PAPER • **OPEN ACCESS**

Non-contact sensing and modelling on the voltage and electric field for powered transmission line

To cite this article: Yuan Fuqing and Jinmei Lu 2020 *J. Phys.: Conf. Ser.* **1453** 012115

Non-contact sensing and modelling on the voltage and electric field for powered transmission line

Yuan Fuqing*1, Jinmei Lu1

1Department of Engineering and Safety, University of Tromsø, 9037, Norway

Yuan.fuqing@uit.no

Abstract. From environmental, ecological or technical perspectives, the measurement of the potential voltage, electric field near the power transmission line is essential to ensure the exposure level does not exceed the limits of the government specified. This paper develops a potential voltage model, designs experiments to verify the developed model and to evaluate the parameters in the model. An electric field model is derived from the voltage model further, which can be used to estimate the electric field strength numerically. The models are found consistent with the observed data in the experiment.

1. Introduction
It has been an issue about the impact of the transmission line on the human health, ecology and environment. Standards have been issued to regulate the maximum exposure rate of electric filed, electromagnetic field on the human being. Referring to the European union exposure guideline, at frequency 50 Hz, the public exposure limit is 5000V/m, the occupational exposure limits is 10,000V/m[1] . Measuring the potential voltage around the transmission line is necessary to ensure it does not exceed the governmental regulation. From technical perspective, determining the components of voltage or current flowing in the power line is very necessary, e.g. one can analyze the harmonics and inter-harmonics from the measured AC (Alternating Current) signal. Moreover, it can help to evaluate the real-time power at a specific region. Another purpose of the measurement is for the EMC (Electromagnetic Compatibility) analysis. For example, for the power transmission line carrying high voltage with 132 kV or 275 kV[2], the nearby environment is highly polluted by the strong electric field. In the railway transportation, the overhead wire induces strong electric field that can interfere telecommunication appliances nearby. For some metro line, the residents living nearby is even exposed to the high electric field [3].

In general, this paper develops electric field model and designs experiment to measure the potential voltage and the electric field. The section 2 defines the problem and describes the developed mathematical electric field model using field theory. Section 3 describes the experiment design and the data measurement method. Section 4 performs data analysis using linear and nonlinear approach to evaluate the unknown parameters. Section 5 presents conclusion.

2. Problem Formulating and Electric Field Modelling
The flowing of AC current in the transmission line induces the change of electric field in the infinite space. This paper assumes the transmission line is one phase with two cables in the line: One is earth line and another is the powered line carrying AC current. It is a realistic assumption as the configuration

of the power cable in the household in some countries follows it. For a point P beneath the power line at length z, as shown in Figure 1, the strength of the electric field will decrease with the z.

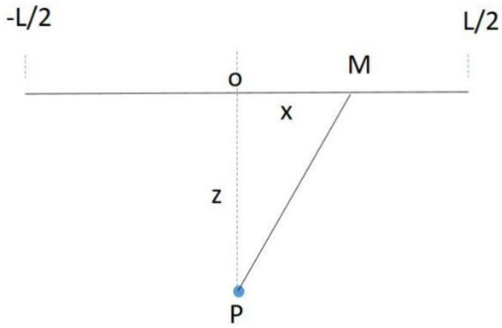

Figure 1. Field near a power line

Assume the radius of the power line is ignored at that moment, and the charges are distributed in the live line evenly without dimension, but is abstracted as a virtual line. Suppose the length of the line is L. The amount of charge at point M for an infinitesimal length is $\lambda\Delta x$. The λ is the amount of charge per unit length. The point of interest is P that locates beneath the middle point of the line (i.e. Point O in Figure 1). According to the Coulomb´s law, the electric field at the point P is [4]

$$E(\boldsymbol{Z}) = \frac{1}{4\pi\epsilon}.\frac{\lambda\Delta x}{Z^2}.\hat{Z} \qquad (1)$$

The bold $\boldsymbol{Z}$ represents vector variable and the $\hat{Z}$ represents a unit direction vector. In the coordinate system of two dimensions, the point P is represented as $\mathrm{z}\hat{Z}$, the M is represented as $\mathrm{x}\,\hat{X}$. The P-M, representing the distance between P and M, is equal to PM=$\mathrm{z}\hat{Z} - \mathrm{x}\,\hat{X}$. The scale of the PM is $\sqrt{z^2 + x^2}$, the unit vector is then $\frac{\mathrm{z}\hat{Z}-\mathrm{x}\,\hat{X}}{\sqrt{z^2+x^2}}$. The total electric field at the point P from -L/2 to L/2 shown in Figure 1 is an integral form as

$$E(\boldsymbol{Z_P}) = \int_{-L/2}^{L/2} \frac{1}{4\pi\epsilon}.\frac{\lambda}{Z^2}.\hat{Z}dx \qquad (2)$$

which is equivalent to

$$E(\boldsymbol{Z_P}) = \int_{-L/2}^{L/2} \frac{1}{4\pi\epsilon}.\frac{\lambda}{z^2+x^2}.\frac{\mathrm{z}\hat{Z}-\mathrm{x}\,\hat{X}}{\sqrt{z^2+x^2}}dx \qquad (3)$$

The (3) is rewritten as

$$E(\boldsymbol{Z_P}) = \frac{\lambda}{4\pi\epsilon}\int_{-L/2}^{L/2} \frac{\mathrm{z}\hat{Z}}{(z^2+x^2)^{3/2}}dx - \frac{\lambda}{4\pi\epsilon}\int_{-L/2}^{L/2} \frac{\mathrm{x}\,\hat{X}}{(z^2+x^2)^{3/2}}dx \quad (4)$$

Readily, it can be found out the $\int_{-L/2}^{L/2} \frac{\mathrm{x}\,\hat{X}}{(z^2+x^2)^{3/2}}dx = 0$, which implies the effect of electric filed in the x direction cancelled on the Point P. Since $\int \frac{\mathrm{z}}{(z^2+x^2)^{3/2}}dx = \frac{\mathrm{x}}{z^2(z^2+x^2)^{1/2}}$, the (4) is hence

$$E(\boldsymbol{Z_P}) = \frac{\lambda}{4\pi\epsilon}.\frac{\mathrm{L}}{z\sqrt{z^2+L^2/4}}\hat{Z} \qquad (5)$$

The electric filed only exists in the z direction. The x direction is zero because they are with opposite directions for line to the life and to the right of the O.

The voltage difference between any point from the line and the point of interest P is same since the wire is a conductor. This can simplify the potential calculation significantly, as the problem can be considered as voltage drop from Point O to point P, owing to the electric field only exists in the z direction. To avoid some mathematical difficulty, we would like to model the difference voltage between P and D, instead to model the potential voltage for the point P.

$$V(\boldsymbol{OP}) - V(\boldsymbol{OD}) = -\int_d^h \frac{\lambda}{4\pi\epsilon}.\frac{\mathrm{L}}{z\sqrt{z^2+L^2/4}}\hat{Z}dz \qquad (6)$$

Since the $\int \frac{\mathrm{L}}{z\sqrt{z^2+L^2/4}}\,dz = -2\ln\frac{\frac{L}{2}+\sqrt{z^2+L^2/4}}{z}$, the (6) is

$$-2\ln\frac{\frac{L}{2}+\sqrt{h^2+\frac{L^2}{4}}}{h}+2\ln\frac{\frac{L}{2}+\sqrt{d^2+\frac{L^2}{4}}}{d}=2\ln\frac{\frac{\frac{L}{2}+\sqrt{d^2+\frac{L^2}{4}}}{d}}{\frac{\frac{L}{2}+\sqrt{h^2+\frac{L^2}{4}}}{h}}=2\ln\left(\frac{\frac{L}{2}+\sqrt{d^2+\frac{L^2}{4}}}{\frac{L}{2}+\sqrt{h^2+\frac{L^2}{4}}}.\frac{h}{d}\right) \quad (7)$$

When the length of power line $L\to+\infty$, which is a realistic assumption as the power line is normally long, the (7) is simplified as $2\ln\frac{h}{d}$. The (6) is equal to

$$V(\boldsymbol{OP})-V(\boldsymbol{OD})=\frac{\lambda}{2\pi\epsilon}\ln\frac{d}{h} \quad (8)$$

The voltage difference between a point P and a point D outside of the power line modelled by the (8). The ϵ is the permittivity of the air is assumed as constant, whereas it would vary with the humidity of the air, wind speed, air pressure, the ambient natural condition and etc.

3. Experiment Setup and Data Acquisition

The transmission line is chosen as one phase cable with 320V amplitude and 50 HZ frequency. It is an in house cable with white plastic cover. The voltage is measured at outside of the cable at the distance of 0 to 20 centimeter with step size 1. As expected, the measured voltage also exhibits as AC signal with frequency around 50 HZ which is also the main component of the signal, as shown in the right figure of Figure 2. In another word, the voltage in (2) is a periodic signal with noise and harmonic frequency where the 50 HZ is the main component. The Formula (2) can still hold when the periodicity and harmonic present. In this experiment, we just record the amplitude of the AC signal. Oscilloscope is used to perform the measurement on the voltage.

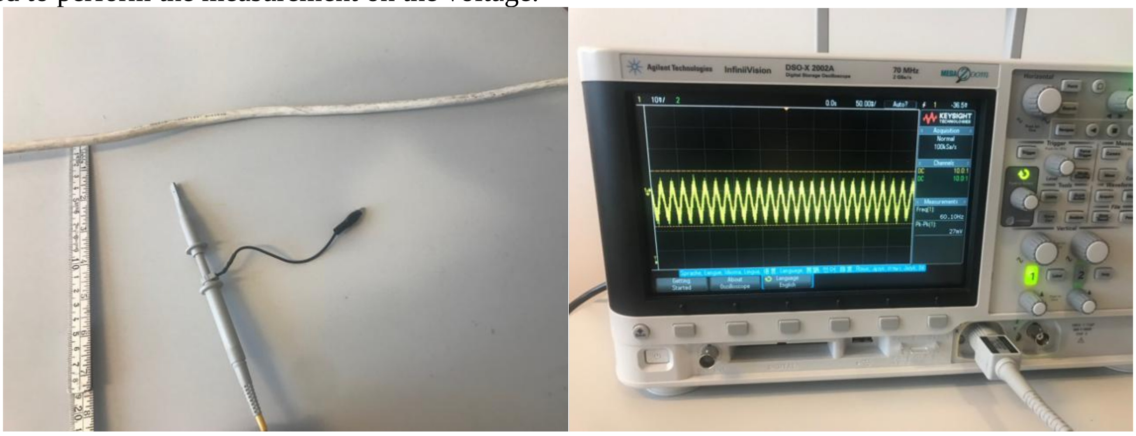

Figure 2. Voltage Measurement

In the model (8), the charges essentially distribute outside the surface of the conductor, instead of stationing in the middle of line with an unrealistic assumption of dimensionlessness. Nonetheless, the charges is still equivalent to the charges residing inside the middle of the wire. Assume the surface to the core of the line is d_0, the thickness of the plastic cover of the power line is d_1. The voltage difference (8) is then rewritten as

$$\Delta V=\frac{\lambda}{2\pi\epsilon}\ln\frac{d_0+d_1}{d_0+d_1+h} \quad (9)$$

Since both the d_0 and d_1 are unknown, the (9) can be simplified into a two parameters function

$$-\Delta V=\frac{\lambda}{2\pi\epsilon}\ln\left(1+\frac{h}{d}\right) \quad (10)$$

where the $\Delta V\geq 0$. In the measurement experiment, the voltage at the points of the distance h from the outside surface of the cable line are measured. The measured data are tabulated in Table 1. The $\Delta V=V_0-V_1$ can be calculated from the Table 1.

Table 1. Raw Measurement Data

Distance h (cm)	Vol (mV)	Distance h (cm)	Vol (mV)
0	360	10	180
1	340	11	160
2	300	12	160

3	280	13	160
4	260	14	140
5	240	15	140
6	240	16	120
7	220	17	120
8	200	18	120
9	200		

4. Data Analysis

The observed data from Table 1 are used to evaluate the parameters in (10). The (10) is a nonlinear function that cannot be converted into linear function by transformation. But from experience, the $\frac{h}{d} \ll 1$ for most points. The $\ln\left(1+\frac{h}{d}\right) \approx \ln\left(\frac{h}{d}\right)$. The (10) is hence

$$\Delta V = \frac{\lambda}{2\pi\epsilon} \ln\left(\frac{h}{d}\right) = k \ln\left(\frac{h}{d}\right) \tag{11}$$

It can be further rewritten as

$$\ln h = \ln d + \frac{1}{k}\Delta V \tag{12}$$

It is of linear function

$$y = \mathrm{m}x + \mathrm{b} \tag{13}$$

Then

$$k = \frac{1}{m}; \quad d = \mathrm{e}^{\mathrm{b}} \tag{14}$$

Using the least square method, the slope and threshold in the linear function can be uniquely determined as m =0.0119, b $= 0.1072$. Then the $k = 84.14$, $d = 1.1132$. The R-Square value for the regression is 98.30, implying the regression fitting the data satisfactorily. A plot for the regression as shown in Figure 3.

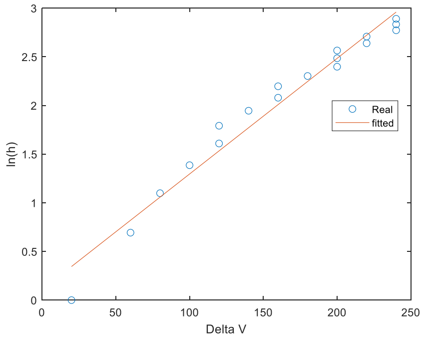

Figure 3. Regression Plot

For the simplified linear model, the data in general fits the model. However, as shown in the Figure 3, an obvious curvature exist from the plotted original data, which implies the linear data does not fit the data at its best. We have to resort a nonlinear regression that directly estimates the parameters from (10). None linear least square is used to minimize the squared error. Trust-region-reflective algorithm is performed to find the optimal values, i.e. $k = 153.6$, $d = 4.45$. Nonlinear regression is normally with multiple local optimums. It is hard to find out which values is best and if the best has been found, but we can plot the fitted lines to check if it fits the data or not. As in Figure 4, the nonlinear regression exhibits a much better fitting than the linear one. For comparison purpose, we plot the nonlinear function for the parameters from linear regression. The linear method shows worse fitting performance, due to the $\ln\left(1+\frac{h}{d}\right) \approx \ln\left(\frac{h}{d}\right)$ does not hold when the $\frac{h}{d}$ is smaller, as shown in the Figure 4, where the “not fitting” appears at the point where the h is small.

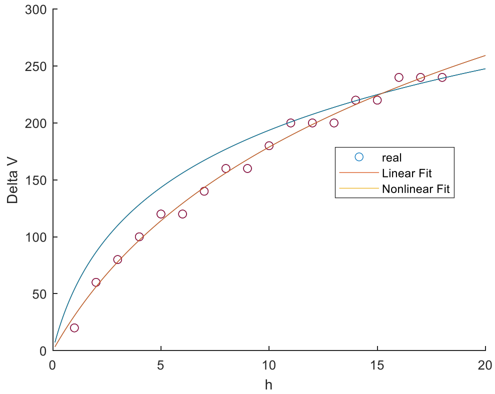

Figure 4. Comparison of fittings

The voltage difference model is $-\Delta V = 153.6\ln\left(1+\frac{h}{4.45}\right)$. The data fits the nonlinear regression from mathematical perspective. Therefore, we can conclude the model (5) and (9) can model the voltage potential. When the k and d are known, the electric filed can be calculated. When $L \to +\infty$, the (5) can be rewritten as

$$E(\boldsymbol{Z}) = \frac{\lambda}{2\pi\epsilon}.\frac{1}{z}\hat{Z} = k.\frac{1}{z}\hat{Z} \qquad (15)$$

The electric field points to or from the power line direction. The scale of the electric field is as

$$E(r) = k.\frac{1}{r} \qquad (16)$$

For the distance from 1cm, the electric field is 15.36 V/m, for r=0.1cm, the electric field is 153.6 V/m, which are much lower than the regulation specified 5000V/m. Figure 5 plots the electric field strength against the distance.

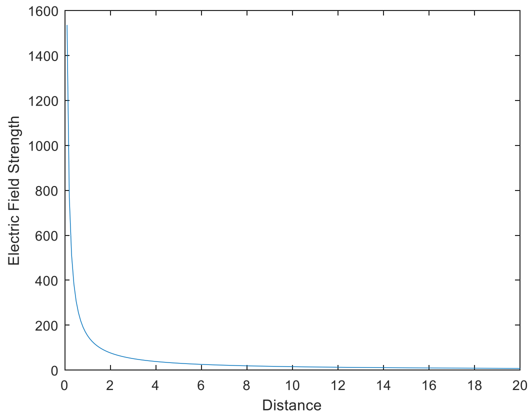

Figure 5. Electric field strength away from power line

The electric field dies out against the distance. It drops significantly when it is away from the power line. When distance comes 10 cm, its electric field will be very weak with only about 1.5 V/m, which implies the strength has almost died out, referring to the electric field event in the air inside a city can be maximum 5 V/m [5].

5. Conclusion

The voltage model is consistent with the observed measurement data, for either the simplified linear method or nonlinear regression method, whereas the nonlinear method has better fitting performance. This model captures the essentials of how the voltage drops when it is away from the transmission cable in the form of $\ln\left(\frac{h}{d}\right)$. The electric field can be numerically computed after the parameters have been estimated from the experimental data. The numerical electric field strength is of importance for the environment or EMC engineering to estimate possible effect on the health, ecosystem or facilities whereabouts.

References

[1] S. M. Hamed Ahmadi, Akmal Akmal, "ELECTROMAGNETIC FIELDS NEAR TRANSMISSION LINES – PROBLEMS AND SOLUTIONS," *Iran. J. Environ. Health. Sci. Eng.,*, vol. 7, no. 2, pp. 181-188, 2010.
[2] S. A. F. R.K.Z. Sahbudin, S. Hitam,M. Mokhtar, "Investigation of Electrical Potential and Electromagnetic Field for Overhead High Voltage Power Lines in Malaysia," *Journal of Applied Sciences,* vol. 10 (22), pp. 2862-2868, 2010.
[3] Y. F. D. Galar, "Harmonic and Inter-harmonic Analysis on Power Signal from Railway Traction Systems," *International journal of COMADEM,* vol. 20, no. 2, 2017.
[4] D. J.Griffiths, *Introduction to Electrodynamics*, 4th ed. New York: Pearson, 2013.
[5] Ç. K. B. K. Engiz, "Measurement and Evaluation of Electric Field Strength in Samsun City Center " *International Journal of Applied Mathematics, Electronics and Computers,* vol. 4, pp. 24-29, 2016.