Communicating mathematics in a real-life context

Saeed Manshadi

UiT The Arctic University of Norway, Norway; saeed.d.manshadi@uit.no

This article presents a case study of communication in mathematics in a seventh-grade classroom of a Norwegian primary school. The main aim of this study is to investigate characteristics of student communication in mathematics in the context of their realities. Video recordings of conversations were analyzed using a framework of various speech acts and interaction patterns that may indicate students' approaches to mathematics. The students explored body movements to describe different rotations in a real-life context. The analyses show that the students' interaction displayed an investigative approach to mathematics that amplifies their voices by facing others' opinions and thoughts. This is important for the development of students' mathematical knowledge and engagement in the learning process. The results also indicate that real-life contexts may influence the flexibility of communication.

Keywords: Communication, speech acts, investigative approach, real-life context.

Background and purpose

Traditional approaches to mathematics teaching are characterized by authoritative communication forms (Mortimer & Scott, 2003), where little space exists for different thoughts and opinions. In contrast, Skovsmose (2001) presented an inquiry approach called ”the landscape of investigation” as a qualitatively different approach to learning mathematics. Contexts in a landscape of investigation invite students to an inquiry process that is often characterized by different speech acts. We can consider conversation and dialogue as communicative actions that students use for different purposes. There is broad consensus in mathematics didactics about the importance of communication in learning mathematics (Nilsson & Ryve, 2010). Sfard (2001) and Lee and Johnston-Wilder (2013) concluded that there is a connection between communication and thinking in mathematics. Students' learning processes often occur in a social context where interactions can make their voices heard. One's voice expresses their thoughts, ideas, perceptions, attitudes, and other personal positions (Dysthe, 1999).

This study is part of a pilot project related to the event, Vitenuka (2014), organized by a research group at UiT, which aims to contribute to science and mathematics recruitment among primary school students in Finnmark, Norway. ”Communication” is a comprehensive term that includes drawing, the use of objects, mimicry, body movements, and short speeches or texts. The analyses in this study cover students' dialogues and texts. By studying communication in each context, we can better understand students' learning processes. The research question is as follows: What characterizes the communication of seventh-grade students in a real-life context?

By ”real-life context,” we refer to a task or problem pertaining to reality. Student engagement and social action are foundational for mathematics learning and form the basis for experience acquisition (Cobb & Bowers, 1999). Therefore, it is crucial that students experience mathematics as relevant and meaningful. Thus, a context with reference to their everyday lives may support their engagement in communicating mathematics. The learner develops mathematical knowledge by constructing concepts through a reference context (Steinbring, 2005). Here, we investigate student communication that expresses thoughts and ideas about mathematical concepts, using motion as a reference context.

Theoretical framework

Pupils encounter the concept of angles in the fourth grade, where they measure angles and examine the properties of geometric figures in two and three dimensions (Directorate of Education, 2020). The concept of angles can mainly be perceived in three ways: static, dynamic, and angular sector (Devichi & Munier, 2013). The static definition of ”angle” expresses two rays with a common point forming the angle. The dynamic aspect involves rotation as a movement. The third aspect is the area bound by two rays with a common vertex (ibid.). Students should gain experience with various aspects of the concept of angles to develop a rich understanding. Argumentation for choosing conversations as a unit of analysis relates to meaning-making as a social process (Bakhtin, 1984; Ernest, 1994).

Mathematics as a field of knowledge involves abstract concepts that can be mediated through semiotic symbols and signs (Steinbring, 2005). The meanings students construct for mathematical concepts (mental objects) often occur in a social setting, such as the classroom or home. Thus, communication in mathematics involves individual’s thoughts and perceptions about these mental objects (Sfard, 2008), where conversation and text are central communicating units. According to Mortimer and Scott (2003), a conversation has a dialogic character if it includes different perspectives and thoughts. They (ibid.) defined a conversation as authoritative if only one perspective was focused and only one voice was heard. The analysis also involves the concept of the interaction pattern, presenting the order of different utterances concerning initiative (I), response (R), and feedback (F) in a conversation. The interaction pattern of a conversation may indicate different approaches to learning in mathematics classrooms. The teacher taking the initiative and the student responding, followed by the teacher’s evaluation (E), may characterize an example of interaction within the task paradigm. This interaction pattern is of the type IRE and reflect the interaction among students (Manshadi & Lysne, 2013).

Alrø and Skovsmose (2002) considered dialogue as a communicative action involving different speech acts that may indicate the investigative characteristics of an interaction. These are getting in touch, locating, arguing, identifying, thinking aloud, reformulating, challenging, and evaluating. For example, through ”getting in touch,” the student tries to convey their ideas, thoughts, and opinions to get a response. The student can make a statement that expresses their opinion on something and is an invitation for dialogue. Therefore, identifying speech acts provides a better picture of students’ approach to learning mathematics. The speech acts as well as interaction patterns form the framework for analyzing conversations.

Research methods

Participants

All schools in the area could register for Vitenuka (2014) via open enrollment. Vitenuka’s management chose three participating schools: Schools A (with a class of seven students), B (with sixteen students), and C (with only two students).

The teachers were mostly observers but could assist students with practical issues in the classroom. This is because some students might be shy when meeting other students, teachers, and adults with whom they were not acquainted. Two external participants, a researcher and a representative from the Mathematics Center in Finnmark, initiated the activity. The conversation regarding rotational movement based on the pupils' own anatomy occurred on day 1, where Schools A and C participated. The total data material (days 1 and 2 combined) consisted of five hours of video recording. Two desktop video cameras were installed in each corner of the classroom, and a third camera was used to record the students' work. This guaranteed that we could capture moments that the other two cameras could not. Each camera had a wireless transmitter, ensuring that the pupils' conversations could be recorded with good quality. Data analysis consisted of the following three steps. An overall interpretation of the video recordings resulted in preliminary interpretations in the first step. Next, data reduction was performed by choosing interactions with explicit mathematical contents. Finally, the interactions were analyzed in depth using speech acts and interaction patterns. Observer triangulation (Robson, 2002) was conducted by the research group by discussing interpretations.

The activity

The activity in this study is based on a sociocultural perspective of learning and knowledge. Cobb and Bowers (1999) argued that students' learning of mathematics is a process that requires active participation in the classroom's learning environment. The activity mostly involves geometry and was designed to focus on the concepts of rotation and angles. It also invites students to explore movements, participate in discussions, and become involved in a problem-solving task. Regarding mathematical content, this will mostly involve proportional reasoning. Initially, the students were asked to identify rotations based on their own movements. They should also determine the center of rotation of the movement. The activity focused on the dynamic and static aspects of the concept of angles. Physical activities involve rotation in most of the movements we perform, making the body relevant as a representation of the dynamic aspect of the concept of angles.

Results and analysis

Discussing body movement

Students were asked to suggest movements that could express rotation. The teacher who had participated in the conversation encouraged her students to begin the activity:

14	Martin:	What is—what is the point? (thinks aloud)
15	Eva:	That's the whole thing, after all.
16	Ole:	To know the body that way.
17	Teacher:	You might bend your stomach.
18	Ole:	You bend your neck, stomach, and arms.
19	Martin:	Butt, tailbone... The tailbone that is not. It's like—it's the tailbone that is in the middle. (Rotated with one hand to visualize body rotation around the hip.)
20	Ole:	It will not be backward.
21	Teacher:	Can you come up with more suggestions—
22	Martin:	The eyes. You cannot spin your hands like that. (Martin moves his arm and indicates the arm's limit of rotation. The rest of the group moves their hands to test the hand's rotation limit, that is, to know the maximum value of the rotation angle.)
23	Martin:	Can you rotate your arm 360 degrees?
24	Simon:	Yes. (rotates the wrist)

25	Simon:	The ankle.
26	Martin:	Kneel, knee.
27	Ole:	The back.
28	Martin:	The back? Can you?
29	Ole:	(moves upper body from side to side)
30	Eva:	When you bounce, you bend down.
31	Martin:	Bounce? Then you do not rotate. You [are] bouncing.
32	Eva:	Now, you're bending down. (smiles)
33	Martin:	Yes.
34	Eva:	Yes, you also bounce.
35	Martin:	Yes, it is bouncing, not rotated — not rotation.
36	Simon:	Yes, but what about the jaw? (moves his jaw)

Martin thought aloud and wondered what the aim of the conversation (activity) was: ”What's the point of this?” (14). Eva may have believed that the whole conversation involved movement, rotation, etc.: ”That's all it is” (15). Ole argued that this is a way of knowing the body. The teacher intervened in the conversation to motivate the students to respond to her statement: ”You might bend your stomach” (17). Ole located rotational movements when the upper body rotates around the hips (18): ”One bends the neck, abdomen, and arms.” Martin identified the axis of rotation, and he was eager to express his thoughts (19). He attempted to reformulate his utterance to convey his argument more clearly. In primary school mathematics, the term ”rotation” is often illustrated with two-dimensional examples. Examples of geometric tasks in a two-dimensional space may be the rotation of a geometric figure on a sheet. Martin may have used his experience of the rotation concept from primary school mathematics to identify the axis of rotation in three-dimensional motion. He says, ”Butt, and tailbone. It is the tailbone that does not move. It's like—it's the tailbone that's in the middle.” Martin expresses his idea of a rotation (salto) and argues that the imaginary line passing through the tailbone is an axis of rotation and that it is invariant. The teacher encourages the students to discover several suggestions, and she is interrupted by Martin, who suggests that eye movement is a rotational motion. Martin challenges others in the group to rotate the arm 360 degrees (23) about the elbow. He is aware that this rotation has a maximum limit of approximately 180 degrees, but Simon moves his wrist and illustrates 360 degrees of rotation. Lines 22–36 are the students' suggestions for rotational movements based on their anatomy. Eva mentions jumping as a rotational motion, with which Martin disagrees (31).

37	Ole:	It's so easy to roll your tongue when we eat.
38	Martin:	One uses the eyes for . . . (Martin moves his eyes. The teacher asks Simon what he thinks. He listens to the discussions but does not participate.)
39	Martin:	The fingers. (Moves his fingers)
40	Eva:	(Bends her index finger)
41	Martin:	There is still rotation. (All three students bend their fingers.)
42	Eva:	Yes, isn't it? You bend down and jump up.
43	Martin:	It's the bouncing. These are fingers.

During the conversation, the students constantly produced several movements that they identified as rotations. Martin believed that bending fingers is also a rotational movement. When he bends his finger, the center of rotation of the movement becomes visible. Eva perceived Martin's statement as support that bouncing is a rotation (42). Martin's response (39) serves as feedback for Eva. To determine a movement as rotation, students must be able to identify the center of rotation. Eva viewed several rotational motions that enable a jumping movement (bouncing), but she likely could not identify a jump as a rotational motion. Martin could not identify any rotation center for the bouncing motion. This may be why he did not accept bouncing as a rotation and resisted Eva's stance (43). The conversation likely had an IRF pattern with a dynamic that differs from the IRE pattern, where an initiative receives a response and is evaluated. The interaction appeared to involve speech acts such as locating, arguing, identifying, thinking aloud, reformulating, and challenging, all of which may indicate that the interaction had investigative characteristics.

Students' different mathematical voices

Students were given a mathematics assignment in PISA 2012 (Kjærnsli & Olsen, 2013, p. 56; OECD, 2012, p. 74) after working on movement with rotational properties. The task is called the "revolving door" and involves the static and dynamic aspects of the concept of angles. It also requires students to imagine rotational movement (revolving door rotation) at a certain speed. The task had a reference to the students' reality as all the students had previously seen and experienced a revolving door. The task involves a rotating door or port with three sections, where each section has a maximum capacity for two people. In Question 1, the students were asked to calculate the angle between the door leaves. Question 2 states that the revolving door rotates four times per minute, and they must determine how many people can pass through the door in 30 min. The task originally consisted of three questions, where one of the questions in geometry concerned the arc length of the revolving door. This part was omitted to adapt the task to the students' knowledge level, because the PISA tasks are aimed at the tenth grade. The students were informed of this edit and were advised to ignore the information about the diameter of the door in the text. Three of the pupils' solutions to Question 2, which is more demanding than Question 1, are mentioned here. Few students explained their solutions because they provided multiple-choice answers. Martin sat quietly in thought. He chose option D (720 people) but did not write anything down. The researcher (R) asked about his argument:

45	R:	And what is the argument?
46	Martin:	The argument . . . If it goes four times a minute and can hold two in each of these (two people in each sector), then there can be six in one of them there, so within a minute, [it can] have twenty-four people. In addition, 24 times 30 is 720.
47	Martin:	Seven hundred twenty people can walk within half an hour here.
48	R:	Okay.
49	Martin:	I might think a little too big.

The pupils' interpretation of the task implies that they could imagine the projection of an object (the door) from a three-dimensional space to a two-dimensional space. Ole has control over the number of turns and the maximum number of people who can pass through the door in 30 min. Simon's solutions differ slightly from those of Ole and Martin.

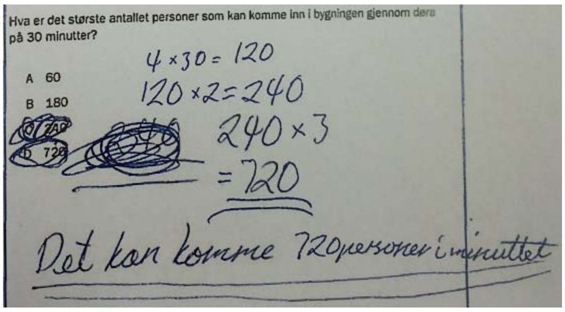

Figure 1. Ole's calculations

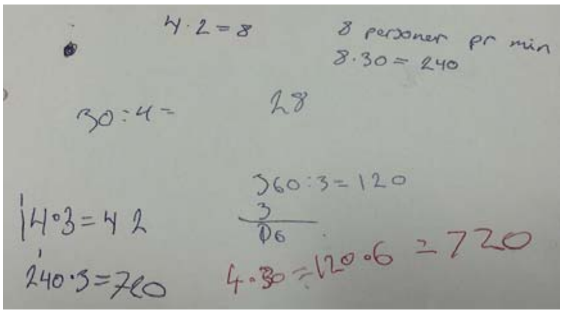

Figure 2. Simon's calculations

Considering the students' answers as their own voices, their expressions of their understanding of the task is reasonable. We may track the indications of proportional reasoning through the students' calculations. This type of reasoning involves multiplicative relationships between different quantities, such as the number of people per turn (rotation), rotation, and time. The students used multiplicative structures and expressed their own strategies in each calculation with different nuances.

Discussion

The students' involvement in the activity is apparent by speech acts that move the conversation forward. These speech acts constructively influence the dynamics of the interaction. This indicates that the conversations had a dialogic (Mortimer & Scott, 2003) character, in which the students' thoughts and ideas were expressed, met, and confronted by one another. This occurred without anyone having an exclusive right to decide what the absolute truth should be. The interaction patterns can be interpreted as IRFRF…, which characterizes the conversation as a dialogue. The absence of evaluations of the form "good, wrong, right . . ." in the conversation can be observed. A combination of various speech acts and interaction patterns may indicate a conversation with investigative characteristics. Students attempted to connect mathematical objects (concepts) and real-life contexts by exploring their anatomical movements. The closeness of the context to the students' real-life experiences may have reinforced this connection. Their statements were open to criticism, and their responses were not negatively loaded. The students supplied several suggestions because they could sense why these movements constituted rotation. The proximity of context may have been important in supporting the students' awareness of a center of rotation that was, in some cases, not physically visible (22, 27, 29, 36). The risk of whether students accept the context's invitation to the learning process is often evident. Lee and Johnston-Wilder (2013) referred to their research involving students as co-researchers to gain knowledge of how schools can improve learning in mathematics. Students had the opportunity to choose their own contexts, which generated a degree of freedom, and their voices became noticeably heard. This also greatly influenced the pupils' involvement and communication in the activities. In the current study, the students used their voices during the problem-solving task (the revolving door task) and expressed their understanding of the problem. Sfard (2008, p. 81) defines thinking as a form of communication: "Thinking is an individualized version of (interpersonal) communicating." Although thinking is an invisible human activity (ibid) for others, one's voice expresses their thoughts and ideas (Dysthe, 1999). The pupils' calculations as an expression for their interpersonal communication can indicate two interesting and important areas in mathematics: proportional reasoning and mathematization of a situation. The development of proportional reasoning is central to students' cognitive development in mathematics (Kastberg, D'Ambrosio & Lynch-Davis, 2012), as proportional reasoning can be traced to several contexts, such as economics, physics (such as the concept of speed), and other sciences. Another important element is the process by which students translate possible connections and structures from a context to mathematical expressions. Using relevant information from a context and then translating it into mathematical expressions can be a demanding process for students. The utterances may indicate that the context influenced the flexibility of the interaction. Sfard (2001, p. 36) considered communication as effective when various interlocutors' statements elicited responses that were consistent with the speaker's expectation. The results suggest that students understood one another during the conversation. They used movements (22, 24, 29, 36, 38, 40) as a statement that supported their verbal expressions. The analysis shows that reformulation as a speech act rarely occurs in conversations. Students' use of body movements may have dampened the need to reformulate their statements. The context's direct reference to the students' reality likely makes their communicative space more flexible and their interaction more efficient.

Conclusion

The students' involvement in the activity reinforced the encounter between their perceptions and thoughts about mathematical concepts. In this case, expressing students' opinions and arguments can be important for constructing a richer understanding of mathematical concepts. Therefore, pupils' voices are an important resource for teachers in the learning process. The choice of a real-life context with reference to the students' reality can have a significant impact on their active participation. The study shows that the interaction displayed investigative characteristics that are reflected by speech acts and the interaction pattern of the conversation. The context likely contributed to the effectiveness of student communication, which, in turn, was beneficial for the visibility of their voices, reflecting their conceptual perceptions. This is the motivation to further study the impact of a variety of contexts on student communication and investigate deep learning processes in elementary school mathematics.

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