

# **A multi-objective mathematical programming for sustainable reverse logistics network design. Part II: Model application and analysis**

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## **Abstract**

Reverse logistics has received more and more attention during the past decade due to the increasing public awareness of sustainable development. Because of the fluctuation in both quantity and quality of the reverse material flow, design and planning of reverse logistics network is much more complicated compared with the forward ones. Therefore, it is important to develop decision support tools for designing reverse logistics network in an economically efficient and environmental-friendly manner. This research proposes a novel multi-objective mixed integer programming model in order to justify the relationship between the cost and sustainability of reverse logistics system, and the weighted sum utility method is employed for combining the two objective functions. This research is presented in a series of two papers. Part I formulates the conceptual framework of reverse logistics network and the mathematical programming for the minimization of the overall system cost and environmental influence. Part II introduces the weighted sum utility method for combining the two objective functions, and the application and analysis are also given in this part.

*Keywords: multi-objective programming, mixed integer programming, reverse logistics, network design.*

## **1 Introduction**

Most practical problems in the real world involve more than one influencing factor [1], so taking into account several criteria concurrently is important for making the most appropriate decisions. During the past few years, multi-criteria optimization techniques have experienced a rapid development and also been



extensively applied in decision-making and support of many different science disciplines in both engineering and management, i.e., product design [2], production scheduling [3], facility location problem [4], transportation and capacity allocation problem [5], to name a few. Generally, conflicts existed in different objectives of a multi-criteria optimization problem, and improving the result of one objective will lead to the sacrifices of others [1]. Therefore, it is of significance to appropriately deal with the trade-off among different objectives in a multi-criteria optimization problem. In the Part I of the study, we proposed a bi-objective mixed integer programming for reverse logistics network in an economically efficient and environment-friendly manner. In this paper, a weighted sum utility method is employed for justifying the trade-off of the two objective functions and determining the optimal network configuration of reverse logistics system. In order to illustrate the applicability of the proposed model and solution method, the calculation and analysis of a fictional case are also presented later in this paper.

## 2 Solution method of multi-objective reverse logistics network design model

Weighted sum is one of the most frequently used methods for combining different objective functions in a multi-criteria optimization problem. The prerequisite for applying weighted sum method is that all the objective functions are measured or can be unified in the same scale. However, this is inapplicable for the case of our model, because of the different measurements of the network configuration cost and carbon emissions. Therefore, a modified weighted sum utility method is employed in this paper for determining the optimal trade-off between the cost and environmental impact of reverse logistics system.

### 2.1 Definition of parameters

Definition of the parameters used in the composite formula of weighted sum utility method is first given as follows:

$CoJ, EoJ$	Value of cost objective and carbon emission objective
$CoJ_{\min}, EoJ_{\min}$	Minimum value of individual cost objective and carbon emissions objective
$\alpha, \beta$	Weight of each objective
$\tau$	Overall composite utility

### 2.2 Composite formula

Equation (1) is the composite formula for the weighted sum utility method. Different from the traditional weighted sum method, the most significant characteristic of weighted sum utility is that the objective functions are first divided by their individual optimal value accordingly and then composite together with the given



weight. The initial step aims at eliminating the problems caused by different scales used and measured in each objective function.

$$\min \tau = \alpha \frac{CoJ}{CoJ_{\min}} + \beta \frac{EoJ}{EoJ_{\min}}. \quad (1)$$

The individual optimal value of each objective function can be found through solving the single objective programming, and sum of the weights of the cost utility and carbon emission utility equals 1. The weighted sum utility formula as well as its derived forms has been used in previous studies and more detailed introduction and application of this method are provided in Nema and Gupta [6], Sheu [7], and Yu et al. [8].

### 3 Model application

In this section, an illustration is given to test the performance of the proposed computational model for reverse logistics network design. The assumption of input parameters is first given in the following part, and discussion of the result and sensitivity analysis are presented later in this section.

#### 3.1 Example

In this example, we are going to determine the optimal network configuration of a reverse logistics system for the reuse and recycling of a single type of product in an economically efficient and environmental-friendly manner. It is noted that the units of parameters and decision variables in this example are not specified for a certain type of measurement because of the generality it aims to represent. Besides, the definition of parameters and decision variables are consistent with the model formulation in Part I.

There are 10 customer locations where used products are required to be collected and properly treated in this example; the annual generation of used product in each customer location is 13,500, 22,300, 12,350, 22,300, 11,500, 14,300, 12,400, 28,600, 21,000, and 15,300, respectively. There is only one landfill for waste disposal in the studied area, and the relevant parameters are given: fixed cost  $V_w = 1,200,000$ , variable cost  $G_w = 10$  and capacity  $OI_w = 200,000$ .

In order to recover the remaining value of used products, a reverse logistics system is going to be established. The locations of repair facility and remanufacturing facility, and the material flow of each itinerary between different levels of facilities, will be determined. The relevant parameters of five candidates for collection centres, three candidates for repair facilities, and two candidates for remanufacturing facilities are given in Tables 1–3, respectively.

Table 4 illustrates the proximity, unit transportation cost, and carbon emission indicator of each itinerary between customer location  $f$  and collection centre  $c$ . Both unit transportation cost and carbon emission factor are proportional to the proximity between customer location and collection centre. However, the relationship between unit transportation cost and carbon emission factor of each itinerary

Table 1: Parameters for candidates of collection centre  $c$ .

Parameters	Candidates of collection centre				
	$c = 1$	$c = 2$	$c = 3$	$c = 4$	$c = 5$
$V_c$	1,350,000	1,580,000	1,700,000	1,450,000	1,380,000
$G_c$	20	15	15	18	20
$OL_c$	120,000	100,000	180,000	120,000	120,000

Table 2: Parameters for candidates of repair facility  $rp$ .

Parameters	Candidates of repair facility		
	$rp = 1$	$rp = 2$	$rp = 3$
$V_{rp}$	1,550,000	1,650,000	2,100,000
$G_{rp}$	25	20	15
$OL_{rp}$	180,000	250,000	450,000

Table 3: Parameters for candidates of remanufacturing facility  $rc$ .

Parameters	Candidates of remanufacturing facility	
	$rc = 1$	$rc = 2$
$V_{rc}$	2,200,000	2,500,000
$G_{rc}$	25	25
$OL_{rc}$	200,000	200,000

Table 4: Parameters of each itinerary between  $f$  and  $c$ .

Parameters	Customers										
	$f = 1$	$f = 2$	$f = 3$	$f = 4$	$f = 5$	$f = 6$	$f = 7$	$f = 8$	$f = 9$	$f = 10$	
$A_{fc}$	$c = 1$	6	14	31	45	18	12	8	11	6	5
	$c = 2$	10	11	12	15	12	24	19	16	17	11
	$c = 3$	22	15	6	5	9	9	17	13	23	20
	$c = 4$	14	30	20	18	9	8	16	16	9	5
	$c = 5$	15	8	9	19	20	22	25	11	12	9
$X_{fc}$	$c = 1$	20	30	100	150	35	30	25	30	20	20
	$c = 2$	50	50	51	70	60	120	100	80	80	50
	$c = 3$	100	70	30	20	40	40	80	60	100	100
	$c = 4$	30	100	70	60	30	30	50	50	30	20
	$c = 5$	80	40	40	100	100	100	120	50	60	40
$C_{fc}$	$c = 1$	18	40	90	140	55	36	24	30	30	30
	$c = 2$	20	20	20	30	25	50	40	35	35	20
	$c = 3$	40	30	12	10	20	20	34	26	40	40
	$c = 4$	50	90	60	60	30	30	50	50	30	15
	$c = 5$	30	20	20	40	40	45	50	22	25	15



is inversely related. This is a reasonable assumption due to the fact that increasing environmental performance always leads to a higher cost. In this example, reducing carbon emissions requires a higher standard of energy consumption of the transport vehicles, so an extra cost for upgrading the transport fleet is necessary and the unit transportation cost will be increased as well. The numerical values of the parameters of the other itineraries between  $c$  and  $w$ ,  $c$  and  $rp$ , as well as  $c$  and  $rc$  are also given in such manner.

### 3.2 Results and discussion

The model is coded and resolved with the help of Lingo solver. The optimal results of the two individual objectives are first calculated, and Table 5 illustrates the selection of the candidates of collection centre, repair facility, and remanufacturing plant in both individual optimal solutions. In order to test the performance of the model in balancing the two conflicting objectives, equal weight is given to both cost utility and carbon emission utility. Table 5 also gives the selection of facilities of the optimal solution of composite objective.

Table 5: Facility selection in each scenario.

Scenario	Collection centre					Repair			Remanufacturing	
	$c = 1$	$c = 2$	$c = 3$	$c = 4$	$c = 5$	$rp = 1$	$rp = 2$	$rp = 3$	$rc = 1$	$rc = 2$
IC <sup>1</sup>		■			■	■			■	
ICE <sup>2</sup>	■	■	■	■	■	■	■	■	■	■
CO <sup>3</sup>	■	■			■	■			■	

<sup>1</sup>IC: Optimal solution of individual cost objective.

<sup>2</sup>ICE: Optimal solution of individual carbon emission objective.

<sup>3</sup>CO: Optimal solution of composite objective.

For the optimal solution of individual cost objective, candidates  $c_2$  and  $c_5$  are selected to open collection centres. The used products collected at customers  $f_1, f_2, f_3, f_6$ , and  $f_9$  are sent to collection centre  $c_2$ , and the used products from the other customer locations are treated at collection centre  $c_5$ . Although the fixed investment and variable processing cost at those two candidate locations are not the lowest ones, the small proximity for lowering the transportation cost makes them becoming very good choices. In this scenario, candidate  $rp_1$  is selected to open repair facility, and candidate  $rc_1$  is chosen to open remanufacturing plant. From the optimal solution of individual cost objective, it is obvious that the selection of the two collection centres will enhance the integration of the transportation of used products and reduce the overall system cost; however, the selection of repair and remanufacturing facilities is significantly influenced by the high fixed investment.

For the optimal solution of individual carbon emission objective, all the candidate locations for collection centre, repair facility, and remanufacturing plant are selected. The reason is that only the carbon emissions related to the transportation

of used products and disassembled components is accounted for in the model, so the itineraries between different facilities are generated based exclusively upon the principle of lowest carbon emissions even if the cost for establishing the reverse logistics system is extremely high. It has been further proved by the allocation of used products and disassembled components as well.

For the optimal solution of composite objective, candidates  $c_1$ ,  $c_2$ ,  $c_5$ ,  $rp_1$ , and  $rc_1$  are selected to open new facilities, and the optimal value of composite utility equals to 1.064554. In this scenario, the optimal result is compromised with both objectives with equal weight. Comparing to the optimal solution of individual cost objective, one more collection centre  $c_1$  is chosen for collecting the used products from customers  $f_1$  and  $f_6$  so as to reduce the overall carbon emissions of the reverse logistics system. Comparing to the optimal solution of individual carbon emission objective, the total number of selected candidates decreased to 5 in order to maintain the overall system cost at an affordable level for the companies of the reverse logistics network.

### 3.3 Sensitivity analysis

In order to test the model's performance with different weights given to the cost utility and carbon emission utility, sensitivity analysis of four scenarios with  $\alpha = 0.1, 0.3, 0.7, \text{ and } 0.9$  is performed. Table 6 illustrates the selection of candidates in different scenarios. As shown in the table, the cost objective has a significant influence on the composite utility even if a small weight is given to the cost utility function. This is mainly due to the large number of candidate locations selected and new facilities opened in the optimal solution of individual carbon emission objective will tremendously increase the overall system cost and then lead to large deviation from the optimal individual cost, which has a great negative impact on the overall composite utility, so the number of selected candidates decreases when cost utility is accounted for.

Table 6: Facility selection in each scenario.

Scenario	Collection centre					Repair			Remanufacturing	
	$c = 1$	$c = 2$	$c = 3$	$c = 4$	$c = 5$	$rp = 1$	$rp = 2$	$rp = 3$	$rc = 1$	$rc = 2$
CO <sup>1</sup>	■	■			■	■			■	
Scenario1	■	■		■	■	■			■	■
Scenario2	■	■		■	■	■			■	■
Scenario3	■	■			■	■			■	
Scenario4		■			■	■			■	

<sup>1</sup>CO: Optimal solution of composite objective.

In weighted sum utility method, the optimal solutions of individual cost objective and individual carbon emission objective, where the composite utility equals 1 (best performance), are set to be the benchmark for evaluating the performance of

reverse logistics network. In accordance with the benchmark, the performance of each scenario can be converted to a relative measurement value that indicates how much percentage of the best performance can be achieved in each scenario. Figure 1 presents the performance measurement of the selected scenarios. As shown in the figure, when the value of the weight of either cost utility or carbon emission utility approaches 1, the overall performance of reverse logistics system improves and approaches the best performance. However, the overall system performance decreases when the weights of both objectives are close to each other. This result has given a clear picture of the conflict of the objectives and the optimal trade-off among them.

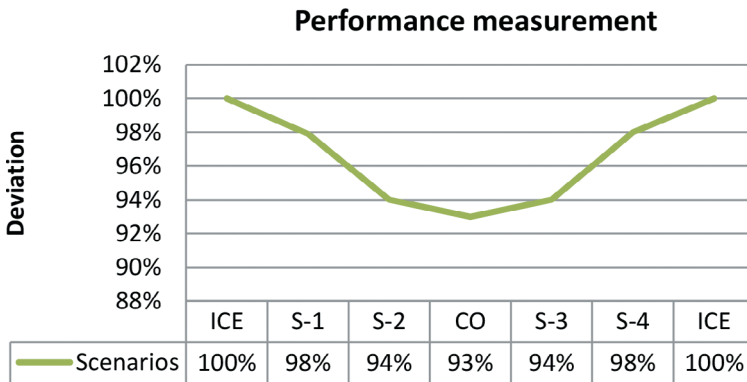


Figure 1: Performance measurement of selected scenarios.

#### 4 Summary and future work

This research has presented a novel multi-objective mathematical programming for reverse logistics network design in an economic-efficient and environmental-friendly manner. An extensive literature review of reverse logistics network design and the model formulation are presented in Part I. In this paper, the method for combining the cost objective function and carbon emission objective function is first introduced, and an illustrative calculation is then conducted to test the performance of the model. From the illustrative calculation, discussion, and analysis of the result, it is obviously that the cost minimization objective and carbon emission minimization objective are usually conflict with one another. To reduce the carbon emissions of reverse logistics system may significantly increase the overall system cost, and the optimal balance of cost and carbon emissions is therefore important in determining the configuration of the reverse logistics network. In addition, a sensitivity analysis with different weights of cost and carbon emission objective is also given so that the comparison of overall system performance in different scenarios can be clearly presented.

For future improvement of our study, three possible directions are suggested. First, we only considered the carbon emissions related to the transportation of goods in reverse logistics system; however, the minimization of carbon emissions of repair and remanufacturing activities may also be considered as an important objective function. Therefore, development of effective measurement and optimization method for minimizing carbon emissions of other relevant activities in reverse logistics network is first suggested. Second, more comprehensive reverse logistics system should be formulated for handling different types of used products. Last but not the least, appropriate treatment of uncertainties related to the quantity and quality of the reverse material flow is also suggested as one of the most promising directions for the future improvement of this study.

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