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Interpreting teaching for conceptual and for procedural knowledge in a teaching video about linear algebra

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The aim of this study is to investigate teaching videos about mathematics, seeking to uncover research-based foundations for their quality. By drawing on the notions of procedural and conceptual knowledge, the research was operationalized by asking professionals in undergraduate mathematics education (n=18) to interpret sections of a teaching video. The video dealt with a topic in linear algebra. The results indicate rather divergent interpretations of conceptual knowledge. This can hinder a reliable evaluation of teaching in terms of aiming for conceptual or procedural knowledge. It is recommended that the notions should be carefully used, defined and explained when used to evaluate the quality of teaching videos in particular, or of teacher's explanations in classrooms in general.

Introduction

On public internet platforms such as YouTube, there are many teaching videos for mathematics. In such videos a single, often invisible speaker teaches about mathematical topics in a confined environment. These videos are meant to assist students in their learning. They can also be resources for other people than learners, for example to seek inspiration for and to compare with one's own production of teaching videos, or to do research on teachers' explanations, whether it is in videos or in classrooms. We belong to the first category, producing videos ourselves. Yet, being researchers of mathematics education for engineers, we want to find research-based foundations for such work. Therefore, we were interested in finding research-based criteria for the quality of these videos.

Research on multimedia learning offers design principles that enhance learning, such as: the use of visualizations, limiting surplus information, personalization (a friendly voice, showing the teacher's face) (Mayer, 2005). However, these guidelines are not didactical, describing how mathematical topics are or can best be taught in a video. By lack of tools for analyzing and evaluating the teaching of mathematics in videos, we turned to the teaching in mathematics classrooms in general, where one can distinguish between different activities, such as activities that involve teacher-student interaction (e.g. probe, evaluate or extend students' ideas) and an activity that does not necessarily involve 2-way-interaction: explaining. Explaining of mathematical topics is a complex activity and research on it is still ongoing (Baxter & Williams, 2010). Explaining aims at supporting students, on the one hand to better understand mathematical ideas, and on the other hand to better carry out tasks. To capture this distinction, we turned to the notions of conceptual and procedural knowledge. These notions are widely used by researchers of mathematics education, based on work by, among others, Hiebert (1986). Backgrounds and definitions of these notions will be explained below. At this stage, it suffices to say that procedural and conceptual knowledge are connected to student's learning and thinking, rather than to teaching, and that "(t)he general consensus, in research on mathematical thinking and in mathematics education, is that having conceptual knowledge confers benefits above and beyond having procedural skill" (Crooks & Alibali, 2014 p. 345). In studying the quality of teaching videos, we can look for whether the teacher is *aiming* at enhancing procedural or conceptual knowledge. As an example, a teacher who aims at procedural knowledge can emphasize how tasks are to be done by demonstrating subsequent steps of the solution process. If a teacher rather aims at conceptual knowledge, he/she can focus on why a procedure works, show different representations, compare procedures or show how classes of problems have similarities.

The purpose of the present study is to support the evaluation of teaching videos, investigating whether the explanations offered in a video are aiming at procedural or conceptual knowledge, and how this can be judged. We operationalized our study by selecting from the web a video on linear algebra, in particular about bases and dimensions of vector spaces. We selected this topic, because (1) it is a topic that is part of many bachelor engineering curricula, and (2) because of the interaction between procedural methods (Gaussian elimination, finding pivots) and a connected network of concepts (vector spaces, bases and dimensions). We watched a dozen YouTube videos on this topic. The majority had an emphasis on the "how", although not one could be indicated as "purely aiming at procedural knowledge". We selected a video with a high didactical quality, clearly aiming at conceptual knowledge, for example by comparing between different solution approaches and by jumping over tedious calculations. We showed it to professionals interested in mathematics education, asking them to judge sections of the video in terms of teaching for conceptual or for procedural knowledge. Would they reach a common agreement? Would their judgment agree with our own? How would they interpret conceptual and procedural knowledge? In this paper we will report on the commonalities and divergences in participants' interpretation of teaching for conceptual and procedural knowledge in mathematics, with respect to the content presented in the chosen video. The judgment could later be useful to evaluate teaching videos on didactical qualities.

We formulated the following research question: What are, according to a group of mathematics-interested professionals, the sections in a teaching video that emphasize conceptual or procedural knowledge?

Theoretical Framework

The notions of conceptual and procedural knowledge in mathematics are widely used by researchers. Hiebert (1986) characterized conceptual knowledge as a set of connecting pieces of knowledge. Kilpatrick, Swafford, and Findell (2001) explain conceptual knowledge as “an integrated and functional grasp of mathematical ideas” (p. 118). Procedural knowledge includes familiarity with symbols and representation systems in mathematics together with knowing rules and procedures that are used to solve a class of tasks in mathematics (Hiebert, 1986).

Researchers agree on a dynamic interplay between conceptual and procedural knowledge, showing that conceptual and procedural knowledge can grow interactively (Baroody, Feil, & Johnson, 2007; Rittle-Johnson & Alibali, 1999): “Linking procedural to conceptual knowledge can make learning facts and procedures easier, provide computational shortcuts, ensure fewer errors, and reduce forgetting” (Baroody et al., 2007, p. 127). However, it is warned not to confuse or equate these notions with deep and superficial knowledge, respectively (Baroody, 2003; Star, 2005). Conceptual knowledge is a basis for procedural *fluency*, which differs from procedural *knowledge*. A superficial procedural knowledge refers to disembodied task preforming procedures, most often algorithmic computations, while procedural fluency may be of a deeper, richer nature, for instance when knowing how to generate solution processes beyond standard problem types (Kilpatrick et al., 2001; Star, 2005). A conceptual knowledge type may be of a superficial quality if the building of schemas for conceptual structures is weak and mainly related to primary level concepts. Bergsten, Engelbrecht, and Kågesten (2015) investigated engineering students’ learning and they created the following working definitions: “Procedural approach: Use and manipulate mathematical skills, such as calculations, rules, formulae, algorithms and symbols. Conceptual approach: Show understanding by e.g. interpreting and applying concepts to mathematical situations, translating between verbal, visual (graphical) and formal mathematical expressions and linking relationships” (p. 932).

Crooks and Alibali (2014) offer a review of research on conceptual and procedural knowledge, explaining that this mainly focuses on students, and the most frequently used instruments are written tests. The more rare studies about deliberate teaching that aims at conceptual knowledge (e.g. Eisenhart et al., 1993; Even & Kvatinsky, 2010) show that this kind of teaching requires, amongst others, flexibility, diligence and conceptual knowledge from a teacher, and it does not necessarily lead to conceptual knowledge with students. These studies were case studies of carefully observed teachers and how they offered the students inquiry-based tasks, used different representations, made connections, asked the students to discuss alternative approaches, and so forth. These studies did not offer categories for the quality of the teaching in terms of conceptual and procedural knowledge, and they did not specify whether a higher quality was reached through student-student interaction, teacher-student interaction, or through teacher's explanations without teacher-student interaction. By studying mathematics teaching videos, we can only observe the latter. We hope that studying the didactical quality teaching videos can also contribute to research on classroom-based explanations that aim at conceptual knowledge.

Methods

Our research design entailed a survey based on a mathematical teaching video. The data collection took place at a Norwegian conference on Undergraduate Mathematics Education. The conference attracted professionals in mathematics education: mathematics education researchers, mathematicians with teaching tasks and teachers of mathematics. Within this conference we conducted a workshop on didactical approaches in teaching videos. Part of the workshop was to show a video and collect judgments by participants in terms of teaching aiming at conceptual or procedural knowledge. Because of time limitations, however, they could only evaluate one video.

The video

From the wealth of videos freely accessible on YouTube, we selected the video "Linear algebra, Basis and dimension" published by Massachusetts Institute of Technology (MIT), available at www.youtube.com/watch?v=AqXOYgpbMBM. We deliberately chose an English video as the Nordic mathematics community is rather small and we run the risk of having the teacher of the video in our workshop. Also, the MIT-video satisfied many guidelines for multimedia (Mayer, 2005): the use of space is well-planned, we see the speaker's face, the video is relatively short (8:09 min.) and the user is activated: after having explained the task (Figure 1, left), the teacher asks users to first hit the stop button and solve the task by oneself.

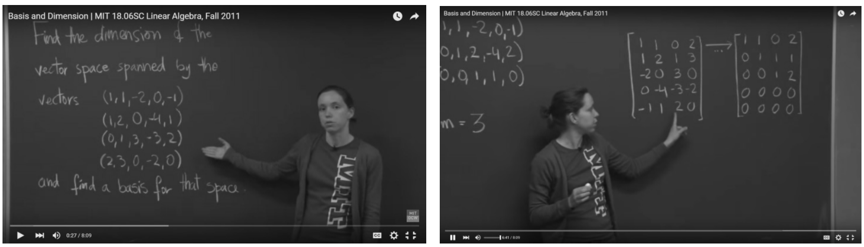

Figure 1: Stills from the video "Linear algebra, Basis and dimension" from MIT

The task in the video is to find the dimension and basis of a vector space spanned by four given 5-dimensional vectors. The solution could be demonstrated step-by-step aiming at procedural knowledge. However, there are several aspects indicating that the teacher aims at conceptual knowledge: at the beginning the teacher links to prior knowledge; before starting calculations, the teacher gives a rough outline of the approach; towards the end she presents an alternative approach for the given problem explaining how the two approaches are related. The procedural aspects, such as carrying out the Gauss operations, are accelerated and the teacher says she will go fast, because *"you must have seen eliminations a million times"*. When she explains the alternative approach, she avoids losing time on calculations and only shows the first and final matrix, indicating the calculations by an arrow and dots (see figure 1, right).

We analysed the video by splitting it into sections and describing these with cognitive steps:

1. Starts by giving the pre-knowledge (linearly independence, spanning, basis, dimension).
2. Gives a rough outline – how to work on the given problem (1st: find basis, 2nd: find dimension).
3. Talks about linear independence (until after 2:00).
4. Takes two minutes to do the elimination of rows. At 3:58: one row of zeros.
5. At 4:05: Circles the pivots and talks for a minute about the last obtained matrix.
6. At 5:04: Writes the basis on the right hand board; talks about alternative bases.
7. At 5:50: Writes down the answer to the question: dim = 3.
8. Summarizes and talks about alternative approach (vectors as columns).
9. At 6:39: Moves to the right, where she had prepared some work (the same vectors, but then as columns + the matrix after the elimination).
10. At 7:25: Stresses that she now cannot use the columns as basis.

Data collection

We created a questionnaire consisting of two pages, on which the above ten video sections were described with 4-5 cm space between, five on each page, in order to provide space for comments. During the workshop, we introduced our interest in the use of videos and gave illustrations of the variety of types of videos available on the web. Then we outlined the content of the MIT-video, defining it as "rather good" and giving the main headlines 'pre-knowledge', 'elimination of rows', 'pivots and basis' and 'another strategy' to describe its progress. The participants were asked to watch the video and indicate about each section whether it was aiming for conceptual or procedural knowledge, and additional comments could also be given. We deliberately did not offer definitions of what is meant by the notions of procedural and conceptual knowledge to avoid funnelling the participants' answers. These notions are frequently used by researchers, often without amplifying their meaning. By not giving the audience definitions, we wanted to get a grip on how the audience interpreted the conceptual and procedural notions - unaffected. Thereafter, we ran the video and the participants filled in the questionnaire. After the video was finished, we initiated a discussion, with questions: “What was good (both procedural and conceptual)?”, “What could have been done differently?” We made field notes of the comments. As the participants left, we collected 18 anonymous responses.

The data analysis process

To analyze the answers on the questionnaire we took advantage of the definition of conceptual and procedural knowledge provided by Bergsten and colleagues (2015). We first tried to organize the responses according to degrees of similarities, this resulted in quite many groups of responses, as few were to a large degree equal. Then, we discovered that most disagreements were on the first page. This made us decide to let the second page on the final five sections of the video be more important for coding. This choice could be supported by the argumentation that (1) in the final sections of the video the teacher was aiming at conceptual knowledge by explaining an alternative approach without losing time on calculations (see figure 1, right), and (2) the participants needed time to get used to the video and the questionnaire, thus the second page better represented their interpretations. This refinement made three categories crystalize: (1) participants who had interpreted most parts of the second half of the video as conceptual - the C-group; (2) participants who had interpreted most parts as procedural - the P-group; (3) participants who had answered either P-P-P-C-C or P-P-C-C-C, which we coded as the PP-□-CC-group. The remaining participants offered blank responses, or responses which were not written in terms of conceptual or procedural knowledge. This group was named “Answering something else or not answering at all”.

We are well aware of methodological limitations of our approach. The participants may have interpreted questions differently from what was intended, and we may have interpreted their answers incorrectly. The participants may not have been well enough prepared to characterize the sections in the video (some did not remember well the linear algebra). The English language in the video, in the workshop and in the questionnaire may have hindered (most participants were Norwegian), and so forth. Therefore, we take our results with caution.

Results

The participants’ responses yielded four groups. Below we will present their additional remarks in the questionnaire and their contributions to the discussion.

The C-group consisted of four participants. Their categorization of the different sections of the second half of the video was ‘conceptual’ or as one participant expressed: “conceptual about ‘what can a basis be?’”. There were also responses stating: “P→C, good: Clear about procedure, link to concepts”. In the group discussion, one of the participants explained this view. He emphasized that since there are linear algebra concepts, on which all the calculations in the video are based, his reading of the video was that most parts were aiming for conceptual knowledge. Here we observed an interpretation of conceptual knowledge as knowledge based on the presence of mathematical concepts - even when presenting only the “how?” of a procedure. Thus, because these participants recognized the underlying concepts, they judged it as aiming for conceptual knowledge.

The P-group consisted of five participants. They interpreted at least four of the five final sections in the video as procedural. One of the participants in this group interpreted nearly all ten sections as procedural writing: “Procedural, less explanation – doing aspect. Non-concept” and “Discussing strategies – not concepts”. In this group, a common view appeared to be that there was something missing: “Presents alternative strategy; - no or little discussion of the general idea behind” and “Procedural (relies on us to remember initial definition introduced)”. In the discussion, several participants stressed that in the video mathematical definitions were missing. They emphasized that definitions should have been given greater attention in the video. The participants in this group considered definitions as important constituents of teaching for conceptual knowledge.

The PP-□-CC-group consisted of four participants. They described the first two sections of the second half of the video as procedural. These sections showed the teacher concluding the first solution approach. The participants in the group did however not have a common interpretation of the ensuing section in the video (section 8), which we cannot explain. The final two sections in the video, referring to how an alternative way of solving the task can be done, was by all participants in this group interpreted as conceptual. An explanation offered was: “C: ‘What if we did something else’”. This indicates that the participant apprehends the variety in methods as a conceptual feature. The responses in this group seem to agree that the alternative solution approach aims at conceptual knowledge.

Group 4 ‘answering something else or not answering at all’ consisted of five participants. Some comments from this group were on quality of the explanations, such as: “Necessary to write how to transform one step to another in elimination process. But explanation was good”. There were also descriptive responses: “explains a little”. Another participant in this group wrote: “General comment: Linear algebra is outside my area, therefore lost focus and understanding of what was going on. Did also lose track of where we were in the video, thus there are not many fruitful comments here.” (translated). These responses could not be analyzed in terms of aiming for conceptual and/or procedural knowledge.

Discussion, conclusion and recommendations

Our research question was: 'What are, according to a group of mathematics-interested professionals, the sections in a teaching video that emphasize conceptual or procedural knowledge?' This question cannot clearly be answered because of diverging apprehensions by the participants of what they recognize as conceptual or procedural knowledge. We can discern several interpretations.

One interpretation is that teaching is judged as aiming for conceptual knowledge, if it is based on mathematical concepts. For the participants who were familiar with the concepts used in the MIT-video it was easy to relate the discussions and processes in the video to the mathematical arguments founding the processes. Thus, because these participants recognized the underlying mathematical concepts, they judged it as conceptual. However, any sequence in the video, whether aiming at procedural or conceptual knowledge, used linear algebra concepts. According to this interpretation then, as there were underlying concepts throughout, all sections were 'conceptual'. With all mathematical thinking and reasoning being based on mathematical concepts, this interpretation of conceptual knowledge will blur any distinction between procedural and conceptual knowledge.

A second interpretation is that a certain approach to teaching is judged as aiming for conceptual knowledge, if it includes formal definitions. Such definitions were lacking in the video, thus connections between concepts and their definitions are up to the viewers of the video to draw themselves. Lack of formal definitions made these respondents interpret the teaching in the video as aiming for procedural knowledge. The importance of formal definitions to mathematicians has been discussed by many researchers (o.a. Van Dormolen & Zaslavsky, 2003; Vinner, 1991), writing that the organization and presentation of mathematical content in textbooks and lectures are often based upon the assumption that concepts should be 'acquired' through definitions. However, the definitions of conceptual knowledge in the research literature do not mention formal definitions. In fact, conceptual understanding may be informal or intuitive, as long as it is rich in connections (Baroody et al., 2007; Hiebert, 1986).

Of the four groups in the study, it was only the PP-□-CC-group that made interpretations of teaching aiming at conceptual knowledge as being about offering relationships between concepts and solution approaches. One of the participants in the PP-□-CC-group put up a definition of what (s)he meant: "Procedural – talks about a method: What is going to be done first and last. How. Conceptual – short about why, (but mostly about what one has to do and the order)" (translated). This interpretation is quite in line with the definitions given in the literature on mathematics education research.

We started with a need for didactical quality descriptors for mathematical teaching videos and chose to study to what extent the explanations in videos can be judged as aiming at conceptual or at procedural knowledge. The dynamic interplay between conceptual and procedural knowledge (Baroody et al., 2007; Rittle-Johnson & Alibali, 1999) may at times make it hard for teachers to distinguish between the approaches. However, at times these are simple to observe: A teacher who just tells about the "how" is clearly procedurally oriented, and one who jumps over a calculation is clearly avoiding procedures. Our study shows that these notions do not yield reliable judgments at all when used by professionals in mathematics education, without first explaining, discussing, defining and explicating these terms. It can be assumed that a number of professionals in mathematics education aren't well aware of the definitions from the research literature. In particular, mathematicians who strongly stick to formal definitions as one of the bases of their explanations, may have misconceptions about conceptual understanding.

What is illuminated by the present project is that there are a number of typical combinations of conceptual and procedural interpretations of a mathematics lecture. The rather diverging interpretations in the first three groups – along with responses in the fourth group that mainly indicate uncertainty – illustrate that the understanding of the notions conceptual and procedural knowledge is rather diverging and, also, that these notions are 'difficult'.

The present project embraces only a small number of responses gained from a small part of the professional community. Thus, it is exploratory. Nevertheless, locating such divergences in a small group of professionals sends a signal of difficulties obtaining a unique apprehension within bigger communities. When studying a teacher explaining mathematics, whether this is within a teaching video or within a live classroom, the judgment of whether it is aiming for conceptual and procedural knowledge should be done. Asking professionals in mathematics education may yield unreliable results if the notions are not carefully defined, explained and discussed.

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