On Multi-Rule and Probability-Dependant Adaptations of Conway's Game of Life and Their Character

Alessandro Cotronei [1]

Abstract

We introduce some novel adaptations of Life-Like Automata. We focus mainly on some modifications of the original rules where the evolution of the system is depending from the step and the zone of the grid. We study also the properties of some novel non-deterministic adaptations.

1 Introduction and Goals

Conway's Game of Life is a discrete cellular automaton where several *cells*, represented as squares, *evolve* in an infinitely wide square grid (or *board*). [1, 3, 4, 6]. In particular, in the original, oldest and most used rule, the cells within the grid follow a simple behavior:

- any empty cells with exactly three living cells in the neighborhood will become alive at the next step (generation);
- any living cells with exactly two or three living cells in the neighborhood will remain alive at the next step;
- any other cells will die or remain dead during the next step.

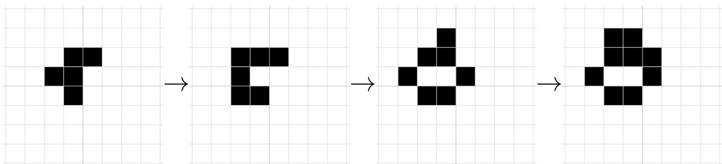


Figure 1: Conway's Game of Life

This rule, called *Life* (from now on, R1), is commonly marked with the notation B3/S23, where the two sets of numbers define the *born* and *surviving* cells. In general any couple of subsets of $\{0, 1, 2, 3, 4, 5, 6, 7, 8\}$ can be used in place of the subsets $\{2\}$ and $\{2, 3\}$ of the first rule. These are called *Life-like* rules (from now on, LL-rules) [2, 7]:

B3678/S34678 (Day and Night, from now on R2), B35678/S5678 (Diamoeba, R3), B3/S023 (DotLife, R4), B37/S23 (DryLife, R5), B3/S12 (Flock, R6), B3457/S4568 (Gems, R7), B36/S23 (Highlife, R8), B38/S238 (HoneyLife, R9), B3/S012345678 (Life Without Death, R10), B2/S0 (Live Free or Die, R11), B36/S245 (Logarithmic Replicator Rule, R12), B345/S5 (LongLife, R13), B368/S238 (LowDeath, R14), B3/S12345 (Maze, R15), B368/S245 (Move/Morley, R16), B1357/S1357 (Replicator, R17), B2/S (Seeds, R18), B3678/S235678 (Stains, R19), B36/S125 (2×2, R20).

There are many ways to generalize the above concept and form other kind of *rules*, but we will focus mainly on the LL ones. However, most of the novel adaptations we discuss can be intuitively adapted to non-LL Rules.

We refer to some structures as *Gun* (pattern that repeats periodically while generating *spaceships*). *Oscillator* (pattern that repeat itself after a certain number of iterations). *Spaceship* (pattern that after a certain number of iterations reappears in different position on the board). *Puffer* (pattern that after a certain number of iterations reappears in different position on the board and leaving a trail of cells behind). *Still life* (pattern not changing between generations). The smallest and

[1] Affiliation: University of Tromsø (Norway). ORCID: https://orcid.org/0000-0002-5495-6863.

most common spaceship (for R1 and other rules) is called *Glider*. We mention also the *Block* pattern, a square of two by two cells, among the still life patterns.

Cellular automata as *Conway's Game of Life* have been used to simulate phenomena in wide-ranging areas as Mathematics, Physics and Chemistry [11, 8]. However, phenomena that can be in principle described by cellular automata are limited by the static determinism of basic automata rules. If we have a phenomenon that changes with time, we might need a rule that changes at each step (Step-Depending Rule), if the phenomenon is not uniform, a nonuniform rule could be needed (Zone-Depending Rule). If the phenomenon is finally driven by random factors, Probability-Dependant Rules might be used in place. Of course, the most general cases might include a combination of two or more of the previously mentioned schemes. One practical example, which is only theoretical at the moment, might be cloud formation. A single rule for cloud formation (with a gridded sky) could be not reliable as different temperature and humidity conditions affect cloud formation, in a similar way, geographical features affect the phenomenon and finally, clouds can form in random places given the correct conditions. Step-Depending, Zone-Depending Rule and Probability-Dependant Rules can respectively and theoretically solve each one of these problems. So our main objective is to increase the simulation possibilities of cellular automata.

2 Step-Depending Rule

In regular settings of totalistic cellular automata, there exists a rule that changes the board at each of the steps of the game:

$$B_0 \overset{R}{\to} B_1 \overset{R}{\to} B_2 \overset{R}{\to} B_3 \ ... \ B_k \overset{R}{\to} B_{k+1} \ ...$$

This global rule could be modified in a way that on each step of the board, a different rule acts:

$$B_0 \overset{R_1}{\to} B_1 \overset{R_2}{\to} B_2 \overset{R_3}{\to} B_3 \ ... \ B_k \overset{R_{k+1}}{\to} B_{k+1} \ ...$$

Both periodical or aperiodical rules can be used in place. This is a generalization of *Alternating Rules* [6].

3 Zone-Depending Rule

Let $\{A_i\}_{i\in\{1,...,r\}}$ a finite partition of the grid ($A_j \cap A_k = \emptyset \ \forall j \neq k$, $\bigcup_{1\leq i\leq r} A_i =$ initial grid) and let $R_i, i \in \{1, ..., r\}$ a finite family of *LL-rules*, we can define a *game* if we consider the rule R_j acting on the partition A_j at each step. If we call R^* the rule defined in a such way, we can describe the evolution of the game as follows:

$$B_0 \overset{R^*}{\to} B_1 \overset{R^*}{\to} B_2 \overset{R^*}{\to} B_3 \ ... \ B_k \overset{R^*}{\to} B_{k+1} \ ...$$

It is sufficient to consider a finite partition of the initial grid as the possible rules considered are finite. As the possible B/S rules depend on twice the number of adjacent cells, is it possible to assume without loss of generality and without discarding trivial rules that $r \leq 2^{18} = 262144$.

4 Step and Zone-Depending Rule

Let $\{A_i^j\}_{i\in\{1,...,r_k\}}$ a finite partition of the grid and let $R^*_{i,j}, i \in \{1, ..., r_k\}$ a finite family of *rules* for each $j \in \mathbb{N}$. We can now define a wider family of games by letting the rule $R^*_{i,j}$ act on A_i^j step number i. If we call R^*_k the rule defined by the union of all rules at step k, we can write:

$$B_0 \overset{R_1^*}{\to} B_1 \overset{R_2^*}{\to} B_2 \overset{R_3^*}{\to} B_3 \ ... \ B_k \overset{R_{k+1}^*}{\to} B_{k+1} \ ...$$

Again, we can consider $r_i \leq 2^{18}$. It is obvious that we can obtain both purely step- and time-dependent rules as particular cases of these set of rules.

4.1 Layered Life

We introduce a particular case of Step and Zone-Depending Rules we call *Layered Life*: Let $R_0, R_1^a, R_2^a, ..., R_{k-1}^a, R_1^d, R_2^d, ..., R_{k-1}^d$ (LL-)rules.

We first assume that there is a bijection of the cells for each couplet of boards. We consider the trivial case obtained by fixing a center and a couplet of orthogonal directions in each board, since each considered grid is squared and orthogonal. At each level and step e define $k+1$ boards rules:

- B_0 is the board where the (LL-)rule R_0 acts;
- B_1 is the board where R_1^a (R_1^d) acts on B_1 in the corresponding alive (resp. dead) cells of B_0;
- B_2 is the board where R_2^a (R_2^d) acts on B_2 in the corresponding alive (resp. dead) cells of B_1;
- ...
- B_k is the board where R_{k-1}^a (R_{k-1}^d) acts on B_k in the corresponding alive (resp. dead) cells of B_{k-1}.

We consider the evolution of B_k as output of the automata. A more general alternative could be to consider some logical functions involving all the boards used. Here we will limit ourselves to the configuration with $k = 1$. Initializing each board with a nonempty state can lead to a nontrivial evolution of the system.

4.2 Parallel Life

We introduce another novel particular case of Step and Zone-Depending Rule: Let $R_1^a, R_2^a, ..., R_{k-1}^a, R_1^d, R_2^d, ..., R_{k-1}^d$ (LL-)rules. If we have k different boards, each one of them initialized with a given configuration of cells, we define for each step :

- B_1, the board where R_k^a (R_k^d) act in the corresponding alive (resp. dead) cells of B_k;
- B_2), the board where R_1^a (R_1^d) act in the corresponding alive (resp. dead) cells of B_1;
- ...
- B_k, the board where R_{k-1}^a (R_{k-1}^d) act in the corresponding alive (resp. dead) cells of B_{k-1}.

We can consider as output of the automata the evolution of a fixed board.

The latter definition allows a mutual perturbation of the boards, while the former only allow perturbations in one sense.

5 Probability-Dependant Rules

Let $\{\alpha_{l,0}, \alpha_{l,1}, ..., \alpha_{l,8}, \alpha_{d,0}, \alpha_{d,1}, ..., \alpha_{d,8}\} \in [0, 1]$, we can define a new rule R_P by means of a *local* definition:

$$R_p(cell) = \begin{cases} \text{living with probability } 1-\alpha_{l,0} \text{ if cell is living and its neighborhood counts 0 cells,} \\ \text{living with probability } 1-\alpha_{l,1} \text{ if cell is living and its neighborhood counts 1 cell;} \\ ... \\ \text{living with probability } 1-\alpha_{l,8} \text{ if cell is living and its neighborhood counts 8 cells;} \\ \text{living with probability } \alpha_{d_0} \text{ if cell is dead and its neighborhood counts 0 cells;} \\ \text{living with probability } \alpha_{d_1} \text{ if cell is dead and its neighborhood counts 1 cell;} \\ ... \\ \text{living with probability } \alpha_{d_8} \text{ if cell is dead and its neighborhood counts 8 cells.} \end{cases}$$

Given the vastity of these possible rules, we consider a weakened yet expressive subfamily of the previous one that is clearly connected to conventional rules:
Let R a (LL-) rule, $\alpha, \omega \in [0, 1]$. We define a new rule R_p with the property that living cells are generated according to R, but they only are alive in the actual board at the next step with probability $1 - \alpha$. In the same way next generation cells are dead according to rule R, except for the fact that they have probability equal to ω of being alive in the actual board. We can consider these additional steps as *filtering* of living and dead cells.
It is clear that if $\alpha = 0$ and $\omega = 0$, then R_p is coinciding with R. If $\alpha = \omega = \frac{1}{2}$, then the automaton is completely random and not depending anymore on the initial rule.
Structures as those mentioned in the first section cannot exist except for the trivial cases corresponding to a purely deterministic rule. Any stable pattern will be indeed perturbated and modified given enough time.

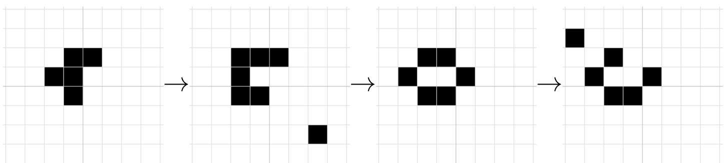
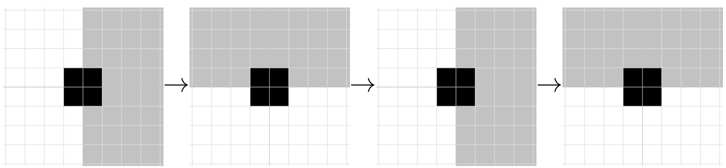


Figure 2: Possible Random Evolution of R1

6 Properties

Statement 1. Let $\{A_i^j\}_{i \in \{1,...,r_k\}}$ a finite partition of the grid and $R^*_{i,j}, i \in \{1, ..., r_k\}$ a finite family of *rules* for each $j \in \mathbb{N}$ (as defined previously), then if an unperturbated (that is not influenced by external cells) pattern is a still life pattern with identical evolution for each of the rules acting on the cells occupied by the pattern for each of the steps $t, t+1, ...t+l$, then the pattern is a still life pattern for the composite rules $R^*_t, R^*_{t+1}, ..., R^*_{t+l}$. For example, if we consider a composite rule where acting primary rules are *Life* (R1, white in pictures below) and *Highlife* (R8, gray in pictures below) that changes every two steps as in the figure below, we can clearly see that the *Block* pattern (which is a still life for both the rules mentioned before) is a still life pattern for the composite rule.



Of course the statement has validity also in case only *Step or Zone-Depending Rules* are used in place, in this case we have:

Statement 2. If in

$$B_0 \xrightarrow{R_1} B_1 \xrightarrow{R_2} B_2 \xrightarrow{R_3} B_3 \; ... \; B_k \xrightarrow{R_{k+1}} B_{k+1} \; ...$$

A *still life* pattern for the rules $R_{i+1}, R_{i+2}, ..., R_{i+r}$ is present in B_i, there will be a still life pattern corresponding to the sequential *evolutions* of the initial pattern in $B_{i+1}, B_{i+2}, ..., B_{i+r}$.

Statement 3. If in

$$B_0 \xrightarrow{R^*} B_1 \xrightarrow{R^*} B_2 \xrightarrow{R^*} B_3 \; ... \; B_k \xrightarrow{R^*} B_{k+1} \; ...$$

A *still life* pattern for the rules $R_{i+1}, R_{i+2}, ..., R_{i+r}$ is present in B_i, there will be a still life pattern corresponding to the sequential *evolutions* of the initial pattern in $B_{i+1}, B_{i+2}, ..., B_{i+r}$.

If we consider the block pattern (which is an example of still life for both the rules *life* and *Highlife*) at the initial step of a game in which the acting rules are $R_{2k-1} =$ *life* and $R_{2k} =$*Highlife* for each $k \in \mathbb{N}$. We have that the block is an example of still life for the Step-Depending rule. A similar and more general statement can be given:

Statement 4. Let $\{A_i^j\}_{i \in \{1,...,r_k\}}$ a finite partition of the grid and $R^*_{i,j}, i \in \{1, ..., r_k\}$ a finite family of *rules* for each $j \in \mathbb{N}$, then if an unperturbated pattern evolves in the same way for each of the rules of the space occupied by it for each of the steps $t, t+1, ...t+l$, then the pattern will evolve in the same way for each of $R^*_t, R^*_{t+1}, ..., R^*_{t+l}$. An analogous statement is valid for purely Time and Step depending rules, but we omit it as it is similar with previously mentioned statements.

If we consider a composite rule where acting primary rules are again *Life* (white in pictures below) and *Highlife* (gray in pictures below) that changes every two steps as in the figure below, we see that the *Glider* pattern (which evolves identically in the two rules mentioned) evolves identically also in the composite rule.

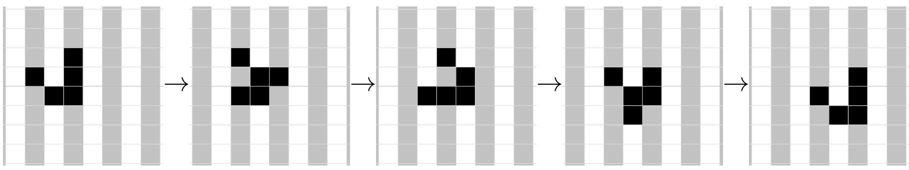


Figure 3: Evolution of Pattern in Zone-depending Grid.

7 Dynamic Behavior

Starting from a commonly used character classification of cellular automata [6]:

- Stable: rules in which patterns tend to stabilize quickly and show little activity;
- Chaotic: rules in which patterns tend to show complex behavior, but do not generally explode;
- Explosive: rules in which patterns tend to grow without bound.

The following table shows a recap of all rules we use, their definition and their behavior:

Rule	Name	B/S	Behaviour
R1	Life	B3S23	C
R2	Day and Night	B3678/S34678	S
R3	Diamoeba	B35678/S5678	S
R4	DotLife	B3/S023	E
R5	DryLife	B37/S23	E
R6	Flock	B3/S12	C
R7	Gems	B3457/S4568	E
R8	HighLife	B36/S23	C
R9	HoneyLife	B38/S238	E
R10	Life Without Death	B3/S012345678	E
R11	Live Free or Die	B2/S0	E
R12	Logarithmic Replicator Rule	B36/S245	S
R13	LongLife	B345/S5	S
R14	LowDeath	B368/S238	C
R15	Maze	B3/S12345	E
R16	Move/Morley	B368/S245	S
R17	Replicator	B1357/S1357	E
R18	Seeds	B2/S	E
R19	Stains	B3678/S235678	S
R20	2×2	B36/S125	C

We consider several configurations to indicate the character of the composite LL-Rules. We remark that since no strict definition for the character exists, it only has to be considered as a soft criterion to understand the behavior of the automata. We used several Python programs to simulate the grids and the plots. Despite the fact that our code was not optimized, an adaptation of hashlife [9] is in theory possible for some of the modifications mentioned. In particular by considering more families of memorized arrays for Step-depending automata, one for each different used rule. For Zone-depending automata if the grid is periodic we can consider small windows having the same rule configuration. In all cases we expect more memory to be required [9].

We discuss the behavior of several particular cases of the above mentioned automata, first of all S1 (or scheme 1) we consider a step-dependent configuration in which two of the LL-Rules alternate periodically:

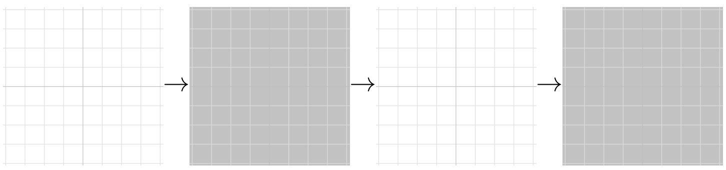


Figure 4: S1 Scheme.

$$B_0 \overset{R_1}{\to} B_1 \overset{R_2}{\to} B_2 \overset{R_1}{\to} B_3 \ \dots\ B_{2k} \overset{R_1}{\to} B_{2k+1} \overset{R_2}{\to} \ \dots$$

Secondly (S2), we consider a purely zone-dependant domain, where even rows follow a LL-Rule and odd rows follow another, we found this character table.

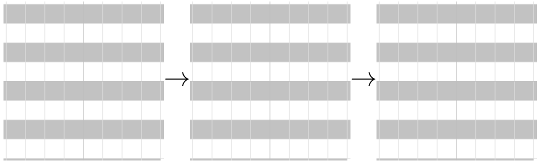


Figure 5: S2 Scheme.

We simulate (S3), another purely zone-dependant domain where points with even row and column follow a LL-Rule and the others (odd row and odd column) follow another LL-Rule from the initial ones given.

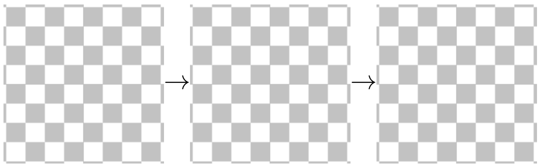


Figure 6: S3 Scheme.

We study (S4) the Step- and Time-Dependant scheme where each of the rules is equivalent to R2, but they alternate at each step: by construction, we know (since both LL-Rules are S2 and when they are reversed the result doesn't change) that the behavior of this scheme is equivalent to S2.

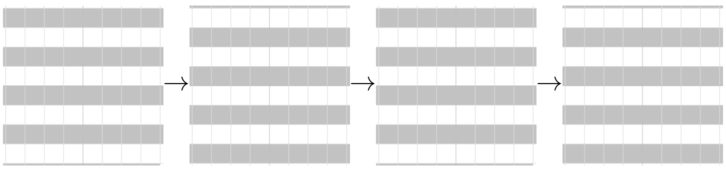


Figure 7: S4 Scheme.

We define (S5) a particular case of Layered life. More precisely if we consider Ra/Rb, alive and dead cells in the secondary board (which evolves according to Rb) define which rules (resp. between Ra and Rb) will be used in the main board. In the secondary subscheme the rule on the secondary board is R1, and the ones on the main board will be again Ra and Rb respectively.

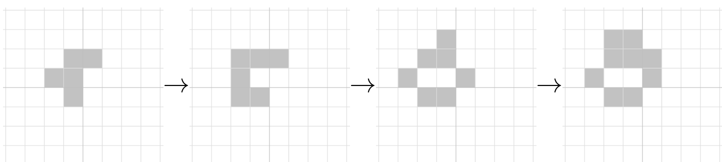


Figure 8: Possible Rule Subdivision of S5 Scheme.

We study (S6) several configurations of Parallel Life. We use in particular two different subschemes, in the first we use only two rules Ra/Rb, cells that not overlap follow rule a, cells that overlap follow rule b. For the second subscheme the living and dead cells define where rule 1 and 4 act on the second board, vice versa cells living and dead cells define where rule a and b will act. Of course the update will be simultaneous and all the process we mentioned is executed at each step.

We noticed that in the last two cases it is not possible to give a classification as in the schemes S1-S4. The underlying grid influences too much the result to have a general scheme. As a rule of thumb we can se that explosive patterns prevail assuming they have enough room in the grid and initial number of cells. We can also state that in the case of nonexplosive automata, the rule with a higher number of cells in the plane, prevales for the behavior, again assuming a proportionate number of initial cells.

Finally (S7), we describe several cases of random automata. We adapt the definitions of dynamics as follow. We will use three different and nonexclusive criteria:

A - Growth

- Collapsing: rules in which patterns tend to stabilize towards a state comparable to the fluctuations of an empty board,
- Stable: rules with patterns that generally stabilize in terms of growth without much change on the border,
- Unstable: rules with patterns that generally stabilize in terms of growth, but with chaotic contour change.
- Growing: rules in which patterns tend to grow by accretion.

B - Internal Turbulence of Compact Pattern

- Quiet: rules showing changes in compact patterns comparable to noise fluctuations,
- Noisy: the activity of the compact patterns is principally due to its internal change, which shows greater activity than pure noise,
- Flickering: the activity of compact patterns is mainly due to the internal noise of more separated subpatterns.

C - Persistant Pattern Generation

We discuss the tendency of a rule (with strong dependency to ω) to generate persisting patterns out of pure noisy background, we will use relative terms and indicators as low (L), medium (M) and high (H). This term includes the Capacity of big patterns to form spontaneously from empty areas, their persistance and their capacity to fill the board.
It is clear that the first criterion covers all the possible cases, as rules that not grow or collapse have a stable number of cells that can have a stable or unstable border. If we have a compact pattern, then if its change is not comparable to the only noise, it can be formed by one compact pattern, or more pieces that change rapidly, and in particular the rule tend to amplify the changes due to noise. The third criterion covers all possible cases by construction. Of course again these have to be considered only as soft criteria, as no rigorous mathematical statement is in place.

We found that in all four cases (S1-S4) the table of charaters is simmetrical, it seems indeed that in the pure time-dependant case the initial rule does not affect significantly the long-term behavior of the automata. In the other two cases of zone-dependant rules we found that the symmetry of the character table depends on the symmetry of the domain (inverting upper and lower direction is equivalent to a traslation upward or downward -resp. diagonally- of one unit). The upper diagonal entries of the matrix represent the behavior of the first, second and third example, where the two rules considered are given from the table. For the main diagonal, in all the four cases R1-R4, the rule is coinciding with the initial one so the behavior is already known [6]. In the S4 case, even though the rules used are identical to the S2 ones, the behavior can radically change, we mark with the apex for the second entry of the matrix the rule that are different in S4 w.r.t S2, with the respective value indicated by the apex letter. We found a main cause for this behavior. With fixed grid the *growing* patterns can be blocked inside (or amplified by) their permanence in a zone with the same rule. When the two rules change place the pattern can be *dragged* in (or destroyed by) another zone of the board recursively. We found that in some cases (as R2-R11 and R3-R11) the patterns in S2 were producing horizontal structures that remained *trapped* between lines of different rules while exploding for S4. However this behavior was not universal as for example R1-R7 and R3-R15 do not generally show this phenomenon.

We also noticed that between S2 and S4 none of the rules with different behavior had a Chaotic charater in S4. We want also to remark that patterns containing R7 or R11 had a bigger inclination to change their behavior w.r.t. other rules.

S1-S4

Rule	R1	R2	R3	R4	R5	R6	R7	R8	R9	R10
R1	C	s,s,s	s,s,s	c,c,c	c,c,c	s,s,s	s,s^e,e	s,c,c	c,c,c	e,e,e
R2		S	s,s,s	s,s,s	s,s,s	s,s,s	e,e,e	s,s,s	s,s,s	e,e,e
R3			S	s,s,s	s,s,s	s,s,s	e,c^e,e	s,s,s	s,s,s	e,e,e
R4				E	c,e,c	s,s,s	s,s^e,e	s,c,c	c,c,c	e,e,e
R5					E	s,s,s	s,s^e,e	c,c^s,c	e,c,c	e,e,e
R6						C	s,s^e,s	s,e,s	s,s,s	s,s^e,e
R7							E	s,s^e,e	s,s^e,e	e,e,e
R8								C	c,c,c	e,e,e
R9									C	e,e,e
R10										E

Rule	R11 (E)	R12 (S)	R13 (S)	R14 (C)	R15 (E)	R16 (S)	R17 (E)	R18 (E)	R19 (S)	R20 (C)
R1 (C)	s,e,e	s,s,s	s,s,s	c,c,c	e,e,e	s,s,s	e,e,e	s,e,s	s,e,e	s,c,s
R2 (S)	s,c^e,s	s,s,s	s,s,s	s,s,s	e,e,e	s,s,s	e,e,e	s,e,s	e,e,e	s,e,s
R3 (S)	s,c^e,s	s,s,s	s,s,s	s,s,s	s,e^s,e	s,s,s	e,c,e	s,e,s	s,e,e	e,s,s
R4 (E)	s,e,e	s,s,c	s,s,s	c,c,c	e,e,e	s,s,c	e,e,e	s,e,e	s,e,e	s,c^e,e
R5 (E)	s,e,e	s,s,s	s,s,s	c,c,c	e,e,e	s,s,s	e,e,e	s,e,s	s,e,e	s,c,s
R6 (C)	e,e,s	s,s,s	s,s,s	s,e^s,s	s,e,e	s,s,s	e,e,e	e,e,s	s,e,e	s,s,s
R7 (E)	s,s^e,e	s,s^e,e	s,e,e	s,s^e,e	e,e,e	s,s^e,e	e,e,e	s,e,e	e,e,e	s,s^e,c
R8 (C)	s,e,e	s,s,s	s,s,s	c,c,c	e,e,e	s,s,s	e,e,e	s,e,s	c,e,e	c,e,s
R9 (C)	s,e,e	s,s,s	s,s,s	e,c,c	e,e,e	s,s,s	e,e,e	s,e,s	c,e,e	s,s,s
R10 (E)	e,e,e	e,e,e	e,e^s,e	e,e,e	e,e,e	e,e,e	e,e,e	e,s^e,e	e,e,e	e,e,e

Rule	R11	R12	R13	R14	R15	R16	R17	R18	R19	R20
R11	E	s,e,s	s,s^e,e	s,e,e	e,e,e	s,e,s	e,e,e	e,e,e	s,e,e	e,e,s
R12		S	s,s,s	s,s,s	e,e,e	s,s,s	e,e,e	s,e,s	e,e,e	c,c,s
R13			S	s,s,s	e,e^s,e	s,s,s	e,e,e	s,e,c	s,e^s,e	s,s,s
R14				C	e,e,e	s,s,s	e,e,e	s,e,s	e,e,e	e,e,s
R15					E	e,e,e	e,e,e	e,s^e,e	e,e,e	c,e,e
R16						S	e,e,e	s,e,s	e,e,e	c,s,s
R17							E	e,e,e	e,e,e	e,e,e
R18								E	s,e,e	e,e,e
R19									S	s,e,e
R20										C

S7

(α,ω)	R1	R2	R3	R4	R5	R6	R7	R8	R9	R10
$(\frac{1}{50},\frac{1}{50})$	G-F-H	U-N-M	G-Q-L	G-F-H	U-F-H	U-F-H	G-N-M	G-F-H	G-F-H	G-Q-H
$(\frac{1}{50},\frac{1}{150})$	C-F-M	C-F-L	G-Q-L	G-F-H	C-F-M	C-F-M	G-N-L	C-F-M	C-F-M	G-H-H
$(\frac{1}{50},\frac{1}{250})$	C-F-L	C-F-L	G-Q-L	G-F-H	C-F-L	C-F-M	G-N-L	C-F-L	C-F-M	G-Q-H
$(\frac{1}{150},\frac{1}{50})$	G-F-H	U-Q-M	G-Q-L	G-F-H	U-F-H	U-F-H	G-N-M	G-F-H	G-F-H	G-Q-H
$(\frac{1}{150},\frac{1}{150})$	G-F-M	C-N-L	G-Q-L	G-F-H	G-F-M	C-F-M	G-N-L	C-F-M	G-F-M	G-Q-H
$(\frac{1}{150},\frac{1}{250})$	C-F-L	C-F-L	G-Q-L	G-F-H	G-F-L	C-F-M	G-N-L	G-F-L	G-F-M	G-Q-H
$(\frac{1}{250},\frac{1}{50})$	G-F-H	G-Q-M	G-Q-L	G-F-H	G-F-H	G-F-H	G-N-M	G-F-H	G-F-H	G-Q-H
$(\frac{1}{250},\frac{1}{150})$	G-F-M	C-Q-L	G-Q-L	G-F-H	G-F-M	C-F-M	G-N-L	G-F-M	G-F-M	G-Q-H
$(\frac{1}{250},\frac{1}{250})$	G-F-L	C-Q-L	G-Q-L	G-F-H	G-F-L	C-F-M	G-N-L	G-F-M	G-F-L	G-Q-H

(α,ω)	R11	R12	R13	R14	R15	R16	R17	R18	R19	R20
$(\frac{1}{50},\frac{1}{50})$	G-F-H	G-F-M	G-F-M	G-F-H	G-Q-H	G-F-M	G-N-H	G-F-H	G-N-H	G-F-H
$(\frac{1}{50},\frac{1}{150})$	G-F-H	C-F-L	G-F-L	C-F-M	G-Q-H	C-F-L	G-F-H	G-F-H	G-F-M	G-F-H
$(\frac{1}{50},\frac{1}{250})$	G-F-H	C-F-L	G-F-L	C-F-L	G-Q-H	C-F-L	G-F-H	G-F-H	G-F-M	U-F-M
$(\frac{1}{150},\frac{1}{50})$	G-F-H	G-F-M	G-F-M	G-F-H	G-Q-H	G-F-M	G-N-H	G-F-H	G-N-H	G-F-H
$(\frac{1}{150},\frac{1}{150})$	G-F-H	C-F-L	G-F-L	G-F-M	G-Q-H	C-F-L	G-F-H	G-F-H	G-F-H	G-F-H
$(\frac{1}{150},\frac{1}{250})$	G-F-H	C-F-L	G-F-L	G-F-L	G-Q-H	C-F-L	G-F-H	G-F-H	G-F-M	G-F-M
$(\frac{1}{250},\frac{1}{50})$	G-F-H	G-F-M	G-F-M	G-F-H	G-Q-H	G-F-M	G-N-H	G-F-H	G-F-H	G-F-H
$(\frac{1}{250},\frac{1}{150})$	G-F-H	C-F-L	G-F-L	G-F-M	G-Q-H	C-F-L	G-F-H	G-F-H	G-F-M	G-F-H
$(\frac{1}{250},\frac{1}{250})$	G-F-H	C-F-L	G-F-L	G-F-L	G-Q-H	C-F-L	G-F-H	G-F-H	G-F-M	G-F-M

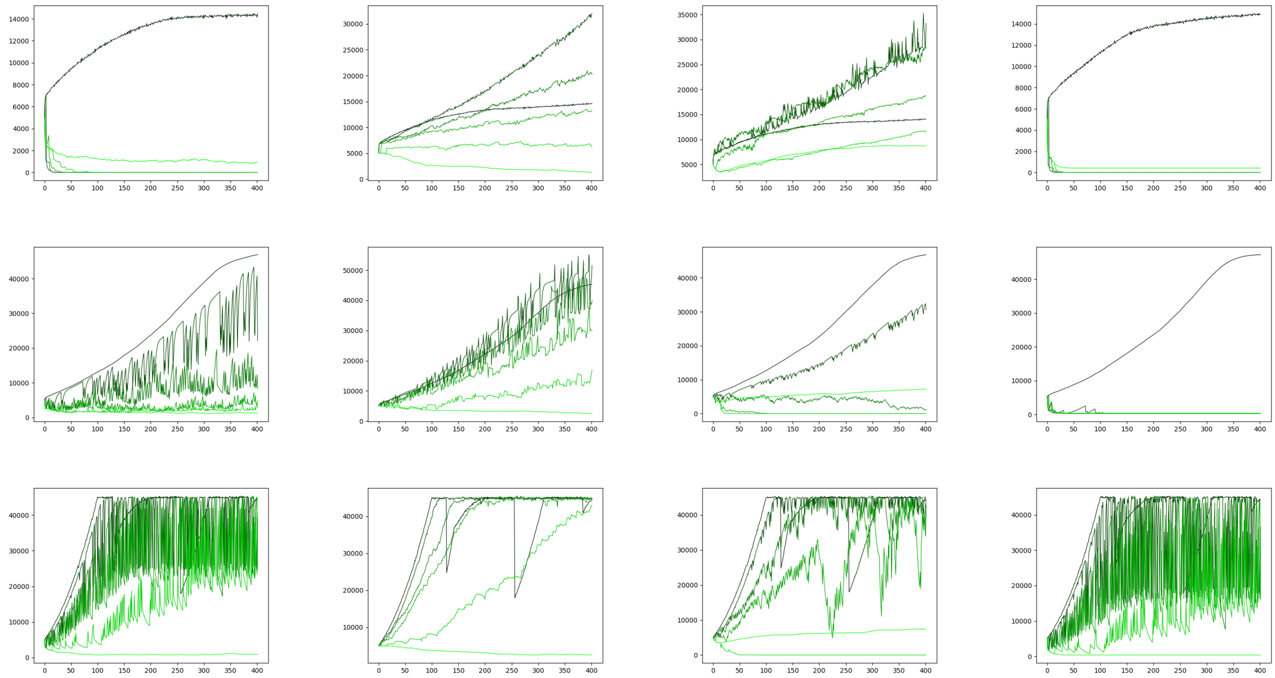
8 Character Mixing

Let a board where a purely Step-Depending (resp. purely Zone-Depending) Rule acts. We want to answer the following question: if one of the rules has a character, which percentage rule steps (resp. percentage of board cells) needs to have a different character so the overall rule is nonexplosive? Again we limit ourselves to the case of two different rules acting on the board. We plot all the combinations where the investigated rule acts on 100%, 80%, 60%, 40%, 20%, 0% of cells (resp. steps). In particular darker green shadings indicate higher concentration of the investigated rule, lighter green indicate lower concentrations. For the explosive case, we consider only R7 (Gems) R15 (Maze) and R17 (Replicator) and their combinations with R1, R2, R3, R6, R8, R9, R12, R13, R14, R16, R19, R20.

These first plots refer to the Step-Depending case:

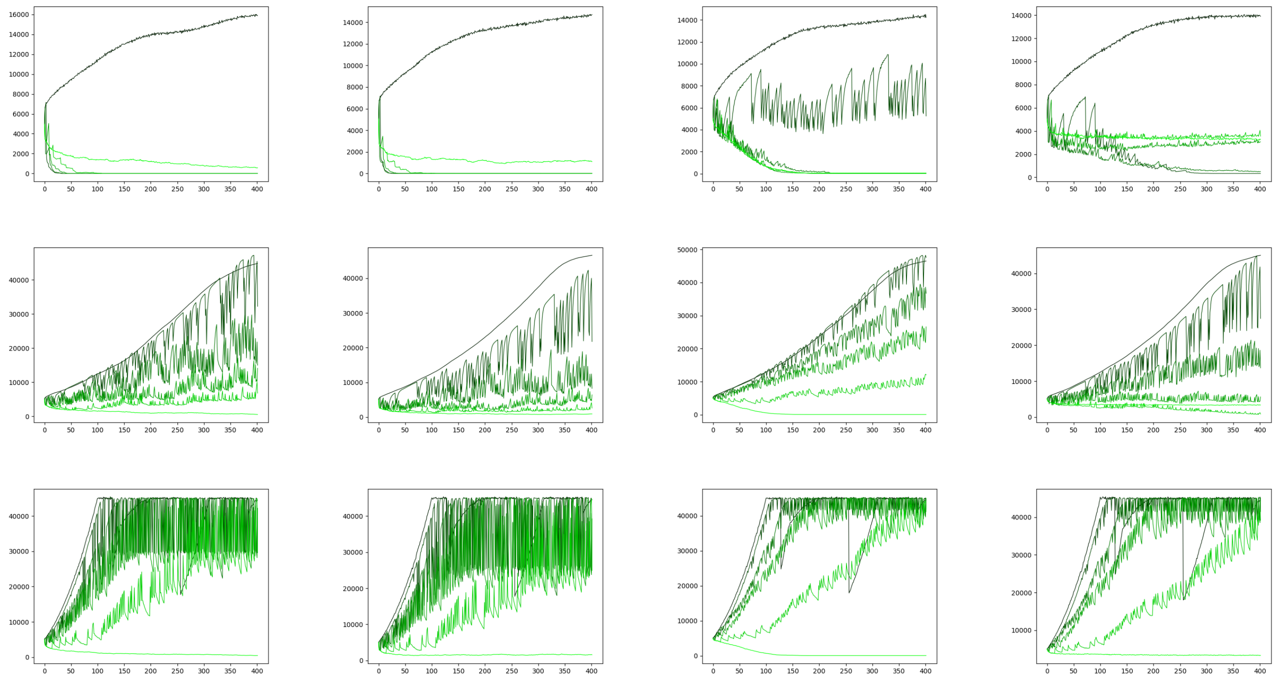
SD-E R1 R2 R3 R6

R7 R15 R17

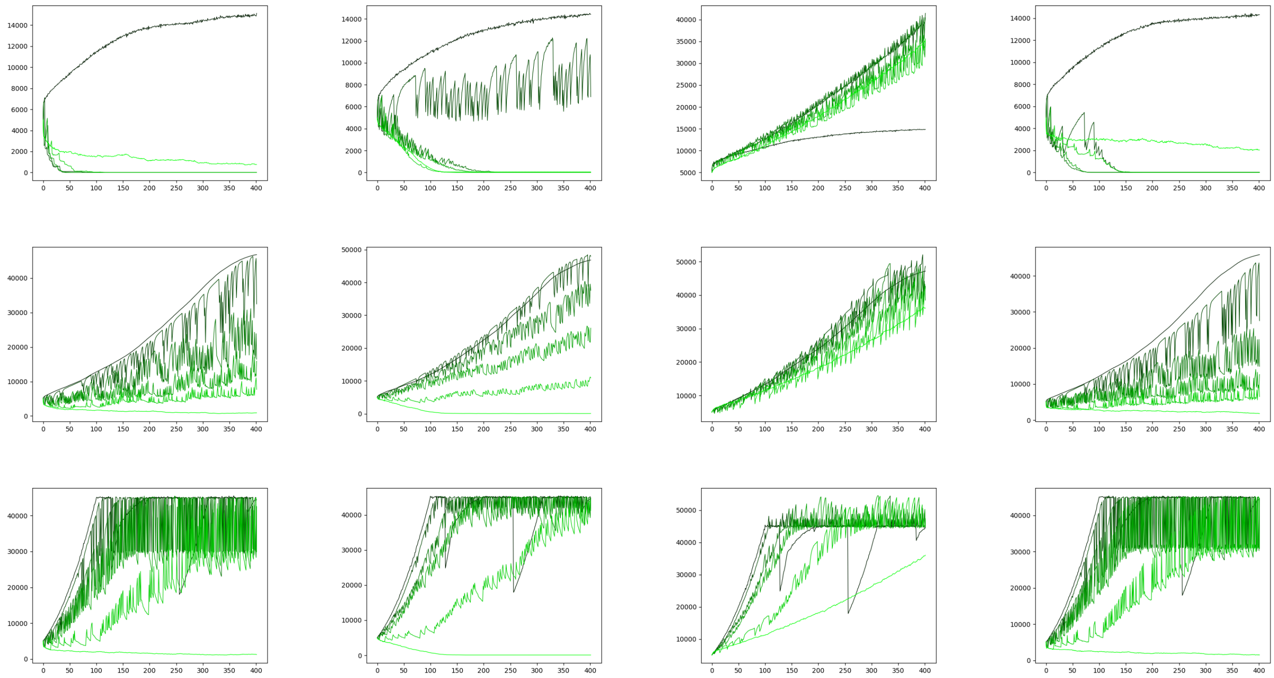


R8 R9 R12 R13

R7 R15 R17



R14 R16 R19 R20



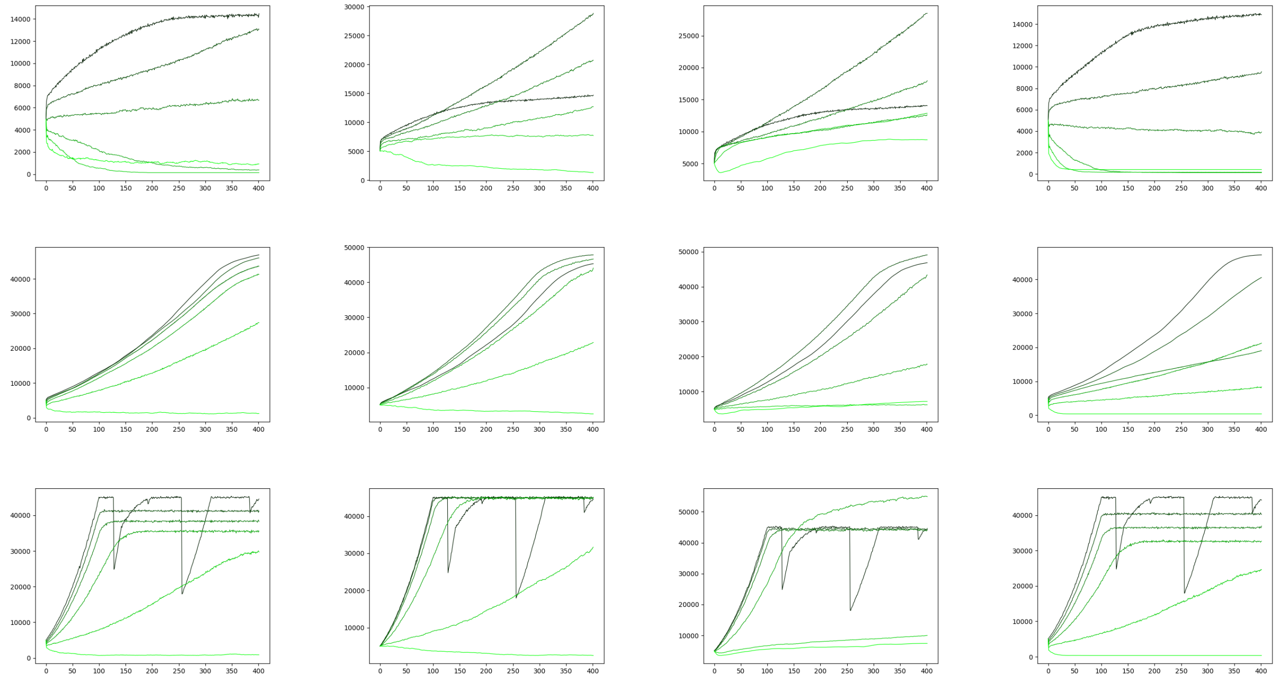
R7

R15

R17

Similarly if different points of the grid have different rules (Zone-depending case).

ZD-E R1 R2 R3 R6

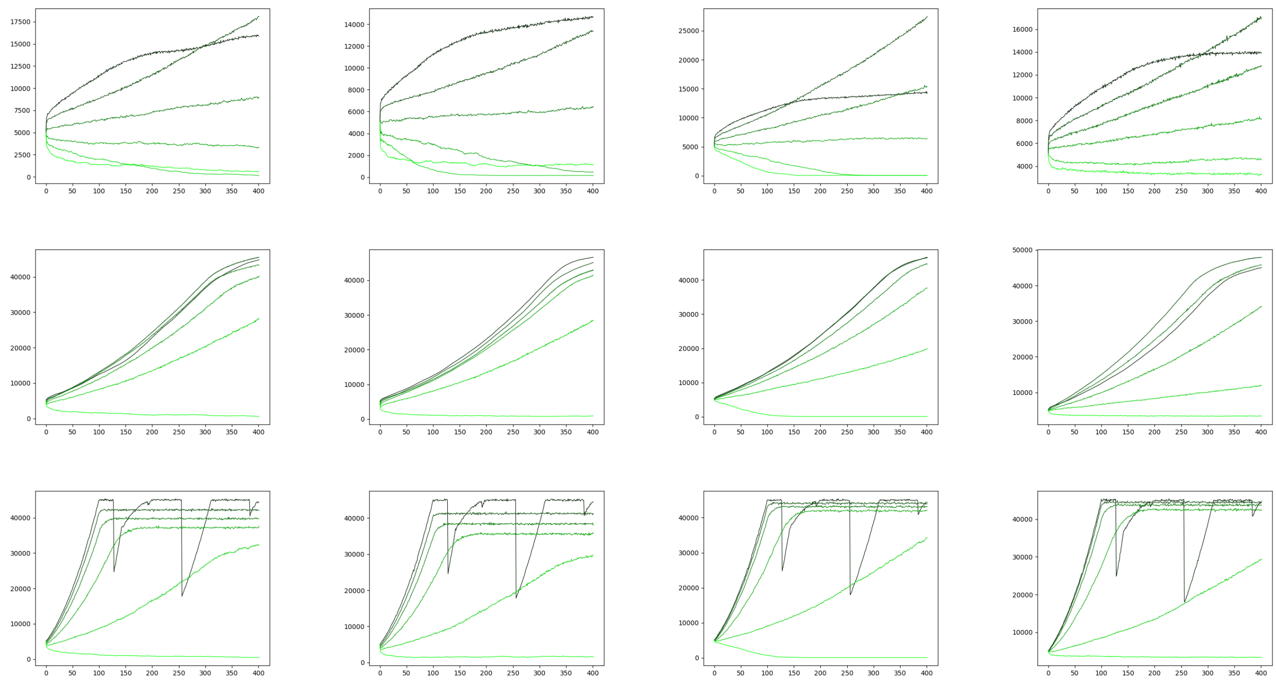


R7

R15

R17

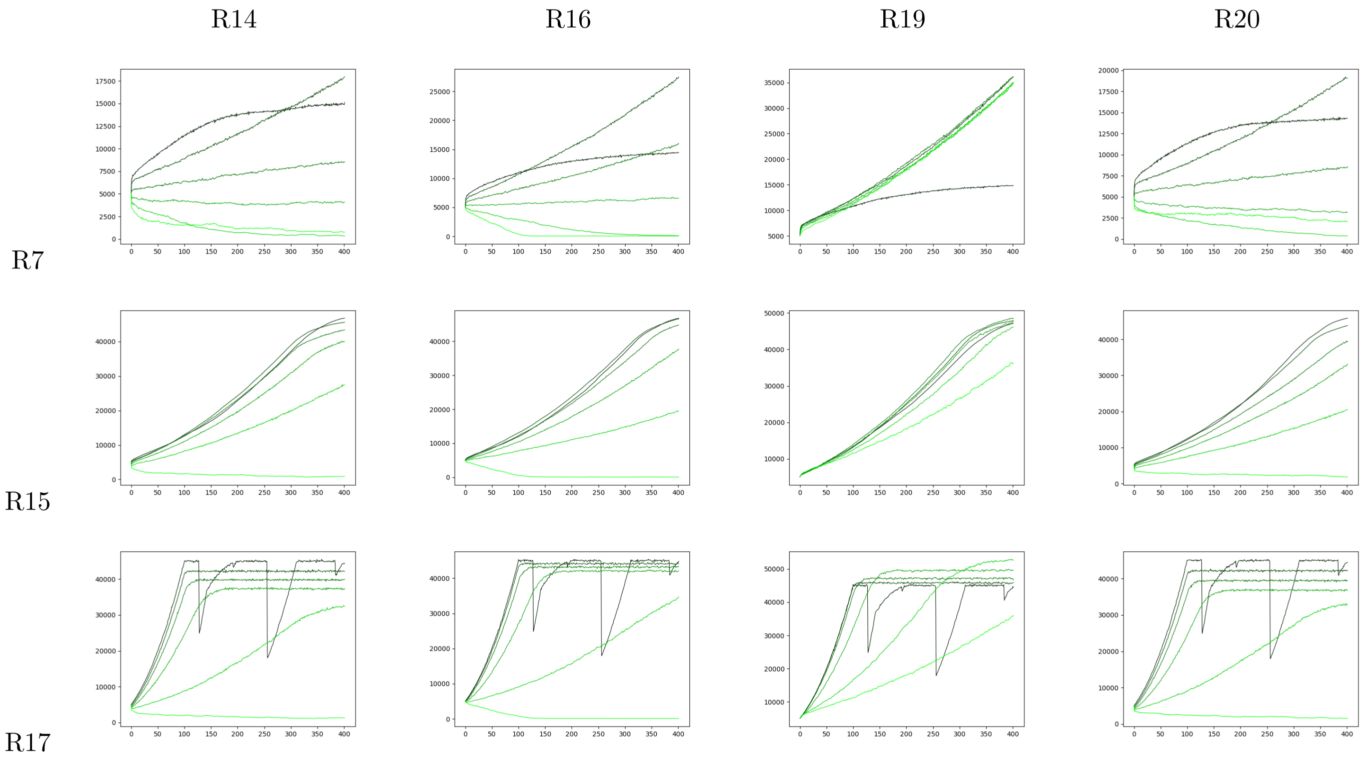
R8 R9 R12 R13



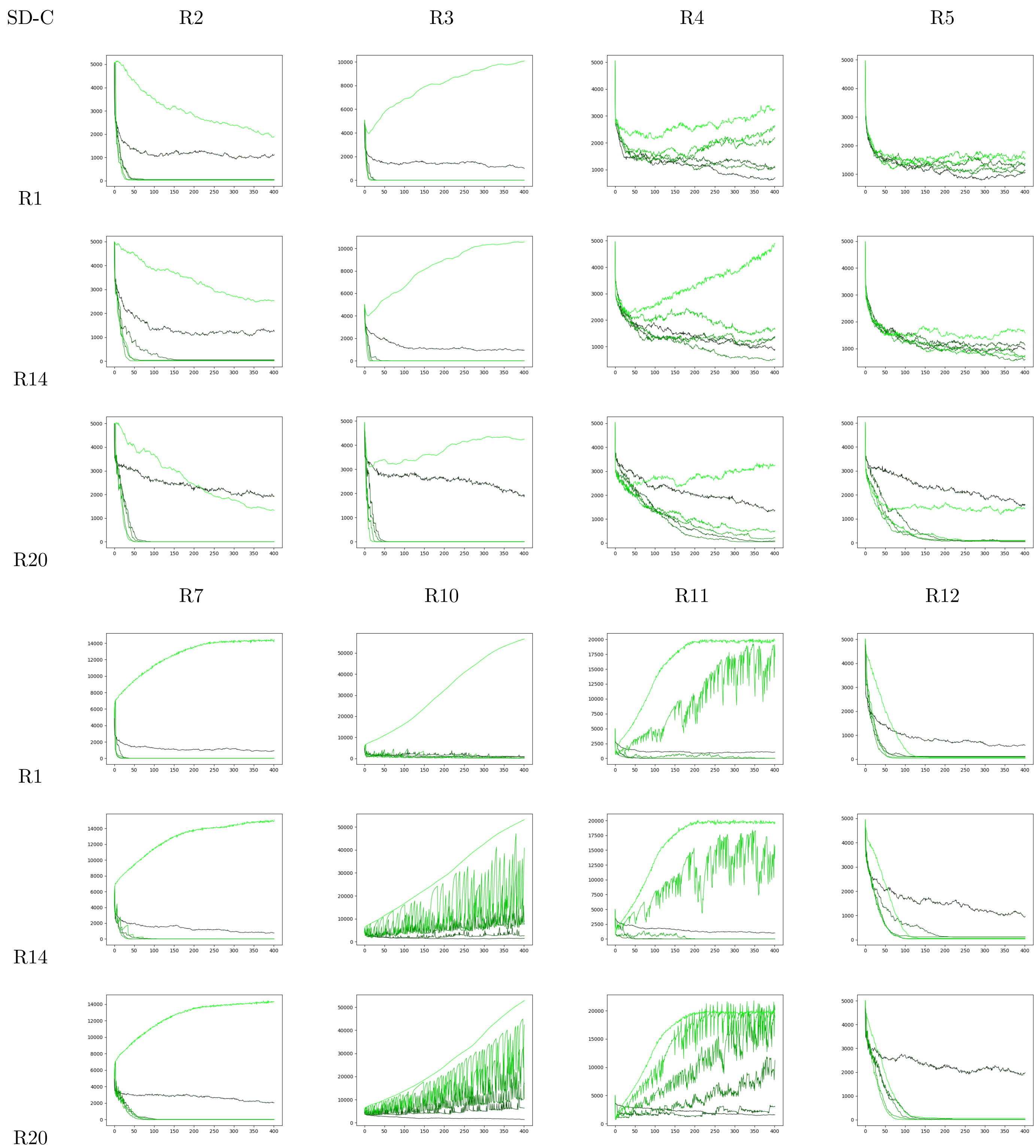
R7

R15

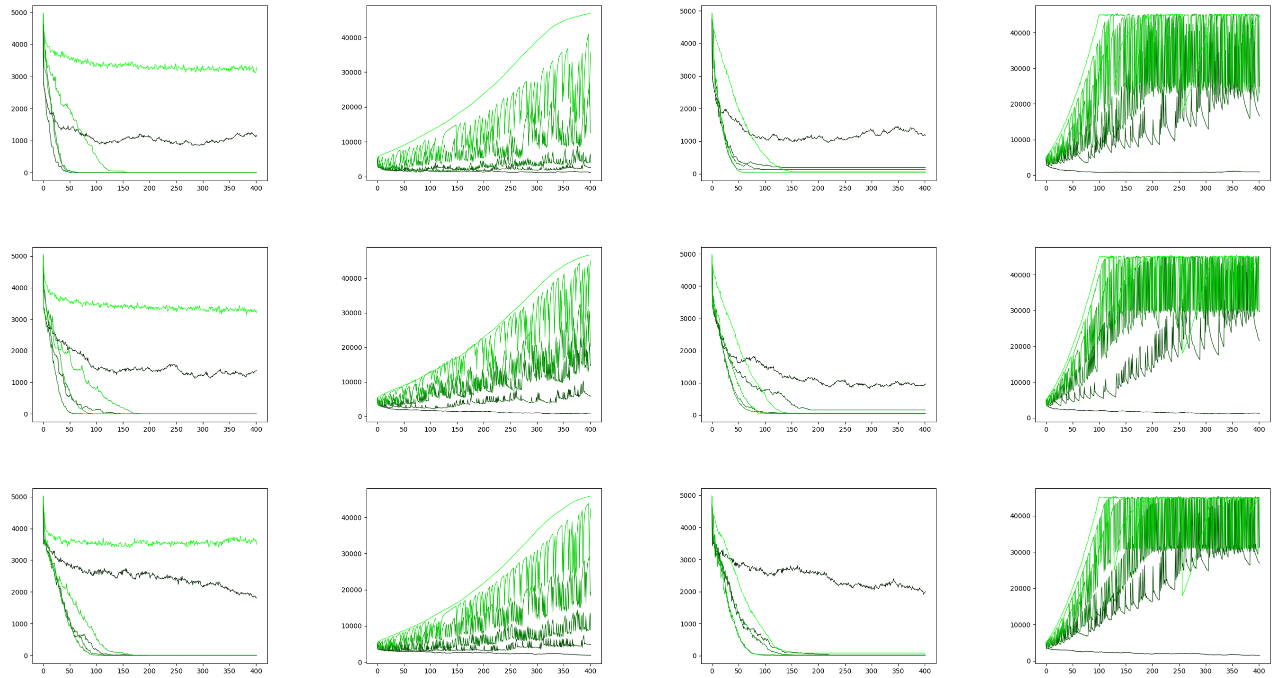
R17



For the chaotic case, we only show plots for R1,R14,R20 (resp. Life, Highlife, 2×2), these plots show again the step-depending case:


SD-C
R2
R3
R4
R5
R1
R14
R20
R7
R10
R11
R12
R1
R14
R20

R13 R15 R16 R17

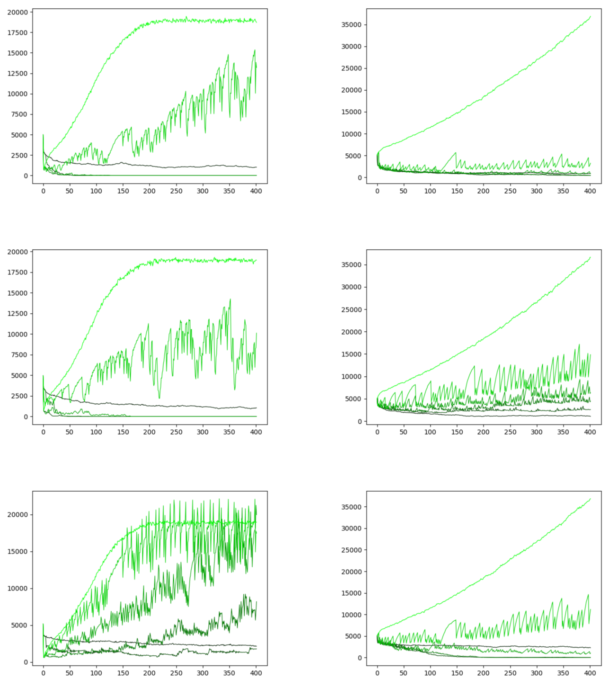


R1

R14

R20

R20

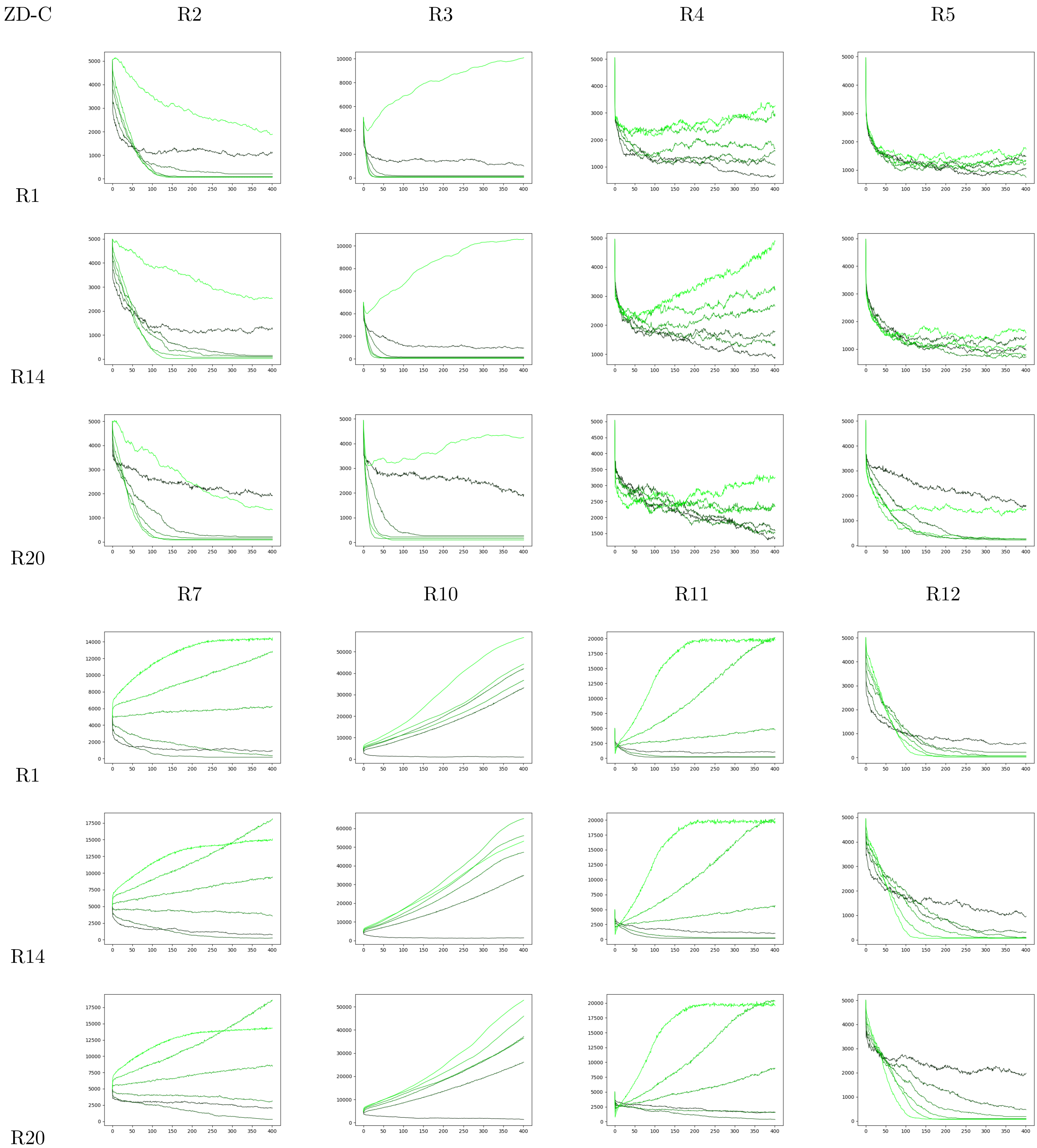


R1

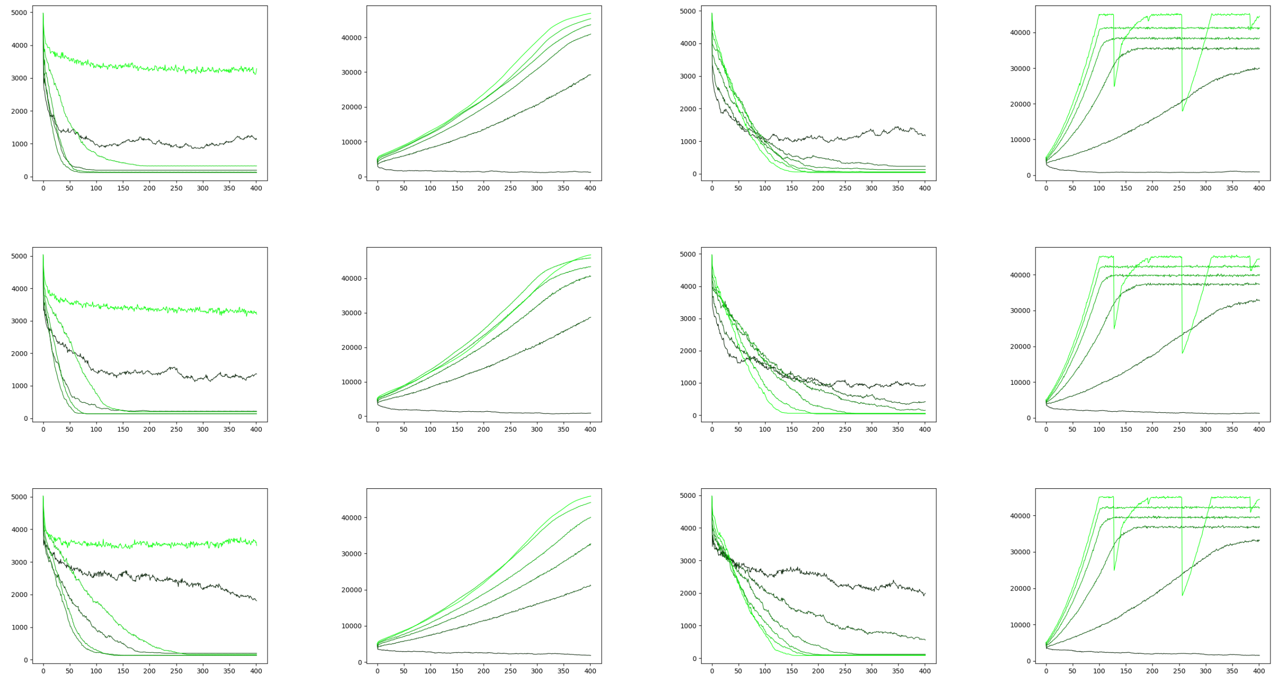
R14

R20

Again we show the Zone-depending case, but corresponding to the chaotic automata:


ZD-C
R2
R3
R4
R5
R1
R14
R20
R7
R10
R11
R12
R1
R14
R20

R13 R15 R16 R17

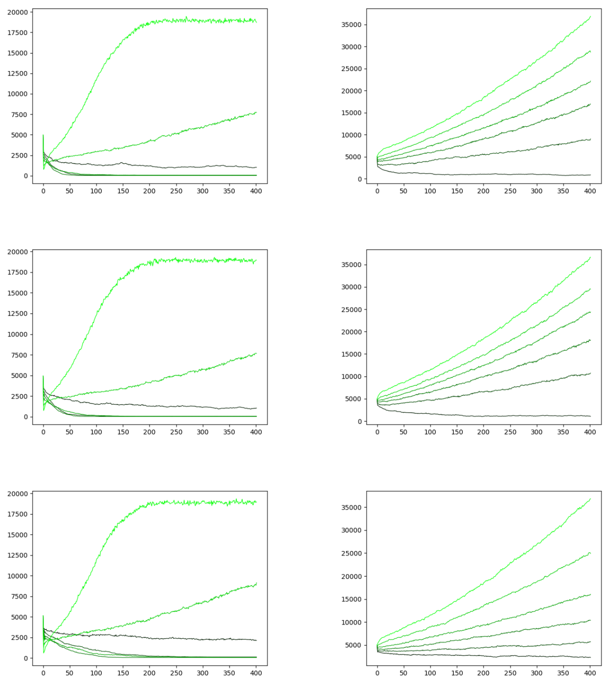


R1

R14

R20

R20



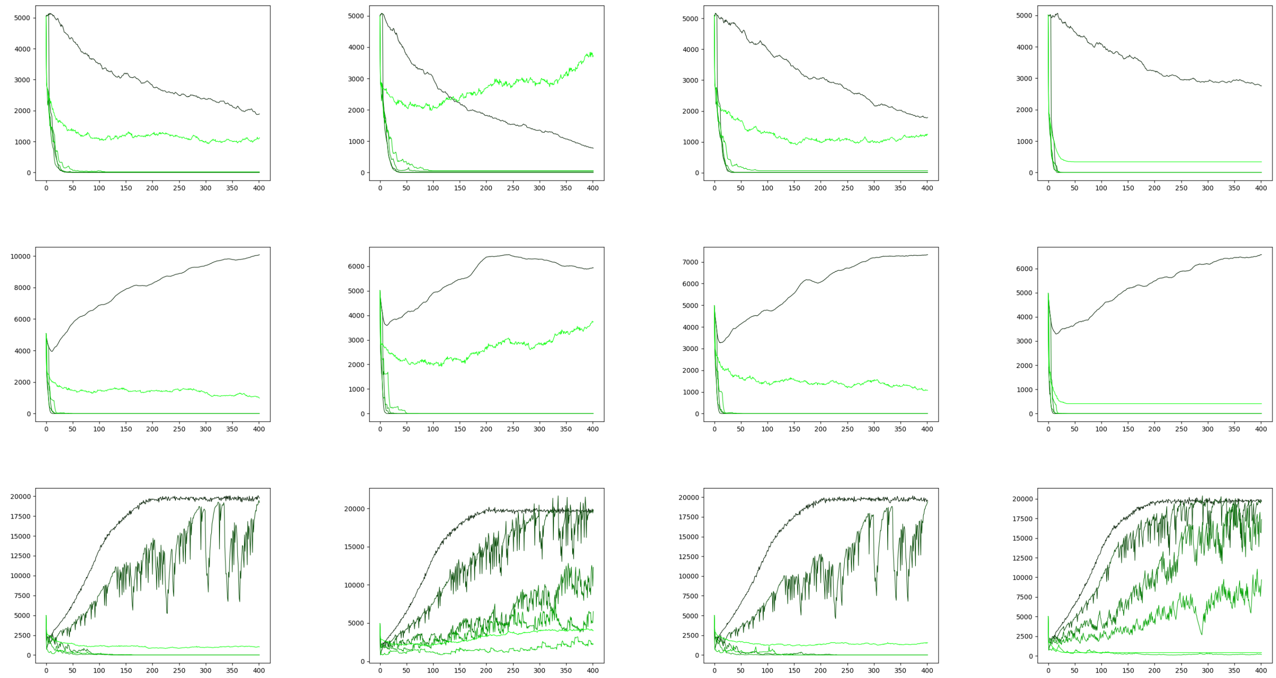
R1

R14

R20

We finally consider R2, R3, R11 (resp. Day and Night, Diamoeba, Live Free or Die) as representants the class of stable automata. We show once again the Step-depending case first:

SD-S R1 R4 R5 R6

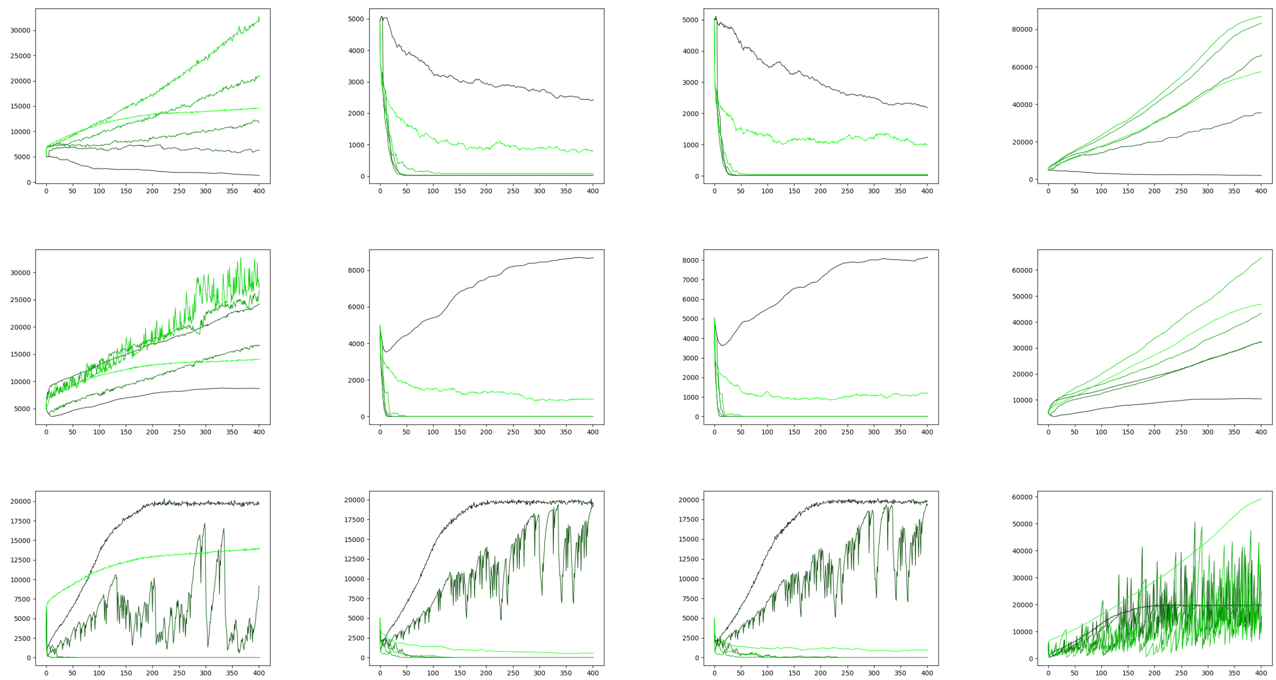


R2

R3

R11

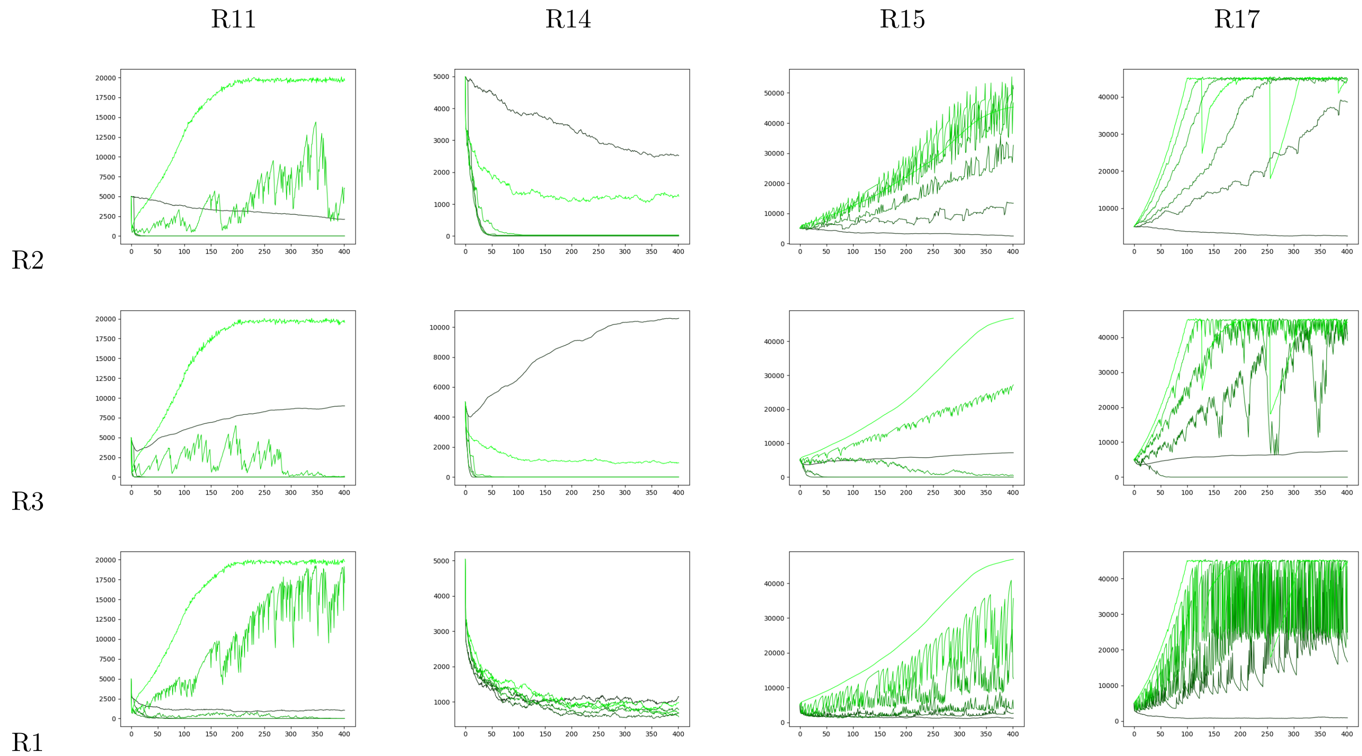
R7 R8 R9 R10

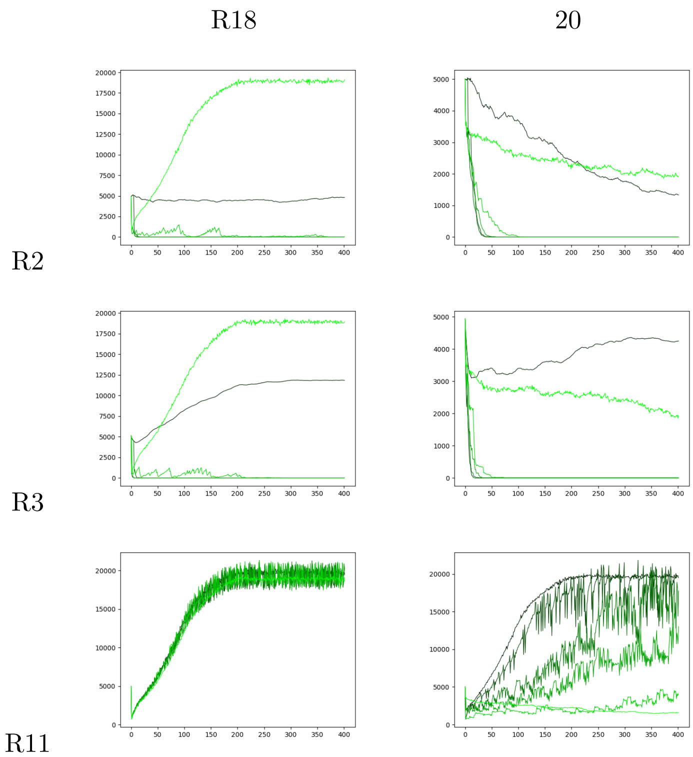


R2

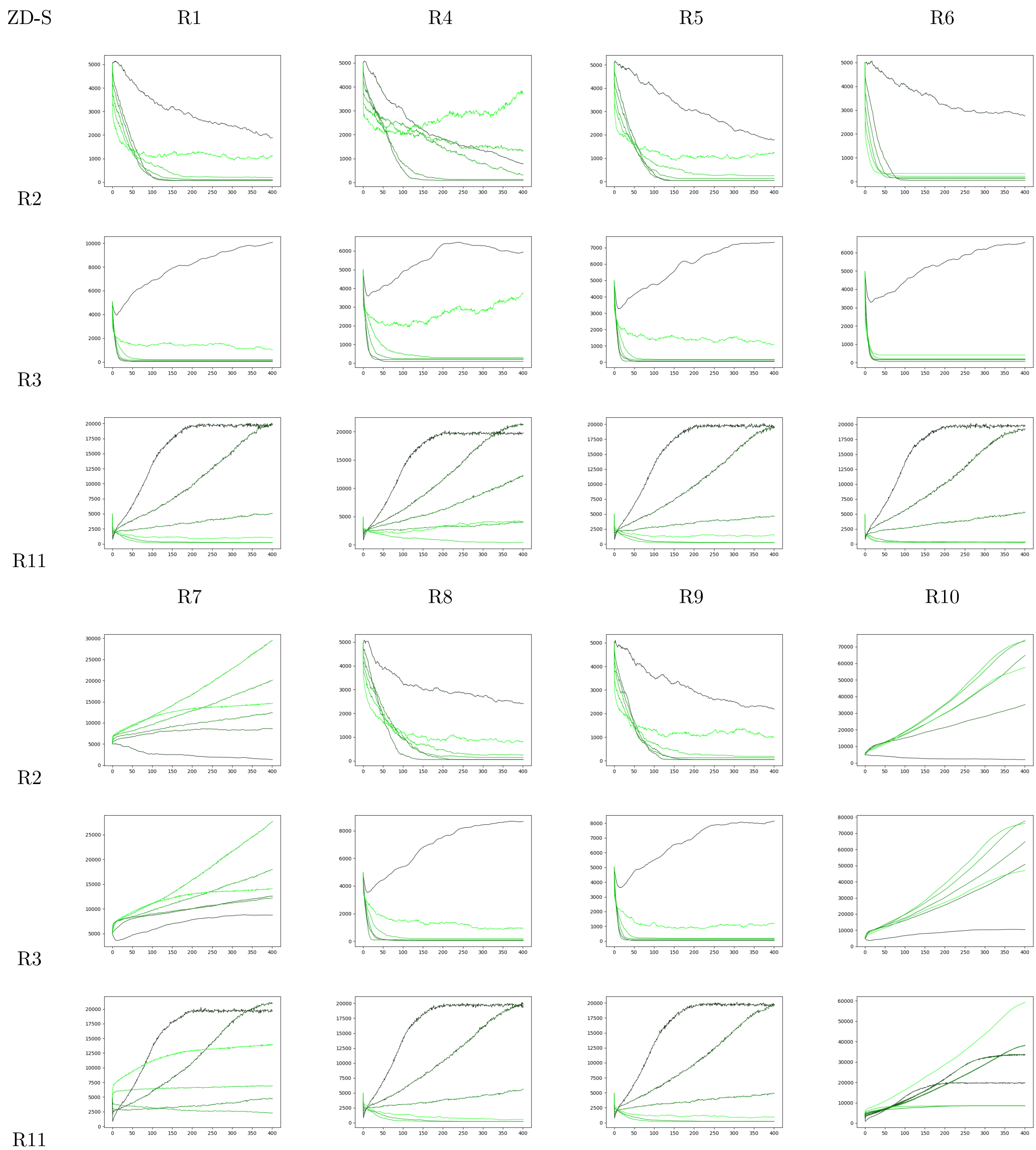
R3

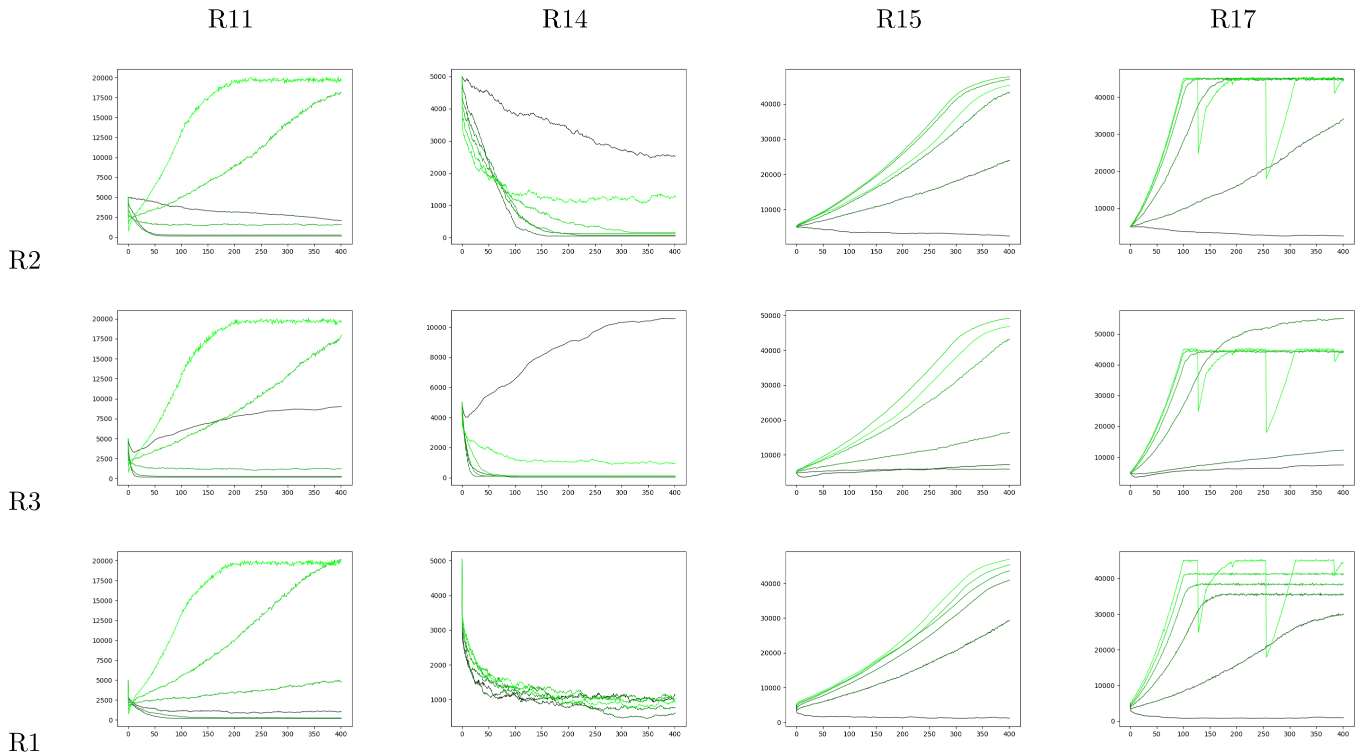
R11

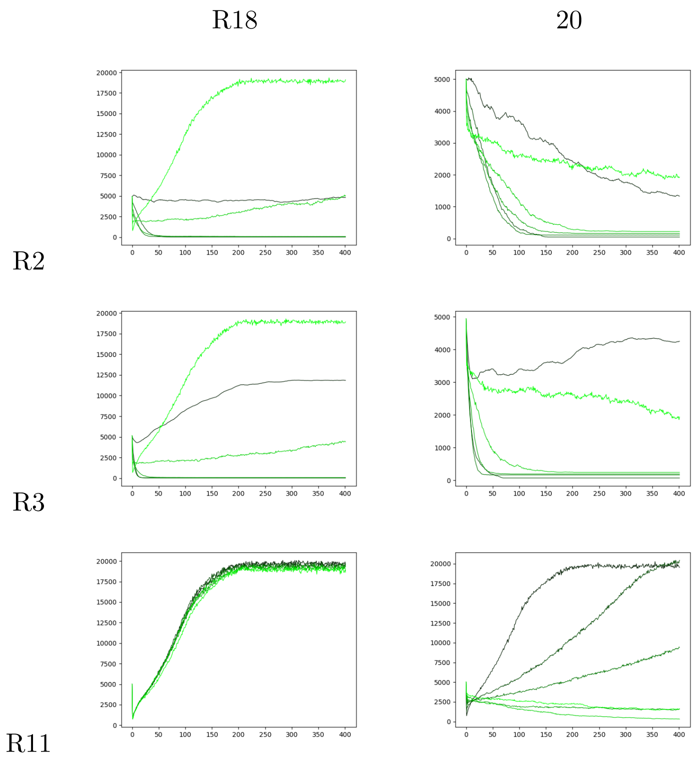




Finally, the mixing of stable and not stable automata for the Zone-depending case:


ZD-S
R1
R4
R5
R6
R2
R3
R11
R7
R8
R9
R10
R2
R3
R11





From an observation of these plots, we notice that in general the transition between one character and another is obtained more or less smoothly as the number of cells changes. We noticed however a number of cases where the character for mixed cells changes drastically, for example we can mention R7-R2 of the tables SD-E and ZD-E, R20-R2 for SD-C and ZD-C and R2-R4 for SD-S and ZD-S. We also noticed that also small amounts of cells with a different behavior can completely change the automaton, for example in the R17-R2 of SD-E and ZD-E we see that spikes of the original automaton (i.e. Replicator, due to the cyclical copying of the pattern across the grid) are not seen even with small concentrations of another automaton. We noticed this phenomenon also in the case of random automata.

9 Structures

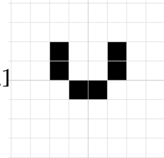
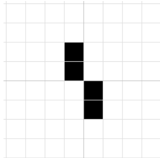
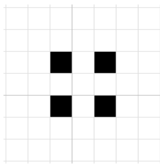
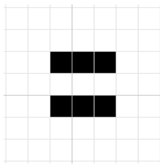
We found several different nontrivial structures, we will list them below. If the behavior is particularly interesting we will show also the respective evolution.

For S1-S6 we introduce the convention of indicating the mixed rule with the notation $\{Ra_1, ..., Ra_i\}/\{Rb_1, ..., Rb_j\}$ (i.e., the pattern is obtained for each of the combinations of $Ra_{\bar{i}}$ and $Rb_{\bar{j}}$, and in particular if $Ra_{\bar{i}} = Rb_{\bar{j}}$ we obtain the original LL-rule). When this is used in the pictures, the white (resp. gray for S5 and S6) refers to the first rule mentioned. This notation has demonstrated a big degree of compactness as we noticed frequently that schemes that repeat share one of the forming rules. In order to reduce the number of degree of freedom of all possible tests of S5 and S6, we consider only some particular cases. For S6, we found a low number of significant patterns, but this is no surprise, as by construction the freedom of the system is reduced, so simple patterns form more hardly, we noticed this behavior, although the reduction of freedom is lower, also for S5.

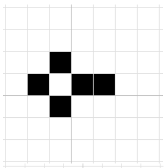
We finally use some modified grids:

Figure 9: Grids with Superposition (Resp. S5, S6).

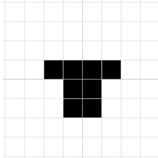
to show the respective superposition of cells of S5 and S6. In the former case the lower level cell is represented by the small square and the higher level cell is represented by the hollow one. In the latter we refer to the output of the automaton as the bottom right section of the square.

Scheme	Pattern Number	Properties/Remarks	Picture
S1	P1	Oscillator for each combination of {R6,R10,R15,R20}/{R1,R4,R5,R8,R9,R1 and {R1,R4,R5,R8,R9,R14,R19}/{R6,R10,R15,R20} (P=8). It is a still life pattern for {R6}/{R6,R10,R15,R20}, {R20}/{R6,R15}.	
S1	P2	Oscillator, P=2 for {R11}/{R1,R2,R3}, {R11}/{R1,R2,R3,R5,R7,R8,R9,R12,R13,R14,R16,R19}, {R18}/{R1,R2,R3,R5,R7,R8,R9,R12,R13,R14,R16,R19}, {R18}/{R2} and (with different behavior) for {R17}/{R2,R3,R7,R13}. It is still life for several Rules (As R4).	
S1	P3	Oscillator, P=2 for {R18}/{R2} and (with different behavior) for {R17}/{R2,R3,R7,R13}. It has P=4 for R4/R18 and R18/R4. It is moreover a still life pattern for {R4,R10}/{R4/R10},{R4}/{R18}.	
S1	P4	Oscillator, P=2 {R2,R3}/{R18} P=2 (different behavior) R17/R2, P=4 {R8,R12,R14,R16,R19}/{R2,R3,R7,R13} P=4 (different behavior) R18/R4.	

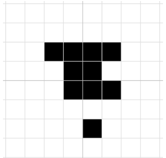
S1 P5 Oscillator, P=4 for {R1,R4,R5,R8,R9,R14,R19}/{R20}.



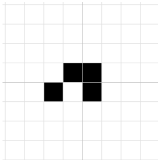
S1 P6 Oscillator with P=2 for R7, spaceship (orthogonal) for {R12,R16}/{R2,R7}, {R7}/{R12,R16}. Interestingly this pattern evolves into the P1 pattern for every of the considered rules such that the P1 pattern is an oscillator.



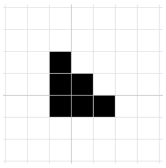
S1 P7 Spaceship (orthogonal), P=2 {R12,R16}/{R7}. It contains the previous pattern.



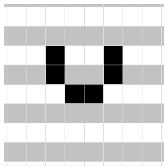
S1 P8 It is still life for {R10,15}/{R10,R15}, it is a puffer with P=12 for R4/R11.



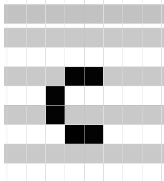
S1 P9 Spaceship, P=7 (diagonal){R12,R16}/{R12,R16}, oscillator with p=12 for R19/R4 and p=5 for R19/R7.



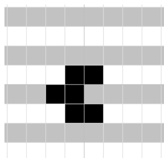
S2 P10 Oscillator P=6 for {R1,R4,R5,R8,R9,R14,R19}/{R6,R20}, still life for {R6,R10,R15,R20}/{R1,R4-R6,R8-R10,R12,R14-R16,R19,R20}.



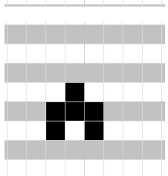
S2 P11 Still life for {R6,R10,R15,R20}/{R6,R10,R15,R20}.



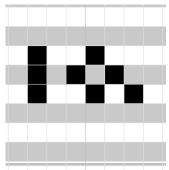
S2 P12 Oscillator, P=2 for {R2}/{R3,R7,R13}, spaceship (orthogonal), P=2 {R6,R20}/{R1,R4,R5,R9,R14}.



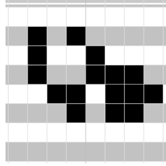
S2 P13 Oscillator, P=5 for {R1,R4,R5,R9,R14}/{R7}, p=2 for {R3,R7,R13}/{R10,R15,R19}.



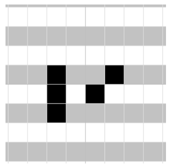
S2 P14 Spaceship (orthogonal), P=2 for {R11,R18}/{R7,R13}.



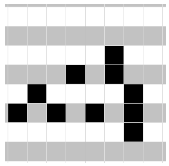
S2 P15 Gun, P=34 for {R1,R5,R9}/{R20}. It emits pattern 11 in both horizontal directions.



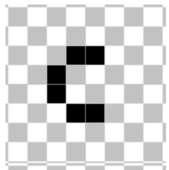
S2 P16 Spaceship (orthogonal), P=1 for {R11,R18}/{R7,R13}.



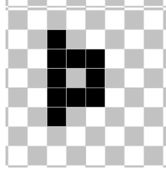
S2 P17 Puffer, P=8 for {R18}/{R7,R13}. It emits pattern P15 in the opposite direction.



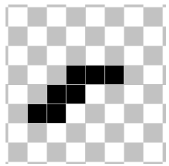
S3 P18 Still life for {R6,R10,R15,R20}/{R6,R10,R15,R20}.



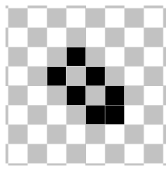
S3 P19 Oscillator, P=8 for {R1,R5,R8,R9,R14}/{R13}, P=10 for {R2}/{R16}.



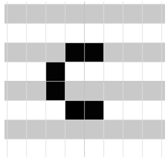
S3 P20 Spaceship (diagonal), P=10 for {R18}/{R1,R5,R8,R9,R14,R19}, P=10 for {R2}/{R16}.



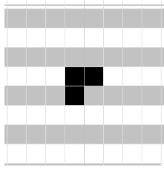
S3 P21 Still life for {R1,R4-R6,R8-R10,R12,R14-R16,R19,R20}/{R1,R4-R6,R8-R10,R12,R14-R16,R19,R20}, oscillator, P=18 for {R1,R4,R5,R8,R9,R14,R19}/{R3}.



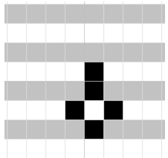
S4 P22 Still life for {R6,R10,R15,R20}/{R6,R10,R15,R20} (as P10 and P17).



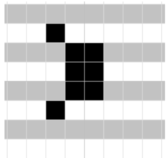
S4 P23 Spaceship (orthogonal), P=2 for {R11,R16}/{R6,R10,R15,R20}.



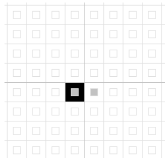
S4 P24 Spaceship (orthogonal), P=8 for {R20}/{R8,R14}.



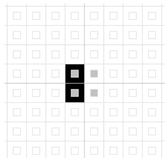
S4 P25 Spaceship (orthogonal), P=4 for {R12,R16}/{R1,R4,R5,R8,R9,R14}.



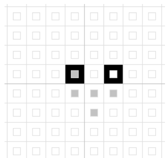
S5 P26 Still life for {R4,R10,R11}/{R6,R10,R15,R20}*, oscillator, P=2 for {R17}/{R6,R10,R15,R20}*. It works similarly if the two background points are diagonally disposed on the plane.



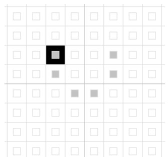
S5 P27 Still life for {R6,R10,R15,R17,R20}/{R1,R2,R4,R5,R8-R10,R14,R15,R19}*, {R6,R10,R15,R17,R20}/{R1-R10,R12-R16,R19,R20}** , oscillator, P=2 for {R11,R18}/{R1,R2,R4,R5,R8-R10,R14,R15,R19}*, {R11,R18}/{R1-R10,R12-R16,R19,R20}**.



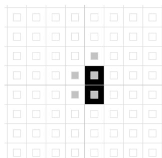
S5 P28 Oscillator, P=12 for {R17}/{R12}*,



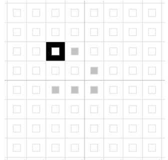
S5 P29 Still life for {R4,R10,R11}/{R6,R10,R15,R20}*, oscillator, P=14 for {R17}/{R6,R10,R15,R20}*,



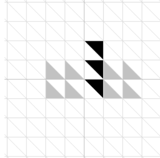
S5 P30 Oscillator, P=12 for {R17}/{R12,R16}*.



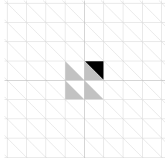
S5 P31 Spaceship, diagonal P=12 for {R17}/{R1,R5,R8,R9,R14,R19}*, {R17}/{R1,R2,R3,R5,R8,R9,R12,R14,R16,R19}**.



S6 P32 Oscillator, P=2 for {R3,R12,R16}/{R1,R4,R5,R8,R9,R14,R19}*, {R3,R6,R12,R16,R20}/{R1,R4,R5,R8,R9,R10,R14,R15,R19}**.



S6 P33 Still life for {R4,R10}/{R1,R2,R4,R5,R8,R9,R10,R14,R15,R19},Oscillator, P=2 for {R17}/{R1,R2,R4,R5,R8,R9,R10,R14,R15,R19}.



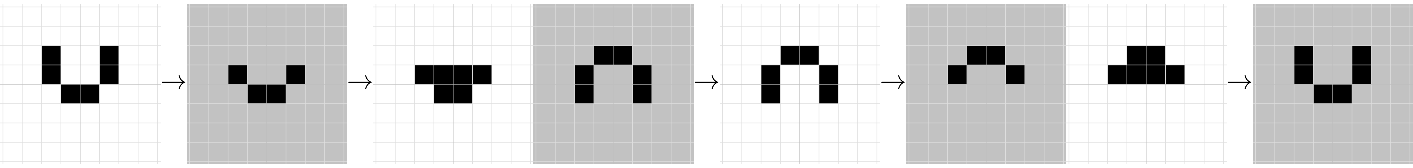


Figure 10: The cyclic evolution for the pattern P1.

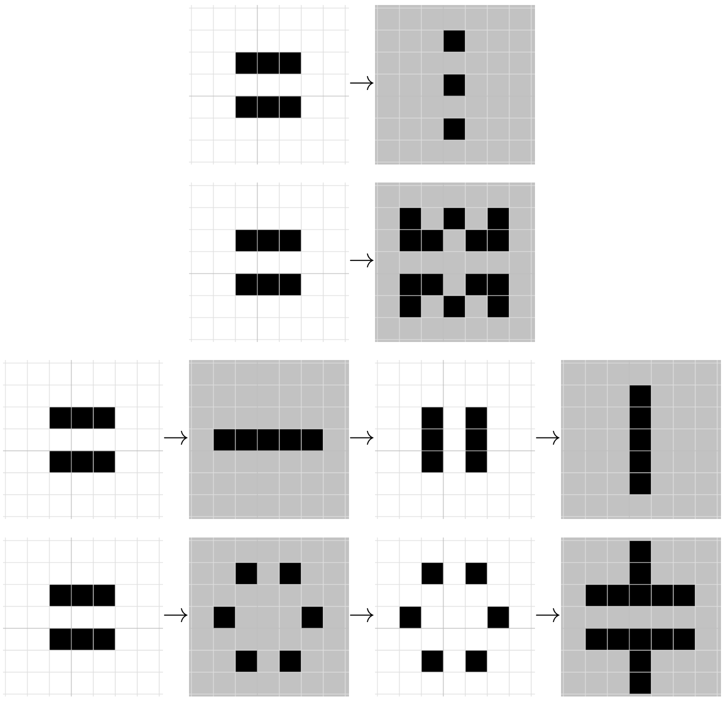


Figure 11: The four different cyclical evolutions detected for P4.

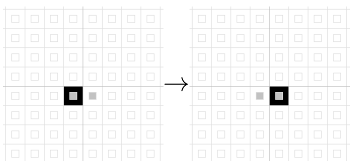


Figure 12: The cyclic evolution for the pattern P26.

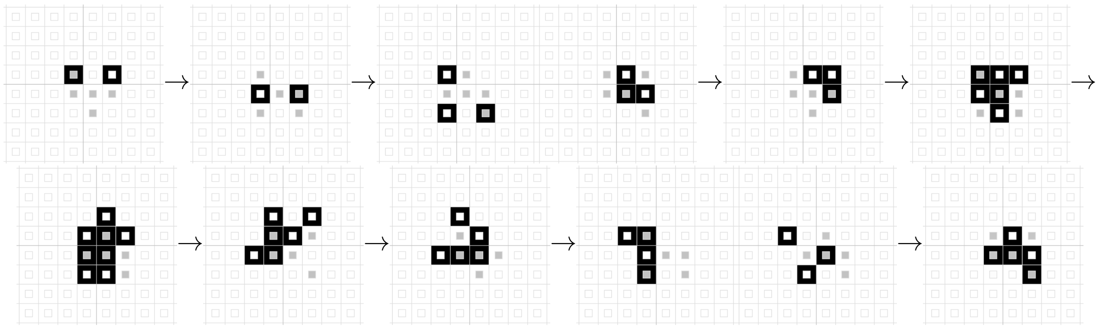


Figure 13: The cyclic evolution for the pattern P28.

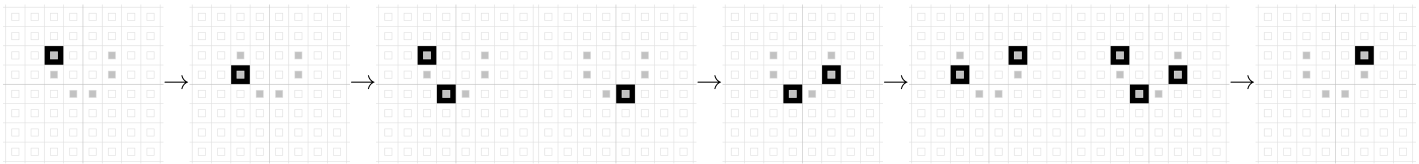


Figure 14: Part of the cyclic evolution for the pattern P29, only half of it is shown as the rest can be obtained by translation.

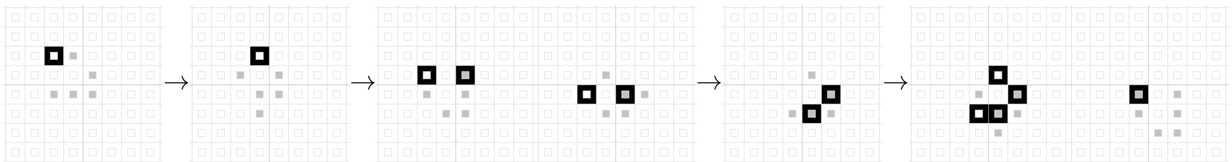


Figure 15: Part of the cyclic evolution for the pattern P31, only half of it is shown as the rest is rotated as the last picture coincides with the first rotated and mirrored.

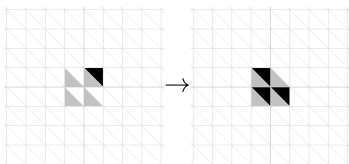


Figure 16: The cyclic evolution for the pattern P33.

Conclusions

We have shown several novels adaptations of Life-Like automata, that are built with the purpose of increasing the expressive capacity of those in the current literature. They are, as many definitions from cellular automata, prone to generalization for non Life-Like rules. It could be an interesting step for future research to study the synthesys of structures [8], or apply them to different grids, dimensions and schemes [10].

Acknowledgements

The author of this paper wishes to thank Daniela Giumbo (University of Messina), Martin Rypdal (University of Tromsø) for the precious suggestions that led to the improvement of this manuscript. This work was supported by the UiT Aurora Centre Program, UiT The Arctic University of Norway (2022).

References

[1] Banks, E. R. *Information processing and transmission in cellular automata*, Thesis, 1971.

[2] Bell, D. I. *Day & Night - An Interesting Variant of Life*,

`http://members.tip.net.au/~dbell/`

[3] Codd, E. F. *Cellular Automata*, WorldCat OCLC number: 637978742, 1968.

[4] Gardner, M. *MATHEMATICAL GAMES The fantastic combinations of John Conway's new solitaire game "life"*, Scientific American 223 (October 1970): 120-123.

`https://web.stanford.edu/class/sts145/Library/life.pdf`

[5] Gardner, M. *On cellular automata, self-reproduction, the Garden of Eden and the game of "life"*, Scientific American, 224 (1971): 112–117.

[6] `https://conwaylife.com/wiki/`

[7] `https://conwaylife.com/wiki/Category:Cellular_automata`

[8] Johnston, N.; Greene, D. *Conway's Game of Life - Mathematics and Construction*, DOI: 10.5281/zenodo.6097284, 2022.

`https://conwaylife.com/book`

[9] Rokicki, T. *An algorithm for compressing space and time*, 2006.

[10] Wolfram, S. *Cellular Automata and Complexity*, DOI: 10.1201/9780429494093, ISBN: 9780429494093, 2018.

[11] Wolfram, S. *A New Kind of Science*, Wolfram Media, ISBN: 1579550088, 2002.

A Evolution of Cell Population

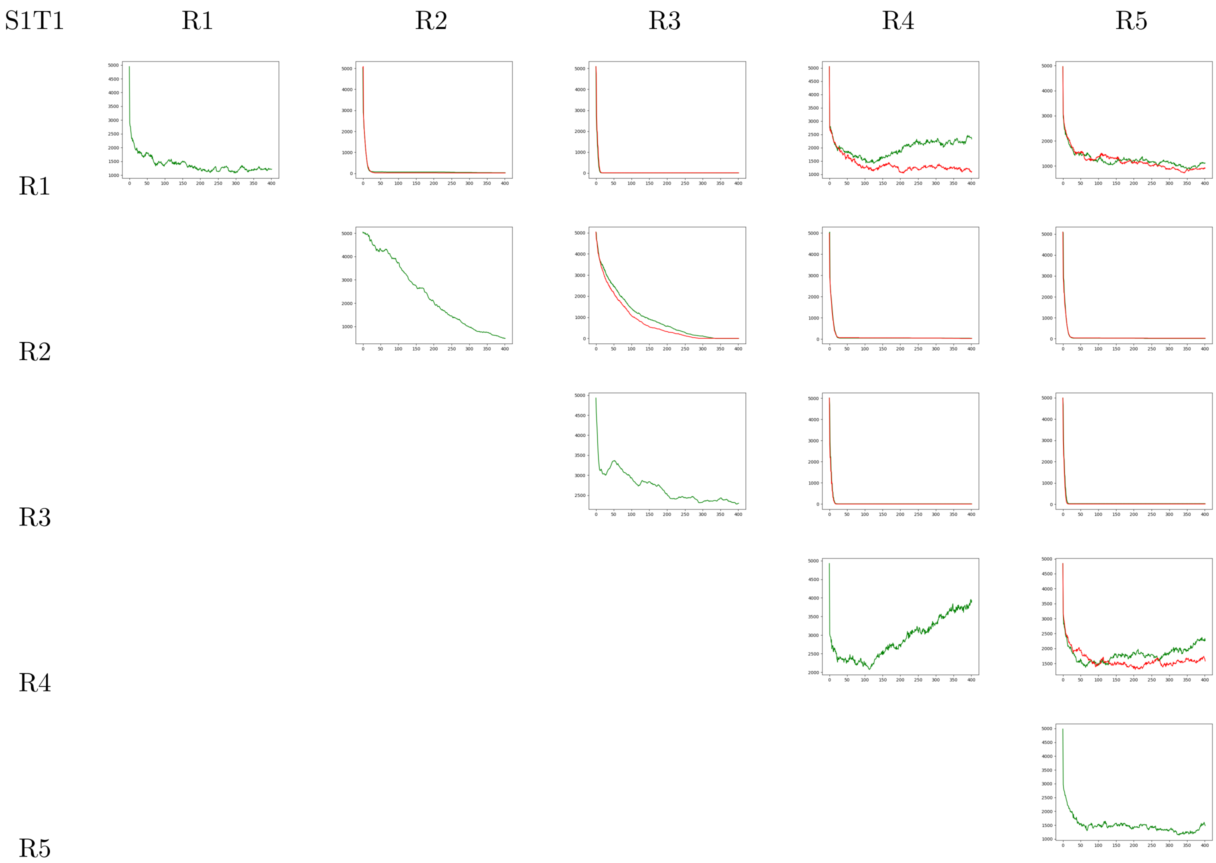
This section includes many plots showing the evolution (in term of number of cells) for all the used configurations S1-S7. For clearer comparison we show both the behaviors (i.e., green (resp. red) plot if the initial rule is on the horizontal (resp. vertical) axis of the table). For S5 and S6, we show also the bottom layer cell counts, in dashed lines. In all cases the considered grid has finite size of 300x300 cells. For S1-S4 and S7, the initial condition is given by an empty table except for the central 100x100 square. We filled this square with random cells having density of 1/2 ($\approx$ 5000 units). For S5 and S6 the central sets of cells have been moved diagonally by 20 cells in opposite directions. This is to have a starting condition with all the possible nonempty states (cell of the first layer only, cell of the second layer only and cells in both layers). For the S5 plots, the entry Ra/Rb is charactreized by having Ra in the first layer and both Ra and Rb in the second one. Similarly for S6, the seed to generate the cells in the central square was generally different in the configurations, but we initialized the configurations in the same plot with the same initial pattern (e.g. alternating R1/R2 and R2/R1) with the same pattern. In all of the cases S1-S4 plots on the main diagonal of the tables are coinciding with the well-known rules mentioned in the first chapter. For better readability we have split the plots in subtables (see following scheme). Symbols in the table are to remark that plots in the main diagonal reflect the structure of the table (i.e., upper triangular or squared). Additional information about these plots can be found in the sections Dynamic Behaviour and Character Mixing of the main text. We also want to remark that in some cases these tables were not containing enough information to determine the characters. For example, in (S1T1, R1/R4) the population plots diverge as a consequence of the specific initial seed in in one of the cases. We performed several runs with seeds having identical characteristics to unveil the effective behavior of the automaton. Finally, we noticed that although these plots might not suffice to determine the behavior of automata, they often contain really important information about the automata character and properties:

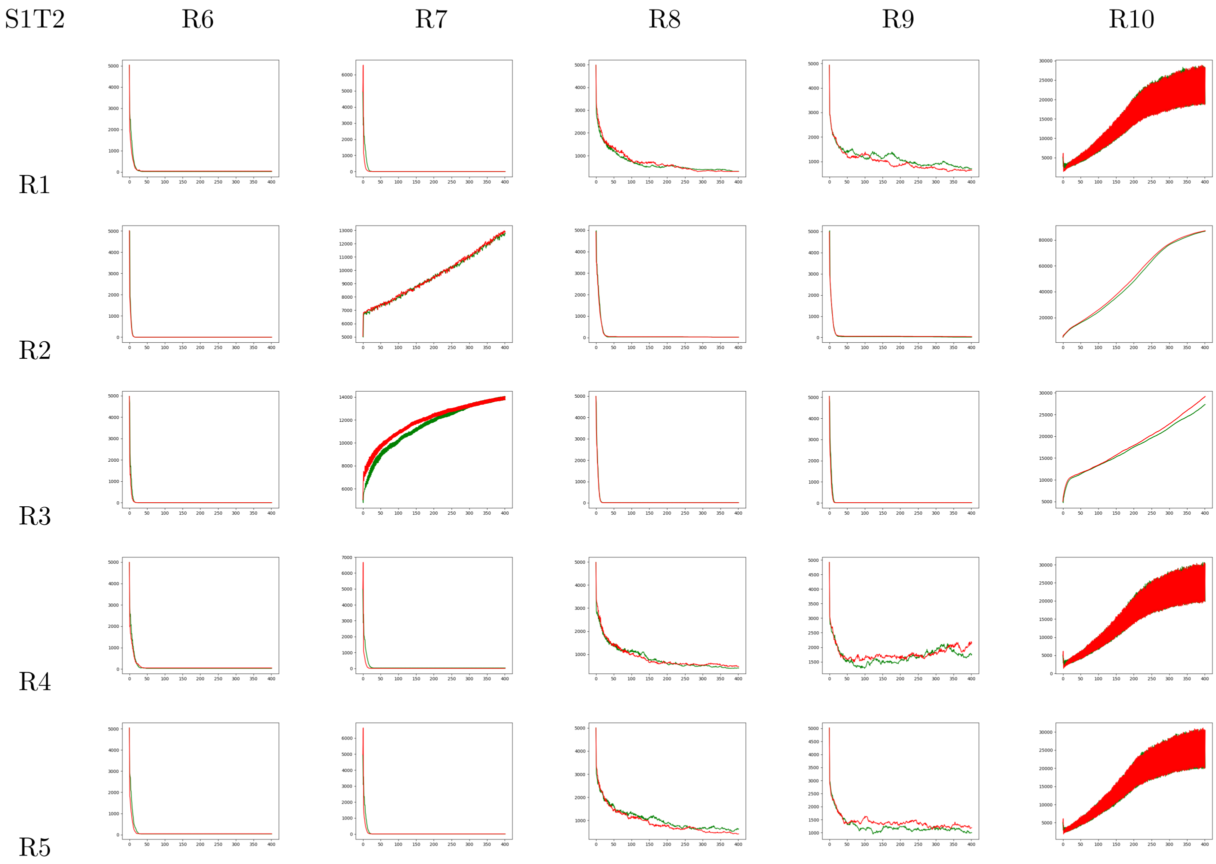
- An increasing graph is generally associated with Exploding patterns (Growing or with higher Persistant Pattern generation in cases of random automata), e.g. S1T4, R2/R17; $\omega = \frac{1}{250}$, R11.
- Noise in later stages of exploding patterns is generally caused by the incresing number of cells of the automaton (e.g. S1T2, R1/R10). In many cases of slowing down of the curve, the filling of the grid and subsequents fluctuations of the automaton are present e.g. S1T2, R2/R10).
- Graph that stabilize quickly are generally associated with stable behaviors (Resp. Stable or Collapsing in the case of random automata) e.g. S1T2, R4/R6; $\omega = \frac{1}{250}$, R12.
- Chaotic and Exploding automata tend to have more fluctuations than Stable ones (S1T3, R1/R13, R1/14, R1/15).
- Chaotic and Explosive rules plot tend to diverge more easily with different initial conditions (R1/R11, R4/R14).
- Explosive patterns can have a drop in their number of cells before actually starting to grow. This means that to grow some pattern organization is needed (S2T3, R4/R11).
- Random automata can vary strongly with small perturbations on α or ω ($\omega = \frac{1}{150}$, R20).

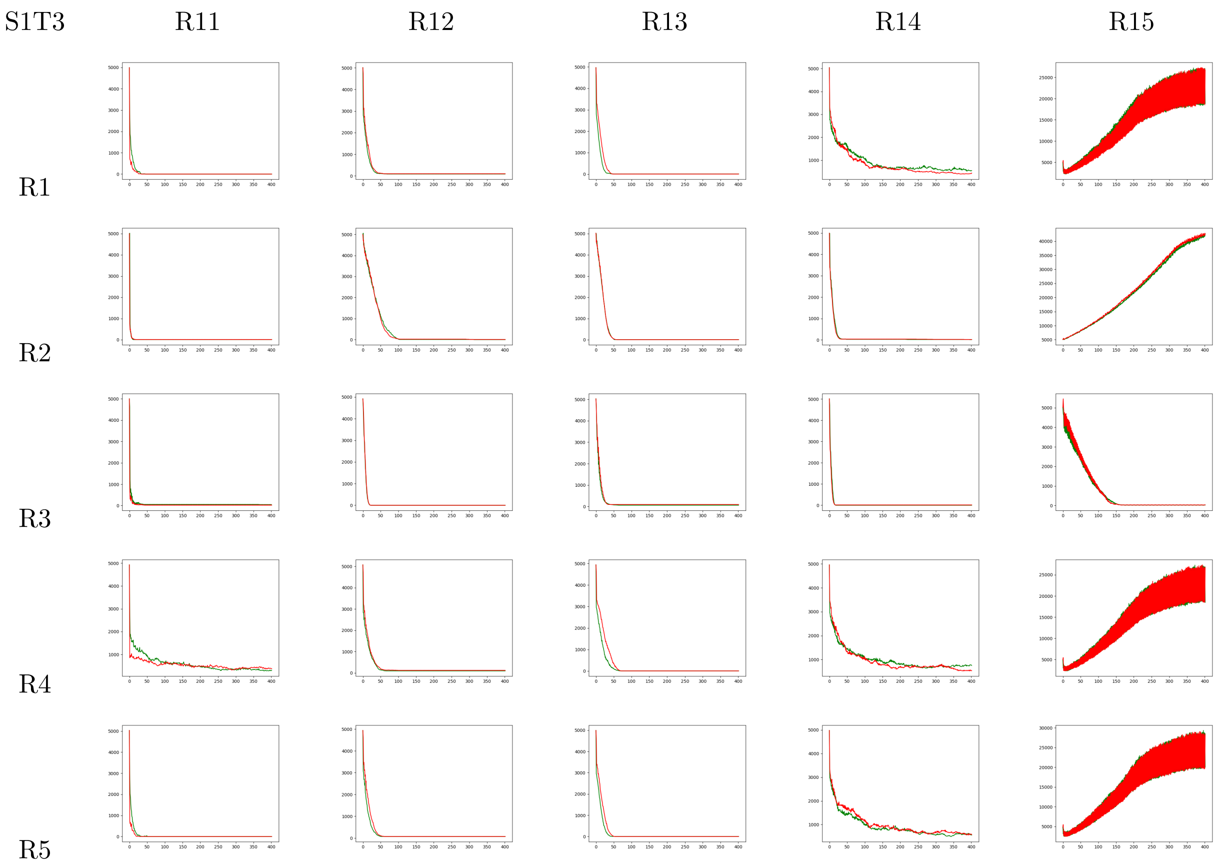
Rules	R1-R5	R6-R10	R11-R15	R16-R20
R1-R5	T1 (▲)	T2 (■)	T3 (■)	T4 (■)
R1-R5		T5 (▲)	T6 (■)	T7 (■)
R1-R5			T8 (▲)	T9 (■)
R1-R5				T10 (▲)

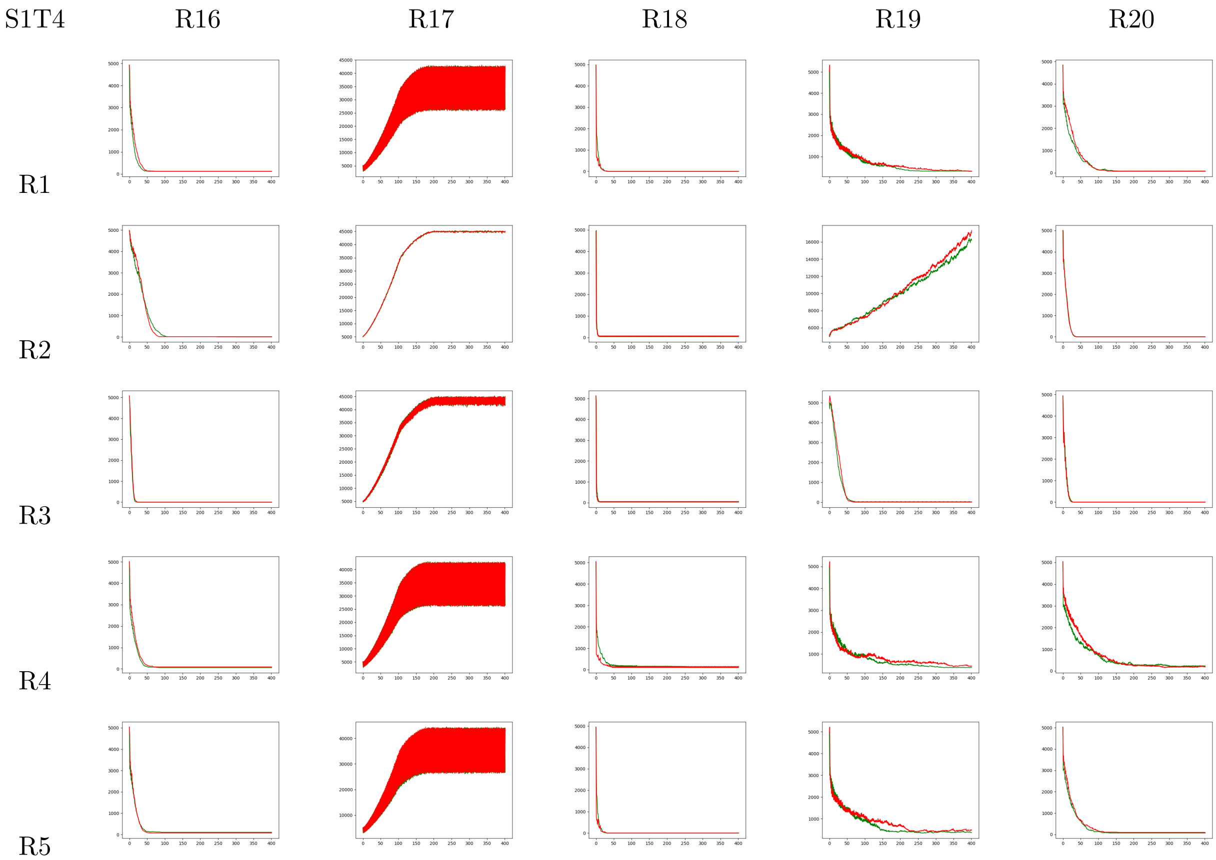
A.1 S1

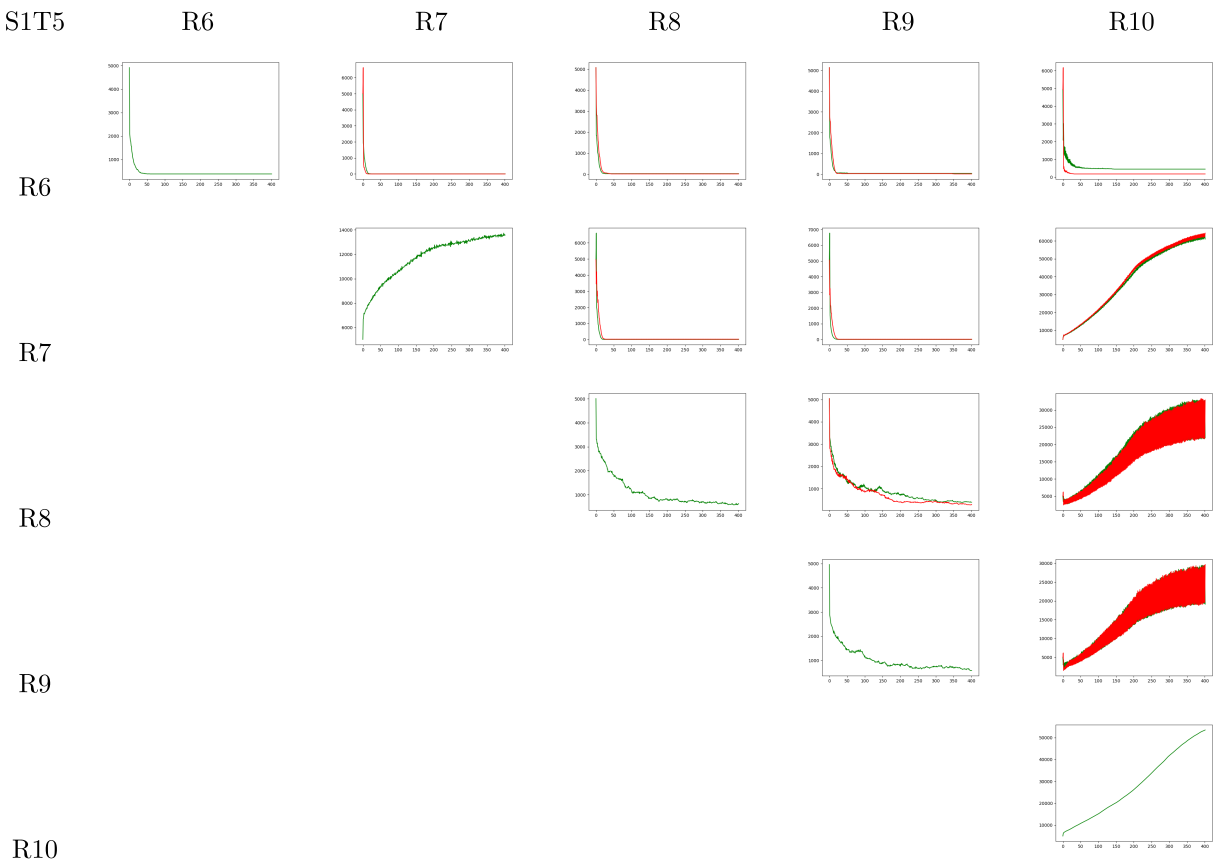
These plots show the cell populations for rules that alternate according to the scheme S1 previously mentioned.




S1T2
R6
R7
R8
R9
R10
R1
R2
R3
R4
R5


S1T3
R11
R12
R13
R14
R15
R1
R2
R3
R4
R5


S1T4
R16
R17
R18
R19
R20
R1
R2
R3
R4
R5


S1T5
R6
R7
R8
R9
R10
R6
R7
R8
R9
R10

S1T6 R11 R12 R13 R14 R15

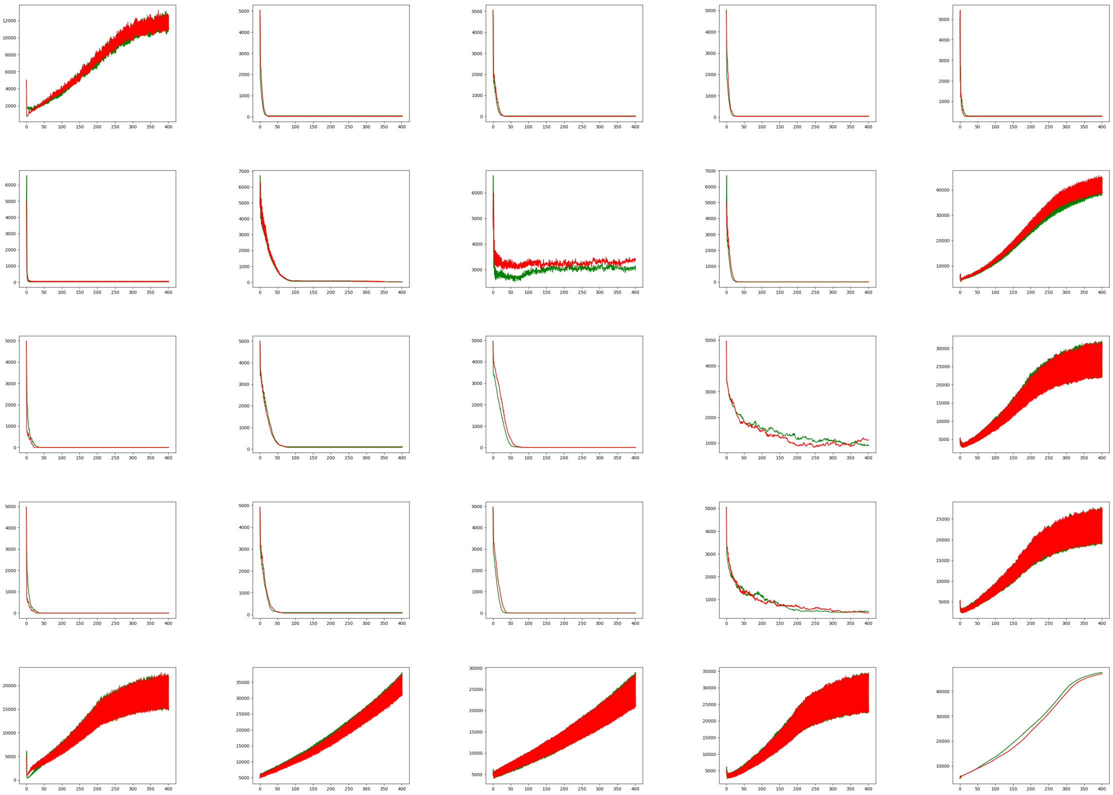
R6

R7

R8

R9

R10



S1T7 R16 R17 R18 R19 R20

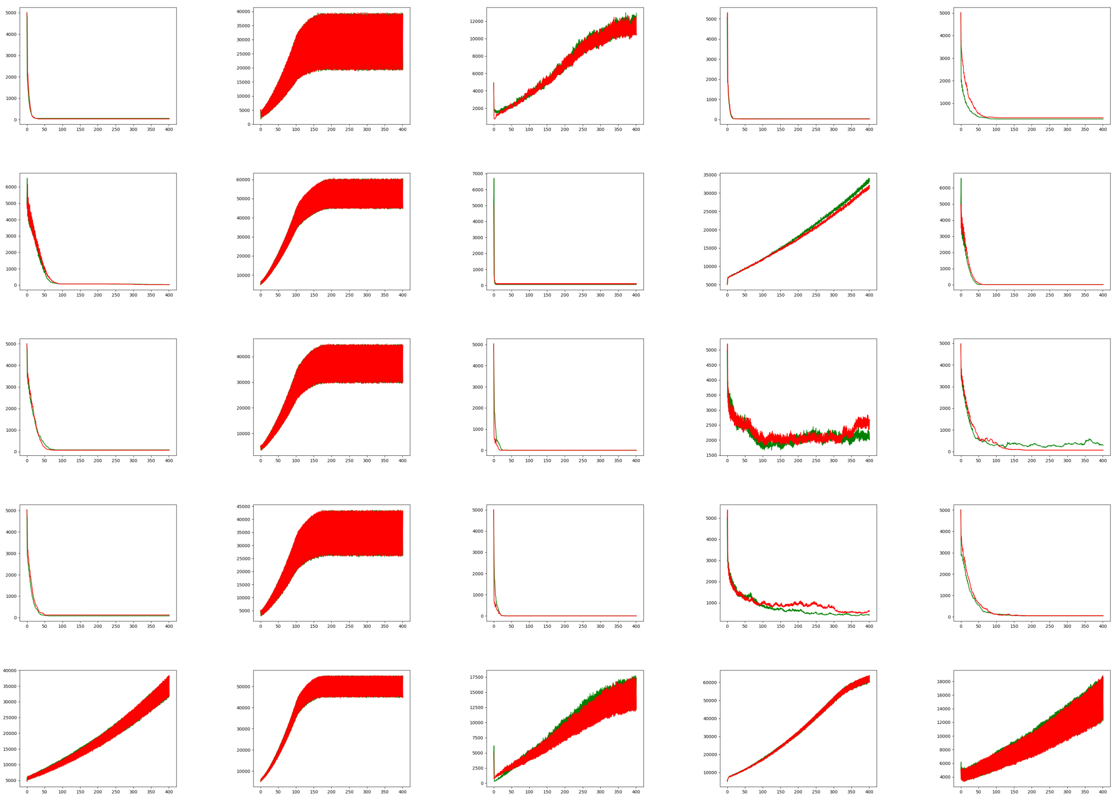
R6

R7

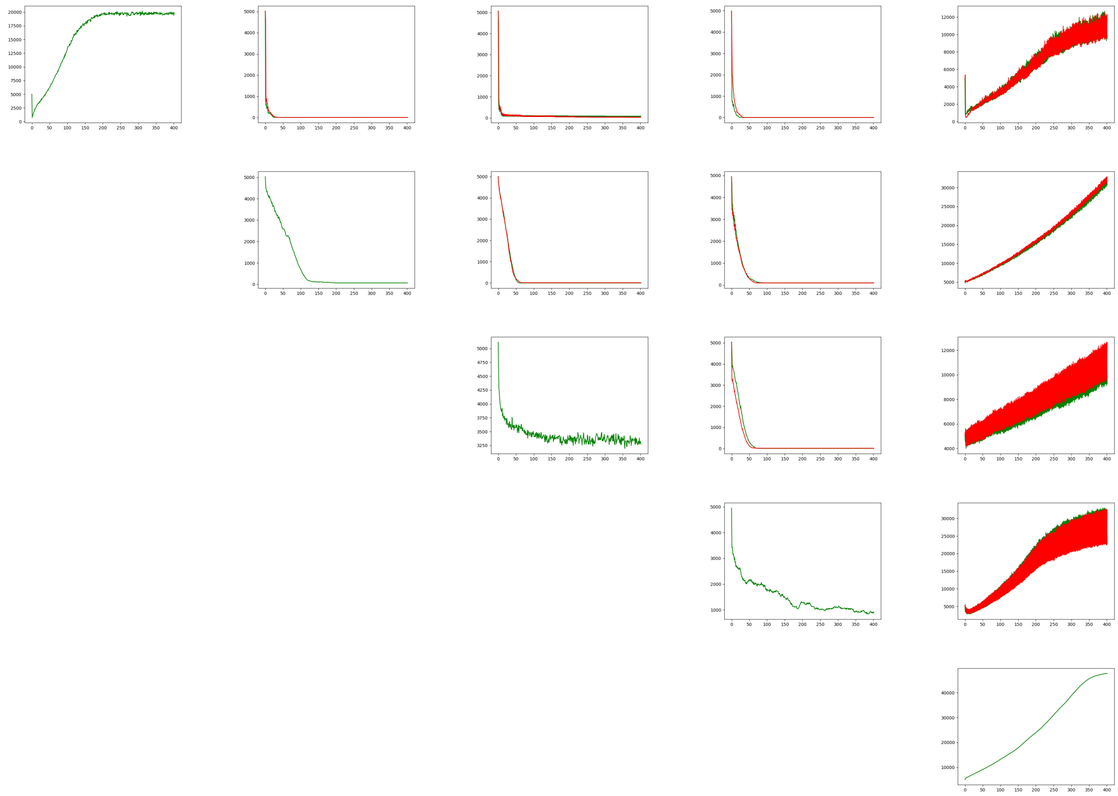
R8

R9

R10

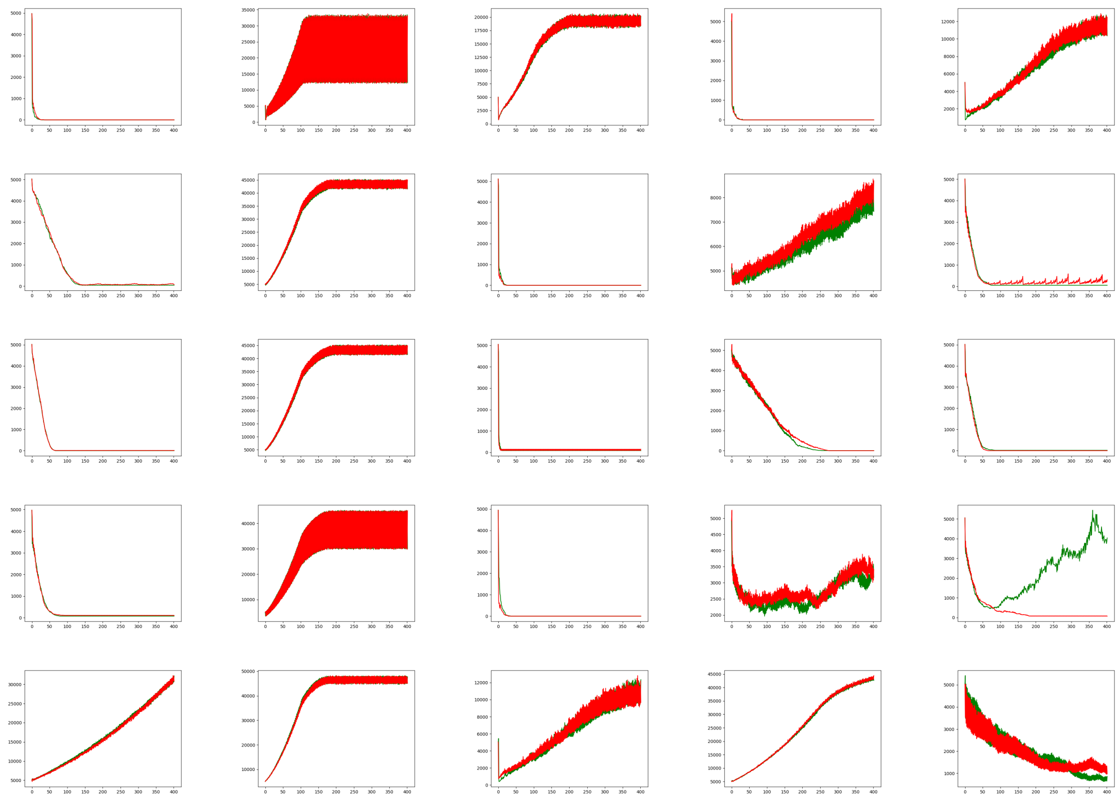


S1T8 R11 R12 R13 R14 R15

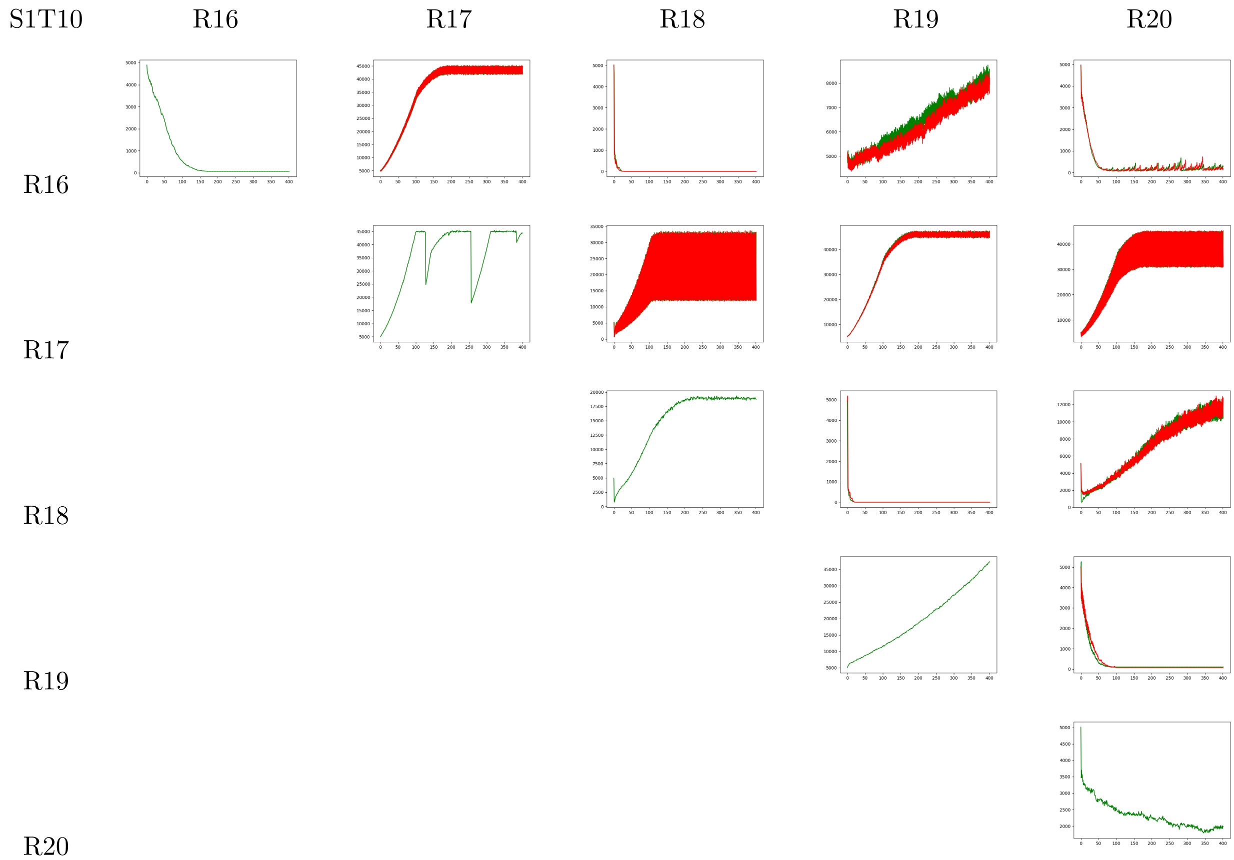


R1
R2
R3
R4
R5

S1T9 R16 R17 R18 R19 R20

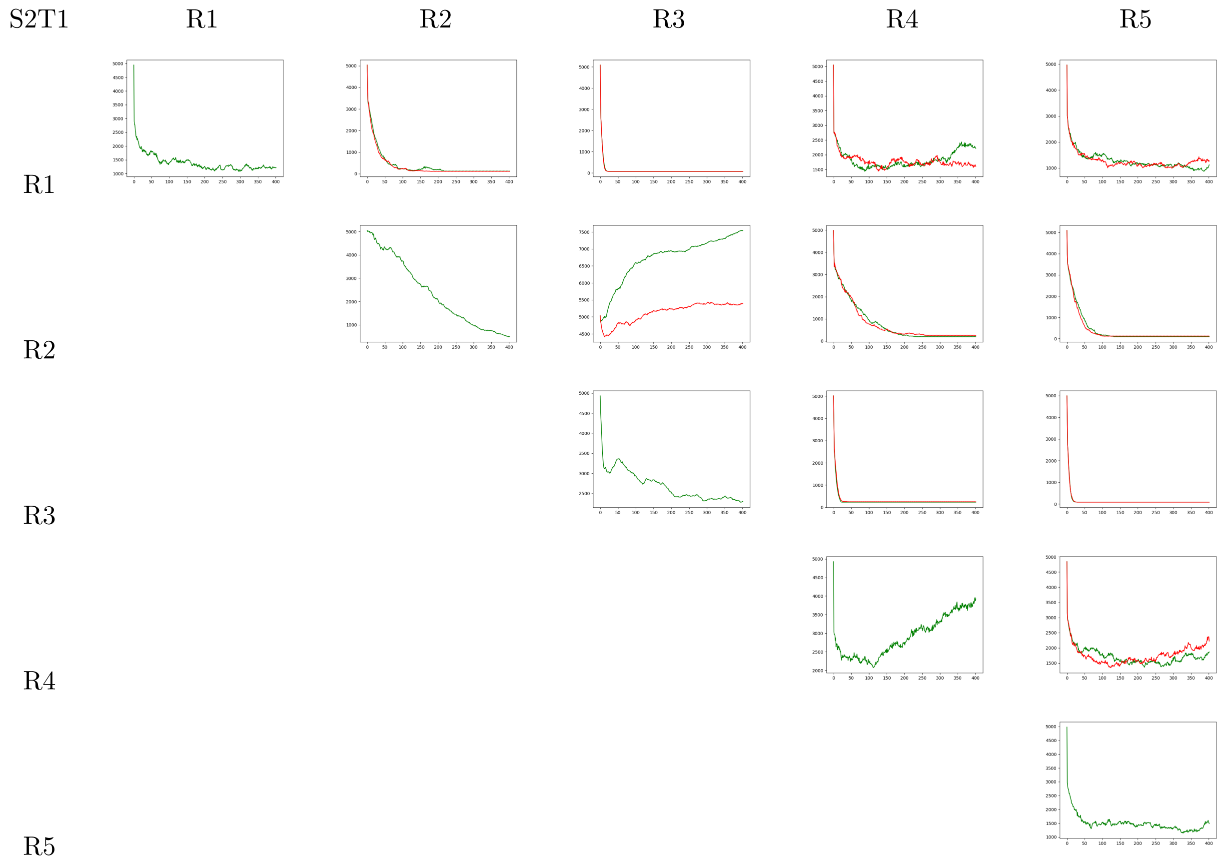


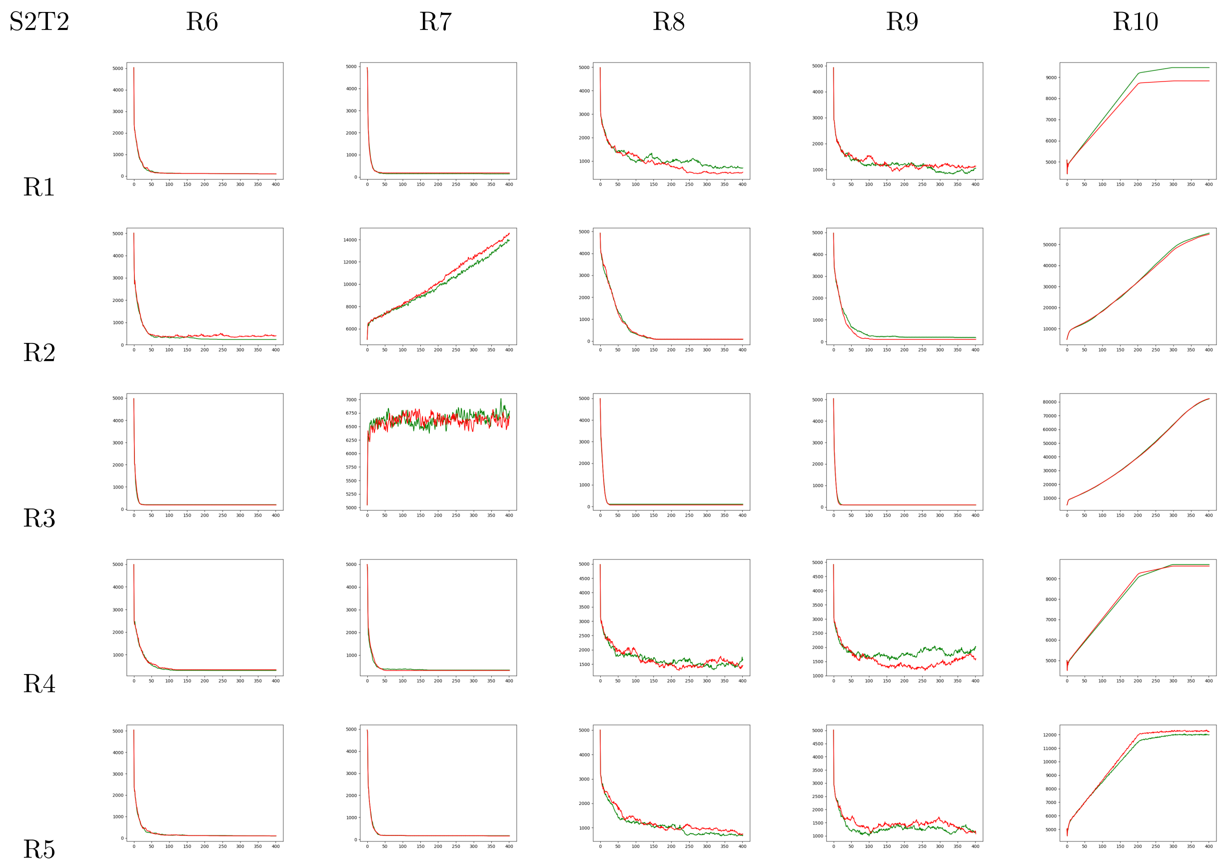
R11
R12
R13
R14
R15



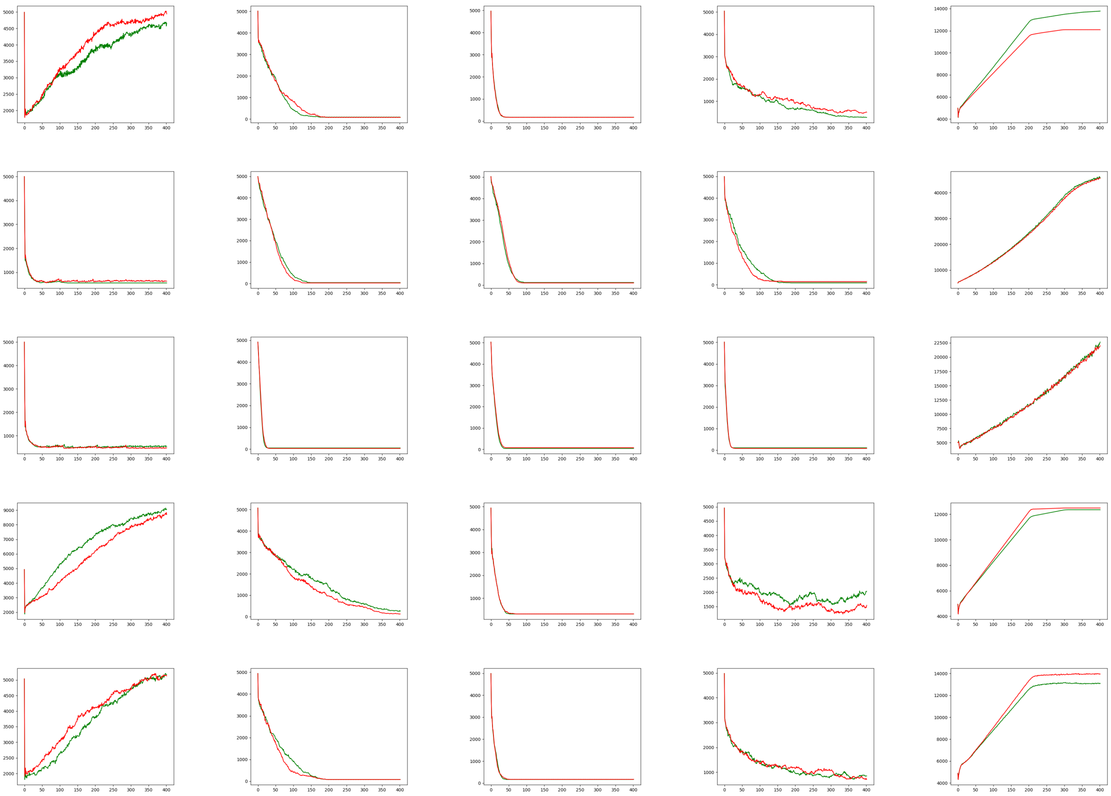
A.2 S2

These plots show the cell populations for rules that act in parallel lines according to the scheme S2 previously mentioned.


S2T1
R1
R2
R3
R4
R5
R1
R2
R3
R4
R5

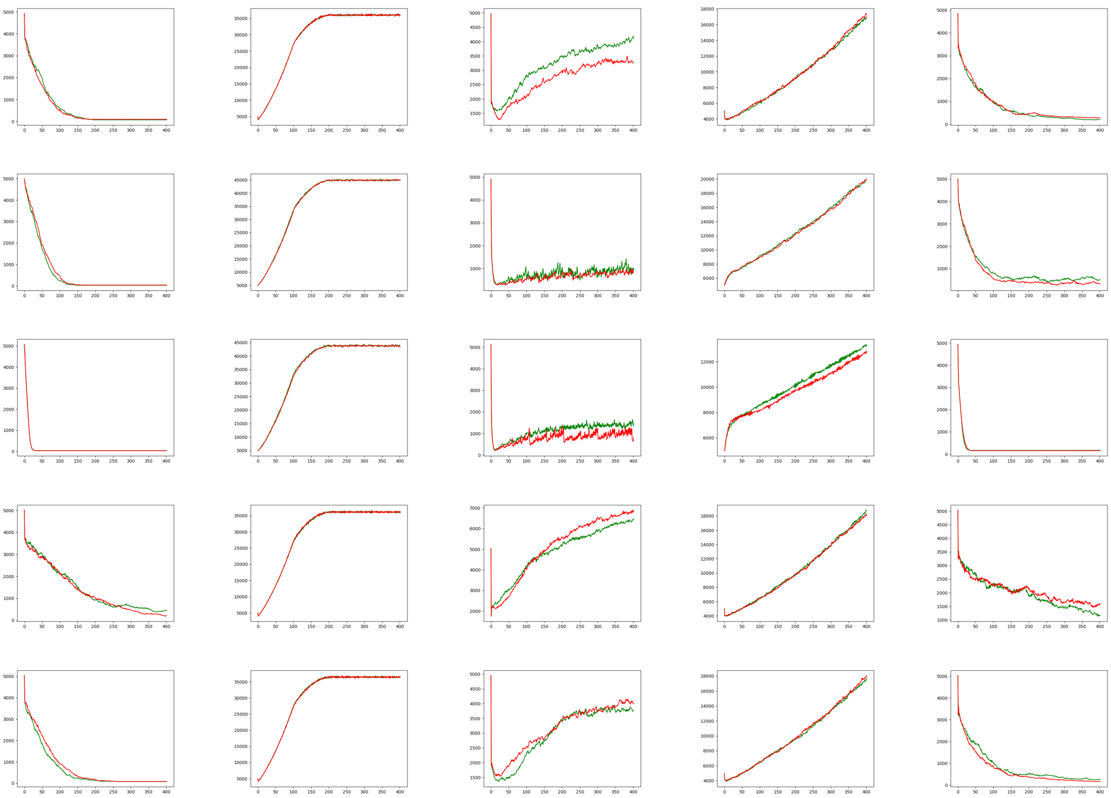

S2T2
R6
R7
R8
R9
R10
R1
R2
R3
R4
R5

S2T3 R11 R12 R13 R14 R15

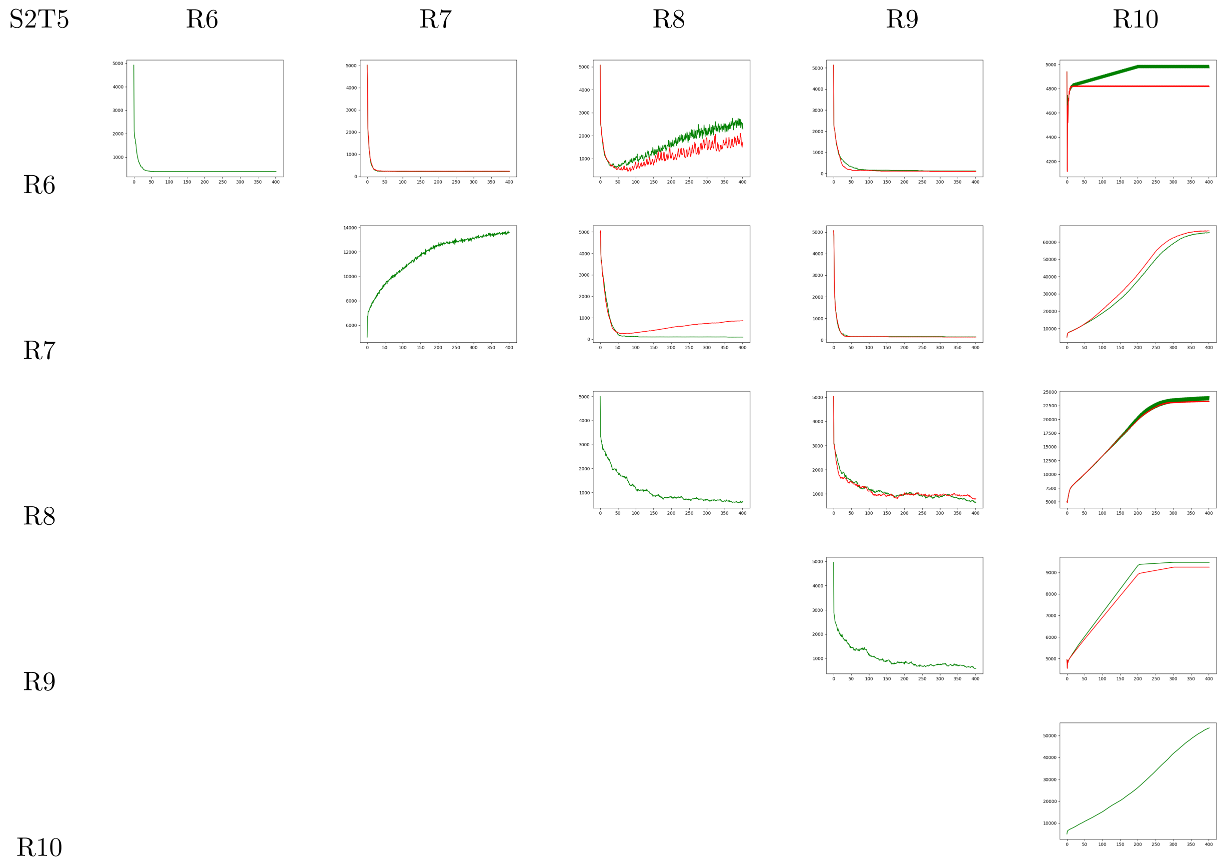


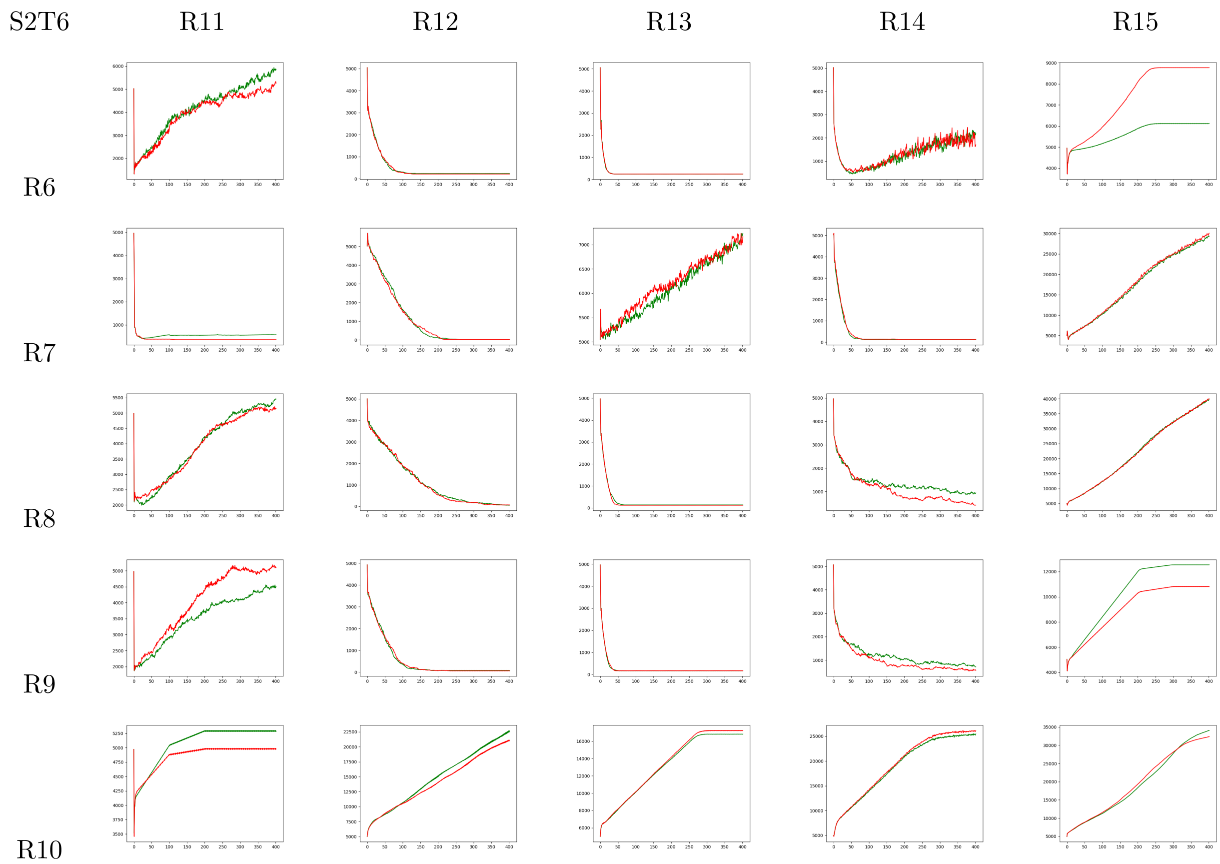
R1
R2
R3
R4
R5

S2T4 R16 R17 R18 R19 R20

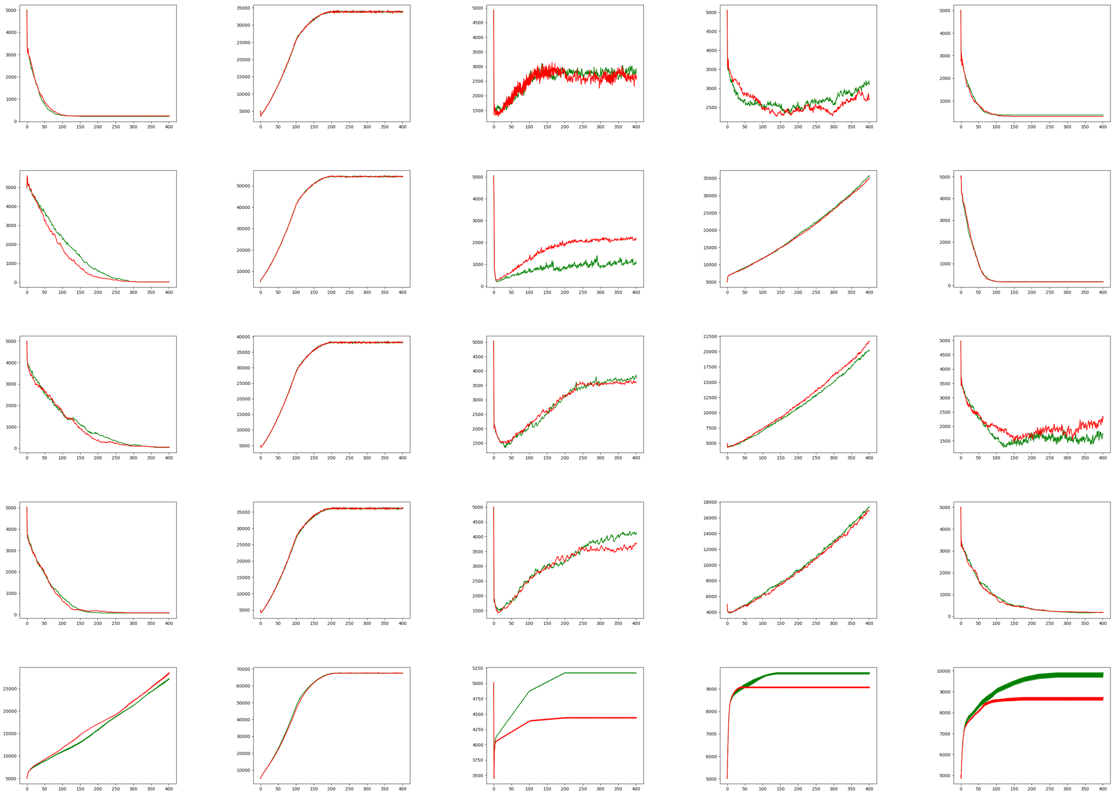


R1
R2
R3
R4
R5


S2T5
R6
R7
R8
R9
R10
R6
R7
R8
R9
R10


S2T6
R11
R12
R13
R14
R15
R6
R7
R8
R9
R10

S2T7 R16 R17 R18 R19 R20



R7

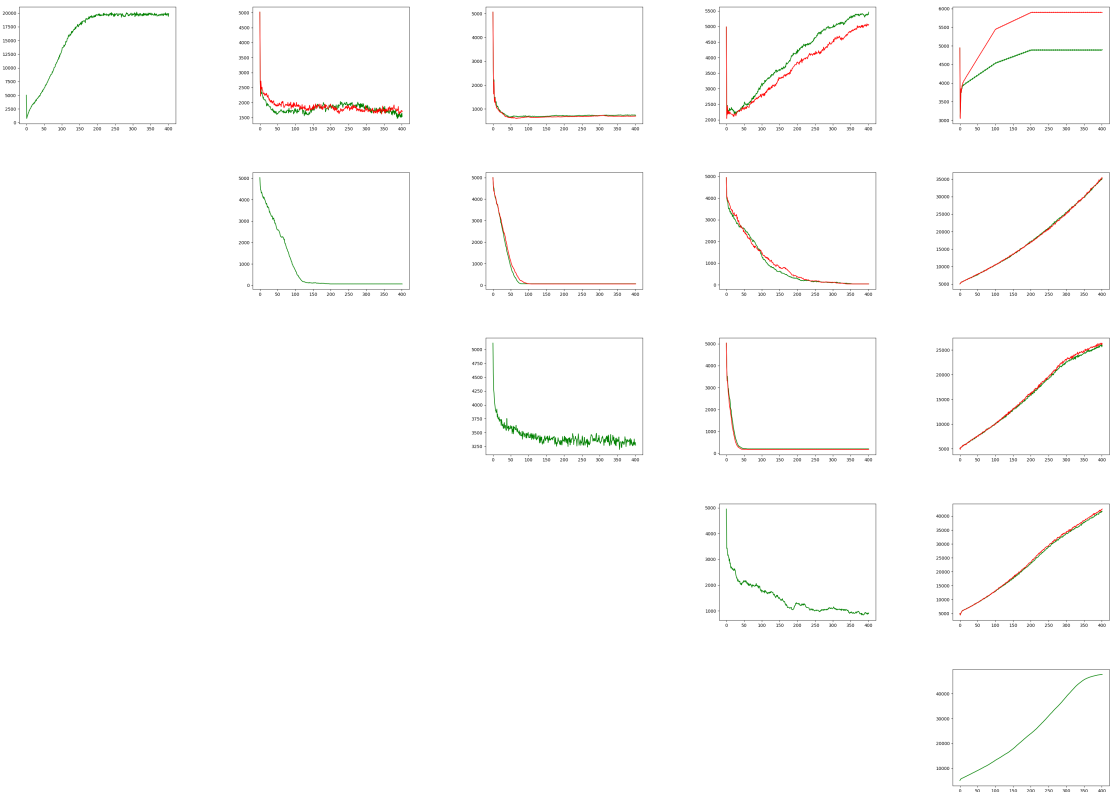
R7

R8

R9

R10

S2T8 R11 R12 R13 R14 R15



R1

R2

R3

R4

R5

S2T9 R16 R17 R18 R19 R20

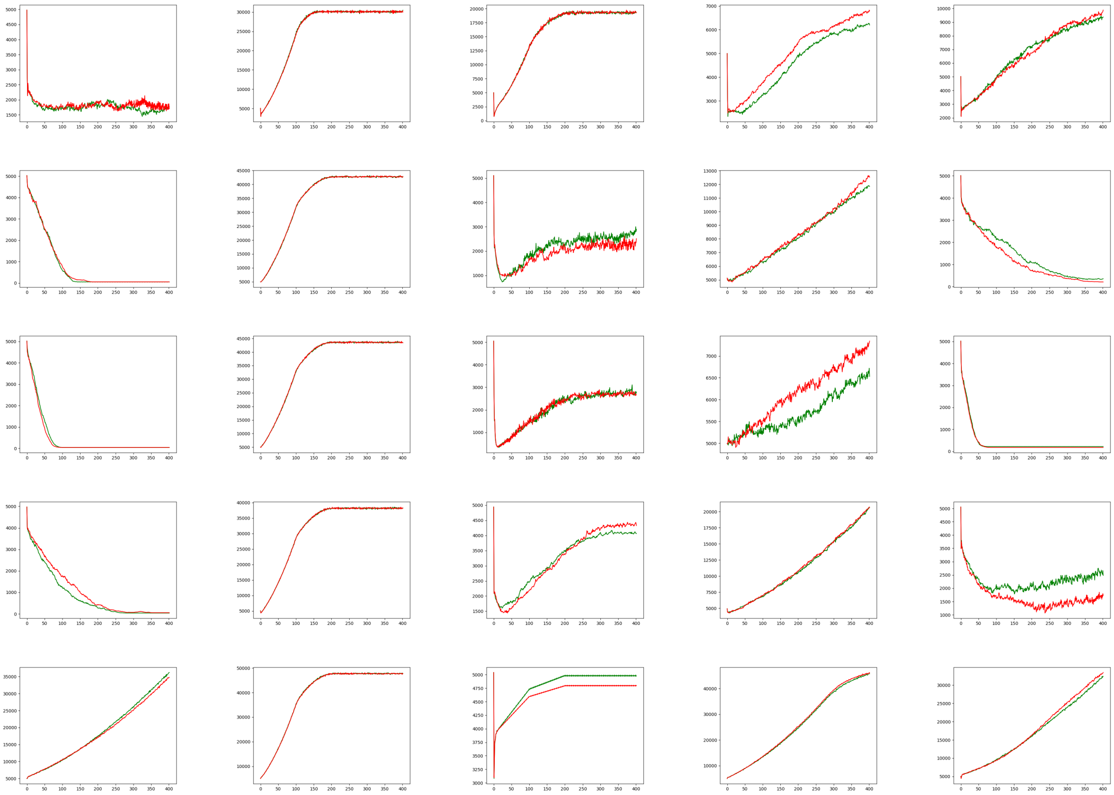
R11

R21

R13

R14

R15



S2T10 R16 R17 R18 R19 R20

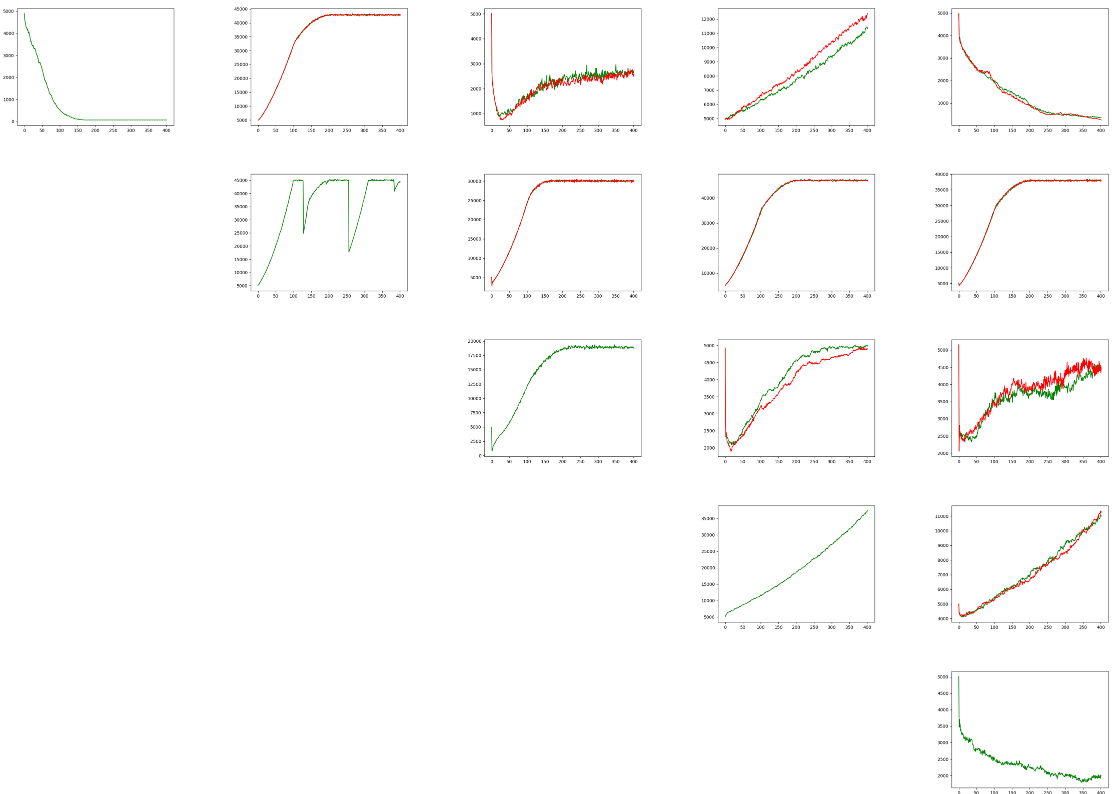
R16

R17

R18

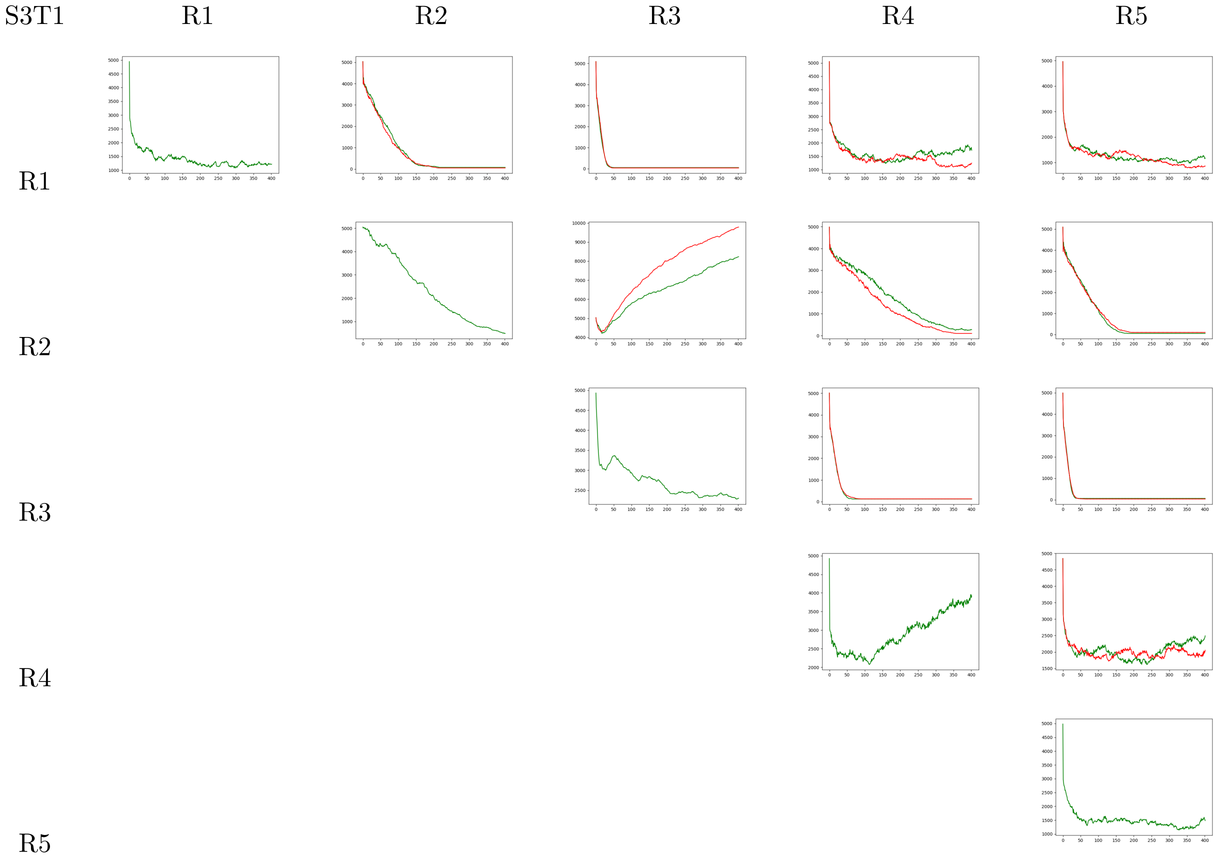
R19

R20

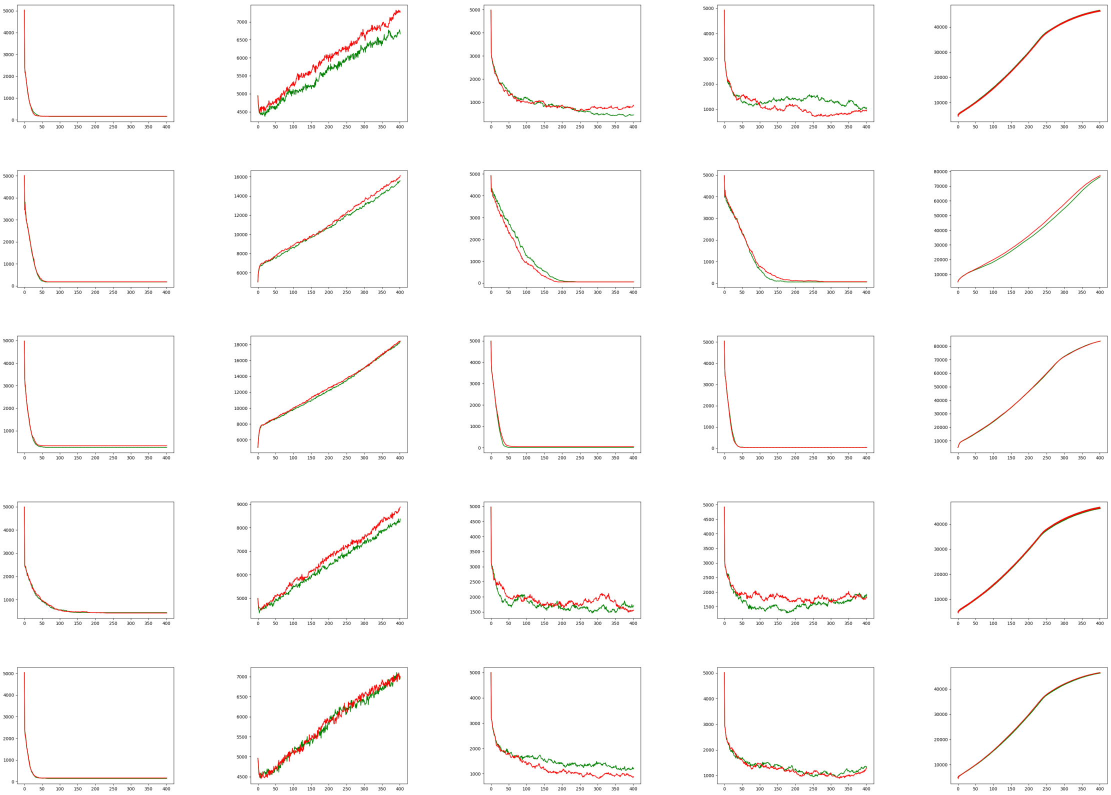


A.3 S3

These plots show the cell populations for rules that act in chessboard-like boards according to the scheme S3.

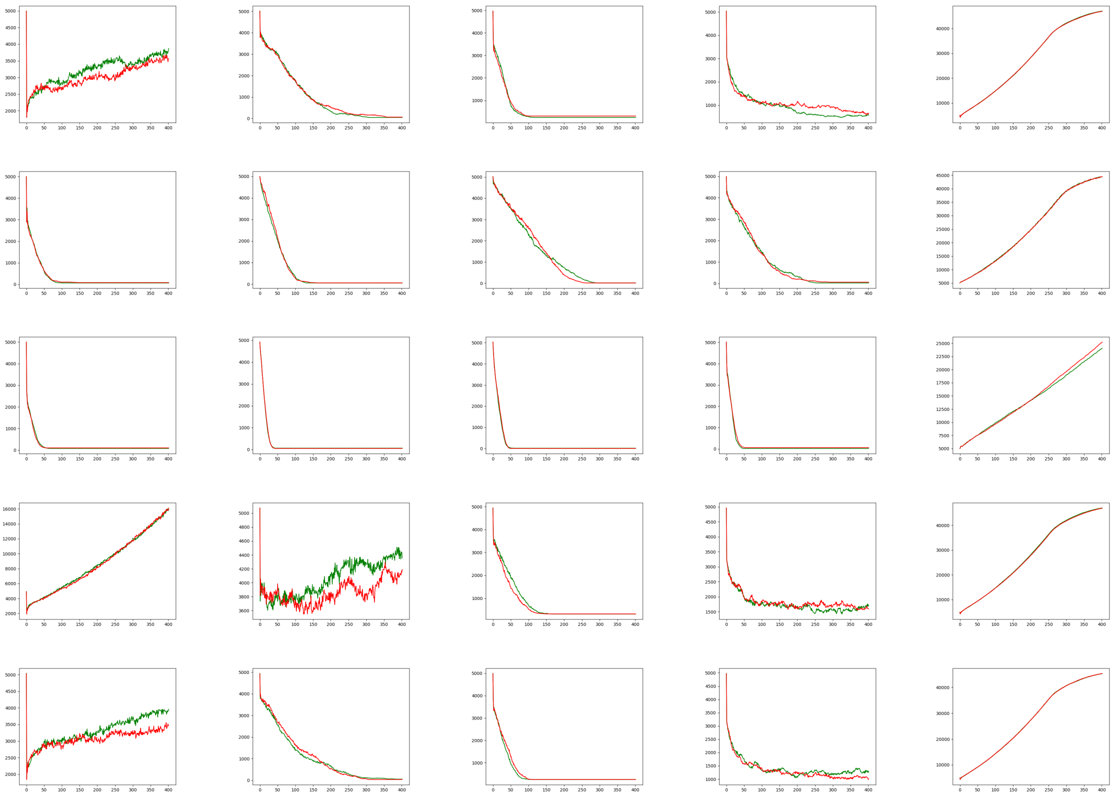


S3T2 R6 R7 R8 R9 R10



R1
R2
R3
R4
R5

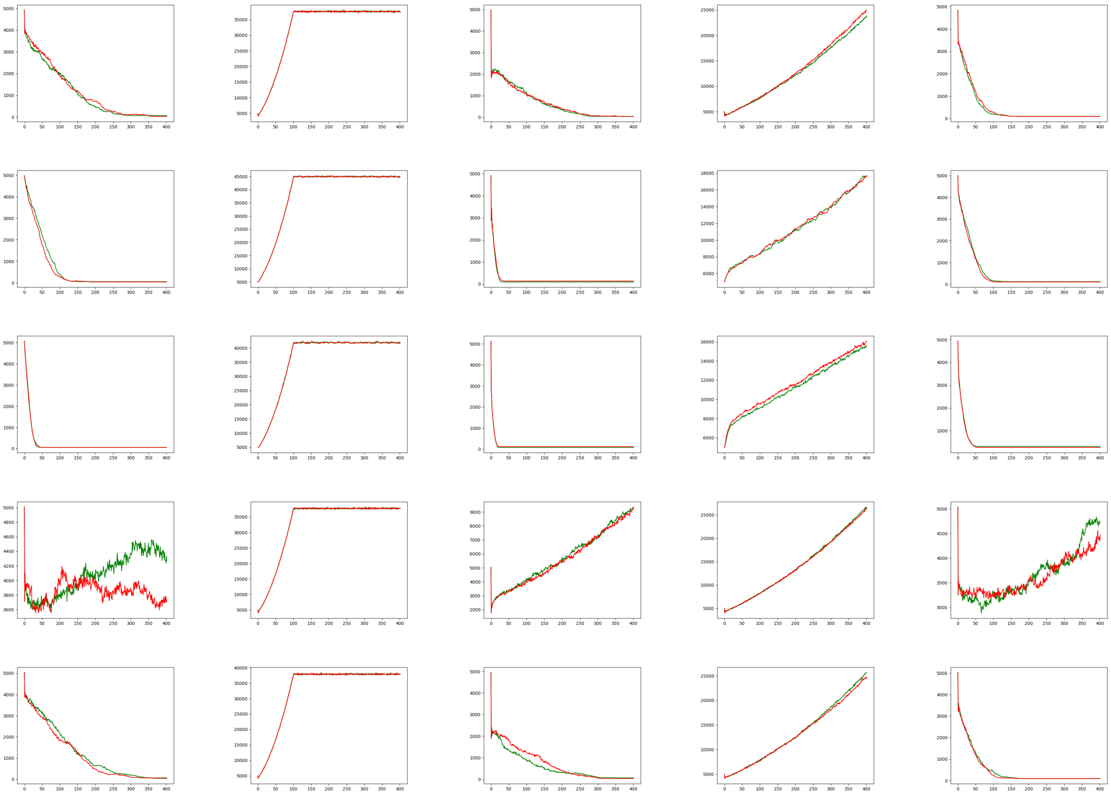
S3T3 R11 R12 R13 R14 R15



R1
R2
R3
R4
R5

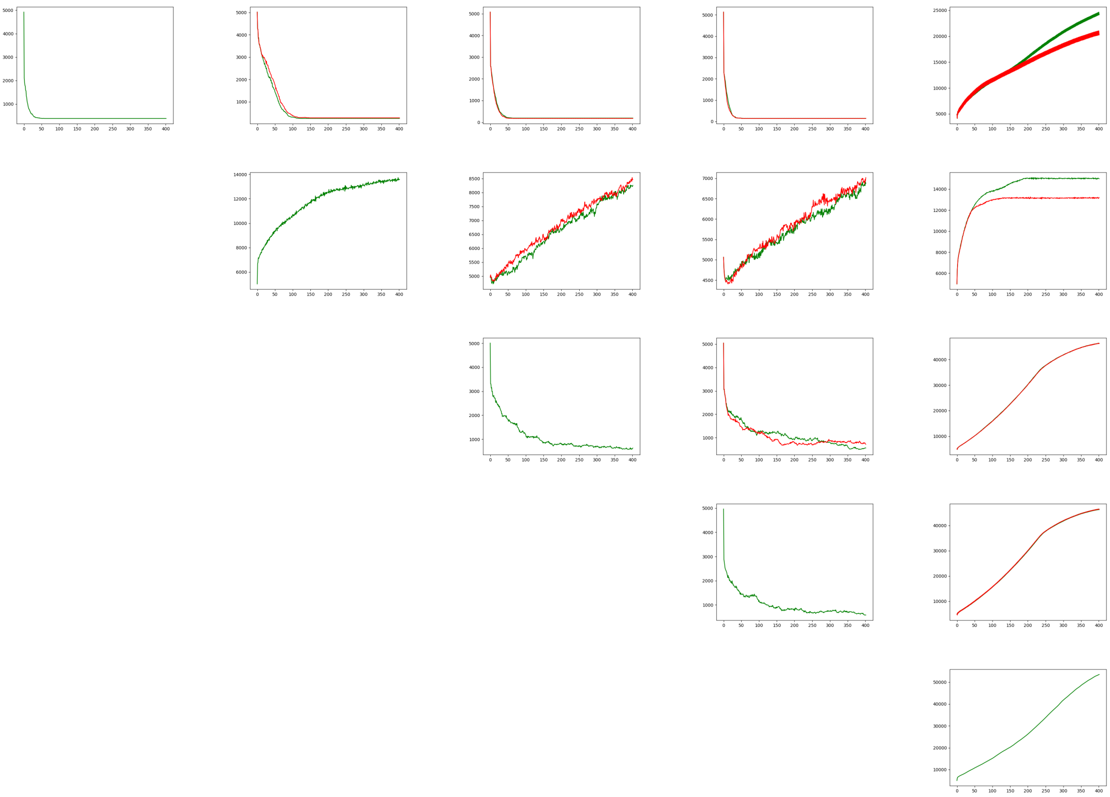
S3T4 R16 R17 R18 R19 R20

R1 R2 R3 R4 R5

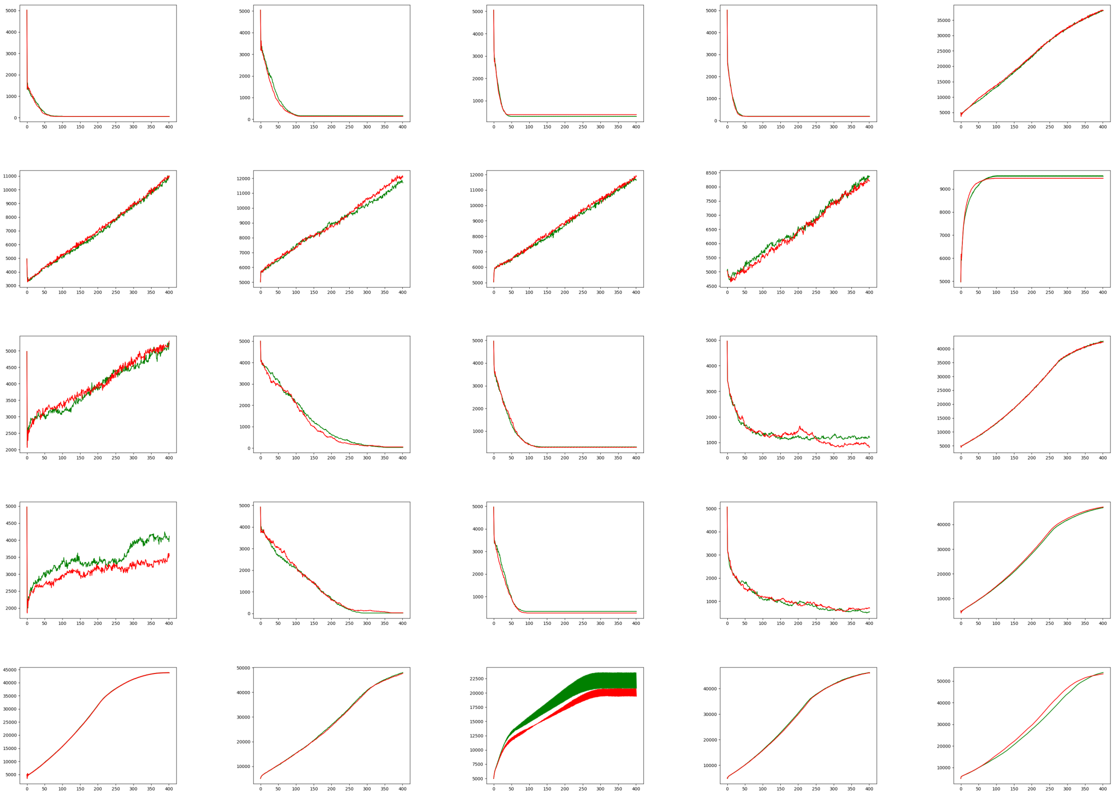


S3T4 R6 R7 R8 R9 R10

R6 R7 R8 R9 R10

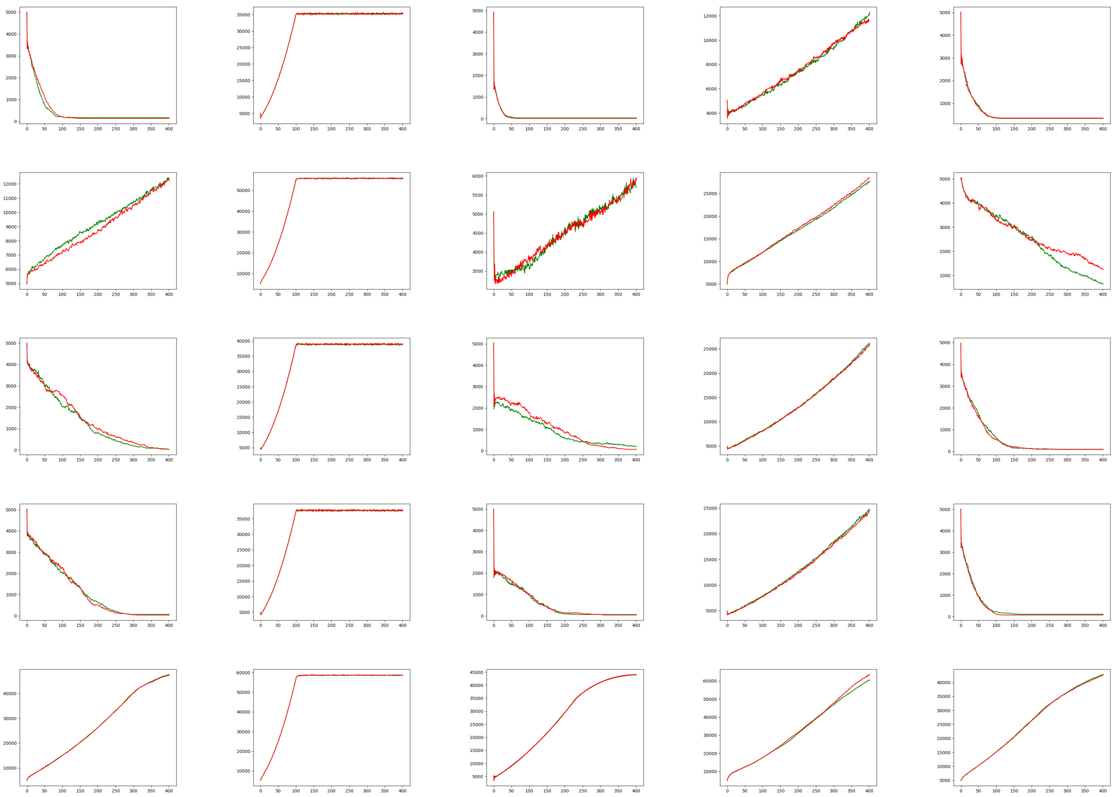


S3T6 R11 R12 R13 R14 R15



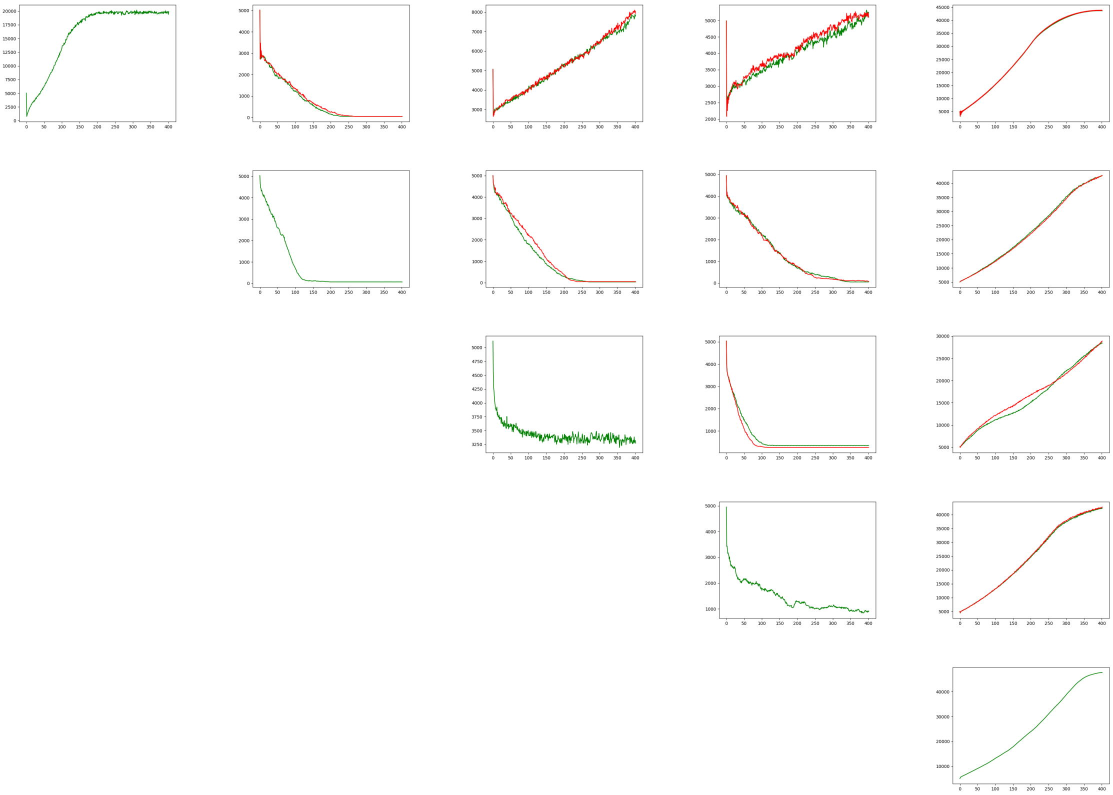
R6
R7
R8
R9
R10

S3T7 R16 R17 R18 R19 R20



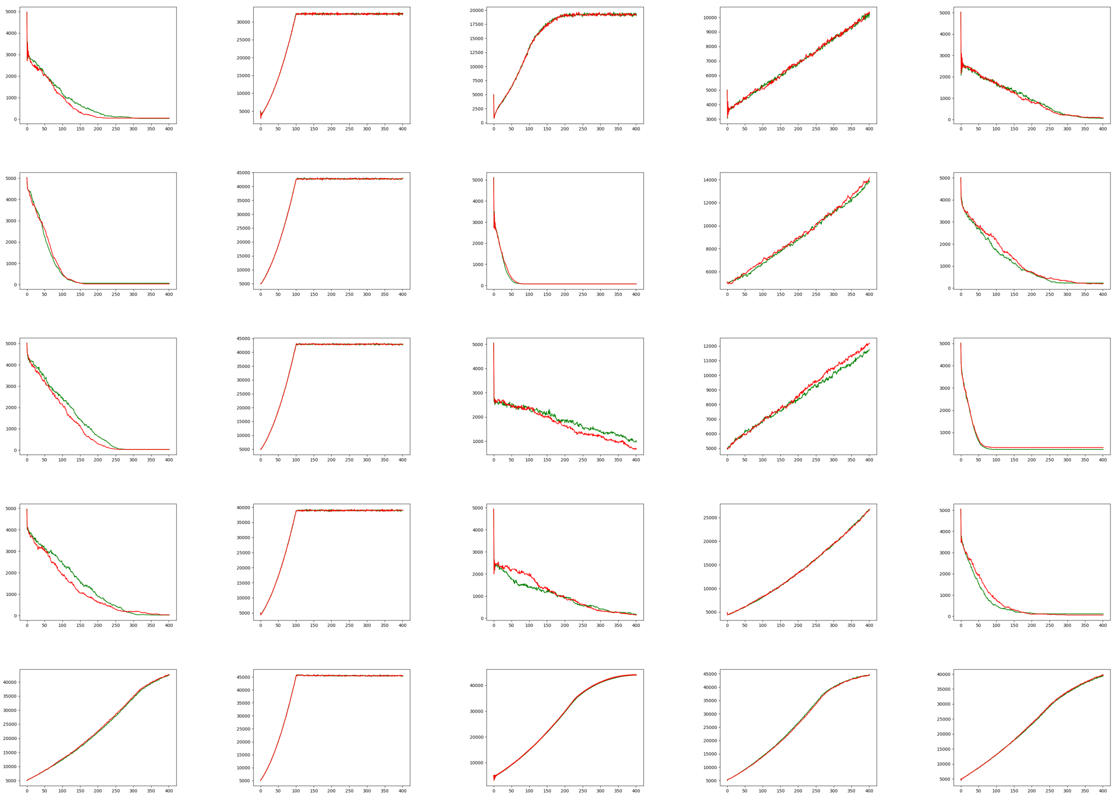
R7
R7
R8
R9
R10

S3T8 R11 R12 R13 R14 R15

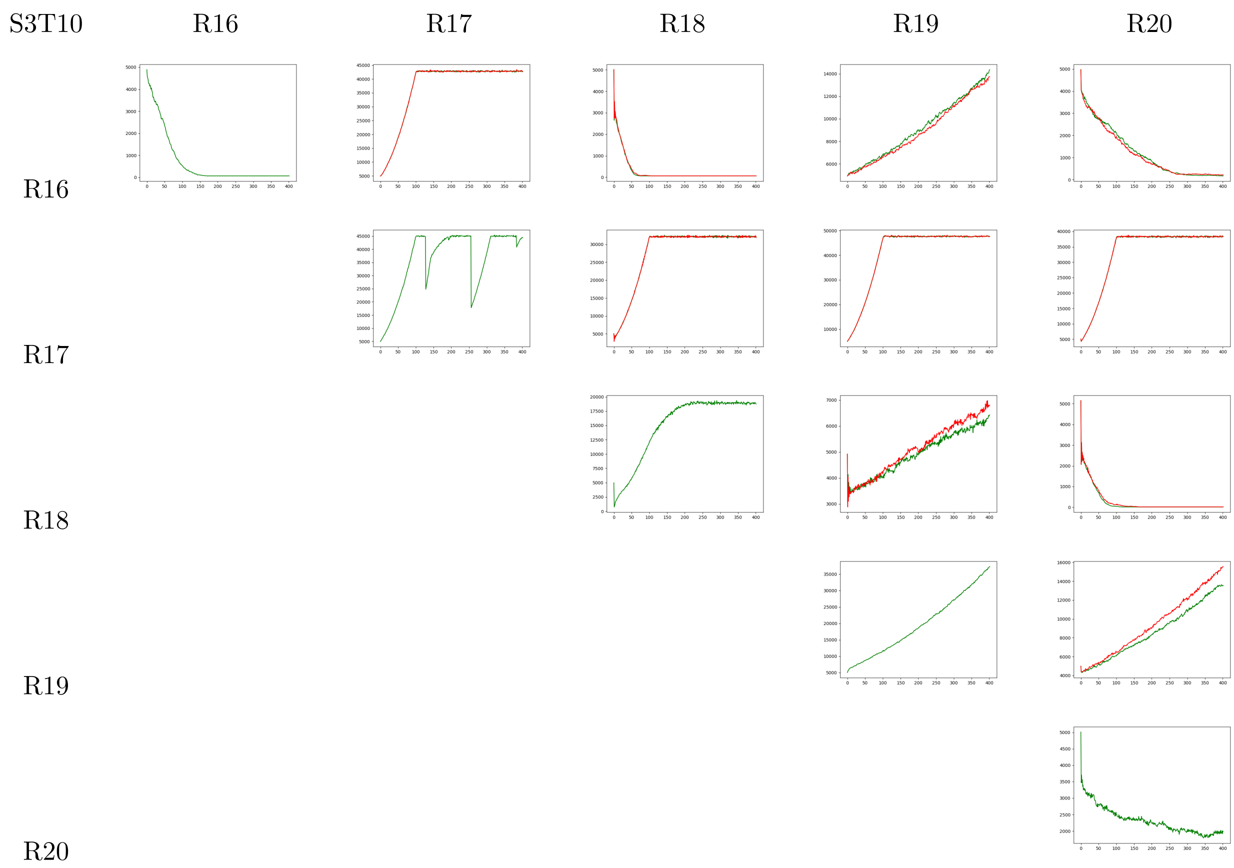


R1
R2
R3
R4
R5

S3T9 R16 R17 R18 R19 R20



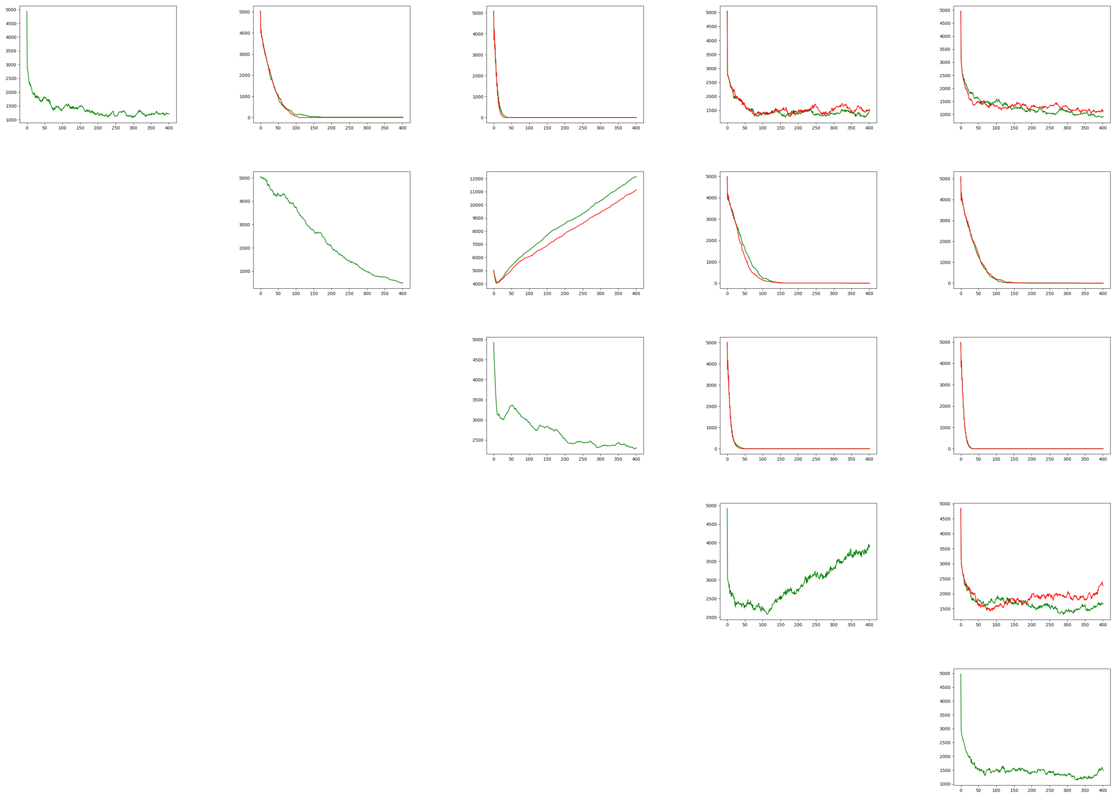
R11
R21
R13
R14
R15



A.4 S4

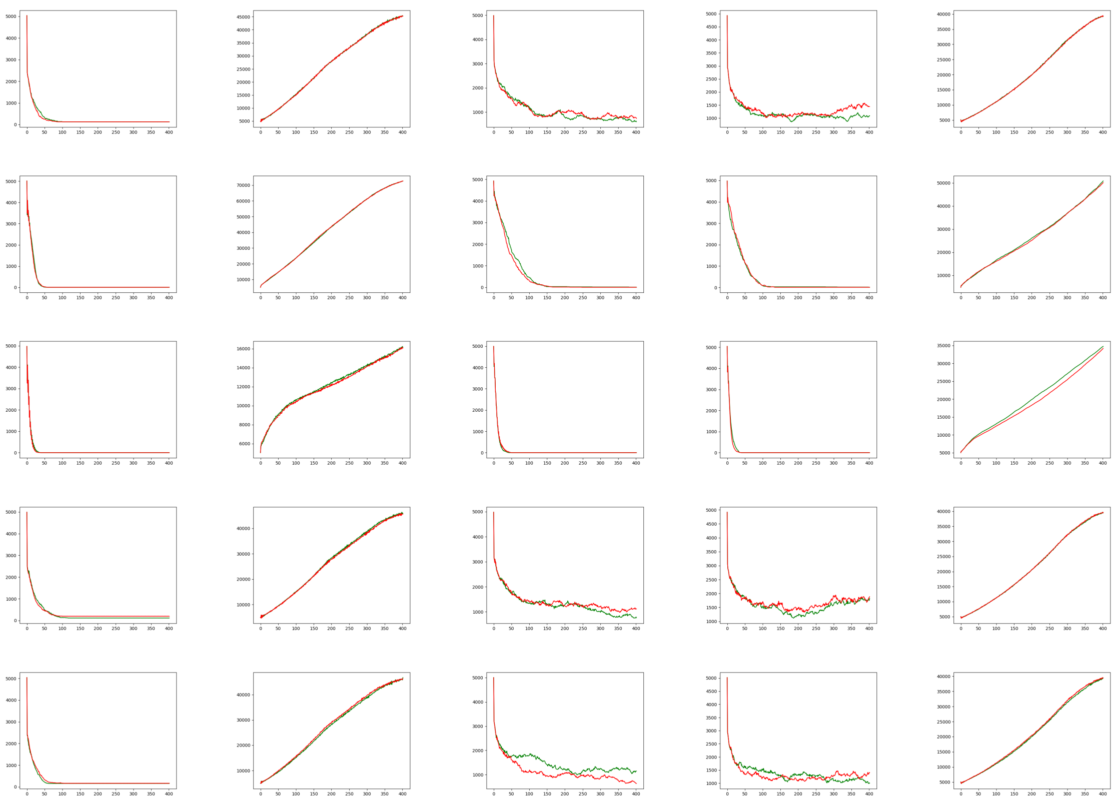
These plots show the cell populations for rules that act in parallel lines that shift periodically according to the scheme S4.

S4T1 R1 R2 R3 R4 R5



R1
R2
R3
R4
R5

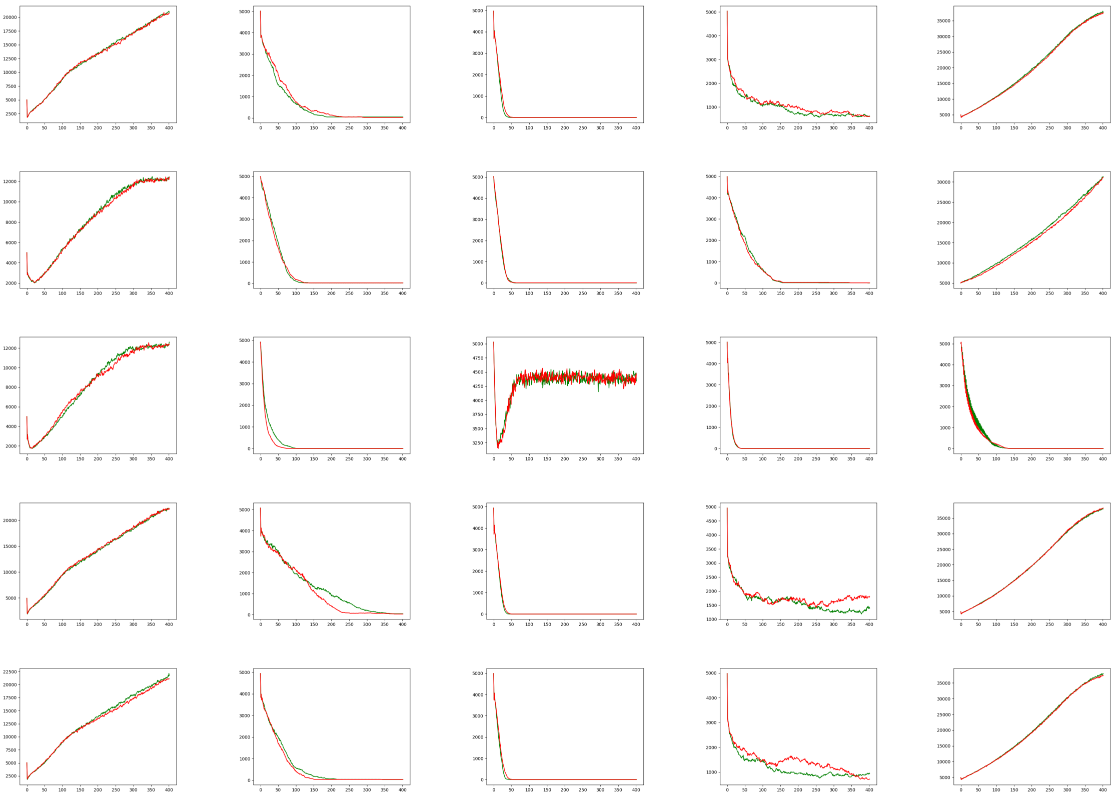
S4T2 R6 R7 R8 R9 R10



R1
R2
R3
R4
R5

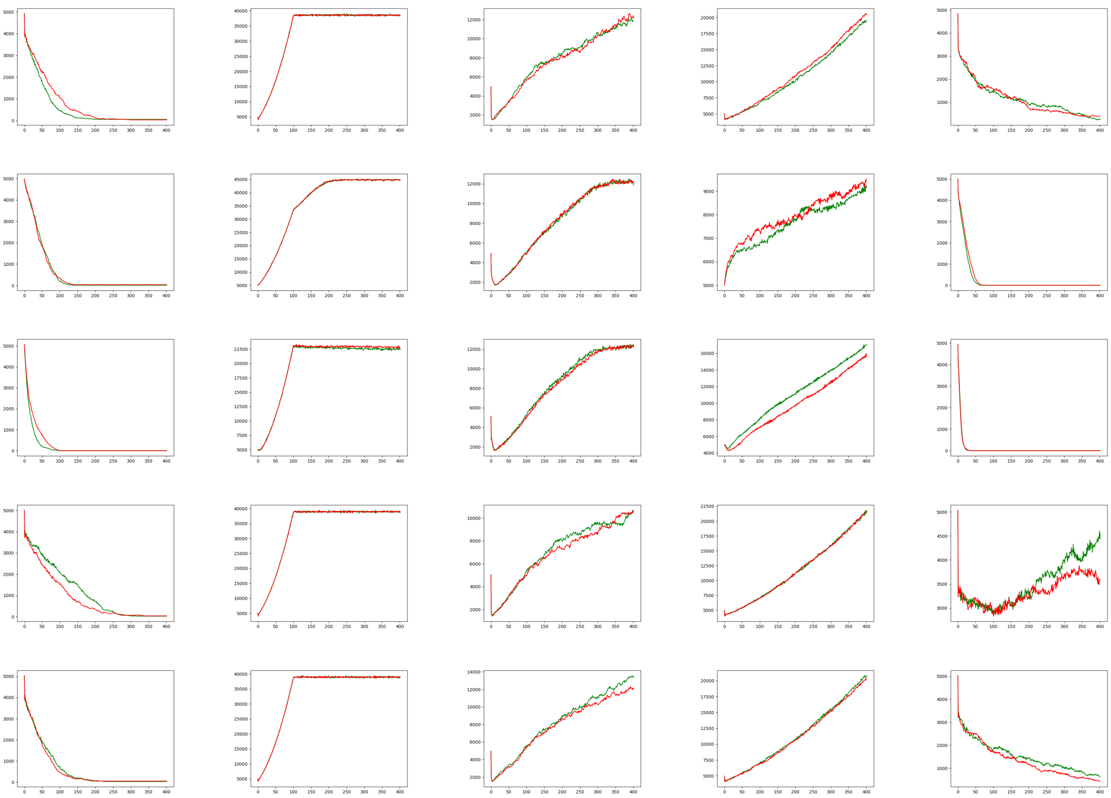
S4T3 R11 R12 R13 R14 R15

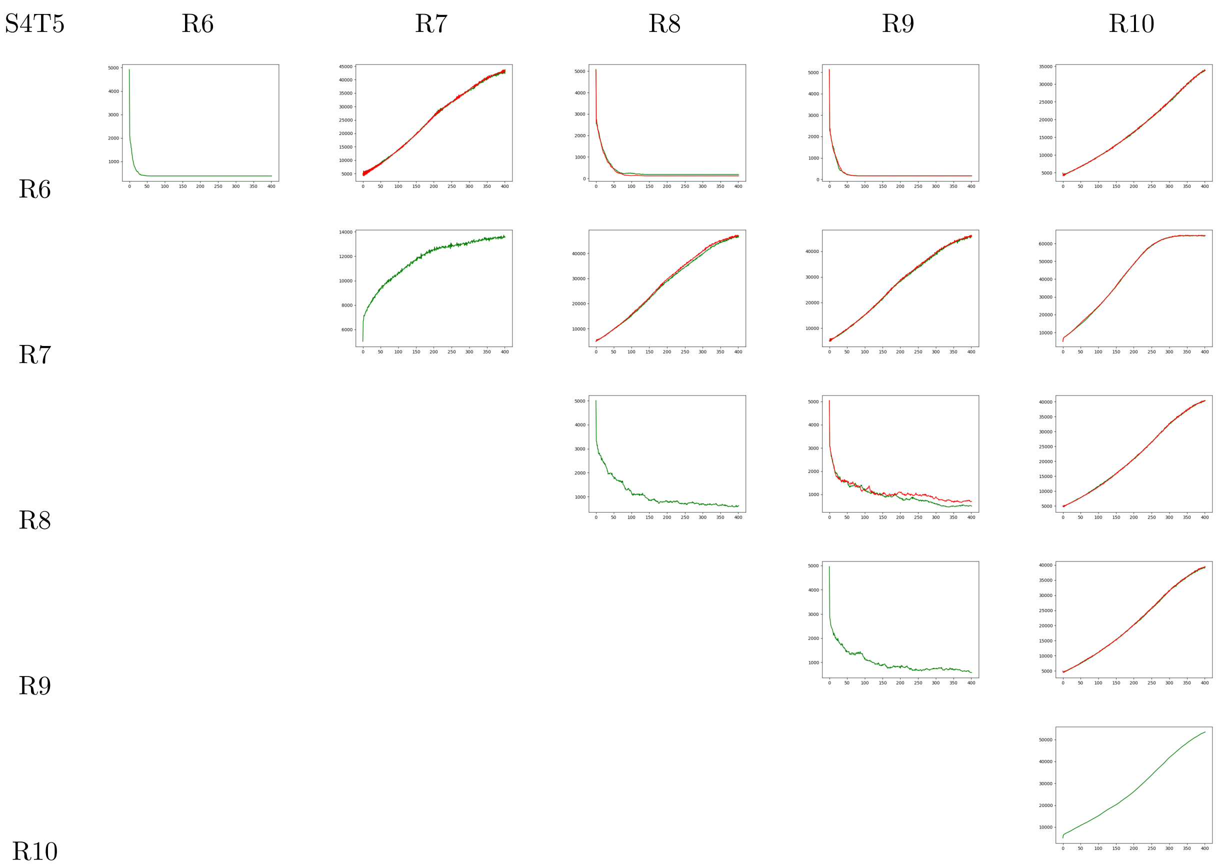
R1 R2 R3 R4 R5

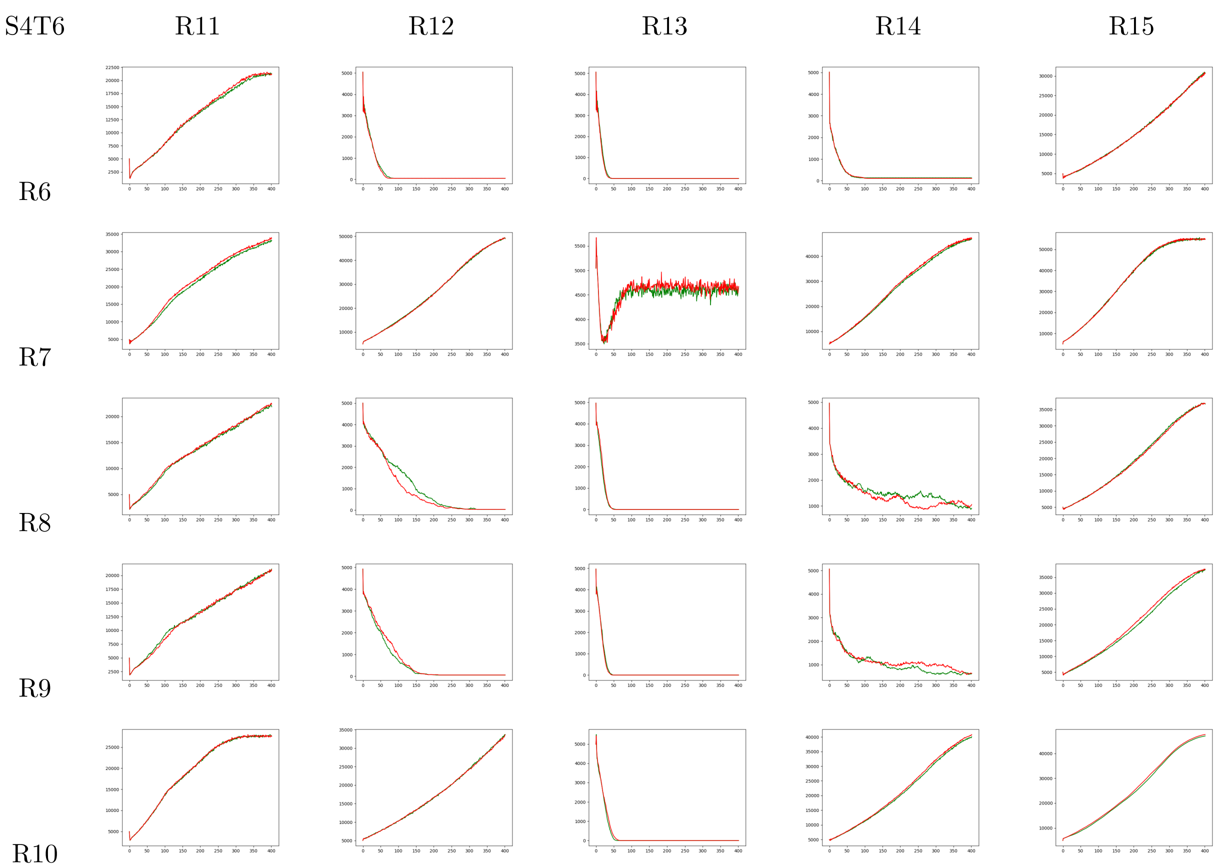


S4T4 R16 R17 R18 R19 R20

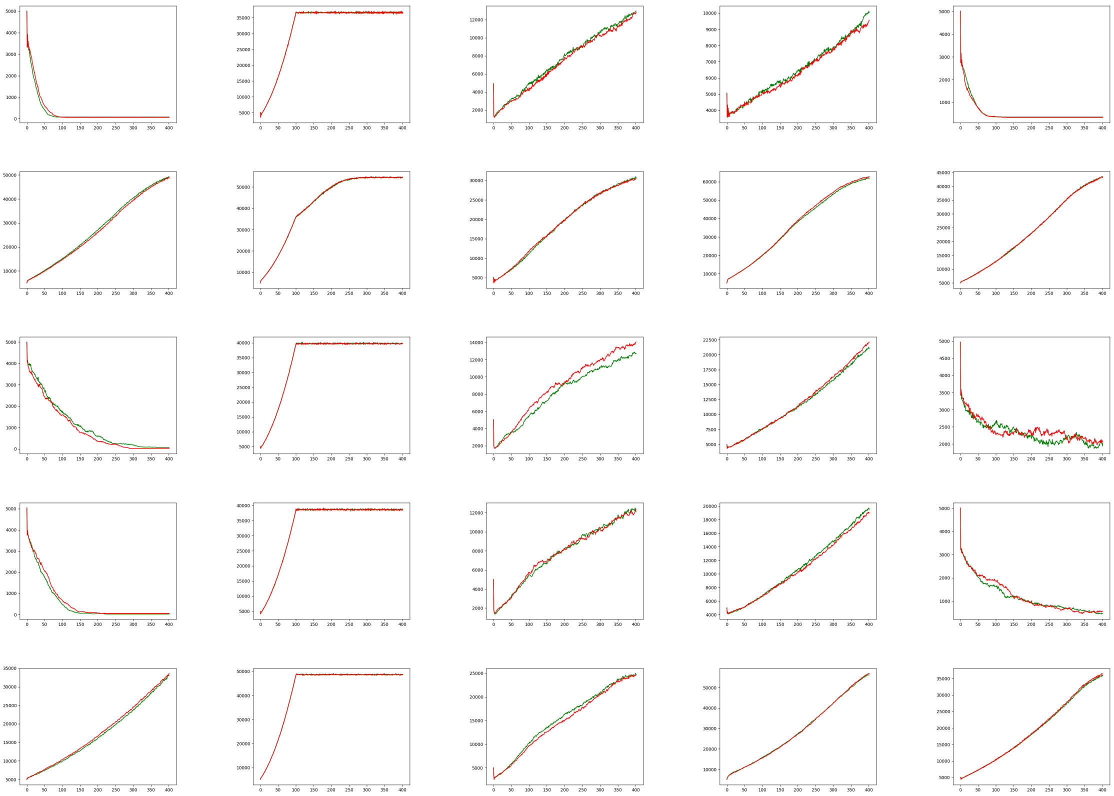
R1 R2 R3 R4 R5




S4T5
R6
R7
R8
R9
R10
R6
R7
R8
R9
R10


S4T6
R11
R12
R13
R14
R15
R6
R7
R8
R9
R10

S4T7 R16 R17 R18 R19 R20



R7

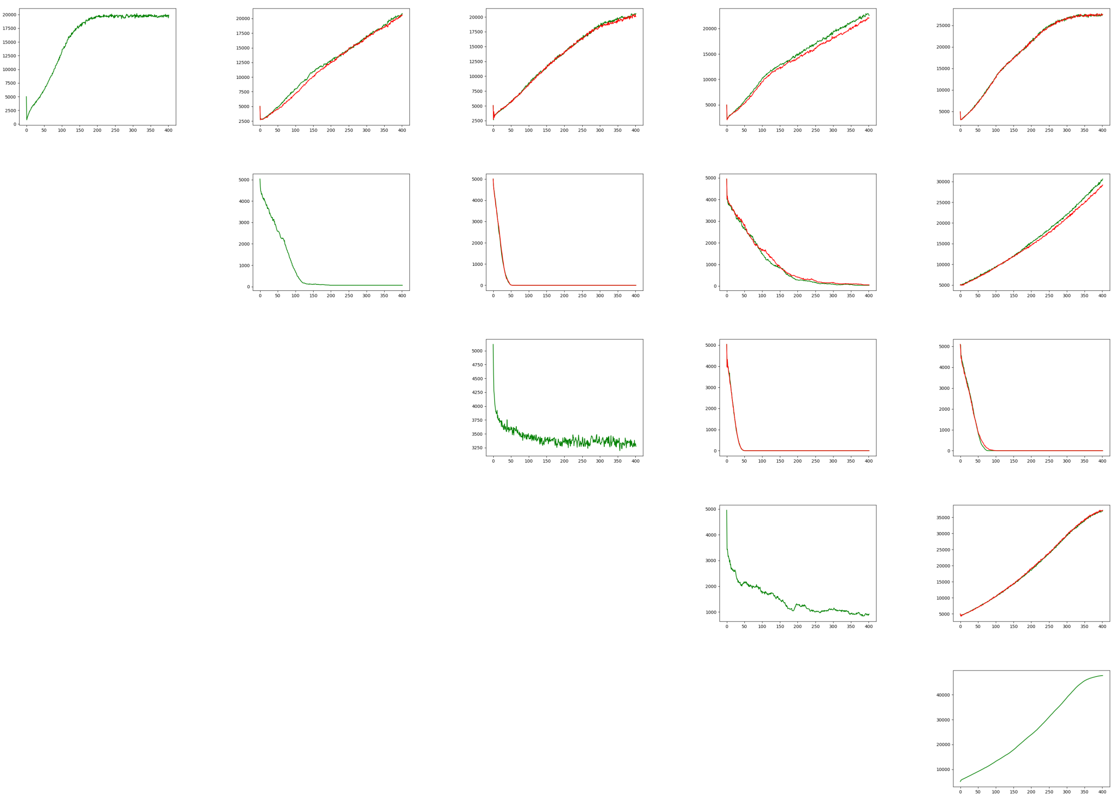
R7

R8

R9

R10

S4T8 R11 R12 R13 R14 R15



R1

R2

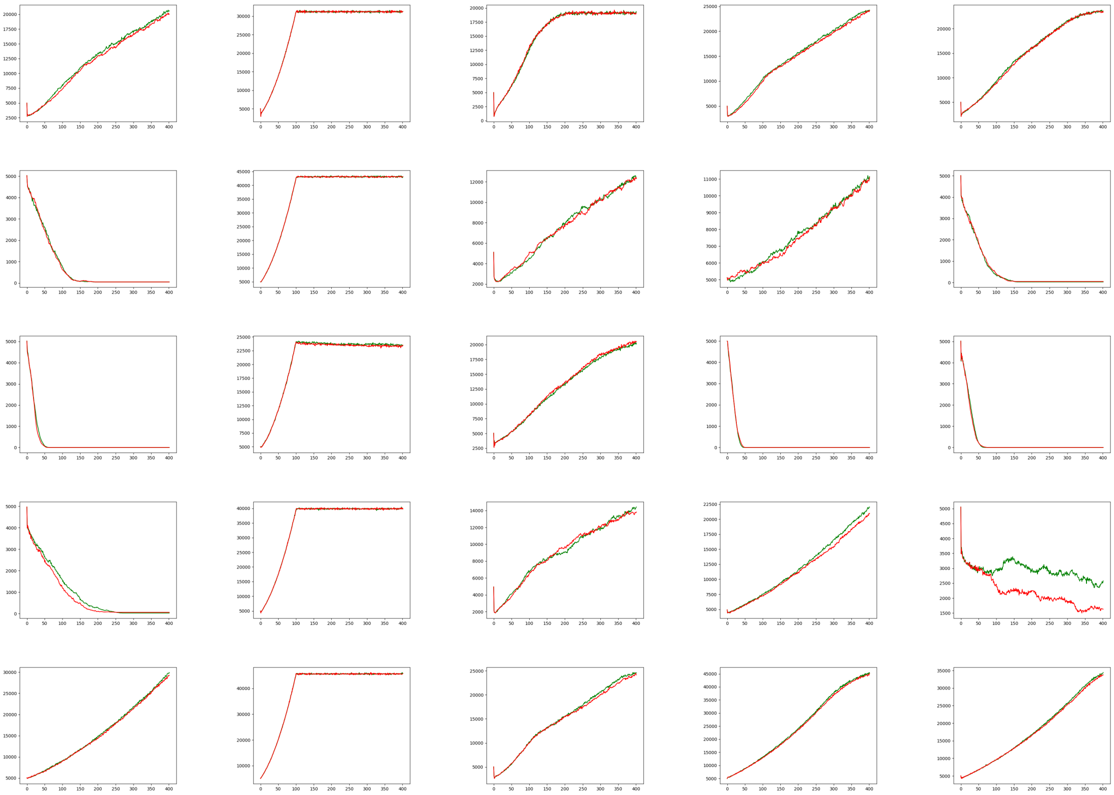
R3

R4

R5

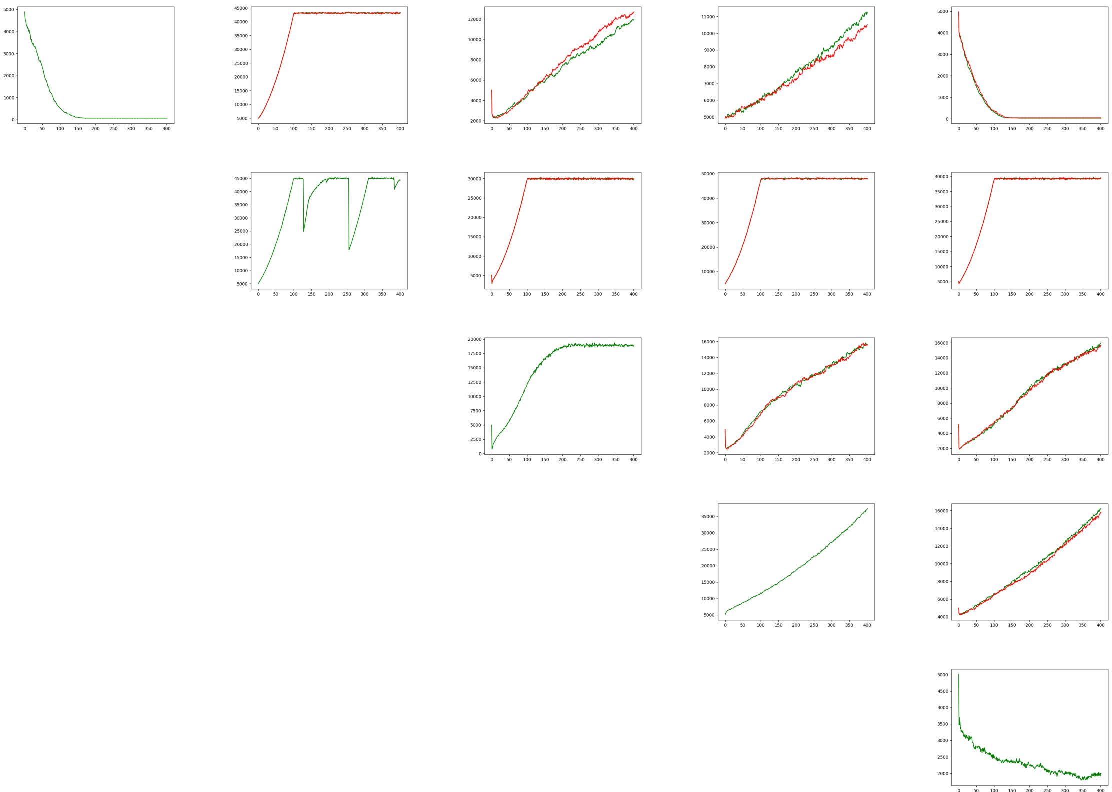
S4T9 R16 R17 R18 R19 R20

R11 R21 R13 R14 R15



S4T10 R16 R17 R18 R19 R20

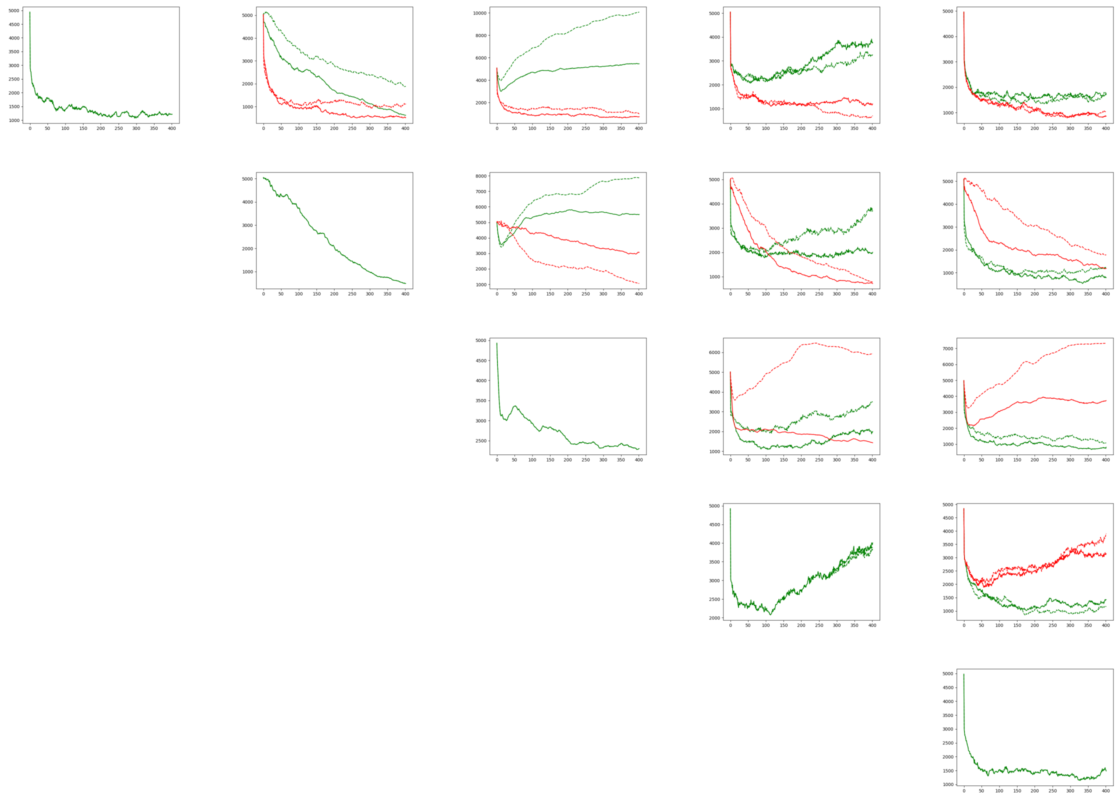
R16 R17 R18 R19 R20



A.5 S5

First subscheme. These plots show the cell populations for rules that act in parallel lines that shift periodically according to the scheme S5. We remark that if we consider Ra/Rb (resp. row and column number rule of the matrix), alive and dead cells in the secondary board (which evolves according to Rb) define which rules (resp. between Ra and Rb) will be used in the main board. The dashed lines show the respective bottom (secondary) layers of the automata.

S5AT1 R1 R2 R3 R4 R5



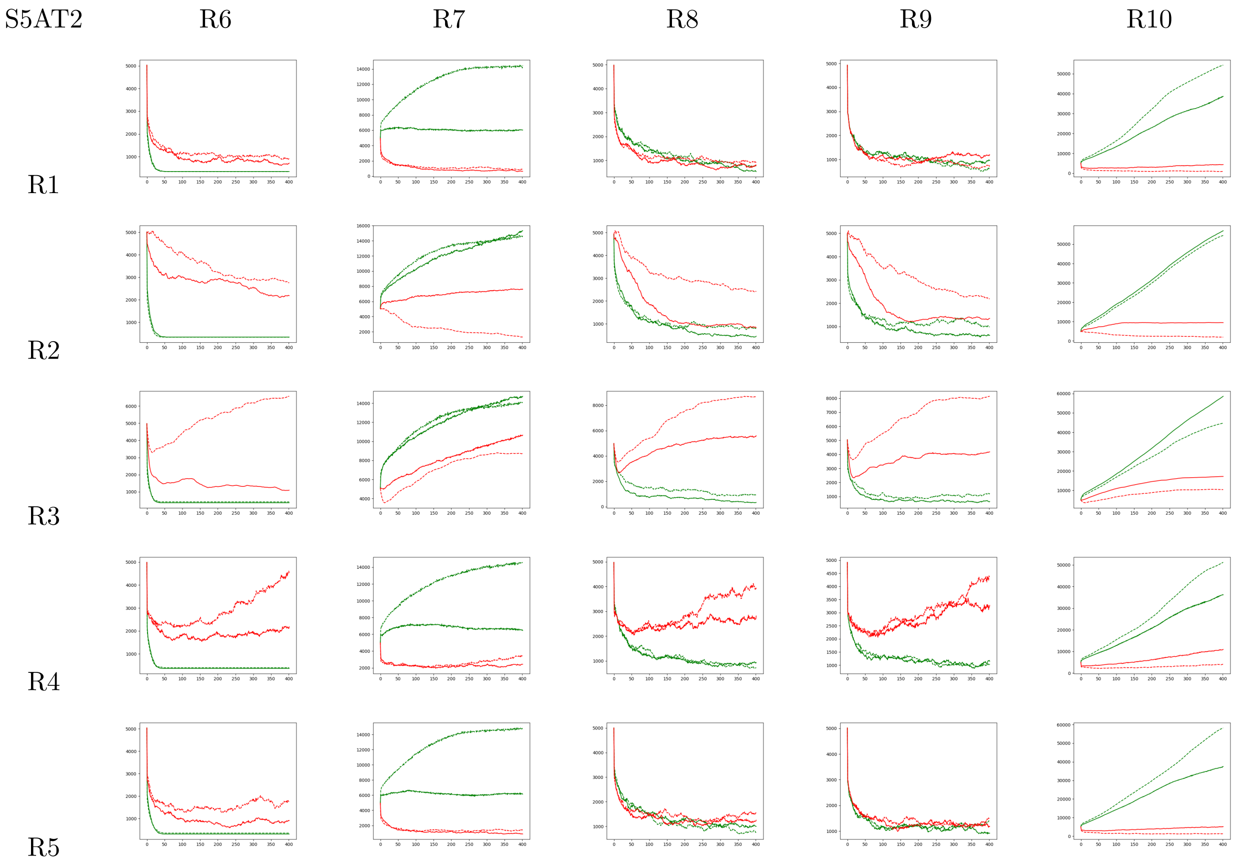
R1

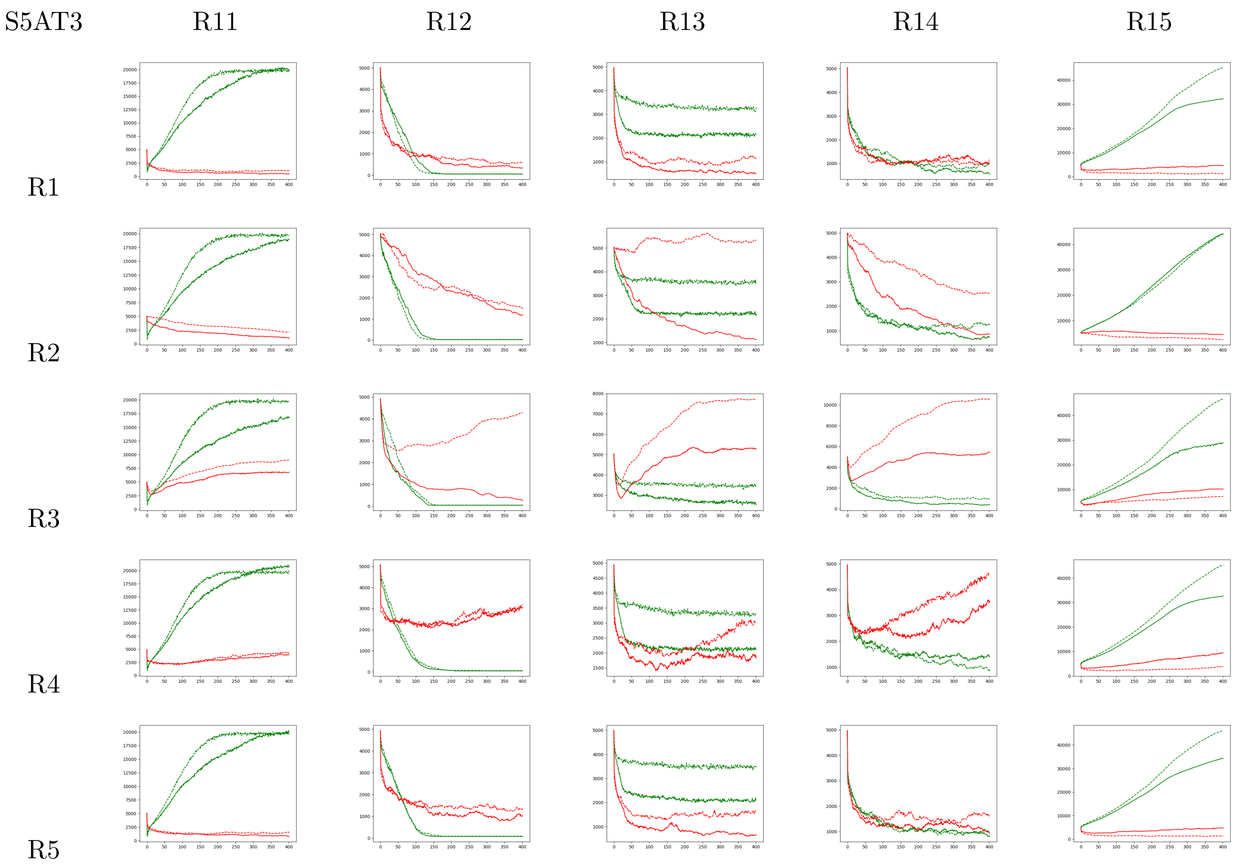
R2

R3

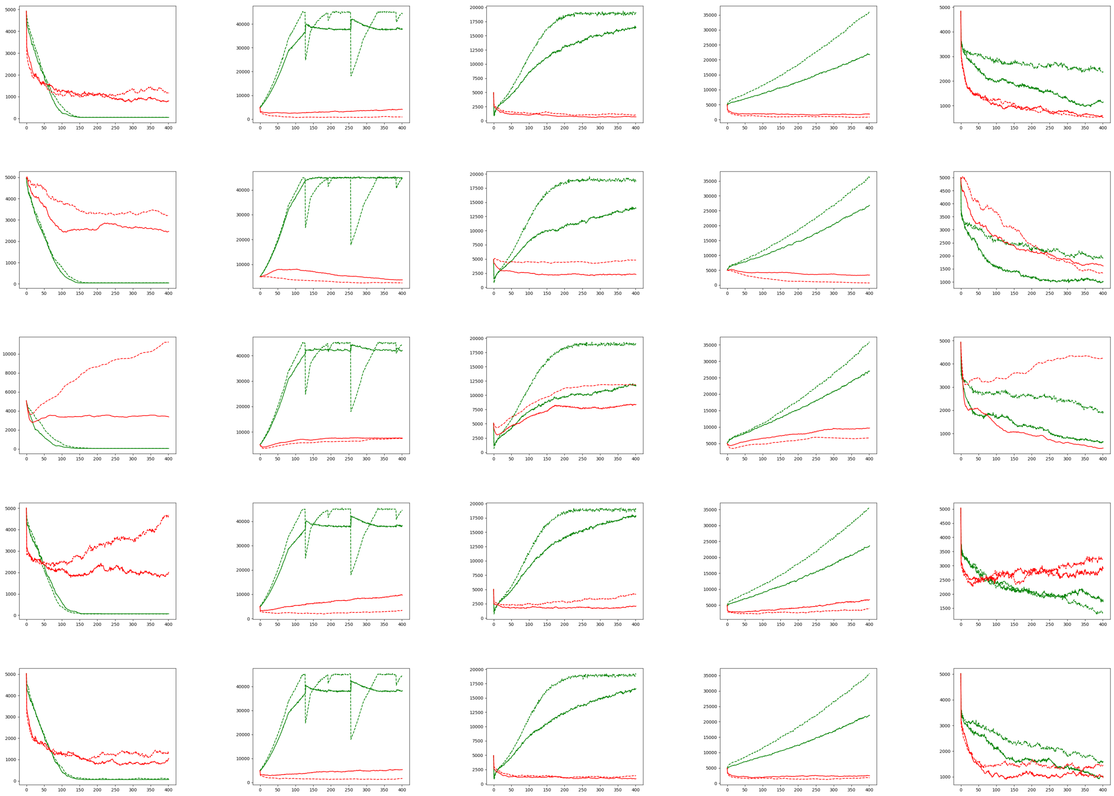
R4

R5


S5AT2
R6
R7
R8
R9
R10
R1
R2
R3
R4
R5


S5AT3
R11
R12
R13
R14
R15
R1
R2
R3
R4
R5

S5AT4 R16 R17 R18 R19 R20



R1

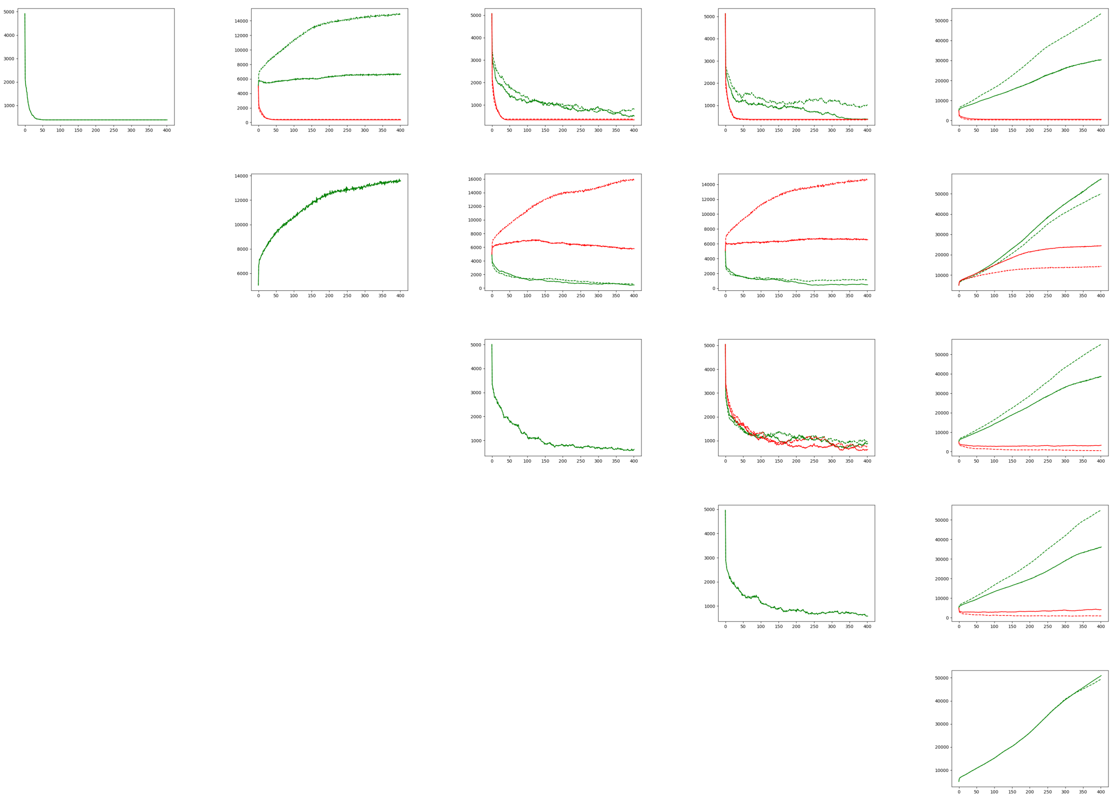
R2

R3

R4

R5

S5AT5 R6 R7 R8 R9 R10



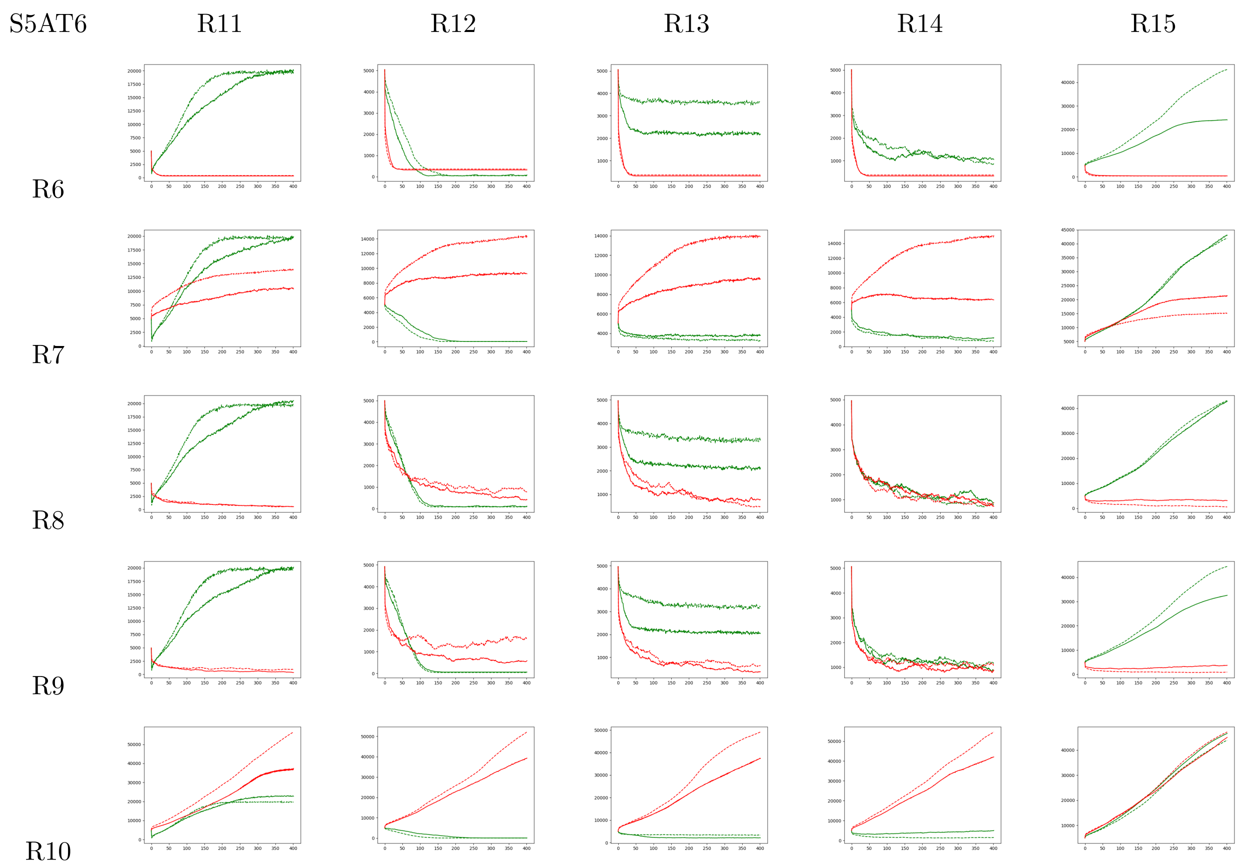
R6

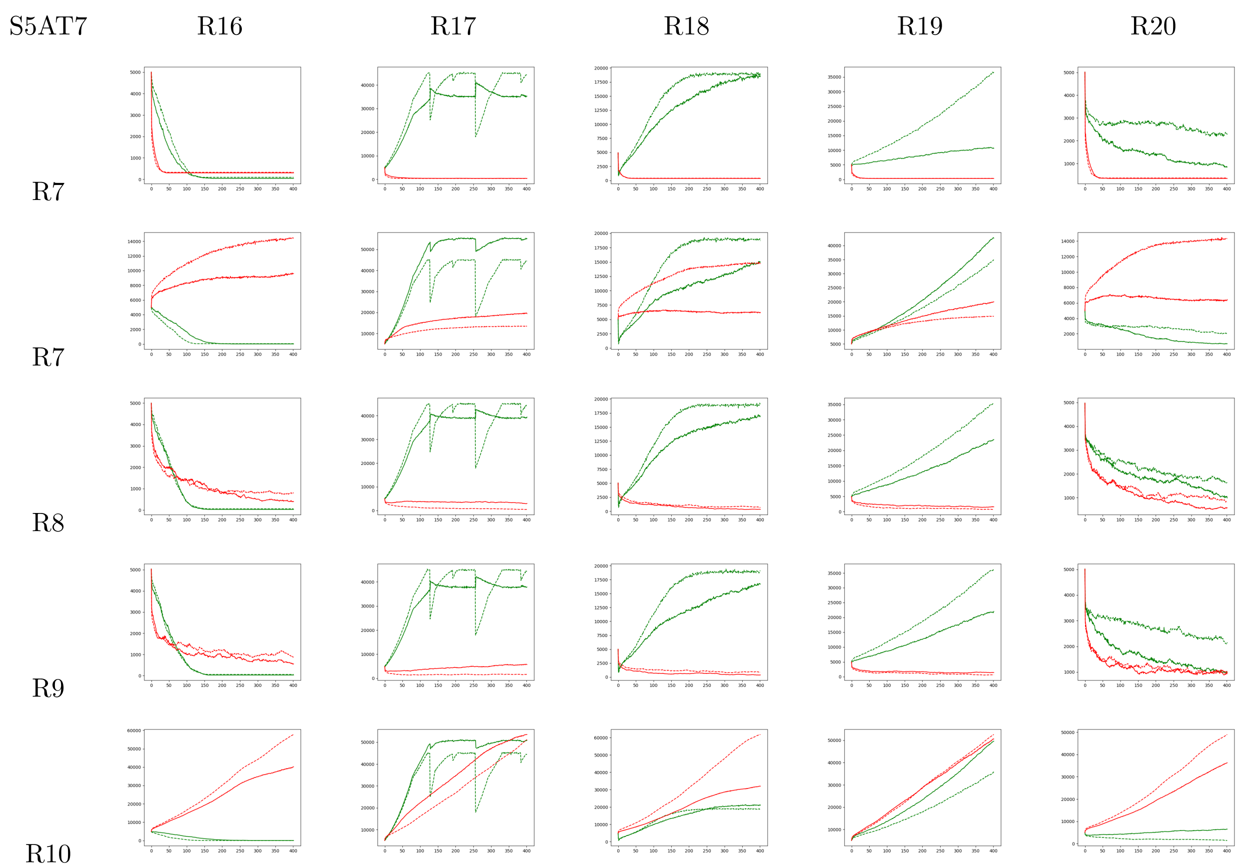
R7

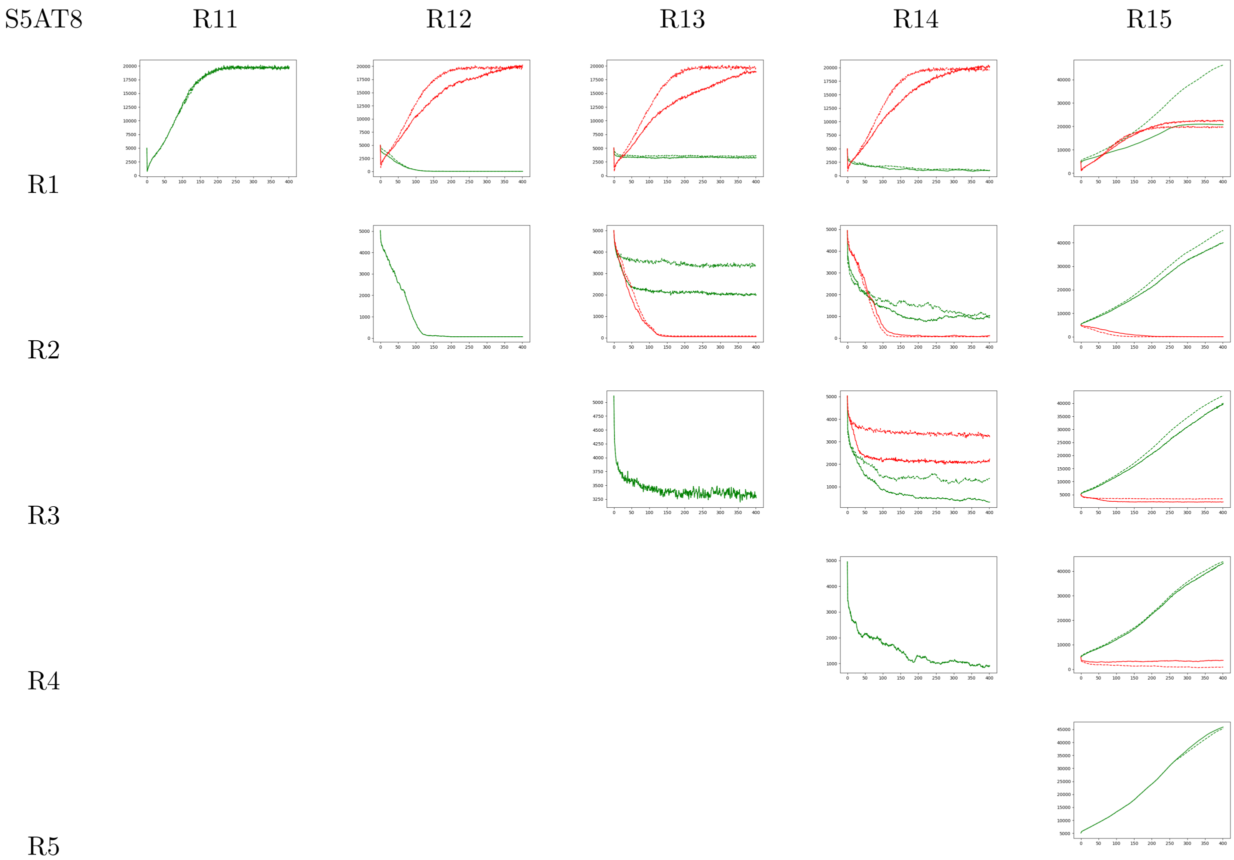
R8

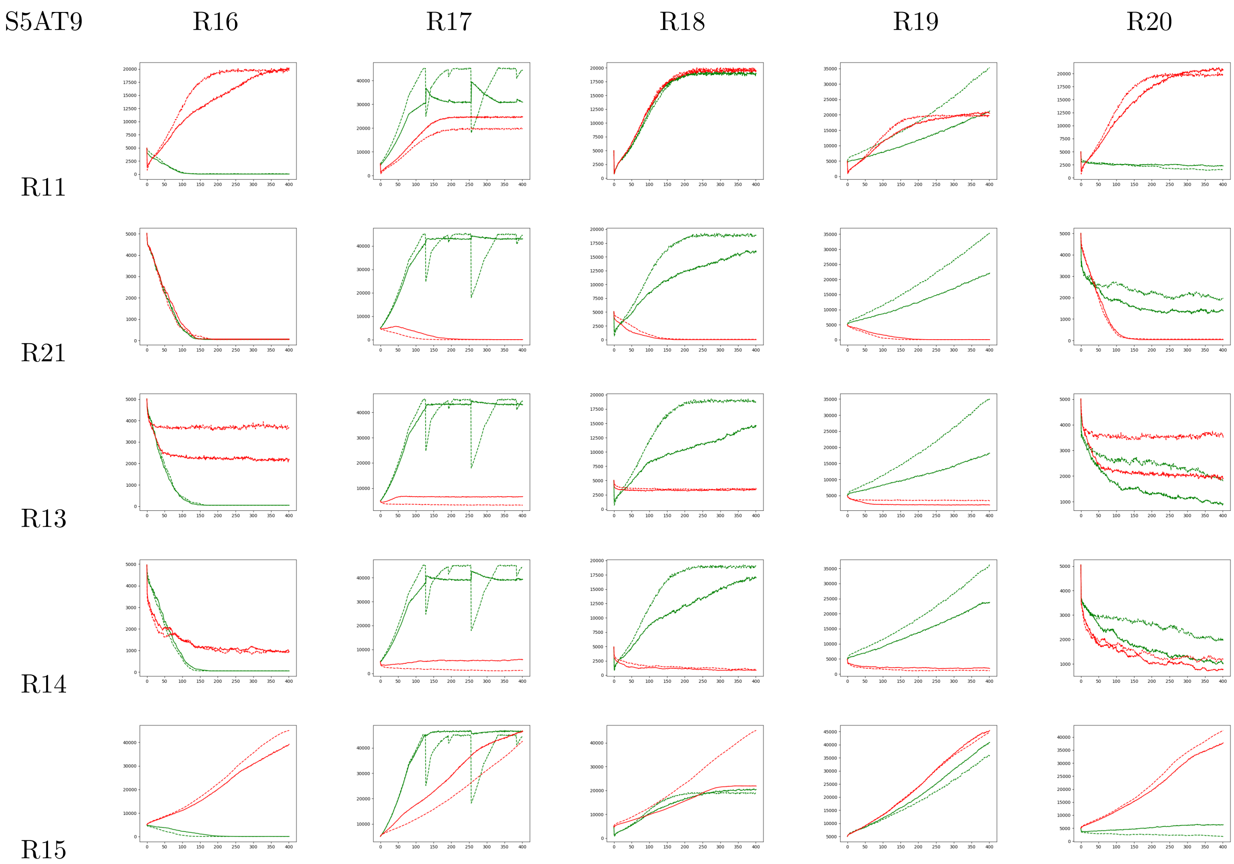
R9

R10

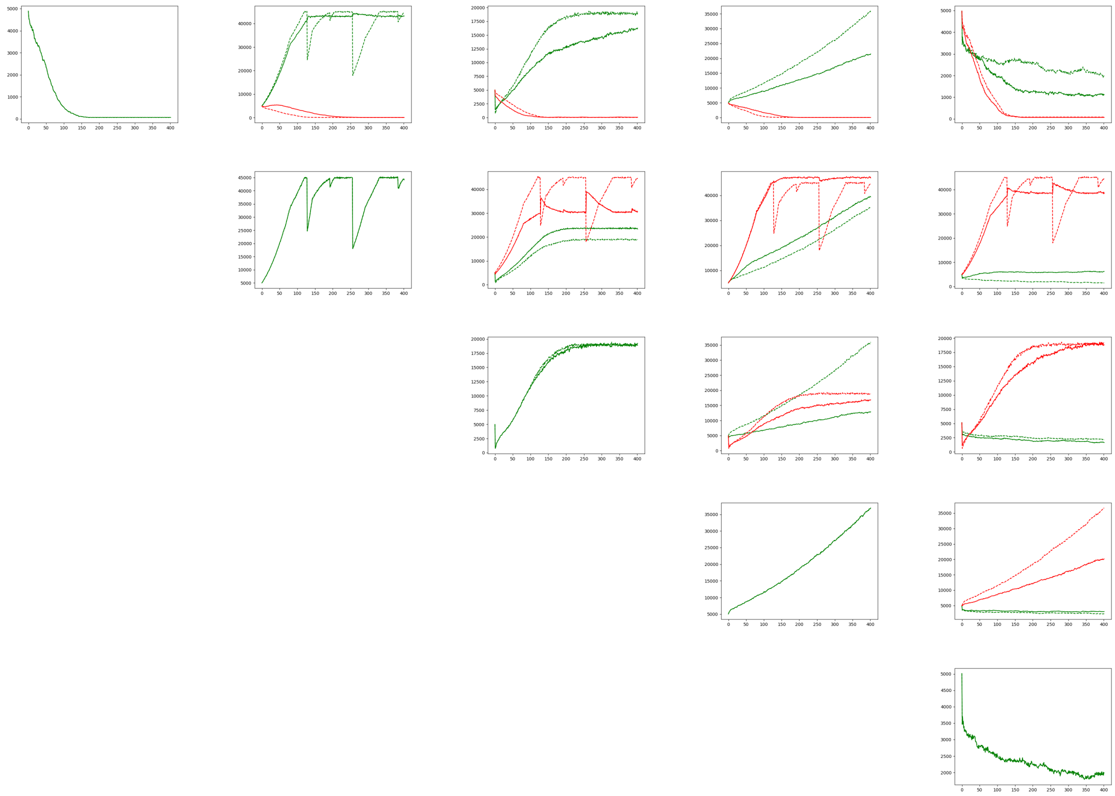

S5AT6
R11
R12
R13
R14
R15
R6
R7
R8
R9
R10


S5AT7
R16
R17
R18
R19
R20
R7
R7
R8
R9
R10


S5AT8
R11
R12
R13
R14
R15
R1
R2
R3
R4
R5


S5AT9
R16
R17
R18
R19
R20
R11
R21
R13
R14
R15

S5AT10 R16 R17 R18 R19 R20



R16

R17

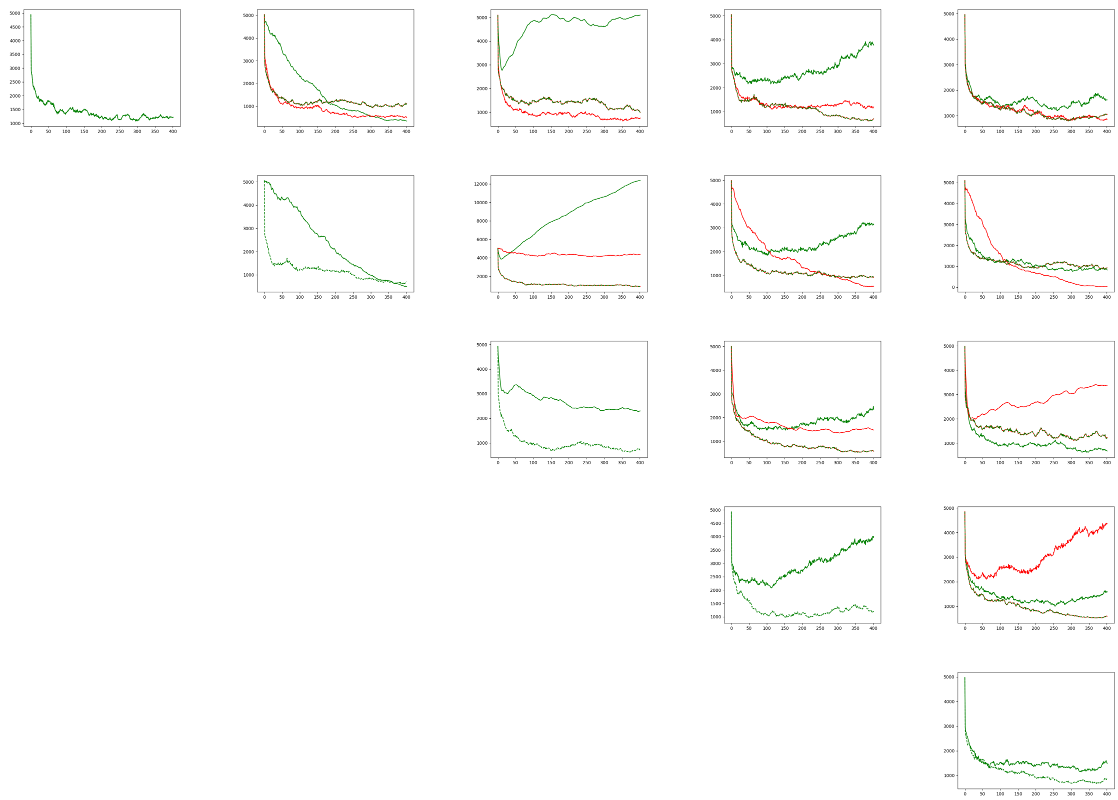
R18

R19

R20

Second subscheme. The rule on the secondary board is R1, and the ones on the main board will be again Ra and Rb respectively. The dashed lines show again the respective secondary layers of the automata.

S5BT1 R1 R2 R3 R4 R5



R1

R2

R3

R4

R5

S5BT2 R6 R7 R8 R9 R10

R1

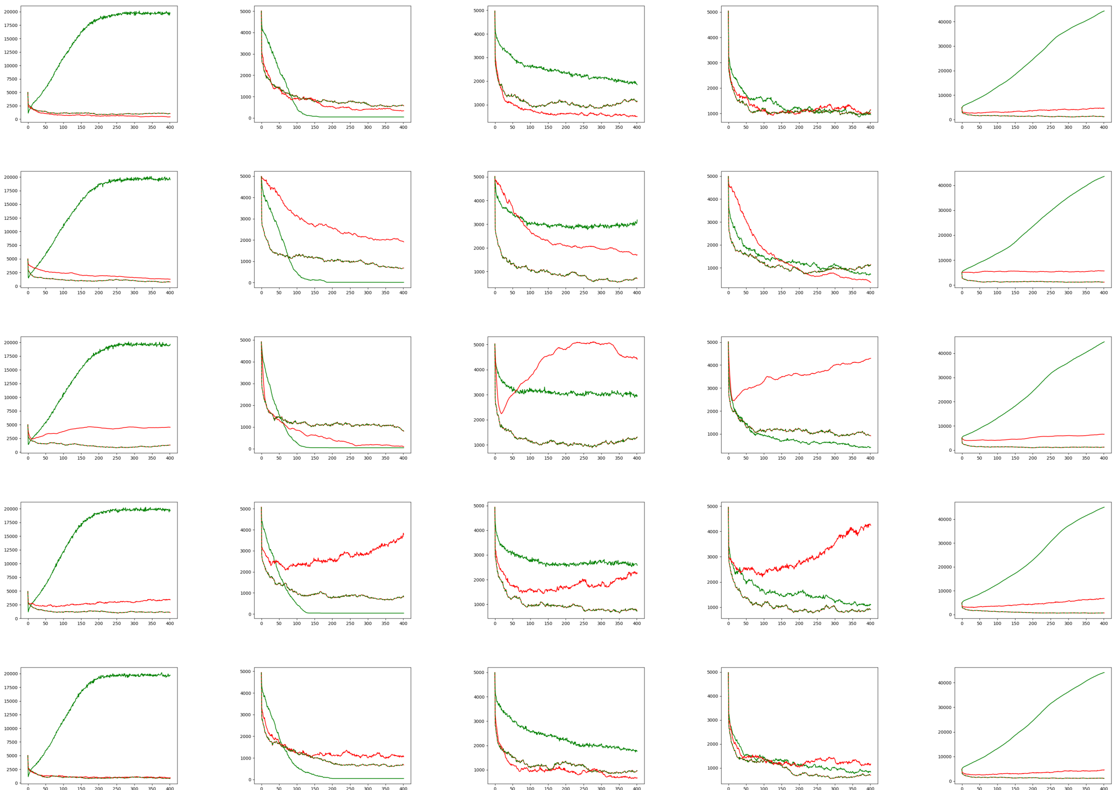
R2

R3

R4

R5

S5BT3 R11 R12 R13 R14 R15



R1

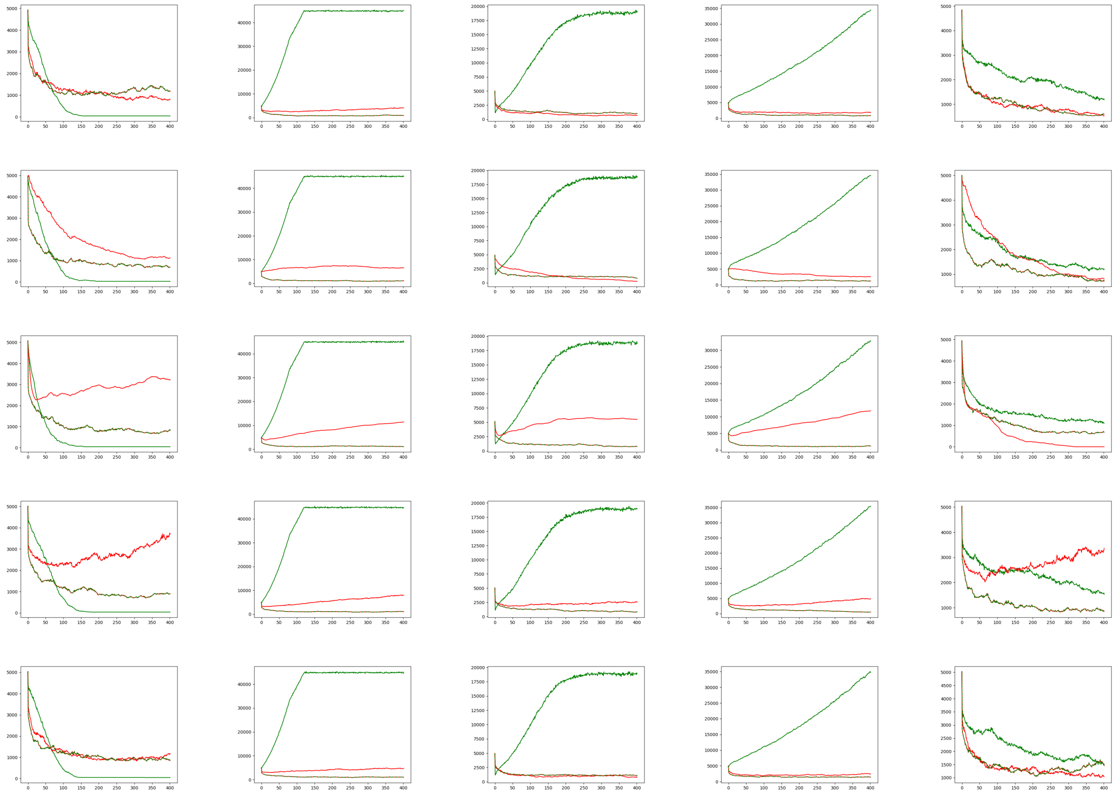
R2

R3

R4

R5

S5BT4	R16	R17	R18	R19	R20



R1

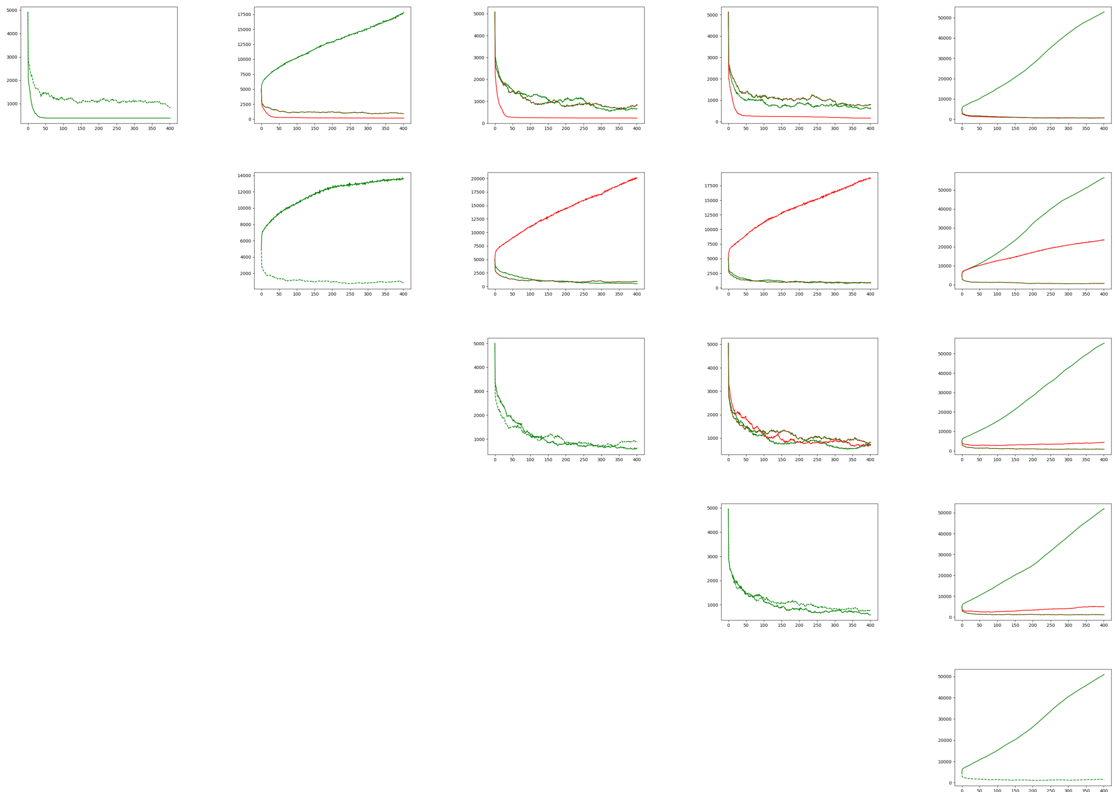
R2

R3

R4

R5

S5BT5	R6	R7	R8	R9	R10



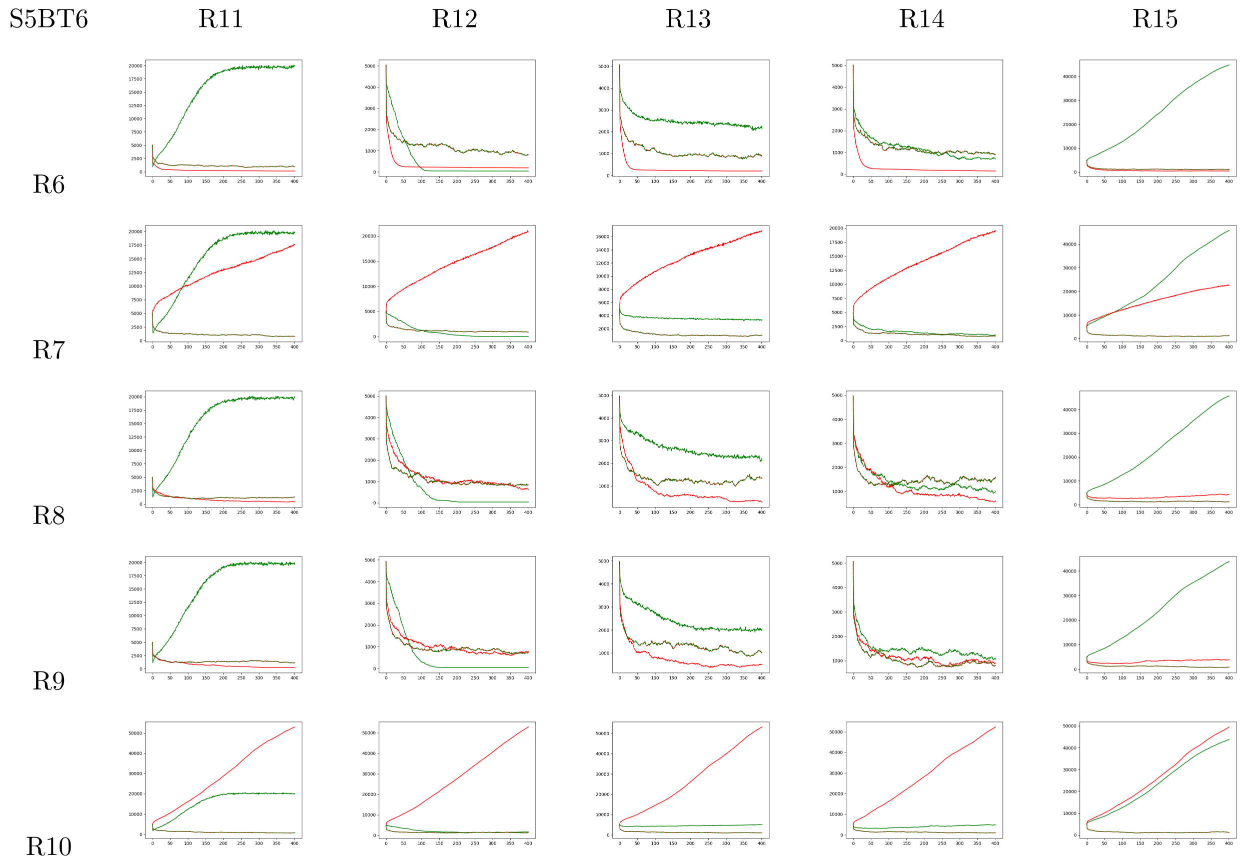
R6

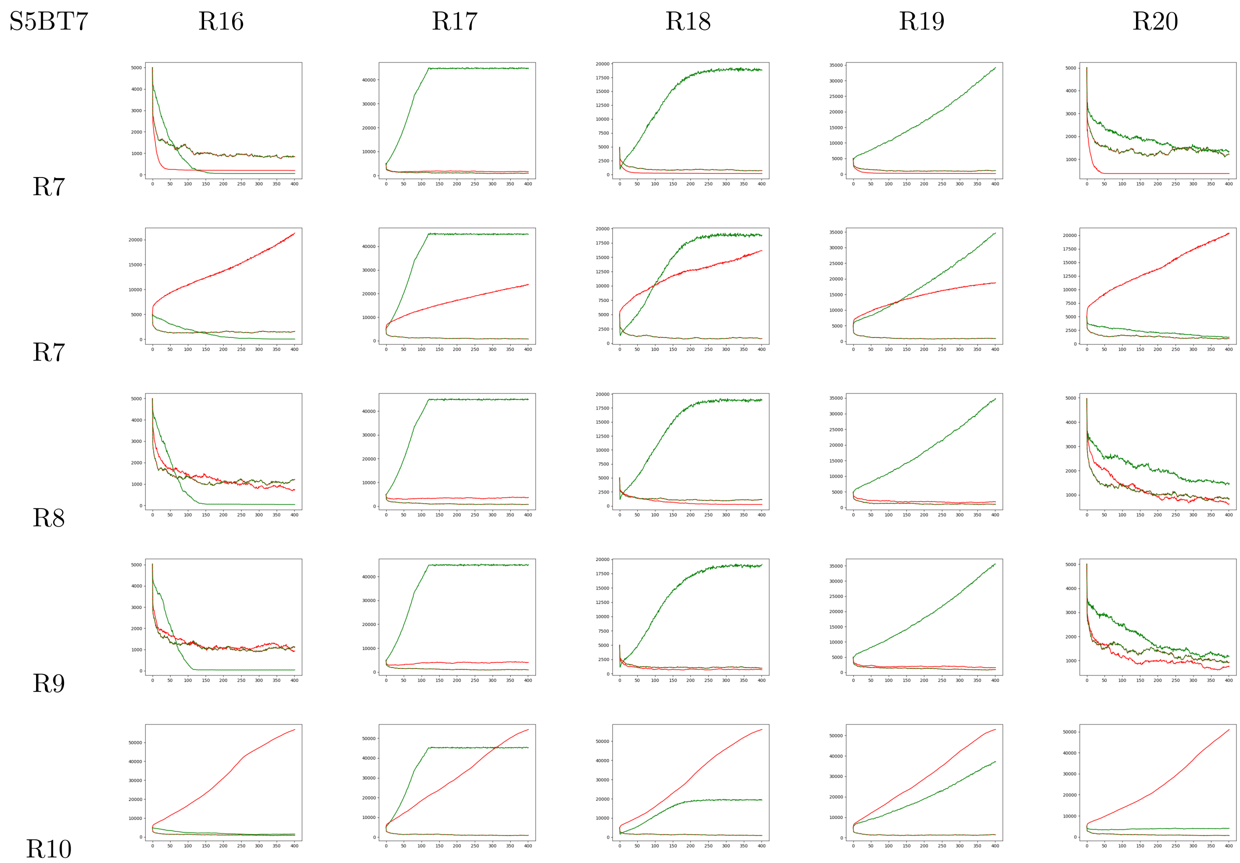
R7

R8

R9

R10


S5BT6
R11
R12
R13
R14
R15
R6
R7
R8
R9
R10


S5BT7
R16
R17
R18
R19
R20
R7
R7
R8
R9
R10

S5BT8 R11 R12 R13 R14 R15

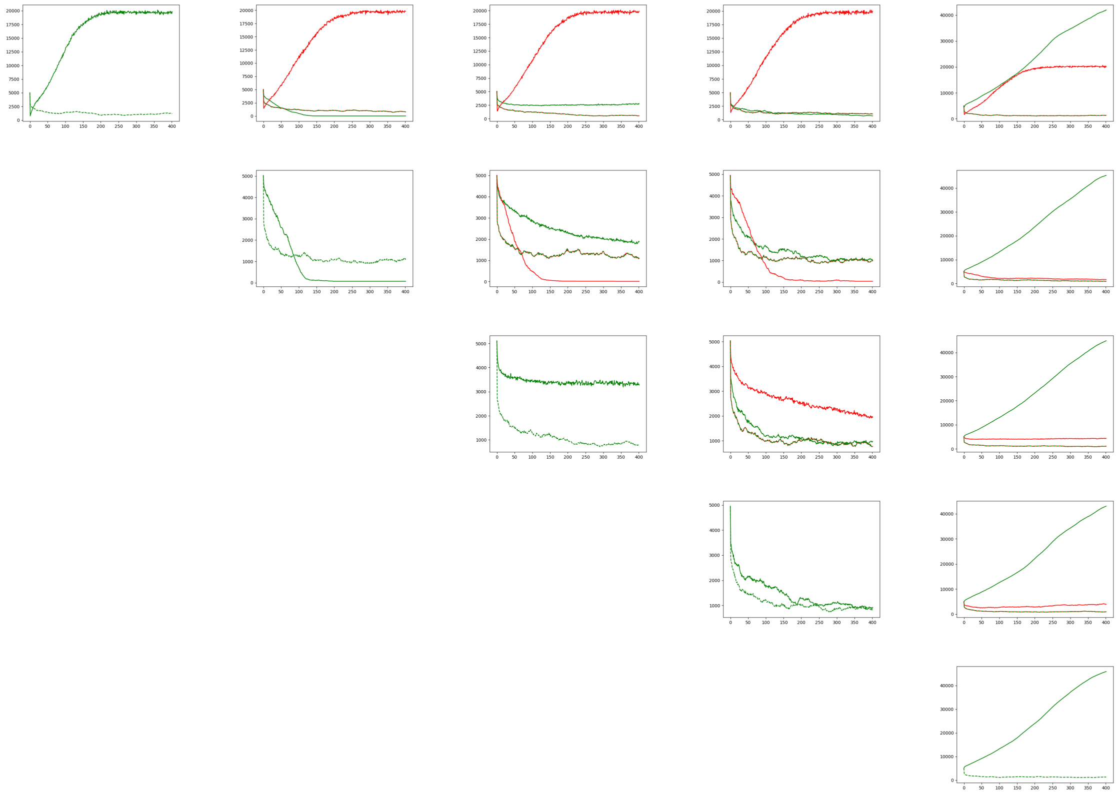
R1

R2

R3

R4

R5



S5BT9 R16 R17 R18 R19 R20

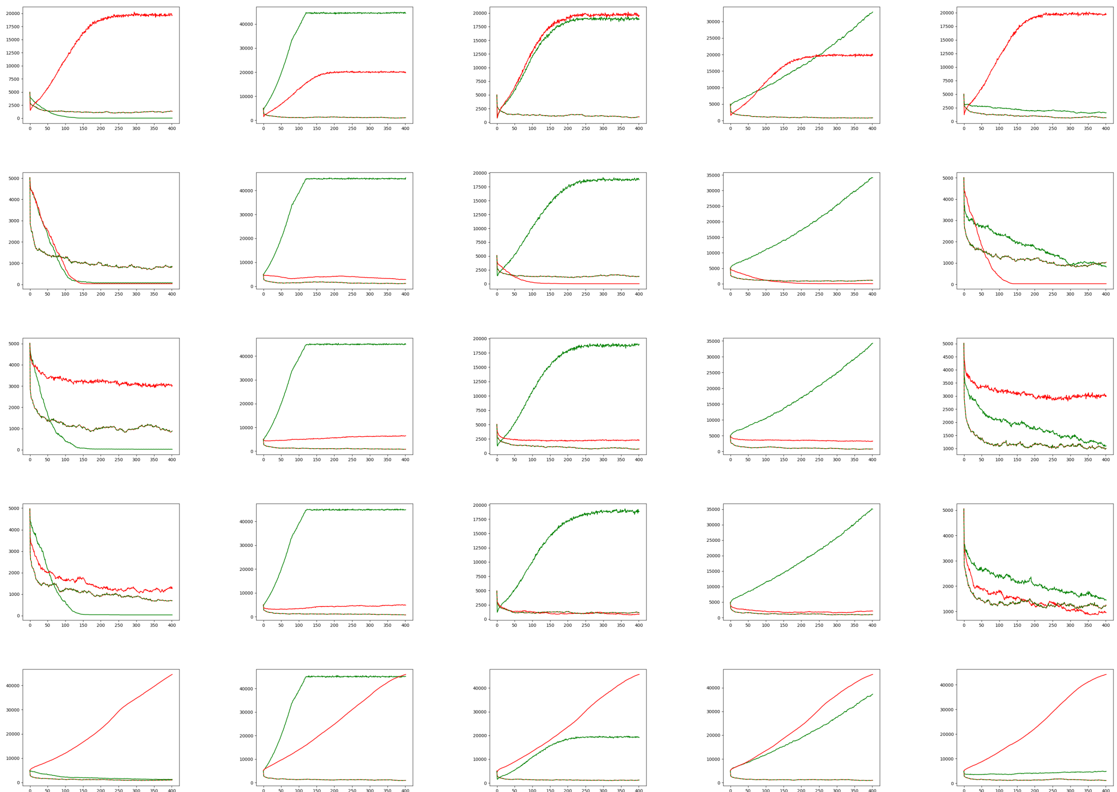
R11

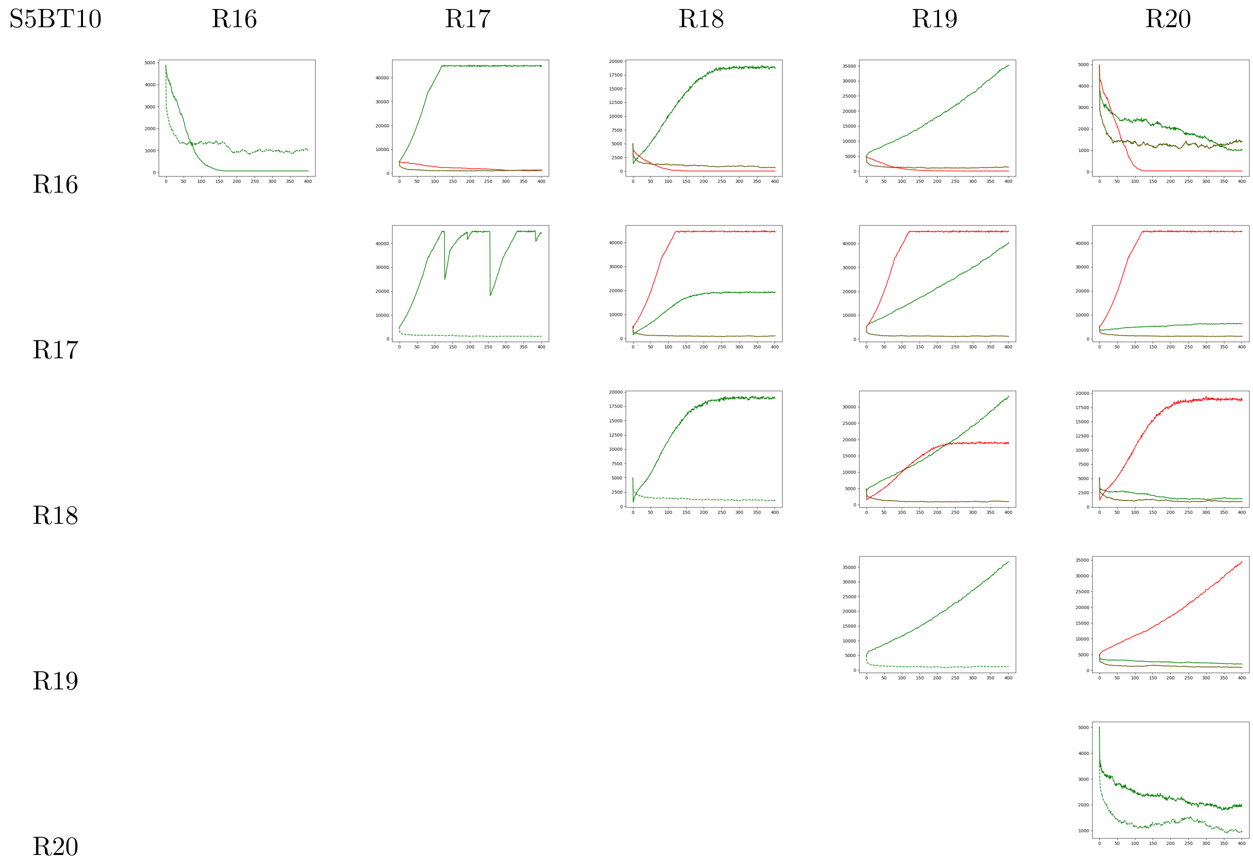
R21

R13

R14

R15



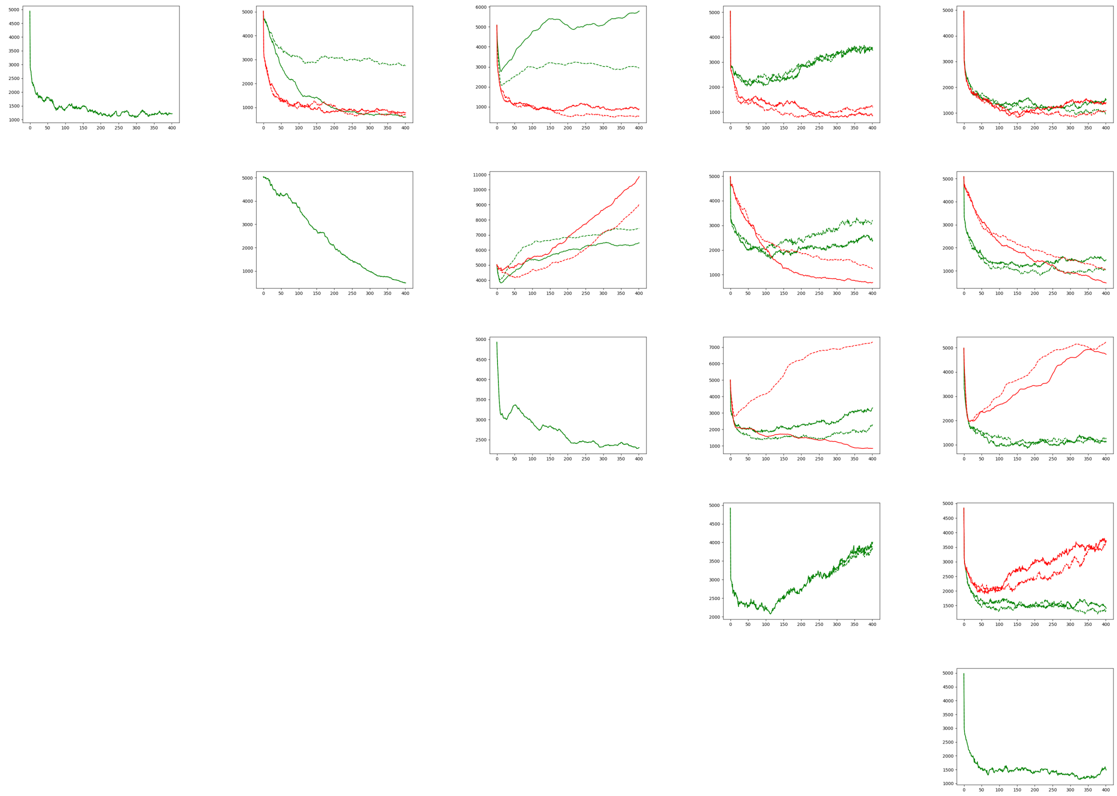


A.6 S6

First subscheme. We use two rules Ra/Rb (resp. row, column in the matrix), cells that not overlap follow rule a, cells that overlap follow rule b. The dashed lines show the respective bottom (secondary) layers of the automata.

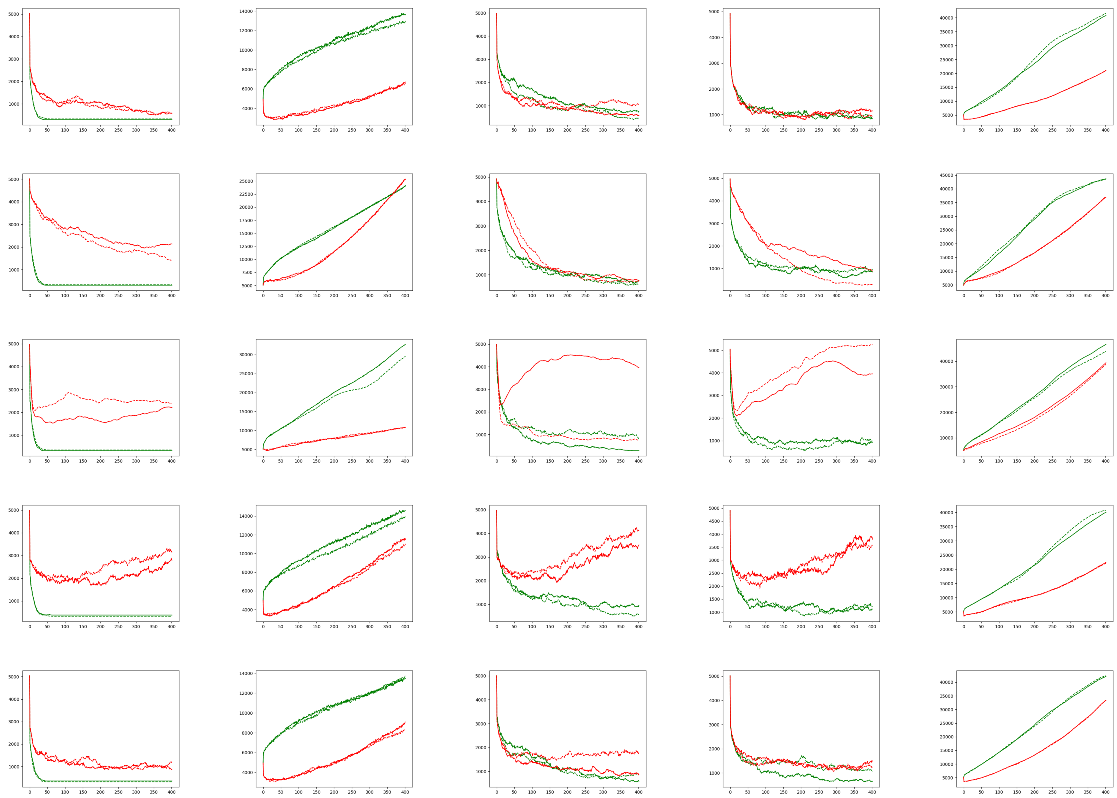
S6AT1 R1 R2 R3 R4 R5

R1
R2
R3
R4
R5



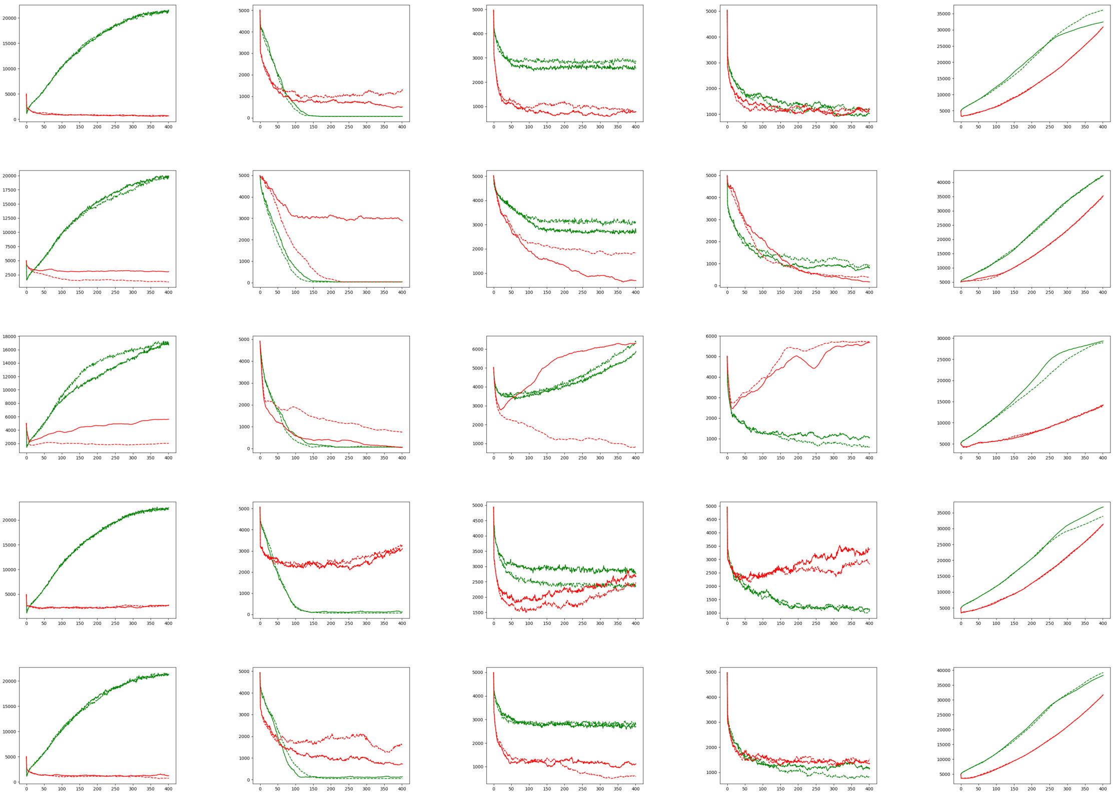
S6AT2 R6 R7 R8 R9 R10

R1
R2
R3
R4
R5



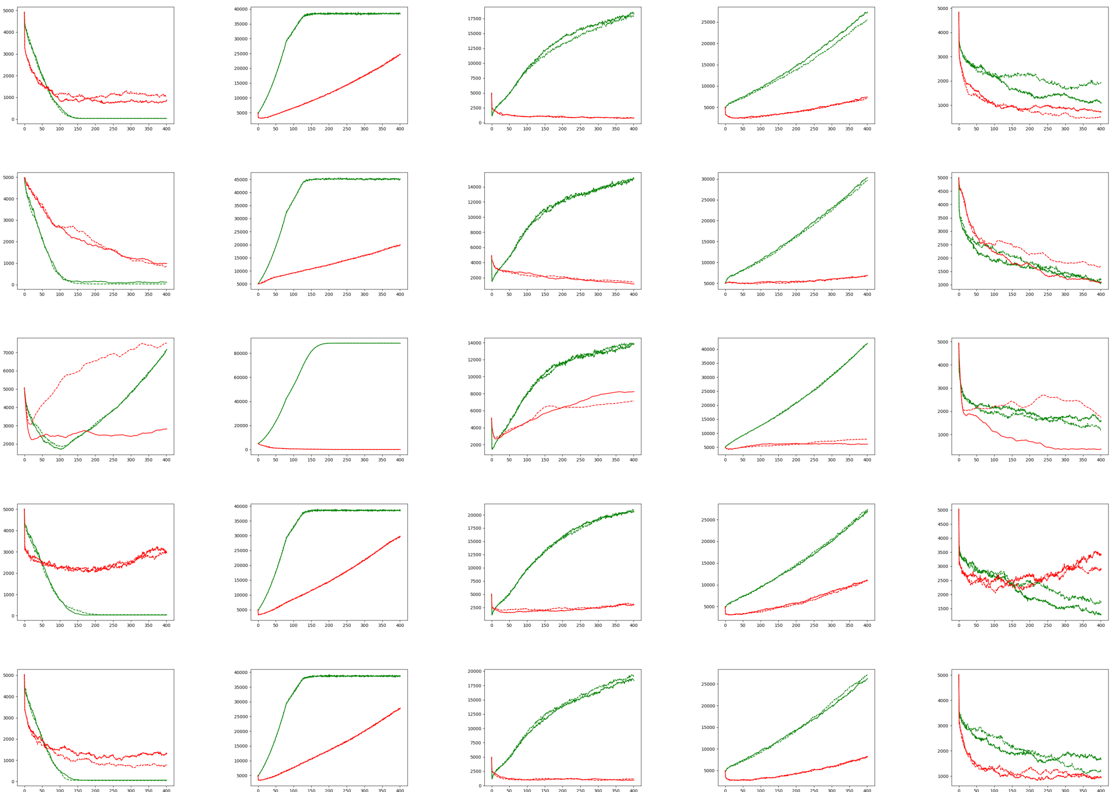
S6AT3 R11 R12 R13 R14 R15

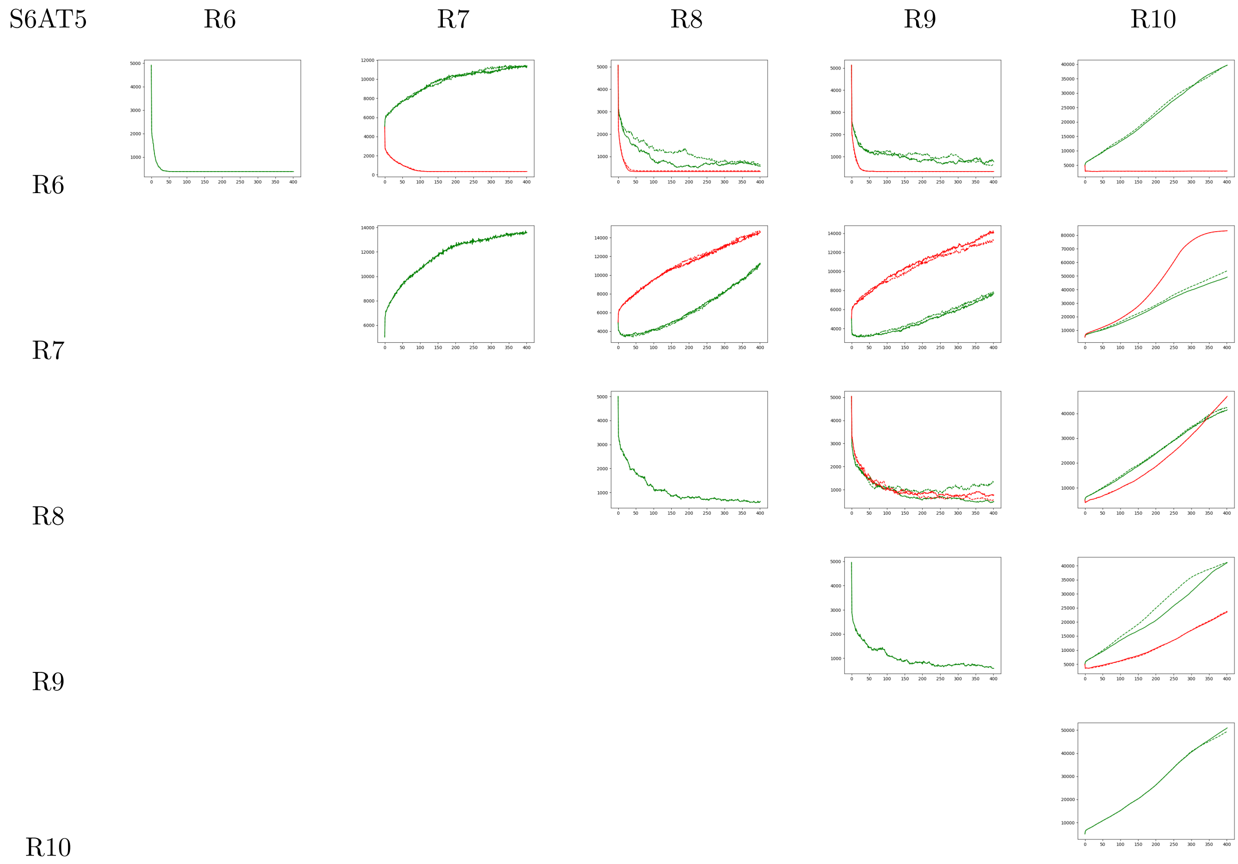
R1 R2 R3 R4 R5

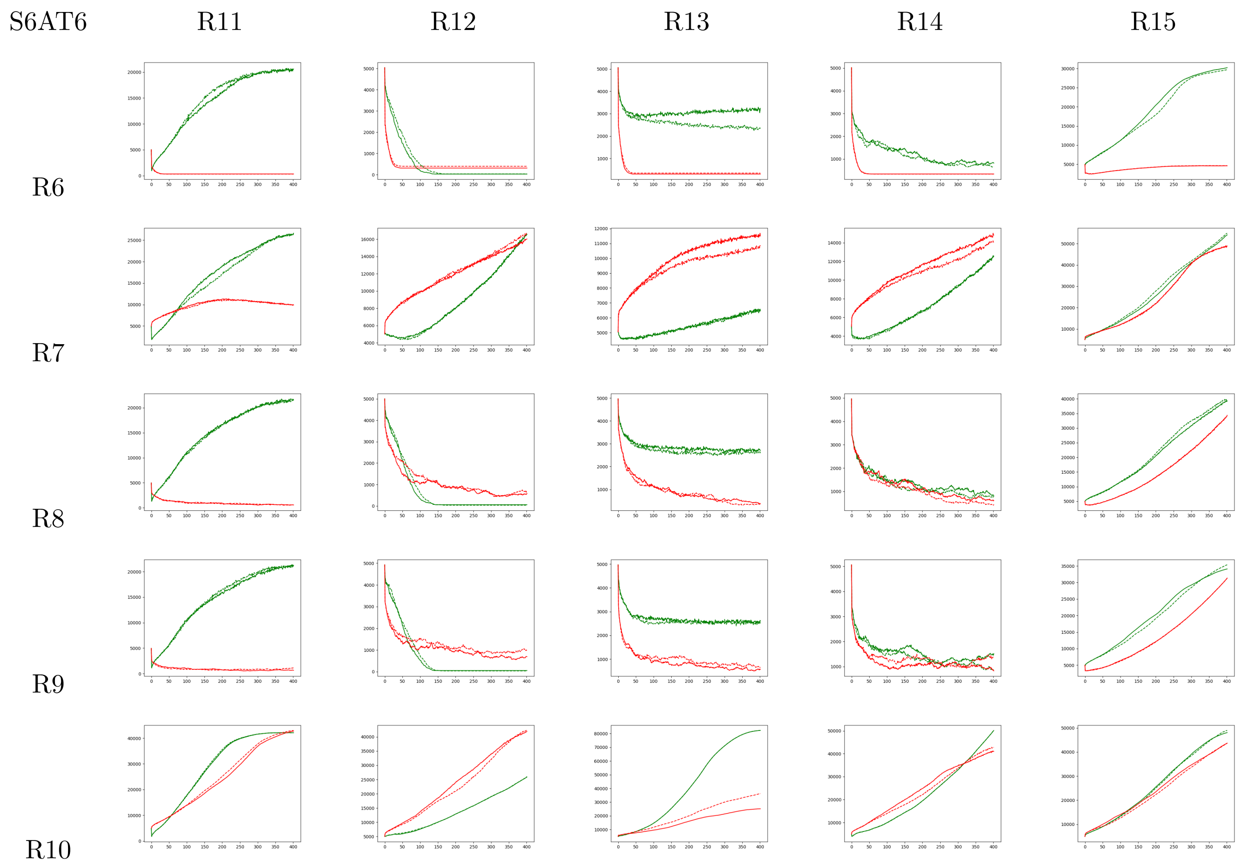


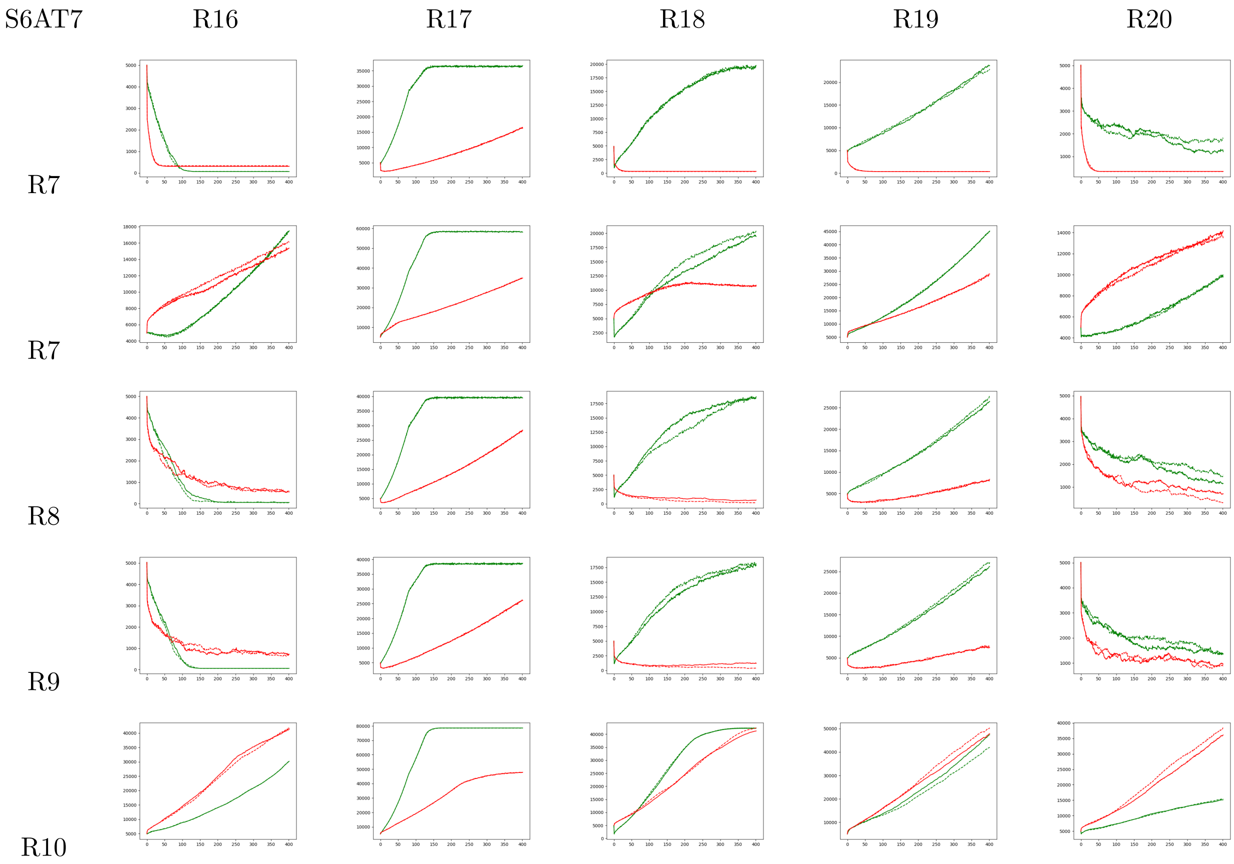
S6AT4 R16 R17 R18 R19 R20

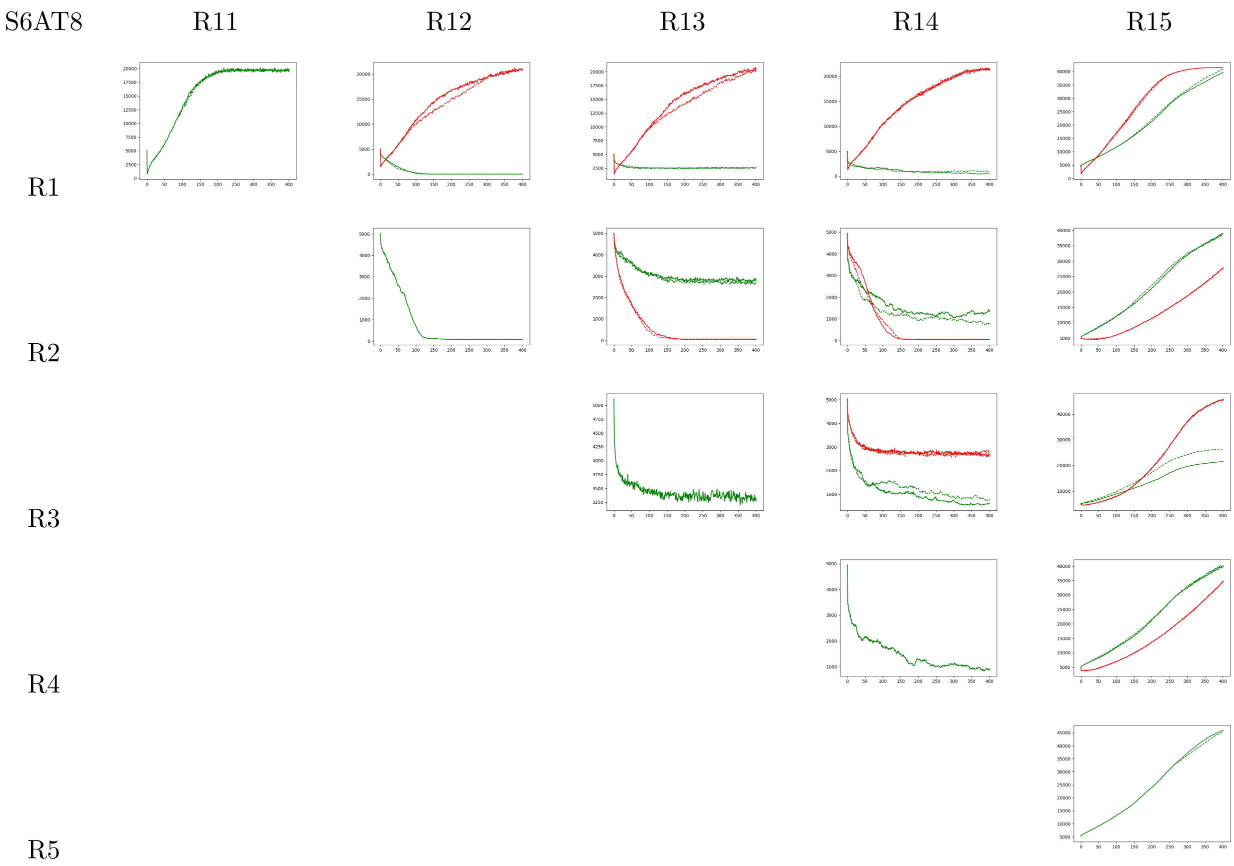
R1 R2 R3 R4 R5

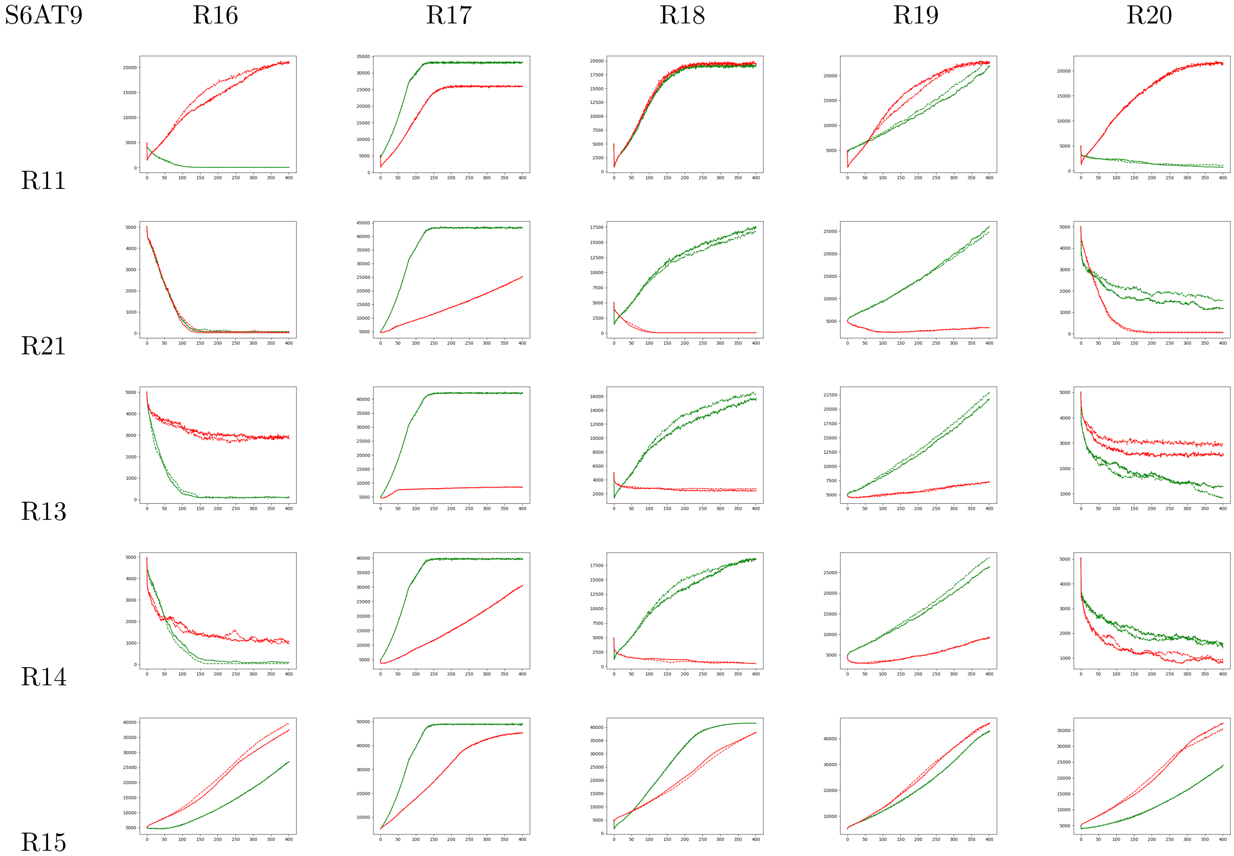


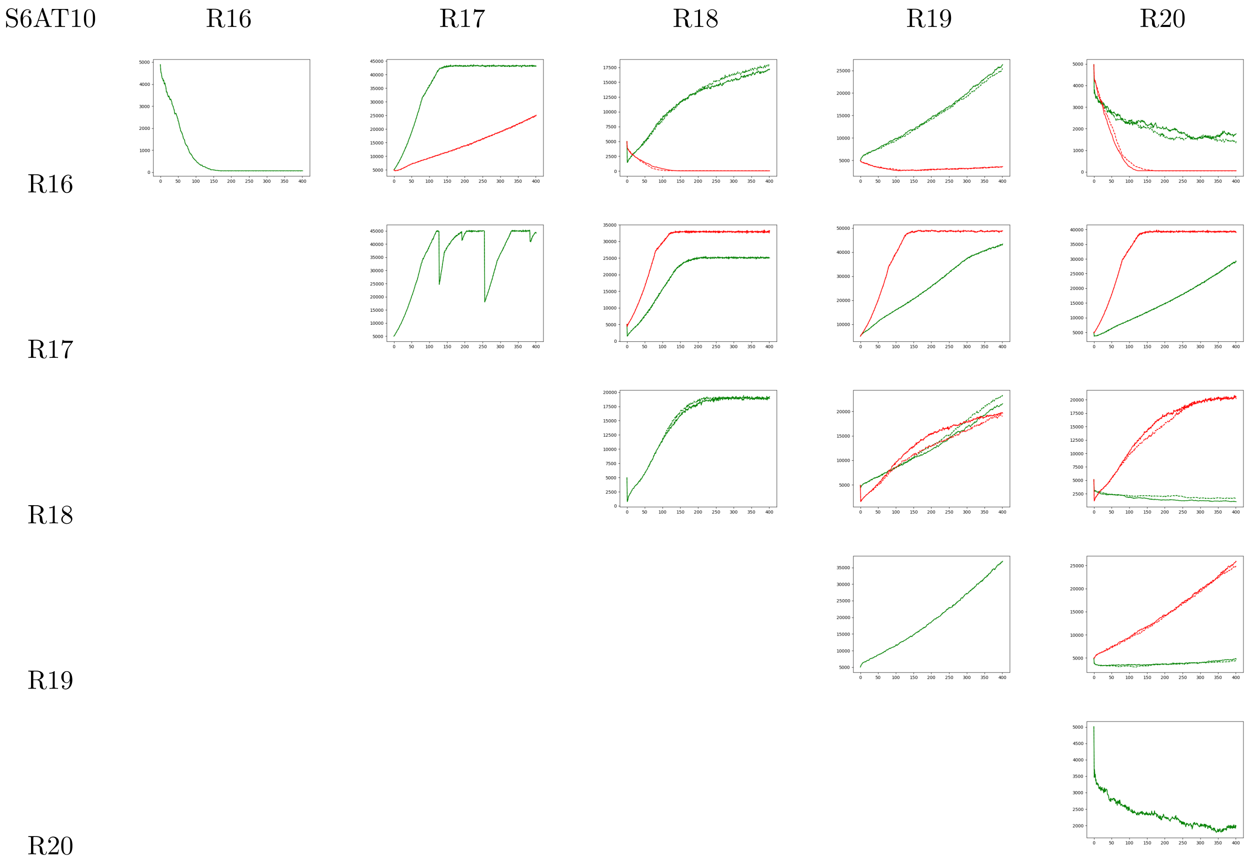

S6AT5
R6
R7
R8
R9
R10
R6
R7
R8
R9
R10


S6AT6
R11
R12
R13
R14
R15
R6
R7
R8
R9
R10


S6AT7
R16
R17
R18
R19
R20
R7
R7
R8
R9
R10


S6AT8
R11
R12
R13
R14
R15
R1
R2
R3
R4
R5


S6AT9
R16
R17
R18
R19
R20
R11
R21
R13
R14
R15


S6AT10
R16
R17
R18
R19
R20
R16
R17
R18
R19
R20

Second subscheme. Living and dead cells define where rule 1 and 4 act on the second board, vice versa cells living and dead cells define where rule a and b will act. The dashed lines show again the respective secondary layers of the automata.

S6BT1 R1 R2 R3 R4 R5

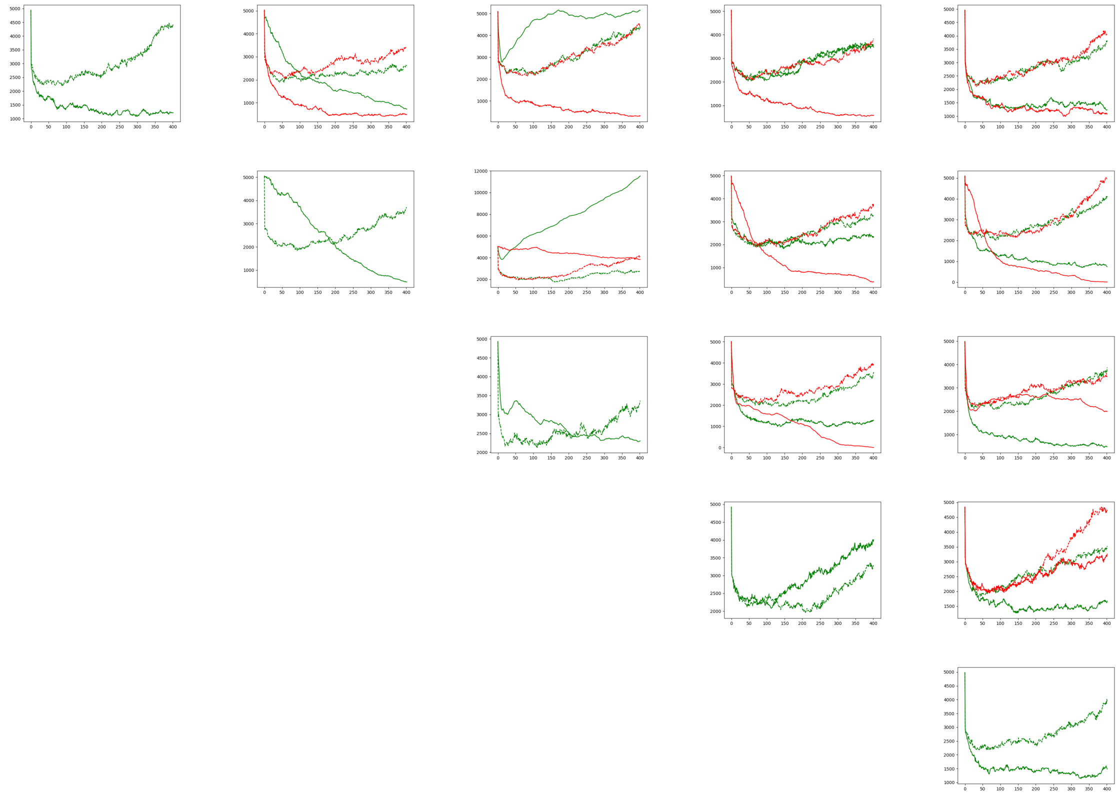
R1

R2

R3

R4

R5



S6BT2 R6 R7 R8 R9 R10

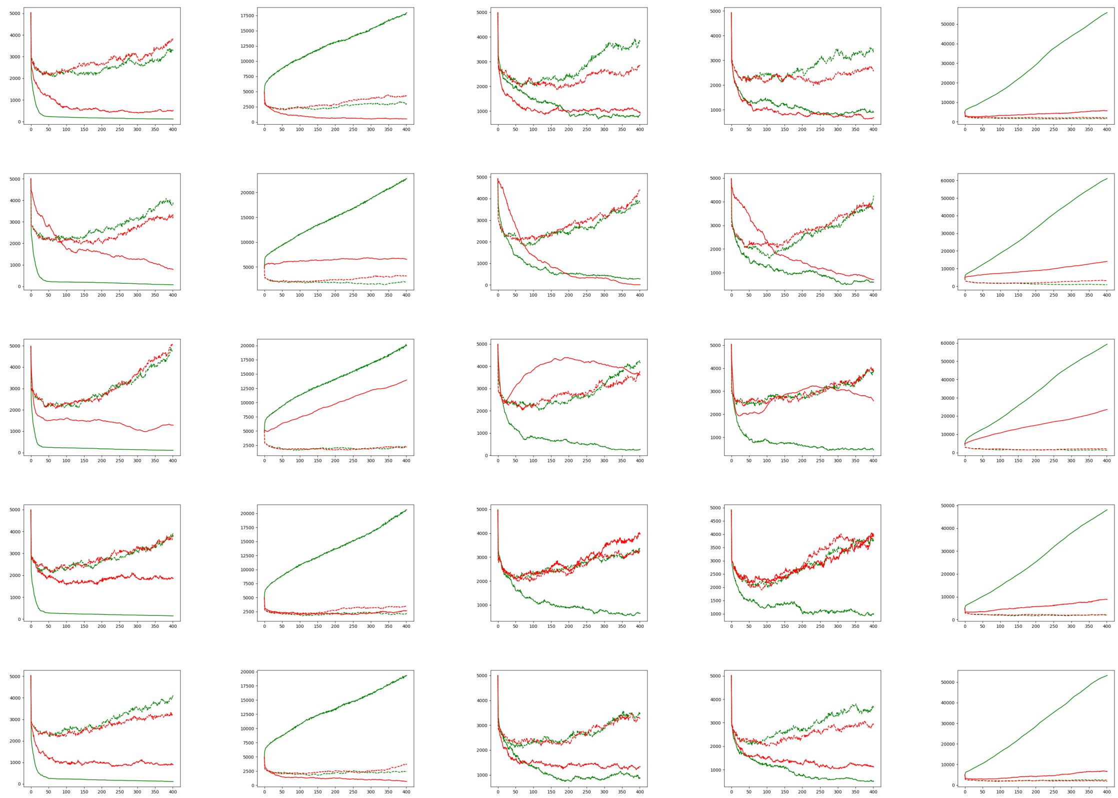
R1

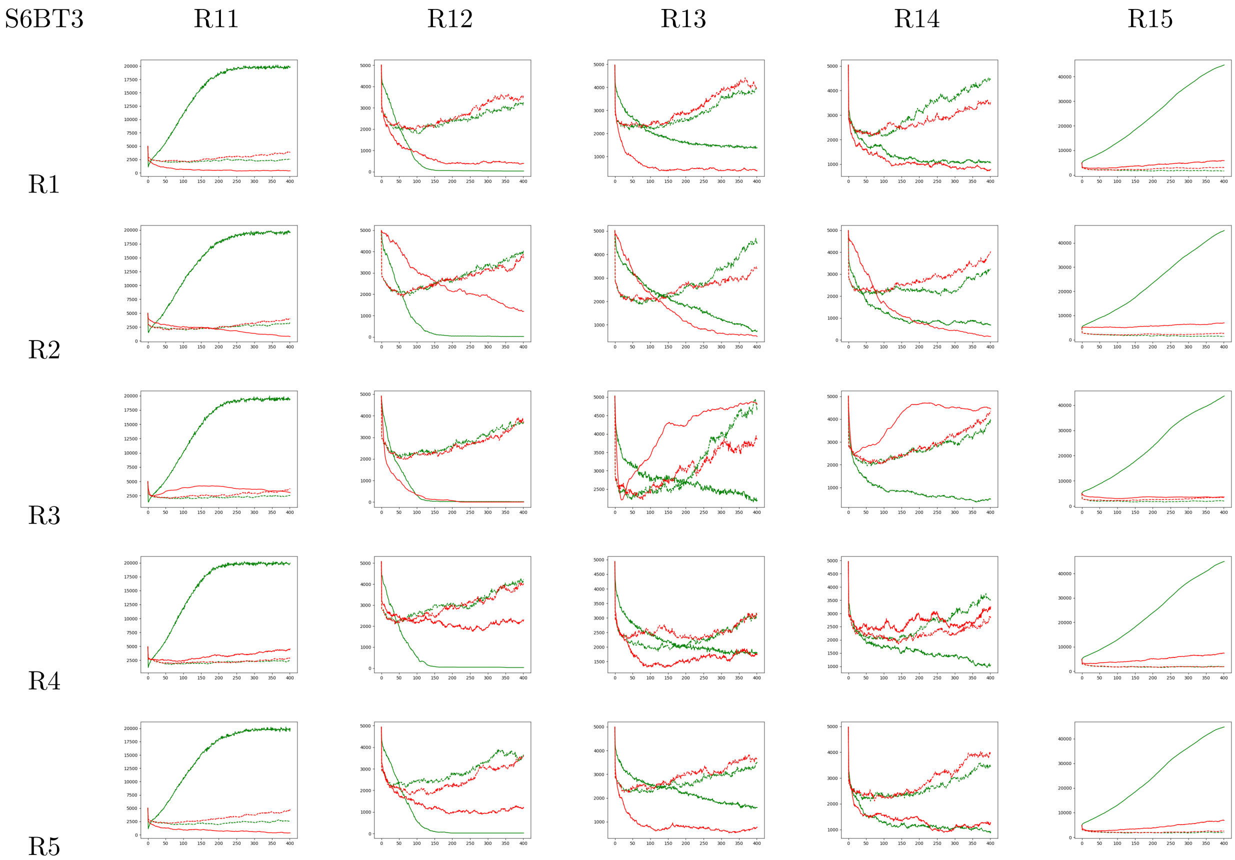
R2

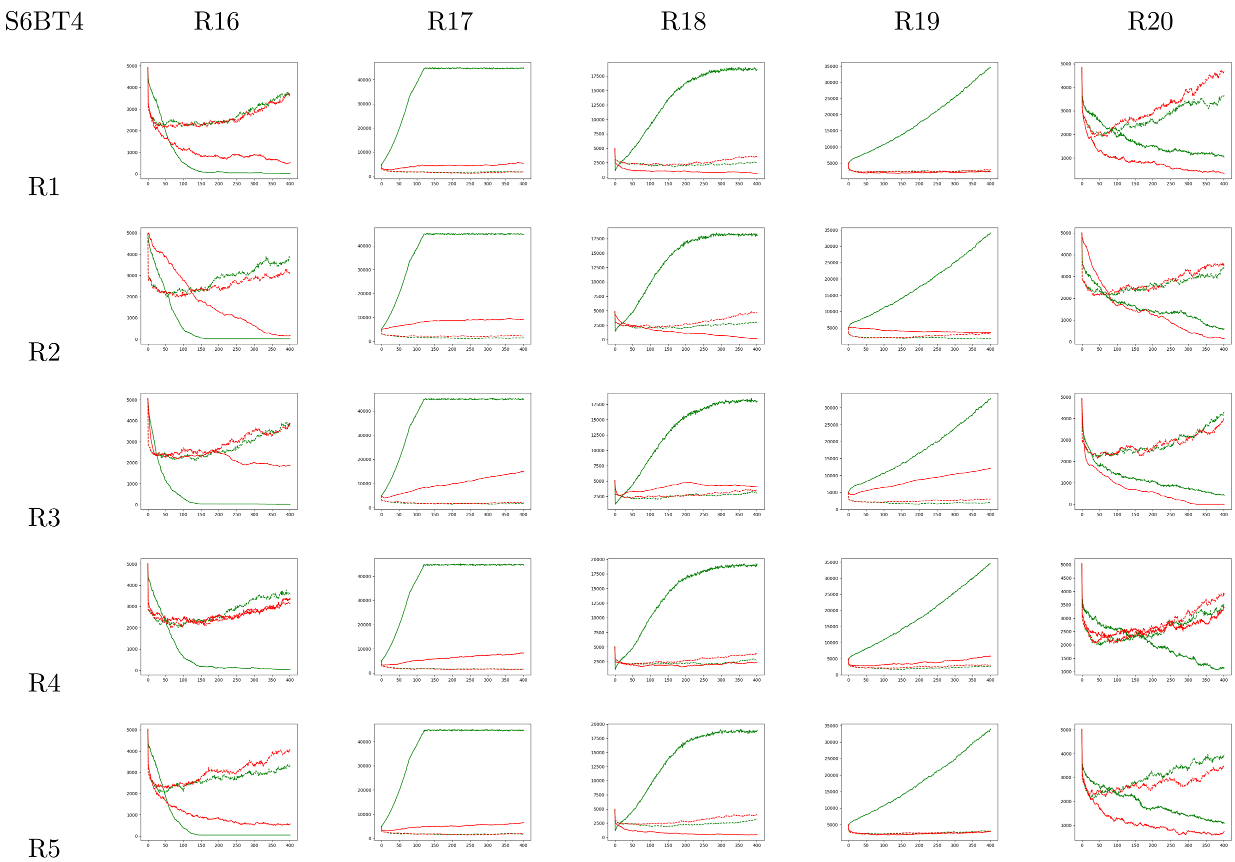
R3

R4

R5




S6BT3
R11
R12
R13
R14
R15
R1
R2
R3
R4
R5


S6BT4
R16
R17
R18
R19
R20
R1
R2
R3
R4
R5

S6BT5 R6 R7 R8 R9 R10

R6

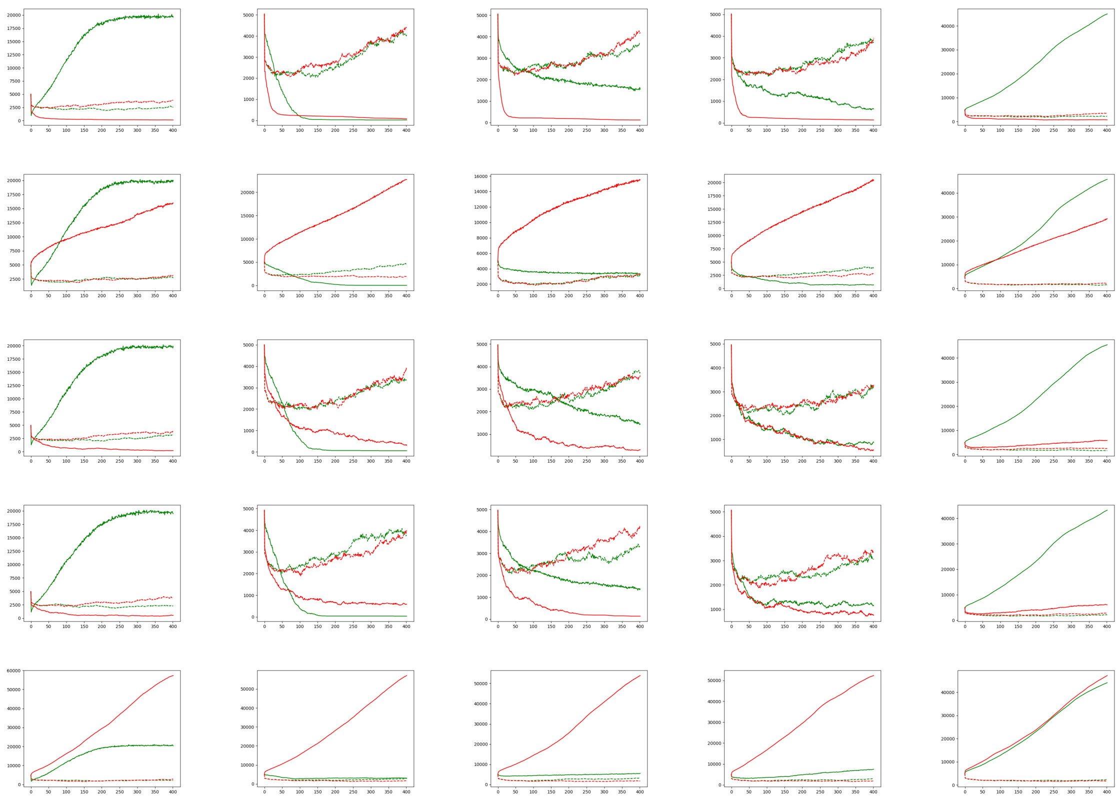
R7

R8

R9

R10

S6BT6 R11 R12 R13 R14 R15



R6

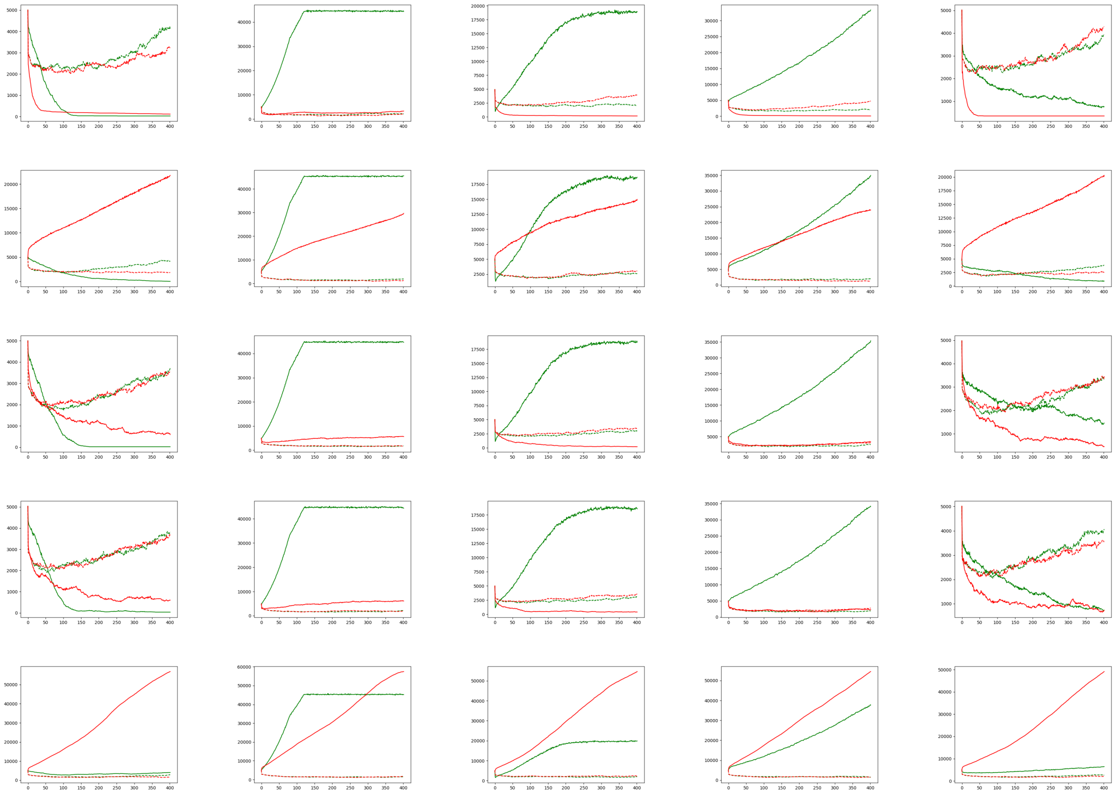
R7

R8

R9

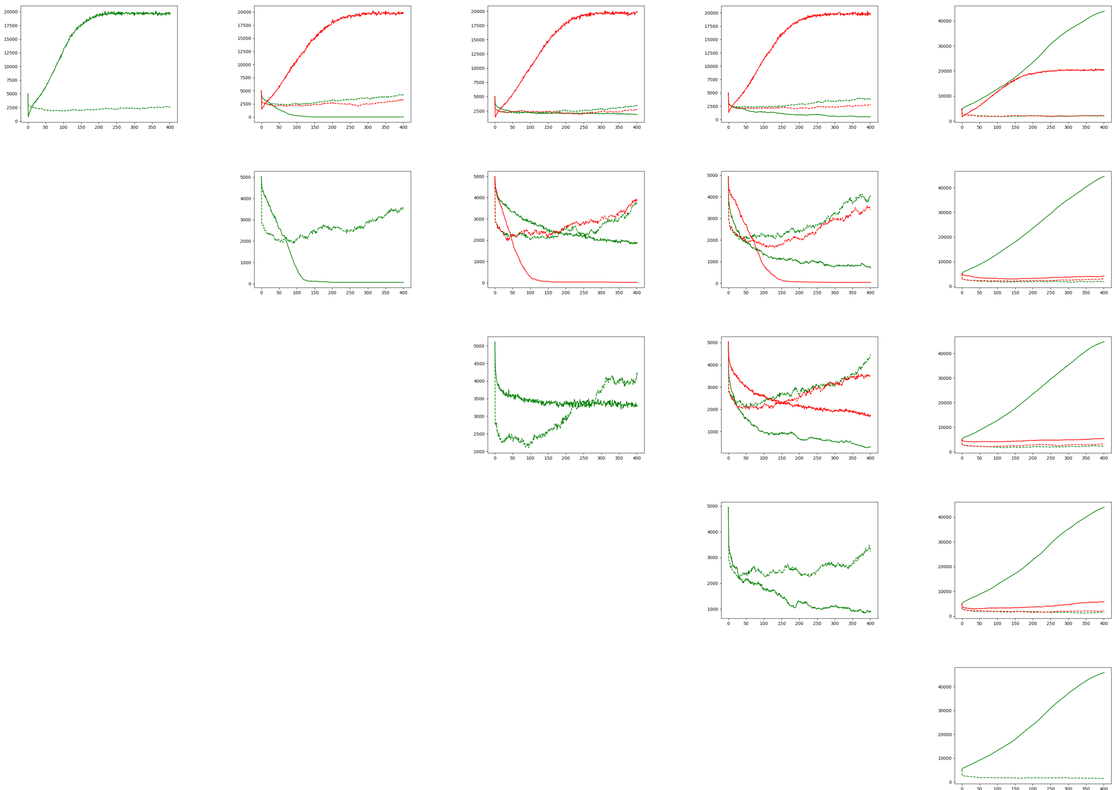
R10

S6BT7 R16 R17 R18 R19 R20

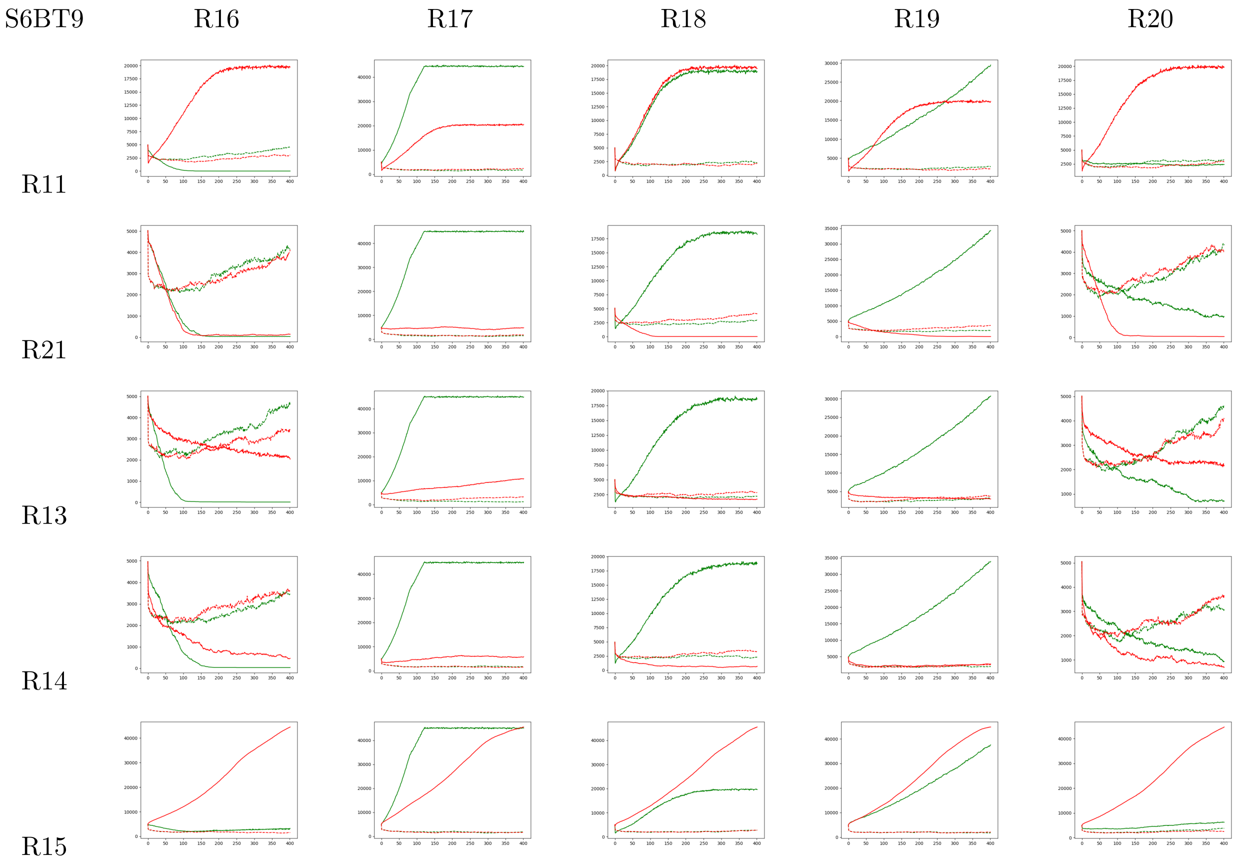


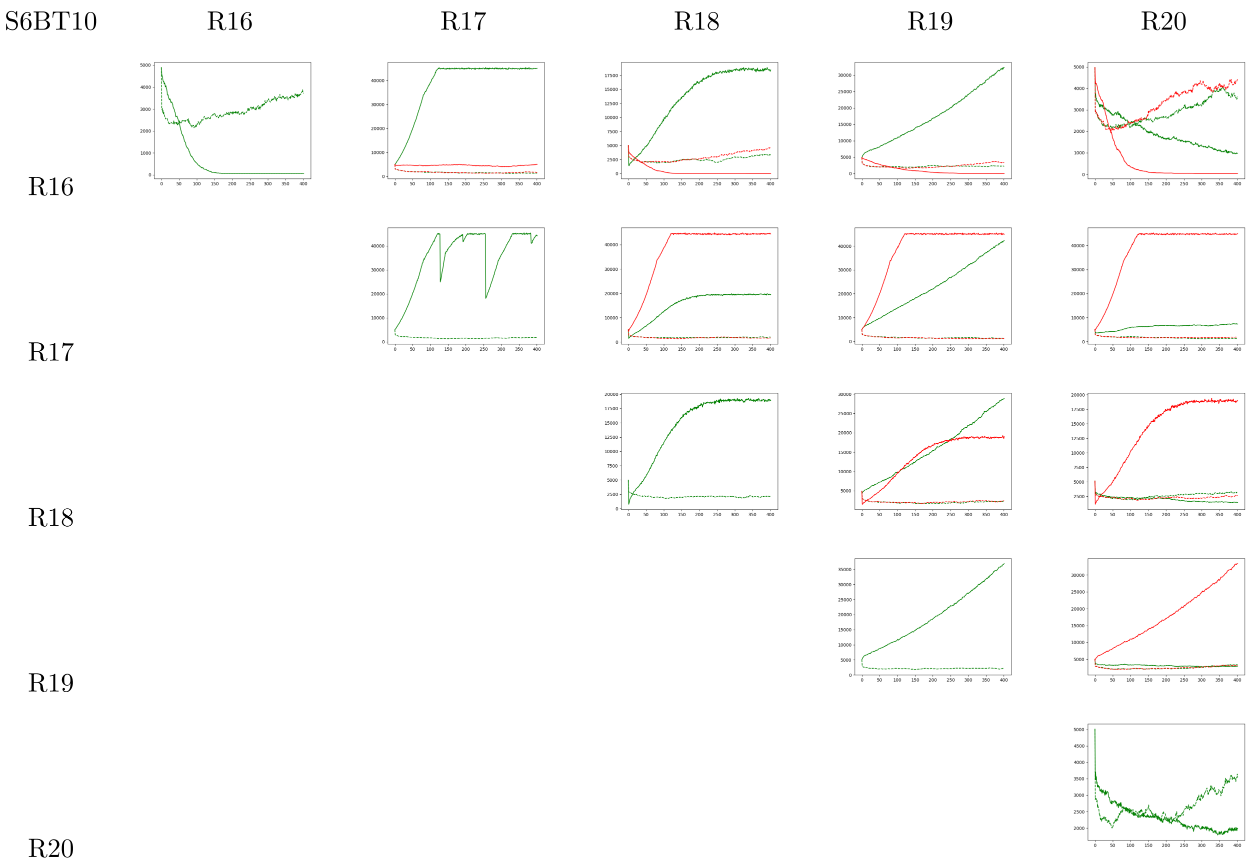
R7
R7
R8
R9
R10

S6BT8 R11 R12 R13 R14 R15



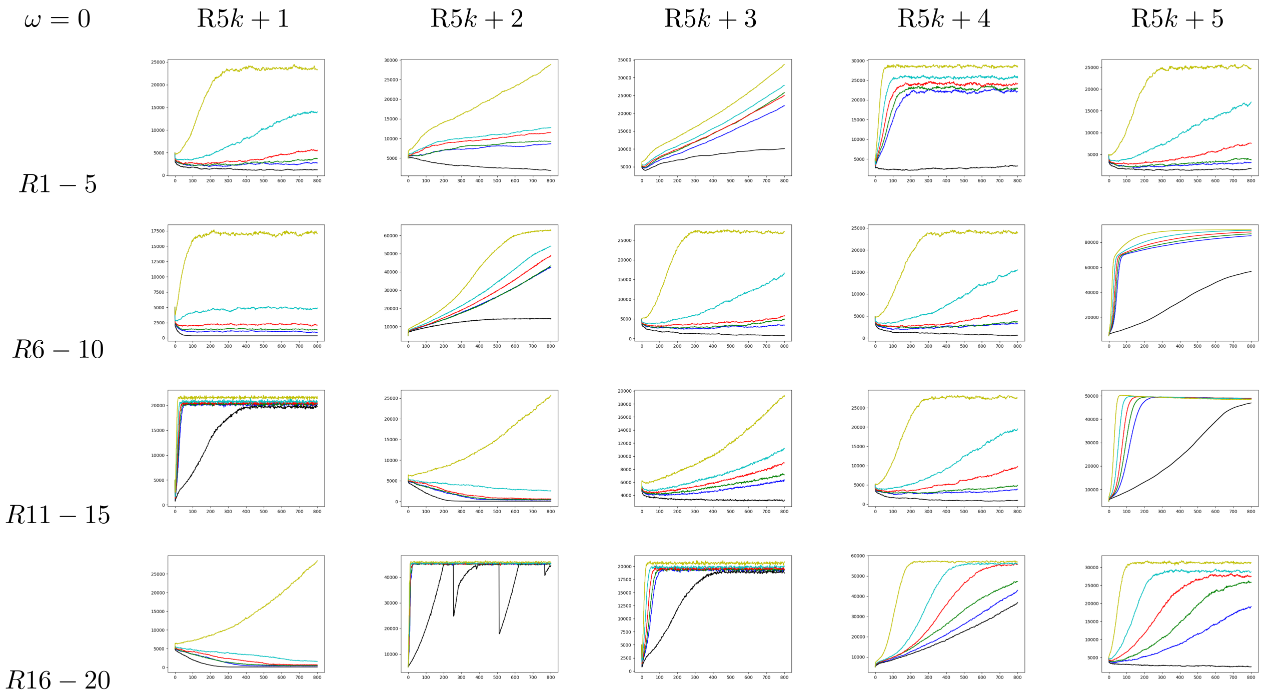
R1
R2
R3
R4
R5

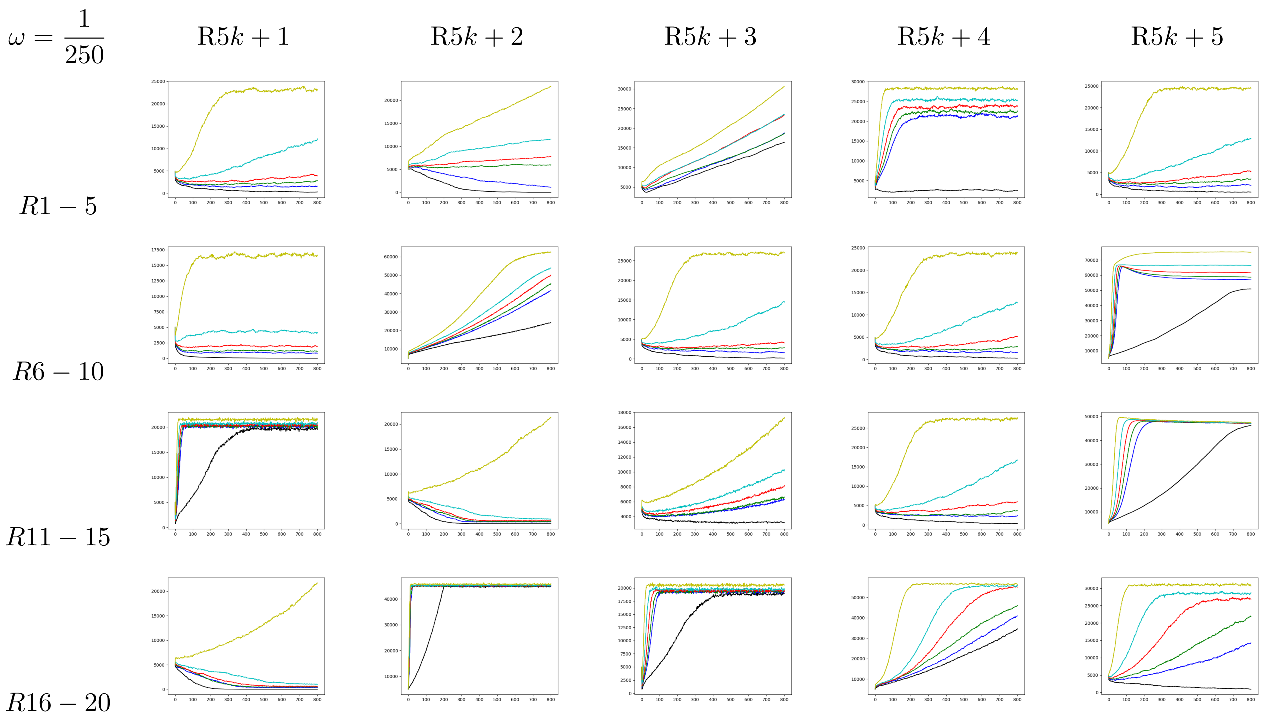

S6BT9
R16
R17
R18
R19
R20
R11
R21
R13
R14
R15

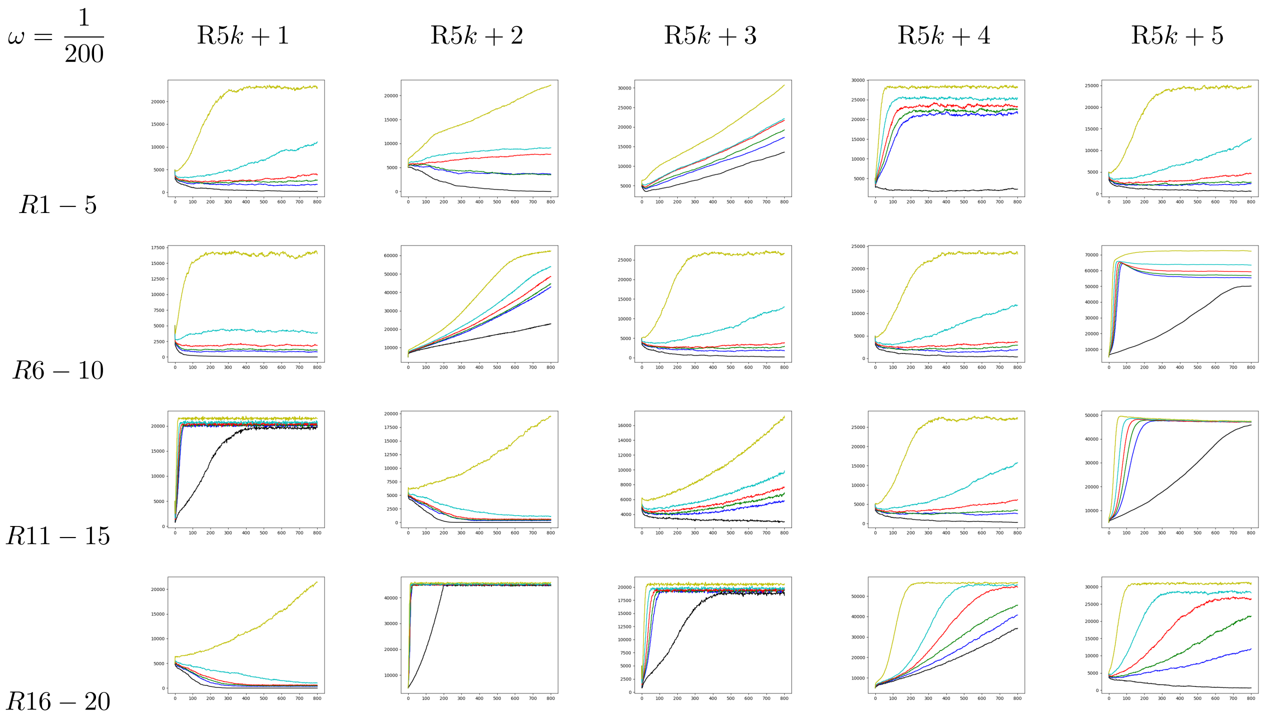

S6BT10
R16
R17
R18
R19
R20
R16
R17
R18
R19
R20

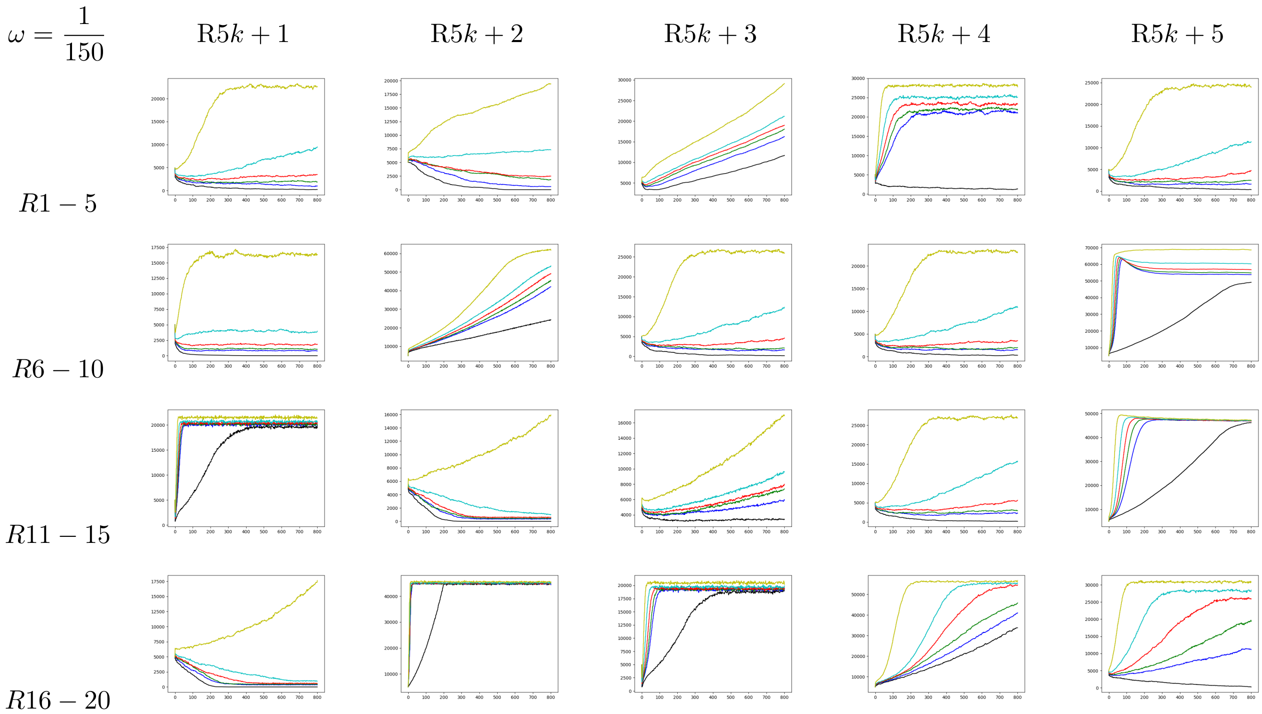
A.7 S7

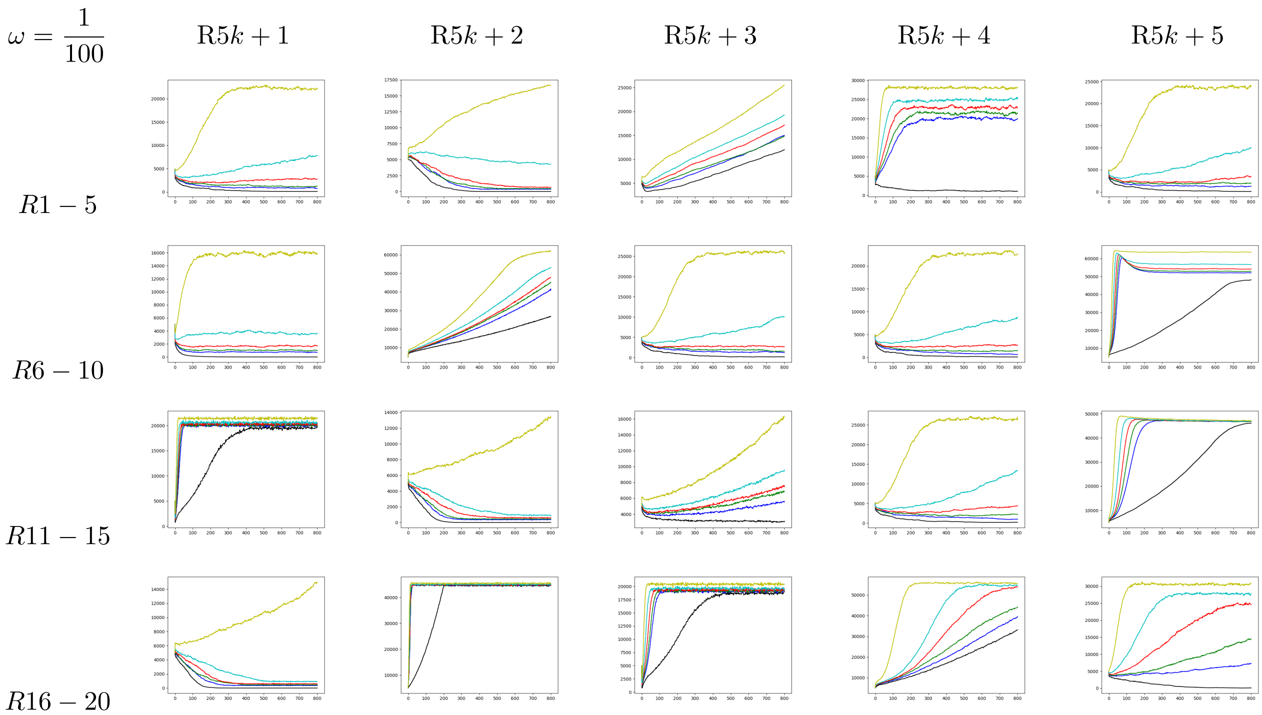
These plots show the cell populations for random cellular automata with several choices for the coefficients α and ω (Scheme S7). To produce the following plots, we have assigned in particular $\omega \in \left\{0, \frac{1}{50}, \frac{1}{100}, \frac{1}{150}, \frac{1}{200}, \frac{1}{250}\right\}$ and shown in each picture the plots for $\alpha = 0$ (black), $\alpha = \frac{1}{250}$ (blue), $\alpha = \frac{1}{200}$ (green), $\alpha = \frac{1}{150}$ (red), $\alpha = \frac{1}{100}$ (cyan), $\alpha = \frac{1}{50}$ (yellow)

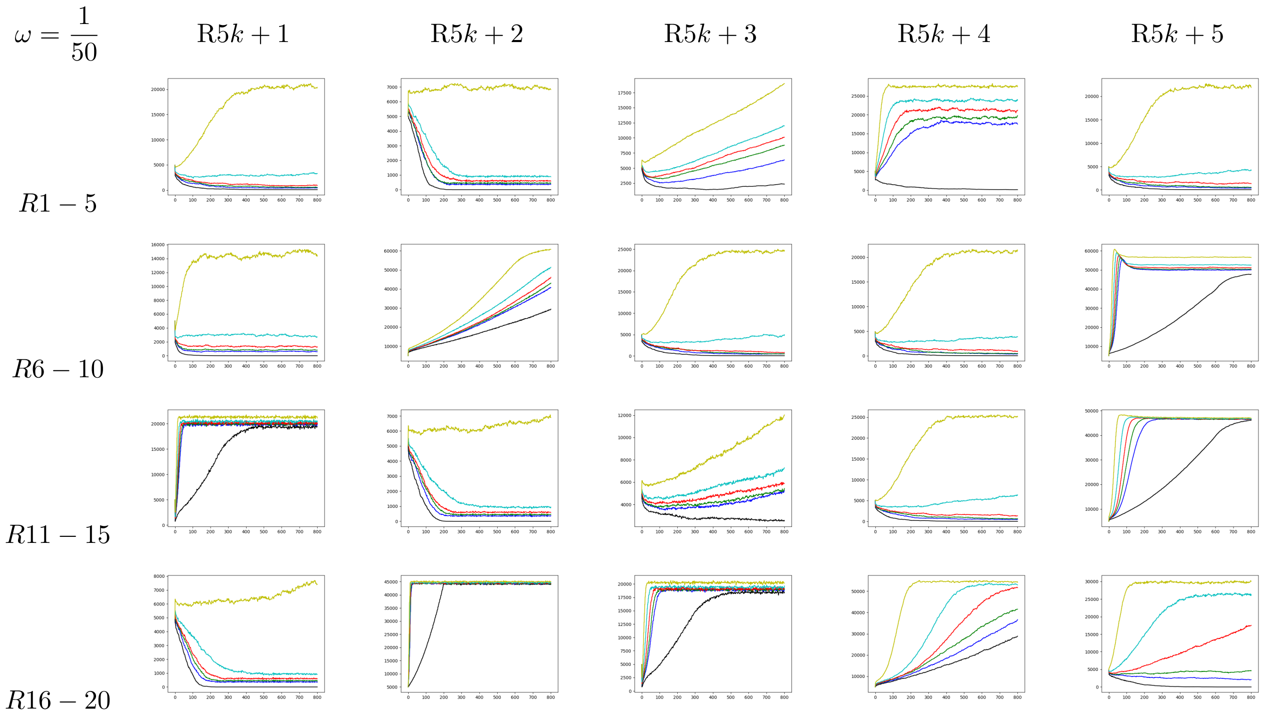





$\omega = \frac{1}{200}$
R$5k+1$
R$5k+2$
R$5k+3$
R$5k+4$
R$5k+5$
$R1-5$
$R6-10$
$R11-15$
$R16-20$


$\omega = \frac{1}{150}$
R$5k+1$
R$5k+2$
R$5k+3$
R$5k+4$
R$5k+5$
$R1-5$
$R6-10$
$R11-15$
$R16-20$





B Open Problems

Open Problem 1. *Find a composite rule, in particular a purely Time-depending one (or prove that such example does not exist) such that a Garden of Eden [5] for one of the composing rules is not a Garden of Eden i.e., configuration that has no predecessor. for the composit rule.*

Open Problem 2. *Find a non trivial pattern (or prove that such example does not exist) that for two different step- and time-depending rules is a spaceship and a gun.*

Open Problem 3. *Find a pattern that evolves in 5 or more differen ways for Step- or Zone-depending rules.*

Open Problem 4. *Find a non trivial gun and spaceship for Parallel life for prove that such example does not exist).*

Open Problem 5. *Find complex structures (as puffers or guns) for S5 and S6.*

Open Problem 6. *Find if it is possible to derive other cellular automata (e.g. Generations [6]) from the rules defined in this paper.*