}},
  {m, 1, 155}, {i, 1, 2}];

In[8401]:= TCRn = Table[AR5new[[i, 4]], {i, 1, 20}];
TCRn = Drop[TCRn, {2}];
TCRn =
  Table[TCRn[[i]], {i, {1, 2, 3, 4, 5, 7, 9, 10, 11, 12, 13, 14, 15, 18, 19, 8, 16, 6, 17}}];
TCRname = {"BCC-CSM1-1", "CanESM2", "CCSM4", "CNRM-CM5",
  "CSIRO-M k3.6.0", "GISS-E2-R", "IPSL-CM5A-LR", "MIROC5", "MPI-ESM-LR",
  "MRI-CGCM3", "NorESM1-M", "Acess10", "bcc-csm1-1m", "inm-cm4",
  "miroc-esm", "HadGEM2-ES", "gfdl-cm3", "GFDL-ESM2M", "gfdl-esm2g"};
Qn = Table[AR5new[[i, 2]], {i, 1, 20}];
Qn = Drop[Qn, {2}];
Qn =
  Table[Qn[[i]], {i, {1, 2, 3, 4, 5, 7, 9, 10, 11, 12, 13, 14, 15, 18, 19, 8, 16, 6, 17}}];
lbc = 0.2;
ubc = 2;
constsn1 = {};
constsn2 = {};
constsn3 = {};
constsn = {};
constsnt = {};
constsncl = {};
constsncl2 = {};
constsncl3 = {};
Do[
  Clear[c1, c2, c3];
  cc = TSTMFITn1[[All, n, 1]];
  dd = TSTMFITn1[[All, n, 2, 1]];

```

```

ee = TSTMFITn1[[All, n, 2, 2]];
ff = ConstantArray[1, 166];
mm = Transpose[{dd, ee, ff}];
ss = LeastSquares[mm, cc[[All, 1]]];
constsn1 = Append[constsn1, ss];
Clear[c1, c2, c3];
funcmin[c1_, c2_, c3_] := Total[(mm.{c1, c2, c3} - cc[[All, 1]])^2];
cval = NMinimize[
  {Total[(mm.{c1, c2, c3} - cc[[All, 1]])^2], {{lbc < c1 * 4.1 + c2 * 219 < ubc},
    {0.02 < c1 < 0.2}, {0 < c2 < 0.006}}}, {c1, c2, c3}];
cval2 = {cval[[2, 1]][[2]], cval[[2, 2]][[2]], cval[[2, 3]][[2]]};
constsn1 = Append[constsn1, cval2];

, {n, 1, 15}]

Do[

cc = TSTMFITn2[[All, n, 1]];
dd = TSTMFITn2[[All, n, 2, 1]];
ee = TSTMFITn2[[All, n, 2, 2]];
ff = ConstantArray[1, 156];
mm = Transpose[{dd, ee, ff}];
ss = LeastSquares[mm, cc[[All, 1]]];
constsn2 = Append[constsn2, ss];
Clear[c1, c2, c3];
funcmin[c1_, c2_, c3_] := Total[(mm.{c1, c2, c3} - cc[[All, 1]])^2];
cval = NMinimize[
  {Total[(mm.{c1, c2, c3} - cc[[All, 1]])^2], {{lbc < c1 * 4.1 + c2 * 219 < ubc},
    {0.02 < c1 < 0.2}, {0 < c2 < 0.006}}}, {c1, c2, c3}];
cval2 = {cval[[2, 1]][[2]], cval[[2, 2]][[2]], cval[[2, 3]][[2]]};
constsn2 = Append[constsn2, cval2];

, {n, 1, 2}]

Do[

cc = TSTMFITn3[[All, n, 1]];
dd = TSTMFITn3[[All, n, 2, 1]];
ee = TSTMFITn3[[All, n, 2, 2]];
ff = ConstantArray[1, 155];
mm = Transpose[{dd, ee, ff}];
ss = LeastSquares[mm, cc[[All, 1]]];
constsn3 = Append[constsn3, ss];
Clear[c1, c2, c3];
funcmin[c1_, c2_, c3_] := Total[(mm.{c1, c2, c3} - cc[[All, 1]])^2];
cval = NMinimize[
  {Total[(mm.{c1, c2, c3} - cc[[All, 1]])^2], {{lbc < c1 * 4.1 + c2 * 219 < ubc},
    {0.02 < c1 < 0.2}, {0 < c2 < 0.006}}}, {c1, c2, c3}];
cval2 = {cval[[2, 1]][[2]], cval[[2, 2]][[2]], cval[[2, 3]][[2]]};
constsn3 = Append[constsn3, cval2];

, {n, 1, 2}]

constsnt = Append[constsn1, constsn2[[1]]];

```

```

constsn2 = Append[constsn2, constsn2[[2]];
constsn3 = Append[constsn3, constsn3[[1]];
constsn3 = Append[constsn3, constsn3[[2]];
constsn = constsn2;
constsn1 = Append[constsn1, constsn2[[1]];
constsn2 = Append[constsn2, constsn2[[2]];
constsn3 = Append[constsn3, constsn3[[1]];
constsn3 = Append[constsn3, constsn3[[2]];
constsn1 = constsn1;
Clear[f, t]
impparn = Table[
  {constsn1[[i, 1]], constsn1[[i, 2]],  $\tau_1$ [[i]],  $\tau_2$ [[i]], Qn[[i]]}, {i, 1, 19}];
Rn = Map[#[[5]] * (((#[[1]] * #[[3]]) / ((2 * Pi * #[[3]] * f)^2 + 1) +
  (#[[4]] * #[[2]]) / ((2 * Pi * #[[4]] * f)^2 + 1))^2 +
  ((2 * Pi * #[[1]] * #[[3]] * #[[3]] * f) / ((2 * Pi * #[[3]] * f)^2 + 1) +
  (2 * Pi * #[[2]] * #[[4]] * #[[4]] * f) /
  ((2 * Pi * #[[4]] * f)^2 + 1))^2)^(0.5) &, impparn];
Rnsj = Map[#[[5]] * (#[[1]] * Exp[-t/#[[3]]) + #[[2]] * Exp[-t/#[[4]])] &,
  impparn]
TCRn = Table[TCRn[[i]], {i, {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 16, 18}}];
TCRname = Table[TCRname[[i]], {i, {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 16, 18}}];
Rn = Table[Rn[[i]], {i, {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 16, 18}}];
Rnsj = Table[Rnsj[[i]], {i, {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 16, 18}}];
Qn = Table[Qn[[i]], {i, {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 16, 18}}];
(*TCRn = Table[TCRn[[i]], {i, {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 16, 18, 12, 13, 14, 15, 17, 19}}];
TCRname = Table[TCRname[[i]], {i, {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 16, 18, 12, 13, 14, 15, 17, 19}}];
Qn = Table[Qn[[i]], {i, {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 16, 18, 12, 13, 14, 15, 17, 19}}];
Rn = Table[Rn[[i]], {i, {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 16, 18, 12, 13, 14, 15, 17, 19}}];*)
korTCSn = Table[{(1.4)^(-i), Correlation[TCRn, Rn /. f -> (1.4)^(-i)]}, {i, 1, 27}]
korTCSnsj = Table[{i, Correlation[TCRn, Rnsj /. t -> i]}, {i, 0, 3, 0.1}]
PLTCS1n = ListLogLinearPlot[korTCSn, Frame -> True, FrameStyle -> Directive[Black, 18],
  PlotStyle -> {PointSize[0.02], RGBColor[0.3, 0.66, 0.69]};
PLTCS2n = ListLogLinearPlot[korTCSn, Frame -> True, FrameStyle -> Directive[Black, 18],
  PlotStyle -> RGBColor[0.3, 0.66, 0.69], Joined -> True];
Show[{PLTCS1n, PLTCS2n}, FrameLabel -> {"Frequency (1/Years)", "Correlation"},
  AspectRatio -> 1, Background -> None, PlotRange -> {{Log[0.0001], Log[1]}, {0.4, 1}},
  PlotLabel -> Style["Correlation TCR - Specific Parameters (Forster)", 20, Black],
  GridLines -> Automatic, Axes -> None]

```

In[8445]=

```

f = 0.01;
TCRn1 = Table[TCRn[[i]] -> i, {i, 1, 13}];
corRn = MapThread[{#1, #2} &, {Rn, TCRn1}];
Correlation[corRn]
corRn1 = Table[corRn[[i]] -> i, {i, 1, 13}];
lincorRn = LinearModelFit[corRn1, x, x]
P3n = Plot[lincorRn[t], {t, 0, 2.4},
  PlotStyle -> {RGBColor[0.3, 0.7, 0.683], Thickness[0.01]};
P4n = ListPlot[corRn1, PlotStyle -> Black, LabelStyle -> {16, Black}];
impparki =
  Table[{constski[[i, 1]], constski[[i, 2]],  $\tau_1$ ,  $\tau_2$ , Mean[Qn]}, {i, 1, 10}];
kistat =
  Map[#[[5]] * (((#[[1]] * #[[3]]) / ((2 * Pi * #[[3]] * f)^2 + 1) + (#[[4]] * #[[2]]) /
  ((2 * Pi * #[[4]] * f)^2 + 1))^2 + ((2 * Pi * #[[1]] * #[[3]] * #[[3]] * f) /
  ((2 * Pi * #[[3]] * f)^2 + 1) + (2 * Pi * #[[2]] * #[[4]] * #[[4]] * f) /

```

```
((2 * Pi * #[[4]] * f)^2 + 1)^2 ^ (0.5) &, impparki];
```

```
s2ki = Variance[kistat];
```

```
sigmaI = Sqrt[s2ki];
```

```
Nn = 13
```

```
s2n = (1 / (Nn - 2)) * Sum[(TCRn[[i]] - lincorRn[Rn[[i]])]^2, {i, 1, 13}]
```

```
sx2n = (1 / Nn) * Sum[(Rn[[i]] - Mean[Rn])^2, {i, 1, 13}]
```

```
forcallg = Table[fac1[[i]], {i, {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}}];
```

```
Fstatncg = {};
```

```
forcallg = Append[forcallg, fac2[[1]]];
```

```
forcallg = Append[forcallg, fac3[[1]]];
```

```
Length[forcallg]
```

```
Do[
```

```
T111[t_] := Sum[forcallg[[n1, s]] * Exp[-(t - s) / tn11[[n1]]], {s, 0, t}];
```

```
T222[t_] := Sum[forcallg[[n1, s]] * Exp[-(t - s) / tn22[[n1]]], {s, 0, t}];
```

```
annualtemp = globaltemp[[ ; ; , 14]];
```

```
years = globaltemp[[ ; ; , 1]];
```

```
T11R = Table[T111[t][[1]], {t, 1, 166}];
```

```
T22R = Table[T222[t][[1]], {t, 1, 166}];
```

```
ccr = Drop[annualtemp, -2];
```

```
ddr = T11R;
```

```
eer = T22R;
```

```
ffr = ConstantArray[1, 166];
```

```
mnr = Transpose[{ddr, eer, ffr}];
```

```
ssr = LeastSquares[mnr, ccr];
```

```
Fstatn = Append[Fstatn, ssr];
```

```
Clear[c1, c2, c3];
```

```
cval =
```

```
FindMinimum[{Total[(mnr.{c1, c2, c3} - ccr)^2], {{1bc < c1 * 4.1 + c2 * 219 < ubc},
{0.02 < c1 < 0.13}, {0 < c2 < 0.006}}}, {{c1, 0.1}, {c2, 0.01}, {c3, 0}}];
```

```
cval2 = {cval[[2, 1]][[2]], cval[[2, 2]][[2]]};
```

```
Fstatncg = Append[Fstatncg, cval2];
```

```
, {n1, 1, 13}]
```

```
In[5043]:=
```

```
Fstatn = {};
```

```
Fstatnc = {};
```

```
Qn = Table[AR5new[[i, 2]], {i, 1, 20}];
```

```
Qn = Drop[Qn, {2}];
```

```
Qn =
```

```
Table[Qn[[i]], {i, {1, 2, 3, 4, 5, 7, 9, 10, 11, 12, 13, 14, 15, 18, 19, 8, 16, 6, 17}}]
```

```
Do[
```

```
T111[t_] := Sum[fac1[[n1, s]] * Exp[-(t - s) / tau1], {s, 0, t}];
```

```
T222[t_] := Sum[fac1[[n1, s]] * Exp[-(t - s) / tau2], {s, 0, t}];
```

```
annualtemp = globaltemp[[ ; ; , 14]];
```

```
years = globaltemp[[ ; ; , 1]];
```

```
T11R = Table[T111[t][[1]], {t, 1, 166}];
```

```
T22R = Table[T222[t][[1]], {t, 1, 166}];
```

```

    ccr = Drop[annualtemp, -2];
    ddr = T11R;
    eer = T22R;
    ffr = ConstantArray[1, 166];
    mmr = Transpose[{ddr, eer, ffr}];
    ssr = LeastSquares[mmr, ccr];
    Fstatn = Append[Fstatn, ssr];
    Clear[c1, c2, c3];
    cval =
      NMinimize[{Total[(mmr.{c1, c2, c3} - ccr)^2], {{1bc < c1 * 4.1 + c2 * 219 < ubc},
        {0.02 < c1 < 0.13}, {0 < c2 < 0.006}}}, {c1, c2, c3}];
    cval2 = {cval[[2, 1]][[2]], cval[[2, 2]][[2]]};
    Fstatnc = Append[Fstatnc, cval2];

, {n1, 1, 15}]
Do[
  T111[t_] := Sum[fac2[[n1, s + 10]] * Exp[-(t - s) / τ1], {s, 0, t}];
  T222[t_] := Sum[fac2[[n1, s + 10]] * Exp[-(t - s) / τ2], {s, 0, t}];
  annualtemp = globaltemp[[ ; ; , 14]];
  years = globaltemp[[ ; ; , 1]];
  T11R = Table[T111[t], {t, 1, 156}];
  T22R = Table[T222[t], {t, 1, 156}];
  ccr = Drop[annualtemp, -2];
  ccr = Drop[ccr, 10];
  ddr = T11R;
  eer = T22R;
  ffr = ConstantArray[1, 156];
  mmr = Transpose[{ddr, eer, ffr}];
  ssr = LeastSquares[mmr, ccr];
  Fstatn = Append[Fstatn, ssr];
  Clear[c1, c2, c3];
  cval =
    NMinimize[{Total[(mmr.{c1, c2, c3} - ccr)^2], {{1bc < c1 * 4.1 + c2 * 219 < ubc},
      {0.02 < c1 < 0.13}, {0 < c2 < 0.006}}}, {c1, c2, c3}];
  cval2 = {cval[[2, 1]][[2]], cval[[2, 2]][[2]]};
  Fstatnc = Append[Fstatnc, cval2];
, {n1, 1, 2}]

Do[
  T111[t_] := Sum[fac3[[n1, s + 11]] * Exp[-(t - s) / τ1], {s, 0, t}];
  T222[t_] := Sum[fac3[[n1, s + 11]] * Exp[-(t - s) / τ2], {s, 0, t}];
  annualtemp = globaltemp[[ ; ; , 14]];
  years = globaltemp[[ ; ; , 1]];
  T11R = Table[T111[t], {t, 1, 155}];
  T22R = Table[T222[t], {t, 1, 155}];
  ccr = Drop[annualtemp, -2];
  ccr = Drop[ccr, 11];
  ddr = T11R;
  eer = T22R;
  ffr = ConstantArray[1, 155];
  mmr = Transpose[{ddr, eer, ffr}];
  ssr = LeastSquares[mmr, ccr];
  Fstatn = Append[Fstatn, ssr];

```

```

Clear[c1, c2, c3];
cval =
  NMinimize[{Total[(mmr.{c1, c2, c3} - ccr)^2], {{lbc < c1 * 4.1 + c2 * 219 < ubc},
    {0.05 < c1 < 0.13}, {0 < c2 < 0.006}}}, {c1, c2, c3}];
cval2 = {cval[[2, 1]][[2]], cval[[2, 2]][[2]]};
Fstatnc = Append[Fstatnc, cval2];
, {n1, 1, 2}]

```

```
Out[5047]= {3.2, 3.8, 3.6, 3.6, 2.6, 3.8, 3.1, 4.1, 4.1, 3.2, 3.1, 3, 3.6, 3, 4.3, 2.9, 3, 3.4, 3.1}
```

```

In[8466]= (*impparFn = Table[{Fstatnc[[i,1]], Fstatnc[[i,2]], τ1, τ2, Qn[[i]]}, {i,1,19}];*)
(*Qn = Table[Qn[[i]], {i, {1,2,3,4,5,6,7,8,9,10,11,16,18}}];*)
impparFng = Table[
  {Fstatncg[[i, 1]], Fstatncg[[i, 2]], tn11[[i]], tn22[[i]], Qn[[i]]}, {i, 1, 13}];
RFn = Map[#[[5]] * (((#[[1]] * #[[3]] / ((2 * Pi * #[[3]] * f)^2 + 1) +
  (#[[4]] * #[[2]]) / ((2 * Pi * #[[4]] * f)^2 + 1))^2 +
  ((2 * Pi * #[[1]] * #[[3]] * #[[3]] * f) / ((2 * Pi * #[[3]] * f)^2 + 1) +
  (2 * Pi * #[[2]] * #[[4]] * #[[4]] * f) /
  ((2 * Pi * #[[4]] * f)^2 + 1))^2)^(0.5)) &, impparFng]
(*RFn = Table[RFn[[i]], {i, {1,2,3,4,5,6,7,8,9,10,11,16,18}}];
RFn = Table[RFn[[i]], {i, {1,2,3,4,5,6,7,8,9,10,11,16,18,12,13,14,15,17,19}}];*)

```

```

In[8468]= muFn = Mean[RFn];
varRFn = Variance[RFn]
sigmatotn = Sqrt[sigmaI^2 + varRFn]
sigmaTCR[xxR_] := s2n * (1 + (1/Nn) + ((xxR - Mean[Rn])^2 / (sx2n * Nn)));
sigmaTCRval = Table[{i, sigmaTCR[i]}, {i, 0, 2, 0.05}];
βn = muFn / sigmatotn^2
αn = muFn^2 / sigmatotn^2
RFnstatpl = Table[{RFn[[i]], 0} → i, {i, 1, 13}];
PRGn = PDF[GammaDistribution[αn, 1/βn], y]
eplo = LineLegend[{Blue, Red},
  {Style["Normal", 18], Style["Gamma", 18]}, LegendMarkerSize → 30];
P8n = ListPlot[RFnstatpl, PlotStyle → {Red, PointSize[0.01]},
  LabelStyle → {Blue, 12}];
P7n = Plot[PDF[SmoothKernelDistribution[RFn], xy], {xy, 0, 2.2},
  PlotStyle → {Red, Thickness[0.005]}];
P6n = Plot[PDF[GammaDistribution[αn, 1/βn], y], {y, 0, 2.4},
  PlotStyle → {Red, Thickness[0.005]}];
P5n = Plot[PDF[NormalDistribution[muFn, sigmatotn], y],
  {y, 0, 2.4}, PlotStyle → {Blue, Thickness[0.005]}];
Pshu = Plot[{lincorRn[y] - Sqrt[sigmaTCR[y]]}, {y, 0, 2.4},
  PlotStyle → {RGBColor[0.3, 0.7, 0.68], Dashed}];
Pshd = Plot[{lincorRn[y] + Sqrt[sigmaTCR[y]]}, {y, 0, 2.4},
  PlotStyle → {RGBColor[0.3, 0.7, 0.68], Dashed}];
Show[P3n, P4n, P5n, P6n, P8n, Pshu, Pshd, PlotLabel → Style["", 22, Black],
  Frame → True, FrameLabel → {"R (f=0.01) [°C]", "TCR [°C] "},
  LabelStyle → {20, Black}, PlotRange → All, GridLines → Automatic,
  Axes → None, Epilog → {Inset[eplo, Scaled[{0.2, 0.8}]]}]
PTCRgn = PDF[NormalDistribution[lincorRn[y], sigmaTCR[y]], s1]
PRn = PDF[NormalDistribution[muFn, sigmatotn], y]

```

```

TST01n1[t_] :=
  Table[Sum[forallsameF[[m, s]] * Exp[-(t - s) / τ1], {s, 0, t}], {m, 1, 15}];
TST02n1[t_] := Table[Sum[forallsameF[[m, s]] * Exp[-(t - s) / τ2], {s, 0, t}],
  {m, 1, 15}];
TST1n1 = Table[TST01n1[m][[i]][[1]], {m, 1, 166}, {i, 1, 15}];
TST2n1 = Table[TST02n1[m][[i]][[1]], {m, 1, 166}, {i, 1, 15}];
TST01n2[t_] :=
  Table[Sum[forallsameF2[[m, s + 10]] * Exp[-(t - s) / τ1], {s, 0, t}], {m, 1, 2}];
TST02n2[t_] := Table[Sum[forallsameF2[[m, s + 10]] * Exp[-(t - s) / τ2], {s, 0, t}],
  {m, 1, 2}];
TST1n2 = Table[TST01n2[m][[i]], {m, 1, 156}, {i, 1, 2}];
TST2n2 = Table[TST02n2[m][[i]], {m, 1, 156}, {i, 1, 2}];
TST01n3[t_] :=
  Table[Sum[forallsameF3[[m, s + 11]] * Exp[-(t - s) / τ1], {s, 0, t}], {m, 1, 2}];
TST02n3[t_] := Table[Sum[forallsameF3[[m, s + 11]] * Exp[-(t - s) / τ2], {s, 0, t}],
  {m, 1, 2}];
TST1n3 = Table[TST01n3[m][[i]], {m, 1, 155}, {i, 1, 2}];
TST2n3 = Table[TST02n3[m][[i]], {m, 1, 155}, {i, 1, 2}];
TSTM1 = Table[{Tmdmatn1[[i, 2, m, 2]]}, {m, 1, 166}, {i, 1, 15}];
TSTM2 = Table[{Tmdmatn2[[i, 2, m, 2]]}, {m, 1, 156}, {i, 1, 2}];
TSTM3 = Table[{Tmdmatn3[[i, 2, m, 2]]}, {m, 1, 155}, {i, 1, 2}];
TSTMFITn1 =
  Table[{TSTM1[[m, i]], {TST1n1[[m, i]], TST2n1[[m, i]]}}, {m, 1, 166}, {i, 1, 15}];
TSTMFITn2 = Table[{TSTM2[[m, i]], {TST1n2[[m, i]], TST2n2[[m, i]]}},
  {m, 1, 156}, {i, 1, 2}];
TSTMFITn3 = Table[{TSTM3[[m, i]], {TST1n3[[m, i]], TST2n3[[m, i]]}},
  {m, 1, 155}, {i, 1, 2}];

```

In[8525]:=

```

TCRn = Table[AR5new[[i, 4]], {i, 1, 20}];
TCRn = Drop[TCRn, {2}];
TCRn =
  Table[TCRn[[i]], {i, {1, 2, 3, 4, 5, 7, 9, 10, 11, 12, 13, 14, 15, 18, 19, 8, 16, 6, 17}}];
TCRname = {"BCC-CSM1-1", "CanESM2", "CCSM4", "CNRM-CM5",
  "CSIRO-M k3.6.0", "GISS-E2-R", "IPSL-CM5A-LR", "MIROC5", "MPI-ESM-LR",
  "MRI-CGCM3", "NorESM1-M", "Acess10", "bcc-csm1-1m", "inm-cm4",
  "miroc-esm", "HadGEM2-ES", "gfdl-cm3", "GFDL-ESM2M", "gfdl-esm2g"};
Qn = Table[AR5new[[i, 2]], {i, 1, 20}];
Qn = Drop[Qn, {2}];
Qn =
  Table[Qn[[i]], {i, {1, 2, 3, 4, 5, 7, 9, 10, 11, 12, 13, 14, 15, 18, 19, 8, 16, 6, 17}}];
lbc = 0.2;
ubc = 2;
constsn1 = {};
constsn2 = {};
constsn3 = {};
constsn = {};
constsnt = {};
constsncl = {};
constsncl2 = {};
constsncl3 = {};

```

Do[

```

Clear[c1, c2, c3];
cc = TSTMFITn1[[All, n1, 1]];
dd = TSTMFITn1[[All, n1, 2, 1]];
ee = TSTMFITn1[[All, n1, 2, 2]];
ff = ConstantArray[1, 166];
mm = Transpose[{dd, ee, ff}];
ss = LeastSquares[mm, cc[[All, 1]]];
constsn1 = Append[constsn1, ss];
Clear[c1, c2, c3];
funcmin[c1_, c2_, c3_] := Total[(mm.{c1, c2, c3} - cc[[All, 1]])^2];
cval = NMinimize[
  {Total[(mm.{c1, c2, c3} - cc[[All, 1]])^2], {{lbc < c1 * 4.1 + c2 * 219 < ubc},
    {0.02 < c1 < 0.13}, {0 < c2 < 0.006}}}, {c1, c2, c3}];
cval2 = {cval[[2, 1]][[2]], cval[[2, 2]][[2]]};
constsn1 = Append[constsn1, cval2];

```

```
, {n1, 1, 15}]
```

```
Do[
```

```

cc = TSTMFITn2[[All, n1, 1]];
dd = TSTMFITn2[[All, n1, 2, 1]];
ee = TSTMFITn2[[All, n1, 2, 2]];
ff = ConstantArray[1, 156];
mm = Transpose[{dd, ee, ff}];
ss = LeastSquares[mm, cc[[All, 1]]];
constsn2 = Append[constsn2, ss];
Clear[c1, c2, c3];
funcmin[c1_, c2_, c3_] := Total[(mm.{c1, c2, c3} - cc[[All, 1]])^2];
cval = FindMinimum[{Total[(mm.{c1, c2, c3} - cc[[All, 1]])^2],
  {{lbc < c1 * 4.1 + c2 * 219 < ubc}, {0.02 < c1 < 0.13}, {0 < c2 < 0.006}}},
  {{c1, 0.1}, {c2, 0.001}, {c3, 0}}];
cval2 = {cval[[2, 1]][[2]], cval[[2, 2]][[2]]};
constsn2 = Append[constsn2, cval2];

```

```
, {n1, 1, 2}]
```

```
Do[
```

```

cc = TSTMFITn3[[All, n1, 1]];
dd = TSTMFITn3[[All, n1, 2, 1]];
ee = TSTMFITn3[[All, n1, 2, 2]];
ff = ConstantArray[1, 155];
mm = Transpose[{dd, ee, ff}];
ss = LeastSquares[mm, cc[[All, 1]]];
constsn3 = Append[constsn3, ss];
Clear[c1, c2, c3];
funcmin[c1_, c2_, c3_] := Total[(mm.{c1, c2, c3} - cc[[All, 1]])^2];
cval = FindMinimum[{Total[(mm.{c1, c2, c3} - cc[[All, 1]])^2],
  {{lbc < c1 * 4.1 + c2 * 219 < ubc}, {0.02 < c1 < 0.13}, {0 < c2 < 0.006}}},
  {{c1, 0.1}, {c2, 0.001}, {c3, 0}}];
cval2 = {cval[[2, 1]][[2]], cval[[2, 2]][[2]]};
constsn3 = Append[constsn3, cval2];

```



```

, {n1, 1, 2}]

constsn1 = Append[constsn1, constsn2[[1]]];
constsn2 = Append[constsn1, constsn2[[2]]];
constsn3 = Append[constsn2, constsn3[[1]]];
constsn4 = Append[constsn3, constsn3[[2]]];
constsn = constsn4;
constsn1 = Append[constsn1, constsn2[[1]]];
constsn2 = Append[constsn2, constsn2[[2]]];
constsn3 = Append[constsn3, constsn3[[1]]];
constsn4 = Append[constsn4, constsn3[[2]]];
constsn = constsn4;
Clear[f]
impparn = Table[{constsn[[i, 1]], constsn[[i, 2]], τ1, τ2, Qn[[i]]}, {i, 1, 19}];
Rn =
  Map[#[[5]] * (((#[[1]] * #[[3]] / ((2 * Pi * #[[3]] * f)^2 + 1) + (#[[4]] * #[[2]]) / ((2 *
    Pi * #[[4]] * f)^2 + 1))^2 + ((2 * Pi * #[[1]] * #[[3]] * #[[3]] * f) /
    ((2 * Pi * #[[3]] * f)^2 + 1) + (2 * Pi * #[[2]] * #[[4]] * #[[4]] * f) /
    ((2 * Pi * #[[4]] * f)^2 + 1))^2)^(0.5)) &, impparn];
(*TCRn = Table[TCRn[[i]], {i, {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 16, 18}}];
TCRname = Table[TCRname[[i]], {i, {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 16, 18}}];
Rn = Table[Rn[[i]], {i, {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 16, 18}}];*)
TCRn =
  Table[TCRn[[i]], {i, {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 16, 18, 12, 13, 14, 15, 17, 19}}];
TCRname = Table[TCRname[[i]],
  {i, {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 16, 18, 12, 13, 14, 15, 17, 19}}];
Rn = Table[Rn[[i]], {i, {1, 2, 3, 4, 5, 6, 7, 8, 9, 10,
  11, 16, 18, 12, 13, 14, 15, 17, 19}}];
korTCSn = Table[{(1.4)^(-i), Correlation[TCRn, Rn /. f -> (1.4)^(-i)]}, {i, 1, 27}]
PLTCS1n = ListLogLinearPlot[korTCSn, Frame -> True, FrameStyle -> Directive[Black, 18],
  PlotStyle -> {PointSize[0.02], RGBColor[0.3, 0.66, 0.69]};
PLTCS2n = ListLogLinearPlot[korTCSn, Frame -> True, FrameStyle -> Directive[Black, 18],
  PlotStyle -> RGBColor[0.3, 0.66, 0.69], Joined -> True];
Show[{PLTCS1n, PLTCS2n}, FrameLabel -> {"Frequency (1/Years)", "Correlation"},
  AspectRatio -> 1, Background -> None, PlotRange -> {{Log[0.0001], Log[1]}, {0.4, 1}},
  PlotLabel -> Style["Correlation TCR - Multimodel Mean (Forster)", 20, Black],
  Axes -> None, GridLines -> Automatic]

f = 0.01;
corRn = MapThread[{#1, #2} &, {Rn, TCRn}]
Correlation[corRn]
corRn1 = Table[corRn[[i]] -> i, {i, 1, 19}];
lincorRn = LinearModelFit[corRn, x, x]
P3n = Plot[lincorRn[t], {t, 0, 2.4},
  PlotStyle -> {RGBColor[0.3, 0.7, 0.683], Thickness[0.01]};
P4n = ListPlot[corRn1, PlotStyle -> Black, LabelStyle -> {16, Blue}];
impparki =
  Table[{constski[[i, 1]], constski[[i, 2]], τ1, τ2, Mean[Q]}, {i, 1, 10}];
kistat =
  Map[#[[5]] * (((#[[1]] * #[[3]] / ((2 * Pi * #[[3]] * f)^2 + 1) + (#[[4]] * #[[2]]) /
    ((2 * Pi * #[[4]] * f)^2 + 1))^2 + ((2 * Pi * #[[1]] * #[[3]] * #[[3]] * f) /
    ((2 * Pi * #[[3]] * f)^2 + 1) + (2 * Pi * #[[2]] * #[[4]] * #[[4]] * f) /
    ((2 * Pi * #[[4]] * f)^2 + 1))^2)^(0.5)) &, impparki];

```

```

s2ki = Variance[kistat];
sigmaI = Sqrt[s2ki]

Fstatn = {};
Fstatnc = {};
Do[
  T111[t_] := Sum[forallsameF[[n1, s]] * Exp[-(t - s) / τ1], {s, 0, t}];
  T222[t_] := Sum[forallsameF[[n1, s]] * Exp[-(t - s) / τ2], {s, 0, t}];

  annualtemp = globaltemp[[;;, 14]];
  years = globaltemp[[;;, 1]];
  T11R = Table[T111[t][[1]], {t, 1, 166}];
  T22R = Table[T222[t][[1]], {t, 1, 166}];
  ccr = Drop[annualtemp, -2];
  ddr = T11R;
  eer = T22R;
  ffr = ConstantArray[1, 166];
  mmr = Transpose[{ddr, eer, ffr}];
  ssr = LeastSquares[mmr, ccr];
  Fstatn = Append[Fstatn, ssr];
  Clear[c1, c2, c3];
  cval =
    FindMinimum[{Total[(mmr.{c1, c2, c3} - ccr)^2], {{lbc < c1 * 4.1 + c2 * 219 < ubc},
      {0.02 < c1 < 0.13}, {0 < c2 < 0.006}}}, {{c1, 0.1}, {c2, 0.001}, {c3, 0}}];
  cval2 = {cval[[2, 1]][[2]], cval[[2, 2]][[2]]};
  Fstatnc = Append[Fstatnc, cval2];

, {n1, 1, 15}]
Do[
  T111[t_] := Sum[forallsameF2[[n1, s + 10]] * Exp[-(t - s) / τ1], {s, 0, t}];
  T222[t_] := Sum[forallsameF2[[n1, s + 10]] * Exp[-(t - s) / τ2], {s, 0, t}];
  annualtemp = globaltemp[[;;, 14]];
  years = globaltemp[[;;, 1]];
  T11R = Table[T111[t], {t, 1, 156}];
  T22R = Table[T222[t], {t, 1, 156}];
  ccr = Drop[annualtemp, -2];
  ccr = Drop[ccr, 10];
  ddr = T11R;
  eer = T22R;
  ffr = ConstantArray[1, 156];
  mmr = Transpose[{ddr, eer, ffr}];
  ssr = LeastSquares[mmr, ccr];
  Fstatn = Append[Fstatn, ssr];
  Clear[c1, c2, c3];
  cval =
    FindMinimum[{Total[(mmr.{c1, c2, c3} - ccr)^2], {{lbc < c1 * 4.1 + c2 * 219 < ubc},
      {0.02 < c1 < 0.13}, {0 < c2 < 0.006}}}, {{c1, 0.1}, {c2, 0.001}, {c3, 0}}];
  cval2 = {cval[[2, 1]][[2]], cval[[2, 2]][[2]]};
  Fstatnc = Append[Fstatnc, cval2];
, {n1, 1, 2}]
Do[
  T111[t_] := Sum[forallsameF3[[n1, s + 11]] * Exp[-(t - s) / τ1], {s, 0, t}];

```

```

T222[t_] := Sum[forallsameF3 [[n1, s + 11]] * Exp[-(t - s) / τ2], {s, 0, t}];
annualtemp = globaltemp[[;;, 14]];
years = globaltemp[[;;, 1]];
T11R = Table[T111[t], {t, 1, 155}];
T22R = Table[T222[t], {t, 1, 155}];
ccr = Drop[annualtemp, -2];
ccr = Drop[ccr, 11];
ddr = T11R;
eer = T22R;
ffr = ConstantArray[1, 155];
mmr = Transpose[{ddr, eer, ffr}];
ssr = LeastSquares[mmr, ccr];
Fstatn = Append[Fstatn, ssr];
Clear[c1, c2, c3];
cval =
  FindMinimum[{Total[(mmr.{c1, c2, c3} - ccr)^2], {{lbc < c1 * 4.1 + c2 * 219 < ubc},
    {0.02 < c1 < 0.13}, {0 < c2 < 0.006}}}, {{c1, 0.1}, {c2, 0.001}, {c3, 0}}];
cval2 = {cval[[2, 1]][[2]], cval[[2, 2]][[2]]};
Fstatnc = Append[Fstatnc, cval2];
, {n1, 1, 2}]

impparFn = Table[{Fstatnc[[i, 1]], Fstatnc[[i, 2]], τ1, τ2, Qn[[i]]}, {i, 1, 19}];
RFn =
  Map[#[[5]] * (((#[[1]] * #[[3]] / ((2 * Pi * #[[3]] * f)^2 + 1) + (#[[4]] * #[[2]]) / ((2 *
    Pi * #[[4]] * f)^2 + 1))^2 + ((2 * Pi * #[[1]] * #[[3]] * #[[3]] * f) /
    ((2 * Pi * #[[3]] * f)^2 + 1) + (2 * Pi * #[[2]] * #[[4]] * #[[4]] * f) /
    ((2 * Pi * #[[4]] * f)^2 + 1))^2)^(0.5)) &, impparFn]
muFn = Mean[RFn];
varRFn = Variance[RFn]
sigmatotn = Sqrt[sigmaI^2 + varRFn]
sigmaTCR[xxR_] := s2n * (1 + (1/Nn) + ((xxR - Mean[Rn])^2 / (sx2n * Nn)));
sigmaTCRval = Table[{i, sigmaTCR[i]}, {i, 0, 2, 0.05}];
βn = muFn / sigmatotn^2
αn = muFn^2 / sigmatotn^2
RFnstatpl = Table[{RFn[[i]], 0} → i, {i, 1, 19}];
PRGn = PDF[GammaDistribution[αn, 1/βn], y]
eplo = LineLegend[{Blue, Green, Red}, {Style["Normal", 18],
  Style["Gamma", 18], Style["Smooth Kernel", 18]}, LegendMarkerSize → 30];
P8n = ListPlot[RFnstatpl, PlotStyle → {Red, Thick}, LabelStyle → {Blue, 12}];
P7n = Plot[PDF[SmoothKernelDistribution[RFn], xy],
  {xy, 0, 2.4}, PlotStyle → {Red, Thickness[0.005]}];
P6n = Plot[PDF[GammaDistribution[αn, 1/βn], y], {y, 0, 2.4},
  PlotStyle → {Green, Thickness[0.005]}];
P5n = Plot[PDF[NormalDistribution[muFn, sigmatotn], y],
  {y, 0, 2.4}, PlotStyle → {Blue, Thickness[0.005]}];
Show[P3n, P4n, P5n, P6n, P7n, P8n, PlotLabel → Style["", 22, Black], Frame → True,
  FrameLabel → {"R(f=0.01)", "TCR [°C]"}, LabelStyle → {20, Black}, PlotRange → All,
  GridLines → Automatic, Epilog → {Inset[eplo, Scaled[{0.2, 0.8}]]}, Axes → None]
PTCRgn = PDF[NormalDistribution[lincorRn[y], sigmaTCR[y]], s1]
PRn = PDF[NormalDistribution[muFn, sigmatotn], y]

```

```

In[2473]:= p11 = LogLinearPlot[{RFn[[6]]}, {f, 0.0001, 1}, Frame → True,
  FrameStyle → Directive[Black, 20], PlotStyle → {{Thickness[0.01], Red}},
  FrameLabel → {"Frequency f (1/Years)", "R(f)"}];
p12 = LogLinearPlot[{RFn[[12]]}, {f, 0.0001, 1}, Frame → True,
  FrameStyle → Directive[Black, 20], PlotStyle → {{Thickness[0.01], Blue}},
  FrameLabel → {"Frequency f (1/Years)", "R(f)"}];
Show[p11, p12, PlotLabel → Style["", 22, Black], Frame → True,
  FrameLabel → {"f [1/Years]", "R(f) [°C] "},
  LabelStyle → {20, Black}, PlotRange → All, GridLines → Automatic,
  Axes → None, Epilog → {Inset[son1, Scaled[{0.6, 0.8}]]}]

LogLinearPlot[{constslol[[1]]}, {f, 0.0001, 1},
  Frame → True, FrameStyle → Directive[Black, 20],
  PlotStyle → {{Thickness[0.01], RGBColor[0.3, 0.67, 0.69]}},
  FrameLabel → {"Frequency f (1/Years)", "R(f)"}]

In[2374]:= np11 = ListPlot[fac1[[3]], Joined → True, PlotStyle → Red];
np12 = ListPlot[fac2[[1]], Joined → True, PlotStyle → Blue, PlotRange → All];
son1 = LineLegend[{Red, Blue},
  {Style["GISS-E2-R (6)", 18], Style["HadGEM2-ES (12)", 18]}, LegendMarkerSize → 30]

Show[np11, np12, PlotLabel → Style["", 22, Black],
  Frame → True, FrameLabel → {"Year after 1850", "Forcing [W/m2] "},
  LabelStyle → {20, Black}, PlotRange → All, GridLines → Automatic,
  Axes → None, Epilog → {Inset[son1, Scaled[{0.25, 0.8}]]}]

In[2801]:= sist1 = constsnrc[[1, 1]] * TST1n1[[All, 1]] +
  constsnrc[[1, 2]] * TST2n1[[All, 3]] + constsnrc[[1, 3]] * ConstantArray[1, 166];
sist2 = c1gof[[1]] * TST1n1[[All, 1]] + c2gof[[1]] * TST2n1[[All, 1]] +
  constsnrc[[1, 3]] * ConstantArray[1, 166];
sist3 = TSTMFITn1[[All, 1, 1]][[All, 1]];
son12 = LineLegend[{Red, Blue, RGBColor[0.3, 0.7, 0.68]},
  {Style["Fit", 18], Style["Fit - Geoffroy parameters", 18],
  Style["Temperature", 18]}, LegendMarkerSize → 30]
sp1 = ListPlot[sist1, Joined → True, PlotStyle → Red];
sp2 = ListPlot[sist2, Joined → True, PlotStyle → Blue];
sp3 = ListPlot[sist3, Joined → True, PlotStyle → RGBColor[0.3, 0.7, 0.68]];
Show[sp1, sp2, sp3, PlotLabel → Style["BCC-CSM1-1", 22, Black],
  Frame → True, FrameLabel → {"Year after 1850", "Temperature Anomaly [°C]"},
  LabelStyle → {20, Black}, PlotRange → All, GridLines → Automatic,
  Axes → None, Epilog → {Inset[son12, Scaled[{0.3, 0.8}]]}]

```

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