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A Slow-Motion Detecting Algorithm using High Order Statistic Approach

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Abstract. Motion detection is vital for consumer electronics and the Internet of things (IOT). For a scenario where the motion is slow and gentle, the resolution of the motion sensor is critical for the detection, while the algorithm development is another critical issue to differentiate the motion signal from noise measurement. This paper investigates the feasibility of using higher order statistics kurtosis as a motion indicator. Statistical hypothesis test has been developed to assess the motion presence. Several experiments are conducted to test the feasibility and performance of the approach. The results show the approach is feasible, but with some limitations.

1. Introduction

The motion detection is vital in the smart devices for motion recognizing such as detecting the rotation of screen, measuring the movement of hand-held devices. A typical example is the step counting commonly used in the smartphone and wearable devices. In the industry, motion detection is applied to measure the acceleration of the machine, the movement of machinery, or to detect the dislocation of components in the infrastructure. For example, motion sensor is used to detect the transmission wire movement in the electricity industry. In ship industry, the motion sensor can be used to detect the stability of the ship.

In recent decade, attributing to the rapid development of Microelectromechanical systems (MEMS) sensor, the sensor becomes low-cost, smaller size, but with high accuracy and high reliability. In addition, the rapid development of Internet of Things (IOT) promotes the demand of the motion sensor. Some chip manufacture companies such as STMicroelectronics provide affordable and miniature MEMS motion sensor. The hardware of motion detection becomes simple and cheap.

The sensor in general, regardless of its physical principle of mechanic, optic or ultrasonic, can measure the acceleration in three dimensions. This paper addresses a scenario with very slow and non-periodic motion. The motion is subtle and the changings in the three dimensions are not significant. If the subtle motion can be identified, the sensitivity of the motion detection can be improved, i.e. detectable for a slight motion. If the detection is successful, the applications will be promising, as the available motion sensor in the market is mostly with resolution as low as to 1 or 2 hz. For slight motion less than 1hz, the sensor is available, but expensive and hard to be procured.

Using common motion sensor, this paper develops algorithms to detect the abovementioned subtle motion. In state of art, the vibration analysis addresses a similar issue. Whereas methods used for vibration analysis such as Fast Fourier Transform would not work. The frequency domain won’t show evident patterns characterizing the motion, since the motion could be transient and not periodic. In addition, as another challenge, the motion signal is not readily discernible as the signal strength could be in the same level as the noise data. In the vibration analysis, when the bearing defect in its early stage, the defect signal is not evident, patterns would be identified in the time domain [1]. Motivated from the time domain analysis on the bearing defect, this paper proposes statistical approach to address the slow-motion detection problem.

2. Statistical Approach

2.1 Sample Kurtosis

When no motion present, the motion measurement would be purely random as white noise. The distribution of this random noise follows Normal distribution. But this assumption should be validated for each measurement. Motion present will induce a systematically increasing, decreasing, cyclic, or anything else embedded in the noise. We can consider the motion signal as additive measurement. The additive measure, as outlier to the Normal distribution, can be identified and therefore the motion can be detected.

Kurtosis is a higher order statistic highly sensitive to outlier. Since the motion signal added to the while noise is equivalent to outliers added into a Normal distributed data, we propose to use the kurtosis to detect the motion. The kurtosis is defines as [2]

$$k = \frac{E(x-\mu)^4}{\sigma^4} \quad (1)$$

where x is a random variable, the σ is the standard deviation of this variable. For a Normal distributed random variable, the kurtosis in (1) is exact valued as 3.

For the sample Kurtosis, state of art has several definitions. This paper defines the sample kurtosis as

$$k_{biased} = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^4}{(\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2)^2} \quad (2)$$

According to (1), the expected kurtosis is 3, while the expectation of k_{biased} is

$$E(k_{biased}) = \frac{3(n-1)}{n+1} \neq 3 \quad (3)$$

The (2) is thus biased. It is a drawback of this estimator. Nevertheless, the biasness is trivial for a large sample size n, as the $\frac{3(n-1)}{n+1}$ is close to 3 when n is large.

It is also desirable to obtain the variance of the k_{biased}, since we will use it for the statistic test. The variance of k_{biased} is

$$Var(k_{biased}) = \frac{24n(n-2)(n-3)}{(n+1)^2(n+3)(n+5)} \quad (4)$$

An corrected estimator of kurtosis is presented as [3]

$$k_{unbiased} = \frac{n-1}{(n-2)(n-3)}\big((n+1)k_{biased} - 3(n-1)\big) + 3 \quad (5)$$

The expectation of $k_{unbiased}$ is readily to be seen as

$$k_{unbiased} = 3 \quad (6)$$

The (6) is unbiased. The corresponding variance of the unbiased (5) is

$$Var(k_{unbiased}) = \frac{24n(n-1)^2}{(n-2)(n-3)(n+3)(n+5)} \quad (7)$$

Since

$$\frac{Var(k_{biased})}{Var(k_{unbiased})} = \frac{\frac{24n(n-2)(n-3)}{(n+1)^2(n+3)(n+5)}}{\frac{24n(n-1)^2}{(n-2)(n-3)(n+3)(n+5)}} = \frac{(n-2)^2(n-3)^2}{(n-1)^2(n+1)^2} < 1 \quad (8),$$

it is seen the $k_{unbiased}$ having larger variance. As less variance is more desirable, the choice of sample kurtosis is a balance between the biasness and variance.

2.2 Hypothesis Proposal

The kurtosis value can be calculated from (2) or (5) for motion detection. In most situations, the sample kurtosis will not be 3, even the data are purely Normal distributed. An issue remains: at which value, the kurtosis value implies a motion. We can solve this problem by using hypothesis test.

The sample kurtosis is asymptotically approaching as sample following Normal distribution when the sample size increase. For our motion sensor data, the data sets can be more than 1000, which is a large data set. If motion occurred, the assumption of Normal distribution of the samples will be violated. The kurtosis is then no longer 3. A statistical test can be proposed to detect the significance of the motion. Suppose the sample kurtosis computed from (2) or (5) is $\breve{k}$. The $\frac{\breve{k}-E(k)}{\sqrt{Var}}$ is approximately a standardized Normal distribution.

$$z = \frac{\breve{k}-E(k)}{\sqrt{Var}} \sim Normal(0,1) \qquad (9)$$

The z can be computed numerically since all its entities can be computed from (2) or (5), (4) or (7). If motion present, the z is expected to locate in the right extreme. Higher z value implies a higher level of the violation on the normality assumption, i.e. higher level of motion presence. The z value can be used as an indicator of the intensity level of the motion. If we set a confidence level, for example 0.95, when the z value locates outside of the 0.95 region, motion is evaluated as detected.

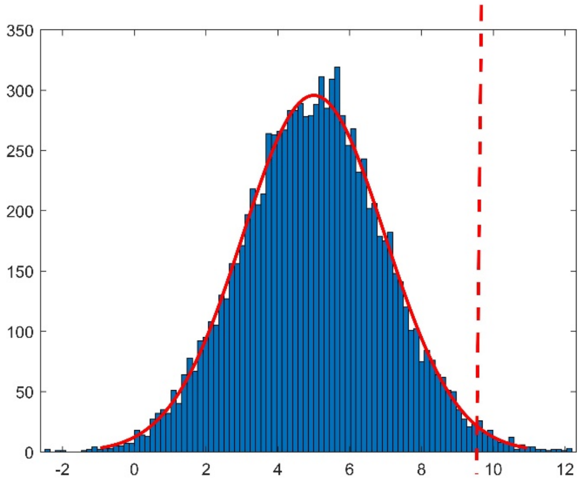


Figure 1. Extreme Level of Sample Kurtosis

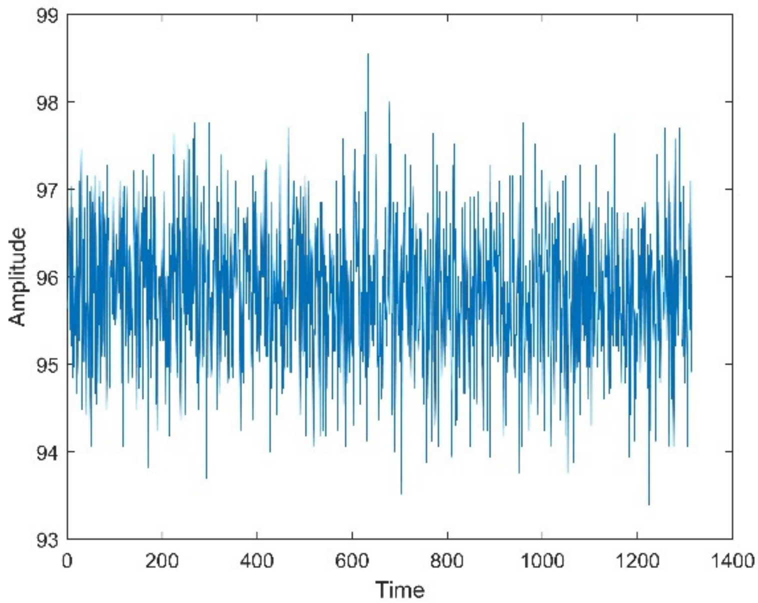
Alternatively, the corresponding probability of the region, i.e. the right region of the dashed line in the Figure 1, can be computed, since the z value follows standard normal distribution. This probability for z can be used as an indicator. This probability is essential the p-Value in the classic statistic test.

A drawback of the probability as indicator is its value might be very small. In classic statistics, the extreme region with 5% or 2.5% or 1% is considered as significant. However, for motion detection, the probability can be extremely small as 10^{-10}. When it is processed by commercial software such as Matlab, this small value is even handled as 0. Nevertheless, we would like to reserve the probability as a motion indicator, since the probability might be able to be used as a potential numerical indication of motion level.

3. Case Study

3.1 Measurement Data without Motion Present

Theoretically, the motion measurement will follow Normal distribution since the signal is pure white noise, as mentioned in previous section. The kurtosis value is hence around 3. The Figure 2 demonstrates a plot drawn from raw data without motion. In the left plot, the distribution of the data is not able to be identified. While from the histogram plot, shown in the right figure, it exhibits a PDF with Normal distribution shape. The raw data exhibit as a Normal distribution.



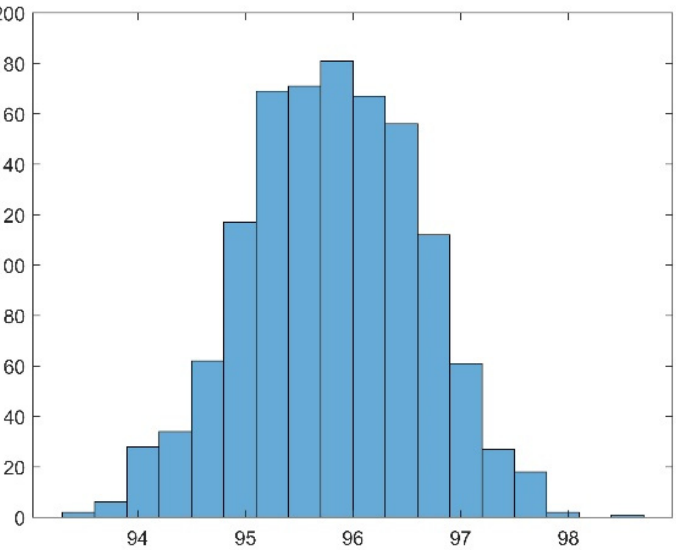


Figure 2. Demonstration of Raw Motion Data

From Eq(2), the biased sample kurtosis value is computed as 2.84, which is near 3. The skewness value, depicting the symmetry of the distribution, is -0.0798. A perfect symmetry PDF like Normal distribution is with skewness value 0. The raw data is evidently following Normal distribution. Table 1 tabulates the calculated skewness and kurtosis value when no motion present. The sensor 1 measures the motion in the x,y and z direction. The motion sensor 2 measures the angular change. For all the measurements, the skewness is near 0 and kurtosis is near 3. The kurtosis is a highly sensitive indicator. Even a few outliers present, the value kurtosis will significantly deviate from value 3. Their values, as shown in the Table 1, are all near 3 showing the raw data is significantly follows Normal distribution.

Table 1.Skewness and Kurtosis for the raw no-motion data

Data	Skewness	Kurtosis	Normal Distribution
Motion Sensor 1 x	-0.0793	2.85	Yes
Motion Sensor 1 y	0.0560	2.91	Yes

Motion Sensor 1 z	0.1021	2.81	Yes
Motion Sensor 2 Rate	0.0098	2.81	Yes

Another finding from Table 1 is, for the purpose of motion detection, it reveals the kurtosis with no motion tends to be less than 3. We will find, when motion presents, the kurtosis will excess 3 from the next experiment.

3.2 Parameter Drift of Motion Sensor
This section evaluates the performance of using mean value of the measurement data as an indicator. A perfect sensor would have a constant value on each motion direction when no motion present. The mean value is thus constant. However, a real situation reveals the mean value differs, sometimes significantly, with time. It is an unexpected adverse phenomenon.

An experimental was conducted to measure the magnitude of drift. Experiment runs several hours, meanwhile data were continuously collected. The mean of each 1500 data sets was calculated. During this period, total 36 mean values were obtained. Figure 3 plot the mean for direction x as a demonstration. In x direction, the mean can differ as large as 1. For y direction, it can differ 0.15 at largest. For the z direction, it can differ 0.45 at largest. The deviation is just for a shorter time period of several hours. If the experiment continues, the difference will be larger. The magnitude of the drift is big. The data fluctuation is inevitable as it is probably due to the parameter drift. A hardware problem exists in most sensors. Using mean as motion indicator is compromised. This paper does not use it as an indicator.

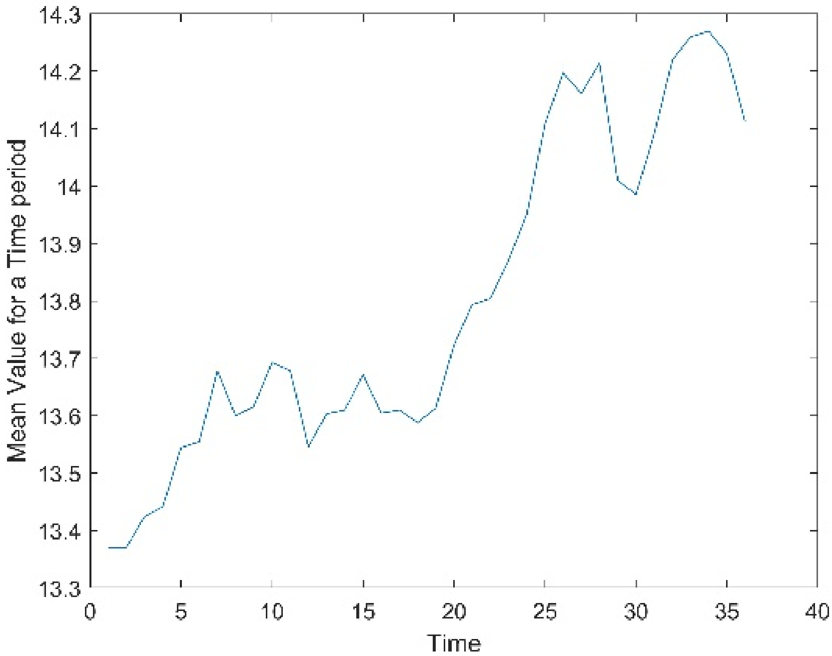


Figure 3. Parameter Drift of Motion Sensor

3.3 Measurement Data with Motion Present
An experiment was conducted by introducing artificial motions on the motion sensor. Sensors are simply locating on a smooth table surface. The severity of the motion was from low to high. The first artificial motion was induced by pressing the table mounting the sensor. The motion induced was not significant. The x and z direction do not show any noticeable change. In the y direction, several higher amplitude points exhibit, but not significantly, as shown in Figure 4. The kurtosis value for the Motion 1 in the x direction and z direction are near 3, almost same as that from pure noise data. For the y direction, the kurtosis value is 3.91. Compared with the kurtosis value from Table 1, Kurtosis value becomes higher.

Table 2.Skewness and Kurtosis Values with Motion Present

Motion	Data	Skewness	Kurtosis	pValue (Biased)	P (UnBiased)
1	Motion Sensor x	-0.1044	2.9541	0.3922	0.3922
	Motion Sensor y	0.0326	3.9129	2.5572*e-10	2.5572*e-10
	Motion Sensor z	0.0881	3.0452	0.3657	0.3657
2	Motion Sensor x	-0.1060	4.0054	1.59*e-12	1.59*e-12
	Motion Sensor y	0.9234	9.6935	0	0
	Motion Sensor z	0.1633	3.9618	1.2889*e-11	1.2889*e-11
3	Motion Sensor x	-2.0321	23.6180	0	0
	Motion Sensor y	-2.8905	29.2543	0	0
	Motion Sensor z	1.5336	26.25	0	0
4	Motion Sensor x	0.3633	15.8504	0	0
	Motion Sensor y	-0.5843	6.5801	0	0
	Motion Sensor z	-0.0193	5.4629	0	0

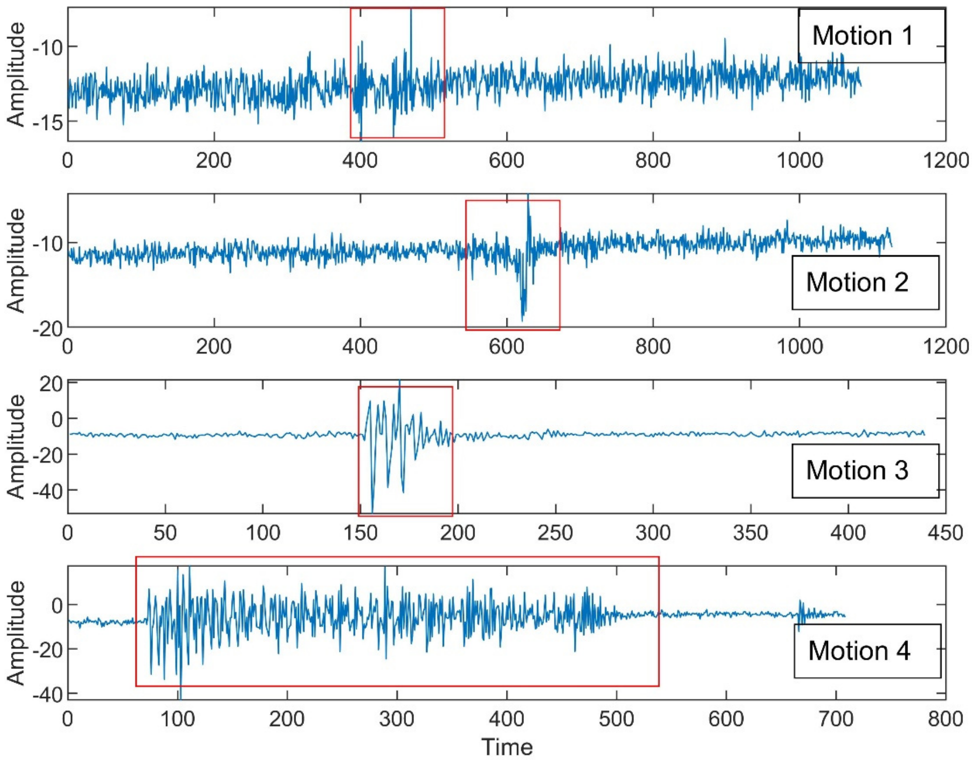


Figure 4. Plot of Original Measurement Data in y Direction

Next the severity of motion increases. The measurement data exhibits more significant change. As shown in the Motion 2 in the Figure 4, obvious peak is shown in the time-series for y direction. As the motion is mainly in the y direction, the measurement data in the x and z for Motion 2, not demonstrated in Figure 4, are still not significant in the time series plot. Nevertheless, the x and z directions show slight changes in their kurtosis values, 4.00 and 3.96 respectively. In the y direction, kurtosis has more significant change as 9.69.

Motion 3 introduced an even higher motion by shaking the table where sensor mounting. For this scenario, both the plot of raw time-series measurement data and the kurtosis exhibit significant change. All x, y and z direction detected motion. The kurtosis value is as high as 29.25 in the y direction. All of them are as expected and desired.

Experiments continued to Motion 4. The table mounting the sensors are shaking frequently and lasted much longer than in the Motion 3. An unexpected result presents. For all 3 directions, the kurtosis values are much lower than Motion 3. But from the measurement time-series data, the motion is shown more significant than Motion 3. It reveals a drawback of using kurtosis as an indicator of the severity of motion.

Further analysis was conducted by using the hypothesis test of Eq (9). The lower p value is designated to indicate higher motion. The results are also tabulated in Table 2. Unexpected results are present: when the kurtosis value is big, the p value is too small and the Matlab handles it as 0. It is a drawback of using the p-value. Nevertheless, the p value can be still be applied for slight motion as shown in the Motion 1. For this experiment, biasness is not necessary to be considered. As shown in Table 2, the p-value from the biased and unbiased kurtosis does not show significant difference.

4. Conclusion

The experiment established in the paper is to simulate the motions in some industrial fields. The experiments show detecting the slow motion from the time-domain is feasible with some limitations. The mean value of measurement data as motion indicator is not working. The kurtosis reveals highly sensitive to the motion. However, a higher kurtosis value is not able to imply a higher intensity of motion, as for periodic motion, the kurtosis does not show a promising performance. Same as the kurtosis, the proposed p-value in the paper can work well for slight motion, but with an additional limitation as it always tends to be zero. Conclusively, the kurtosis and its p-value can be applied to detect the motion presence with high sensitivity, but their performances are compromised when motion is periodic.

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