A Stochastic Closed-Loop Supply Chain Network Optimization Problem Considering Flexible Network Capacity

Hao Yu[1], Wei Deng Solvang[1] and Xu Sun[1]

Abstract. Nowadays, due to the concern of environmental challenges, global warming and climate change, companies across the globe have increasingly focused on the sustainable operations and management of their supply chains. Closed-loop supply chain (CLSC) is a new concept and practice, which combines both traditional forward supply chain and reverse logistics in order to simultaneously maximize the utilization of resource and minimize the generation of waste. In this paper, a stochastic CLSC network optimization problem with capacity flexibility is investigated. The proposed optimization model is able to appropriately handle the uncertainties from different sources, and the network configuration and decisions are adjusted by the capacity flexibility under different scenarios. The sample average approximation (SAA) method is used to solve the stochastic optimization problem. The model is validated by a numerical experiment and the result has revealed that the quality and consistency of the decision-making can be dramatically improved by modelling the capacity flexibility.

Keywords: Closed-loop supply chain, Network design, Location problem, Stochastic optimization, Sample Average Approximation

1 Introduction

In today's global market, the competition is not only between different individual enterprises but also largely between different supply chains. The effectiveness and efficiency in handling material flow, information flow and capital flow within a supply chain will determine the profitability and success of a company. Supply Chain Management (SCM) aims, through decision-makings at both strategic level and operational level, at properly managing different players and flows within a supply chain in order to maximize the total supply chain surplus or profit [1].

Network design is one of the most essential strategic decisions in SCM, which formulates the configuration of a supply chain through facility selection and determines the operational strategies. Traditionally, the design of a supply chain only focuses on

[1] Hao Yu (✉), Wei Deng Solvang and Xu Sun
Department of Industrial Engineering, Faculty of Engineering Science and Technology, UiT The Arctic University of Norway, Narvik, Norway
{hao.yu, wei.d.solvang, xu.sun}@uit.no

the forward direction from raw material supplier towards end customer. However, due to the concern of environmental challenges, global warming and climate change from the whole society, increasing attention has been paid to the value and resource recovery through reverse logistics activities [2, 3]. Closed-loop supply chain (CLSC) is a new concept and practice, which combines both traditional forward supply chain and reverse logistics in order to simultaneously maximize the utilization of resources and minimize the generation of wastes. Compared with the traditional supply chain network design, the planning of a CLSC is more complicated due to the involvement of more players. Furthermore, reverse logistics involves more uncertainties compared to the forward supply chain [3], and this needs to be appropriately treated in a CLSC network design problem.

Due to the aforementioned complexity of the CLSC network design problem, significant efforts have been given in order to develop advanced optimization models and algorithm for a better decision-making. Yi et al. [4] developed a mixed integer linear program for minimizing the total cost of a retailer oriented CLSC for the recovery of construction machinery. The model was solved by an enhanced genetic algorithm and was validated through a case study in China. Özceylan et al. [5] proposed a linear program for maximizing the total profit generation of an automotive CLSC. The model was solved by CPLEX solver and was validated by a real world case study in Turkey. Taking into account of the recovery options, Amin et al. [6] investigated a tire manufacturing CLSC network optimization problem, which was validated by a real world case study in Canada.

In addition to the economic incentives from incorporating reverse logistics activities, some research works considered the overall environmental performance of a CLSC. Hasanov et al. [7] formulated a mathematical model for the optimization of a CLSC network design problem considering remanufacturing options. The model aims at minimizing the total cost and emission cost of greenhouse gas (GHG) through optimal decision-making on both production planning and inventory management. Taleizadeh et al. [8] investigated a bi-objective optimization model for CLSC network design considering the balance between total cost and total CO_2 emission. A fuzzy Torabi-Hassini (TH) method was used to solve the multi-objective optimization problem.

Due to the complexity of the proposed mathematical models, significant computational efforts may be required to solve the optimization problems of CLSC network design. Therefore, several research works have been conducted to develop improved algorithm. Soleimani and Kannan [9] developed a hybrid genetic algorithm (GA) and particle swarm optimization (PSO) for improving the computational efficiency of a multi-period and multi-level CLSC network optimization model. Chen et al. [10] investigated a location-allocation problem for the CLSC network design of cartridge recycling, which was solved by an enhanced two-stage GA. Hajipour et al. [11] formulated a non-linear mixed integer program for maximizing the profit generation in CLSC network design. Two metaheuristics: PSO and greedy randomized adaptive search procedure (GRASP) were employed to solve the proposed mathematical model.

The treatment of uncertainty within the life cycle of a CLSC is another focus of the recent modeling efforts. Zhen et al. [12] proposed a two-stage stochastic optimization model for optimizing the decision-making of facility location and capacity allocation in a CLSC, and an enhanced Tabu search algorithm was developed to solve the model. Jeihoonian et al. [13] formulated a two-stage stochastic model for CLSC network design considering uncertain quality. Mohammed et al. [14] proposed a stochastic optimization model for minimizing the total cost of CLSC network design. The model was further incorporated with different carbon policies in order to test their effectiveness in carbon reduction.

In this paper, we developed a new two-stage stochastic mixed integer program for CLSC network optimization. Compared with the existing models, the main difference is the capacity flexibility is taken into account in order to improve the stability and consistency of the objective values under different scenarios. In addition, the sample average approximation (SAA) method is used to test the performance of the proposed mathematical model.

2 Mathematical Model

In this paper, we considered a network optimization problem of a single-product multi-echelon CLSC. As shown in Fig.1, the forward supply chain consists of manufacturer, wholesaler/distributor and customer. The reverse logistics activities are performed at collection center, disposal center and recycling center. The material flows between different facilities are given in Fig.1.

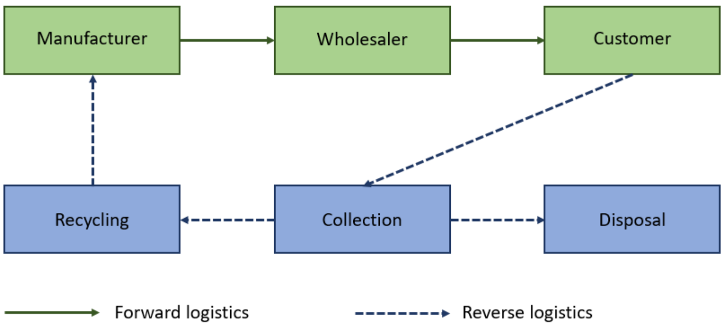

Fig. 1. Network structure of a CLSC.

In this paper, the flexible network capacity is taken into account and is formulated in the mathematical model. The capacity limitation in a traditional facility location model may lead to unstable objective values and sub-optimal decisions under a stochastic environment [3, 15]. For example, because of the rigid capacity constraint, one more facility may be opened for dealing with a small increase on the customer demand in some scenarios, which results in unreasonable decisions and inefficient use of capacity opened. Due to this reason, the flexible network capacity is formulated as a penalty in the objective function in order to solve the problem and generate reasonable decisions. In practice, the inclusion of the flexible network capacity is a more realistic representation of the decision-making problem of CLSC network design, which enables different interpretations under different conditions, i.e., increase of facility capacity, outsourcing options, hire of temporary or seasonal workers, or even loss of sales. In addition, the uncertainty related to the customer demand in the forward supply chain and the rate of waste generation and the quality level in the reverse logistics are taken into consideration and are formulated as stochastic parameters.

$$\begin{aligned}
\boldsymbol{Min}\ \text{cost} =& \Big(\sum_{m=1}^{M} F_m i_m + \sum_{w=1}^{W} F_w i_w + \sum_{c=1}^{C} F_c i_c + \sum_{r=1}^{R} F_r i_r\Big) \\
&+ \sum_{s=1}^{S} U_s \Bigg(\Bigg(\sum_{m=1}^{M} P_m \left(Ap_m^s + \sum_{r=1}^{R} A_{rm}^s\right) + \sum_{w=1}^{W} P_w \sum_{m=1}^{M} A_{mw}^s \\
&+ \sum_{c=1}^{C} P_c \sum_{v=1}^{V} A_{vc}^s + \sum_{r=1}^{R} P_r \sum_{c=1}^{C} A_{cr}^s\Bigg) \\
&+ \Bigg(\sum_{m=1}^{M} \sum_{w=1}^{W} C_{mw} A_{mw}^s + \sum_{w=1}^{W} \sum_{v=1}^{V} C_{wv} A_{wv}^s + \sum_{v=1}^{V} \sum_{c=1}^{C} C_{vc} A_{vc}^s \\
&+ \sum_{c=1}^{C} \sum_{r=1}^{R} C_{cr} A_{cr}^s + \sum_{c=1}^{C} \sum_{d=1}^{D} C_{cd} A_{cd}^s + \sum_{r=1}^{R} \sum_{m=1}^{M} C_{rm} A_{rm}^s\Bigg) \\
&+ \sum_{m=1}^{M} Pu_m Ap_m^s + \sum_{v=1}^{V} O_v A_v^s + \sum_{v=1}^{V} Or_v Ar_v^s \\
&+ \sum_{d=1}^{D} P_d \sum_{c=1}^{C} A_{cd}^s\Bigg)
\end{aligned} \tag{1}$$

Subject to:

$$D_v^s \leq \sum_{w=1}^{W} A_{wv}^s + A_v^s, \forall s, v \tag{2}$$

$$\vartheta^s D_v^s \leq \sum_{c=1}^{C} A_{vc}^s + Ar_v^s, \forall s, v \tag{3}$$

$$Ap_m^s + \sum_{r=1}^{R} A_{rm}^s = \alpha \sum_{w=1}^{W} A_{mw}^s, \forall s, m \tag{4}$$

$$\sum_{m=1}^{M} A_{mw}^s = \sum_{v=1}^{V} A_{wv}^s, \forall s, w \tag{5}$$

$$\sum_{v=1}^{V} A_{vc}^s = \sum_{r=1}^{R} A_{cr}^s + \sum_{d=1}^{D} A_{cd}^s, \forall s, c \tag{6}$$

$$q^s\beta\sum_{v=1}^{V}A_{vc}^s=\sum_{r=1}^{R}A_{cr}^s\,,\forall s,c \tag{7}$$

$$\gamma\sum_{c=1}^{C}A_{cr}^s=\sum_{m=1}^{M}A_{rm}^s\,,\forall s,r \tag{8}$$

$$Ap_m^s+\sum_{r=1}^{R}A_{rm}^s\leq Cap_m i_m,\forall s,m \tag{9}$$

$$\sum_{m=1}^{M}A_{mw}^s\leq Cap_w i_w,\forall s,w \tag{10}$$

$$\sum_{v=1}^{V}A_{vc}^s\leq Cap_c i_c,\forall s,c \tag{11}$$

$$\sum_{c=1}^{C}A_{cr}^s\leq Cap_r i_r,\forall s,r \tag{12}$$

$$\sum_{c=1}^{C}A_{cd}^s\leq Cap_d,\forall s,d \tag{13}$$

$$\sum_{v=1}^{V}A_v^s\leq Uo,\forall s \tag{14}$$

$$\sum_{v=1}^{V}Ar_v^s\leq Uro,\forall s \tag{15}$$

Objective function (1) minimizes the total cost that is comprised of fixed facility cost, processing cost, transportation cost, purchasing cost, flexible network capacity cost and disposal cost. Besides, the model includes 14 constraints. Constraints (2) and (3) require the CLSC system should be capable to deal with the customer demands in both forward and reverse directions. Constraints (4) and (5) specify the relationship between the input and output amount in the forward channels. Constraints (6)-(8) balance the material flows in the reverse logistics. Constraints (9)-(13) are capacity requirements of respective facilities. Constraints (14) and (15) give the upper limits of the flexible network capacity in both forward and reverse logistics. Besides, the decision variables fulfill their respective binary and non-negative requirements.

3 Algorithm

Eq. (16) defines a generic form of a two-stage stochastic optimization problem, which has the same structure as a CLSC network optimization problem. The first stage-decisions should be robust to withstand the change of the external environment under which the system is operated, and the second-stage decisions should be flexible to adapt those changes and can be easily altered in order to maximize the system performance.

Solving a stochastic programming model is a complex optimization problem that may require large computational efforts. In this paper, a sample average approximation (SAA) method is employed in order to obtain the optimal objective value of a large stochastic optimization problem with a great number of scenarios.

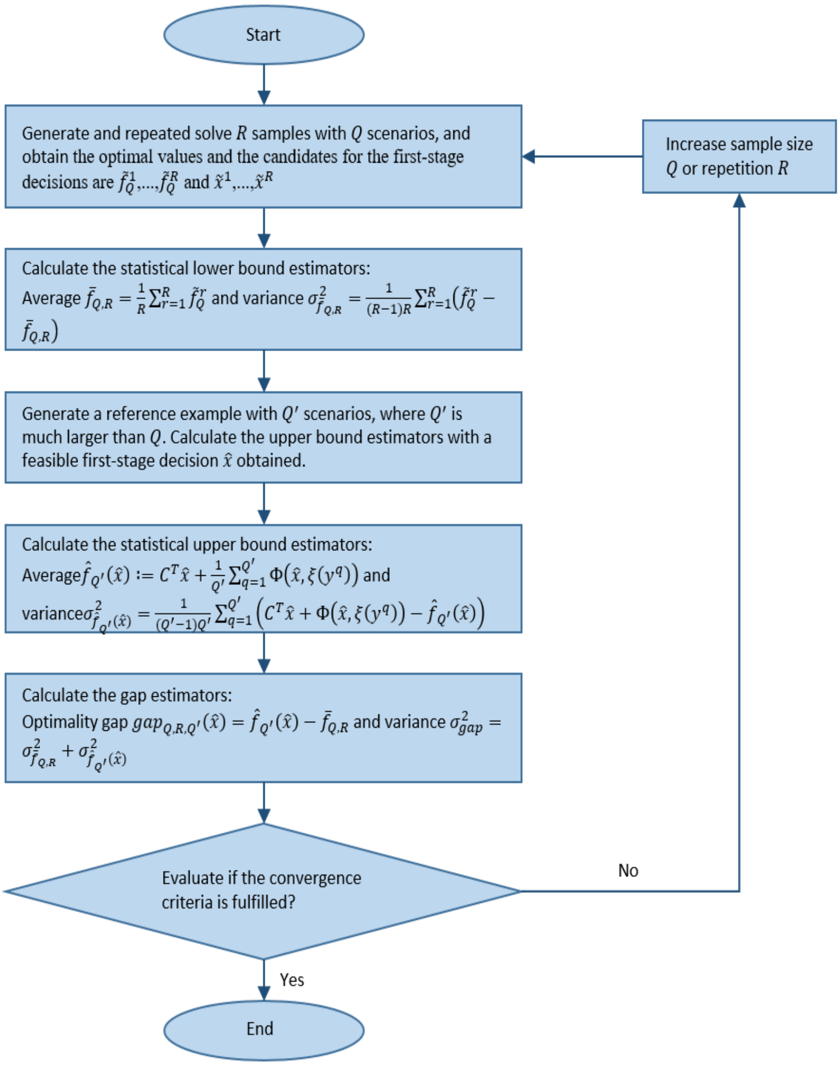

Fig. 2. Algorithmic procedures of the SAA.

$$\min_{x,y\,\in\,\Theta}\left\{f(x,y) := C^T x + \mathbb{E}_{\mathcal{P}}\left[\Phi\left(x,\xi(y)\right)\right]\right\} \tag{16}$$

$$\min_{x, y \in \Theta} \left\{ \tilde{f}_Q(x,y) := C^T x + \frac{1}{Q}\sum_{q=1}^{Q} \Phi\big(x, \xi(y^q)\big) \right\} \tag{17}$$

With the SAA, the optimal objective value is approximated by solving a set of randomly generated small problems repeatedly instead of solving the original problem directly, as shown in Eq. (17). In such a way, the computational efforts required is manageable. Fig.2 illustrates the algorithmic procedures of the SAA method. For more details of the solution method, the research works given by Verweij et al. [16] and Kleywegt et al. [17] can be referred.

4 Experiment and Discussion

In order to illustrate the application of the proposed model for CLSC network optimization, this section presents a computational experiment based on a set of randomly generated parameters. The stochastic parameters are generated from uniform distribution of respective parameter intervals. Besides, we investigated the performance of three different sample sizes: 10, 30 and 50, respectively. All the optimizations were performed with Lingo 18.0 solver. The results are presented in Fig. 3 and Fig. 4.

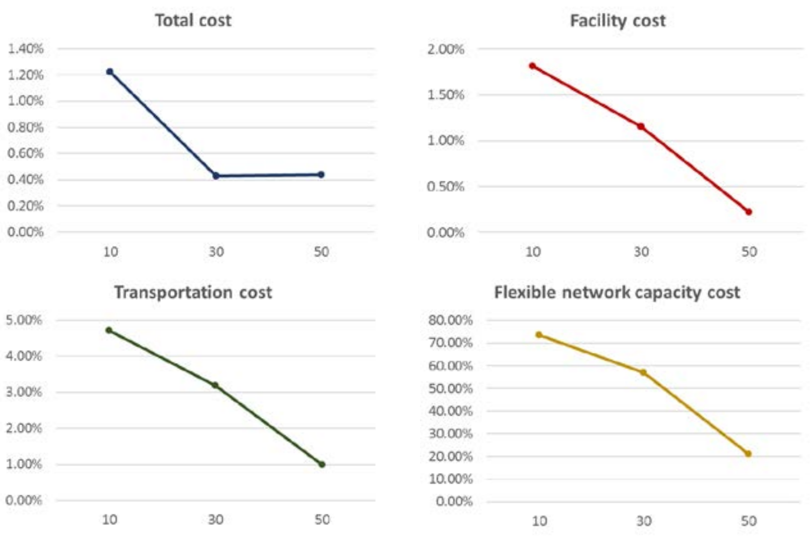

Fig. 3. CV of the total cost, facility operating cost, transportation cost and flexible network capacity cost.

First, the in-sample stability is tested with coefficient of variation (CV) that is obtained using $CV = {}^{\sigma}/_{\mu}$. Fig. 3 illustrates the CVs of the total cost as well as different cost components. With the increase of sample size, the CVs of all relevant cost components reduce, which reveals an improvement on the in-sample stability. When the sample size increases from 30 to 50, the improvement on the in-sample stability of the total cost is negligible. In addition, compared with the CVs of other cost components, the CV of flexible network capacity cost is extremely high. This can be explained that the flexible network capacity can be used as an adjustment factor for mitigating the negative impact on the first-stage network decisions and the objective values from uncertainty. In such a way, the unsatisfied demand in some scenarios can be fulfilled by the flexible network capacity, i.e., outsourcing, instead of opening new facilities, which may dramatically reduce the in-sample stability and result in a low capacity utilization.

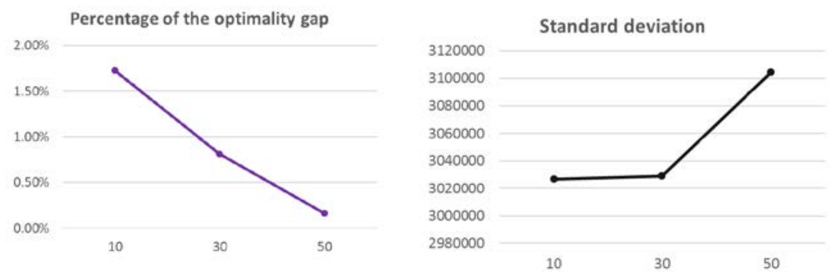

Fig. 4. Percentage of the optimality gap and standard deviation.

Then, the quality of the SAA solutions are tested with the reference example. As shown in Fig. 4, the optimality gap reduces significantly with the increase of sample size. Compared with 10 scenarios, the optimality gap is decreased by 90.6% when 50 scenarios are used. However, in this case, the combined standard deviation will be increased by 2.51%. Thus, the solution quality of the stochastic optimization problem can be improved drastically with the increase of the sample size. It is noteworthy that the selection of the sample size is based upon a trade-off analysis between quality of solution and computational efforts required.

5 Conclusions

In this paper, a novel two-stage mixed integer programming model is formulated for the network optimization of a single-product multi-echelon CLSC. The model aims at minimizing the total cost for opening and operating the CLSC through optimal decision-makings on both facility locations and transportation strategies. Compared with the existing optimization models, this model takes the flexible network capacity into account and thus formulates a penalty in the objective function. In order to solve the proposed model, the SAA method is used. The result of the computational experiment has shown that the inclusion of the flexible network capacity can significantly improve the in-sample stability, and the increase on sample size will improve the quality of solution of a large stochastic optimization problem.

For further improvement of the current research, two suggestions are given. First, the environmental impact and policies, i.e., different carbon policies or strategies [3, 14], may be formulated in the CLSC network optimization problem under an uncertain environment. Second, different alternatives may be tested in order to increase the network flexibility [18].

Notations

Set and Parameters	
M	Index of manufacturer, *m*=1,...,*M*
w	Index of wholesaler, *w*=1,...,*W*
v	Index of customer, *v*=1,...,*V*
C	Index of collection center, *c*=1,...,*C*
D	Index of disposal center, *d*=1,...,*D*
R	Index of recycling center, *r*=1,...,*R*
S	Index of scenario, *s*=1,...,*S*
F_m, F_w, F_c, F_r	Fixed opening cost of respective plants
P_m, P_w, P_c, P_d, P_r	Unit processing cost at respective plants
U_s	Probability of occurence
Pu_m	Purchasing cost of materials
O_v, Or_v	Flexible network capacity cost in both forward and reverse logistics
D_v^s	Customer demand from respective locations
ϑ^s	Conversion rate to used products
α	Materials required for assembling one product
q^s	Quality level
β, γ	Conversion fraction at respective plants
Cap_m, Cap_w, Cap_c, Cap_d, Cap_v	Capacity of respective plants
Uo, Uro	Upper limits on flexible network capacity in both forward and reverse logistics
Variables	
i_m, i_w, i_c, i_r	Binary decision variables for the location decision on respective candidates
A_{mw}^s, A_{wv}^s, A_{vc}^s, A_{cr}^s, A_{cd}^s, A_{rm}^s	Amount of products transported on respective links
Ap_m^s	Amount of materials purchased
A_v^s, Ar_v^s	Amount of flexible network capacity used in both forward and reverse logistics

Reference

1. Chopra, S., and Meindl, P.: 'Supply chain management: Strategy, planning, and operation', 2016

2. Yu, H., and Solvang, W.D.: ‘A general reverse logistics network design model for product reuse and recycling with environmental considerations’, The International Journal of Advanced Manufacturing Technology, 2016, 87, (9-12), pp. 2693-2711
3. Yu, H., and Solvang, W.D.: ‘A carbon-constrained stochastic optimization model with augmented multi-criteria scenario-based risk-averse solution for reverse logistics network design under uncertainty’, Journal of Cleaner Production, 2017, 164, pp. 1248-1267
4. Yi, P., Huang, M., Guo, L., and Shi, T.: ‘A retailer oriented closed-loop supply chain network design for end of life construction machinery remanufacturing’, Journal of Cleaner Production, 2016, 124, pp. 191-203
5. Özceylan, E., Demirel, N., Çetinkaya, C., and Demirel, E.: ‘A closed-loop supply chain network design for automotive industry in Turkey’, Computers & industrial engineering, 2017, 113, pp. 727-745
6. Amin, S.H., Zhang, G., and Akhtar, P.: ‘Effects of uncertainty on a tire closed-loop supply chain network’, Expert Systems with Applications, 2017, 73, pp. 82-91
7. Hasanov, P., Jaber, M., and Tahirov, N.: ‘Four-level closed loop supply chain with remanufacturing’, Applied Mathematical Modelling, 2019, 66, pp. 141-155
8. Taleizadeh, A.A., Haghighi, F., and Niaki, S.T.A.: ‘Modeling and solving a sustainable closed loop supply chain problem with pricing decisions and discounts on returned products’, Journal of Cleaner Production, 2019, 207, pp. 163-181
9. Soleimani, H., and Kannan, G.: ‘A hybrid particle swarm optimization and genetic algorithm for closed-loop supply chain network design in large-scale networks’, Applied Mathematical Modelling, 2015, 39, (14), pp. 3990-4012
10. Chen, Y., Chan, F., and Chung, S.: ‘An integrated closed-loop supply chain model with location allocation problem and product recycling decisions’, International Journal of Production Research, 2015, 53, (10), pp. 3120-3140
11. Hajipour, V., Tavana, M., Di Caprio, D., Akhgar, M., and Jabbari, Y.: ‘An Optimization Model for Traceable Closed-Loop Supply Chain Networks’, Applied Mathematical Modelling, 2019
12. Zhen, L., Sun, Q., Wang, K., and Zhang, X.: ‘Facility location and scale optimisation in closed-loop supply chain’, International Journal of Production Research, 2019, pp. 1-19
13. Jeihoonian, M., Zanjani, M.K., and Gendreau, M.: ‘Closed-loop supply chain network design under uncertain quality status: Case of durable products’, International Journal of Production Economics, 2017, 183, pp. 470-486
14. Mohammed, F., Selim, S.Z., Hassan, A., and Syed, M.N.: ‘Multi-period planning of closed-loop supply chain with carbon policies under uncertainty’, Transportation Research Part D: Transport and Environment, 2017, 51, pp. 146-172
15. King, A.J., and Wallace, S.W.: ‘Modeling with stochastic programming’, Springer Science & Business Media, 2012.
16. Verweij, B., Ahmed, S., Kleywegt, A.J., Nemhauser, G., Shapiro, A.: ‘The sample average approximation method applied to stochastic routing problems: a computational study’, Computational Optimization and Applications, 2003, 24, (2-3), pp. 289-333
17. Kleywegt, A.J., Shapiro, A., and Homem-de-Mello, T.: ‘The sample average approximation method for stochastic discrete optimization’, SIAM Journal on Optimization, 2002, 12, (2), pp. 479-502
18. Yu, H., and Solvang, W.D.: ‘Incorporating flexible capacity in the planning of a multi-product multi-echelon sustainable reverse logistics network under uncertainty’, Journal of Cleaner Production, 2018, 198, pp. 285-303