

# Security Constrained Unit Commitment and Economic Dispatch by AC Sensitivity Factors

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**Abstract**— The dispatch schedule of the electrical power plant units is the result of the solution to the unit commitment optimisation problem, and it minimises the cost of production considering pre-defined technical limits. The security-constrained unit commitment problem has been defined as including the network constraints in the previously mentioned optimisation problem to obtain a feasible power system solution. The traditional security-constrained unit commitment methods are based on the DC-power flow model, where the network losses and voltage magnitudes are neglected in the problem formulation. This paper proposed a bi-stage security-constrained unit commitment with an economic dispatch (SCUCED) optimisation problem. A merit-order-based zonal day-ahead market problem is solved in the first stage to define a preliminary generation commitment. In the second stage, the SCUCED is solved based on the AC-power flow model and sensitivity factors to embed the full network representation in the optimisation problem. In this paper, the proposed method is illustrated by an application to a modified version of the IEEE 39-bus test system.

**Keywords**—Day-ahead market, security-constrained unit commitment and economic dispatch, IEEE 39-bus system.

## I. INTRODUCTION

One of the most critical problems of power system operation is the secure and economic scheduling of the power production of the generation units over time. This problem is typically referred to as the unit commitment (UC) problem. The current UC are mixed-integer programming problems, and they minimise the cost to supply the forecasted electrical load considering the power plants' technical constraints (e.g., minimum and maximum power, minimum up-/down-time, etc.). The concept of security-constrained UC (SCUC) has been introduced in [1] to obtain a feasible solution from the network perspective, including the network constraints in the problem formulation. Despite the difficulty of the mathematical problem, due to the complexity of the objective function, the number of decision variables, the length of the time horizon, the number of system constraints and operational requirements, it must be solved in a small-time [2]. Also, the higher penetration of renewable energy sources (RES) has increased the difficulty of UC problem, mainly due to the complexity related to the uncertainty and the high variability of RES. However, a current challenge of the modern transmission system operator is to solve the UC problem by using an efficient optimisation formulation that offers the best possible scheduling and secures the electrical power system's reliability. In the scientific literature, many different techniques have been applied to find the solution to the SCUC problem. Those techniques have been improving over time, from early ones based on Lagrangian Relaxation to the current most used ones based on mixed-integer linear programming [3]. Likewise, in the literature, the system network DC-power flow model is a well-known method to solve the SCUC more straightforward than the AC formulation [4]-[6]. The DC model is a linearisation of the entire AC model, in which transmission losses and voltage magnitude are neglected. But the DC model also has drawbacks because its simplification might provide unrealistic results [7].

This paper proposes a bi-stage optimisation problem to develop a SCUC with economic dispatch (SCUCED) optimisation. Initially, a zonal day-ahead market (ZDAM) optimisation problem is solved during the first stage. Then, it considers the interzonal flow bounds and generators' rated power, aiming at minimising the generation production costs. The dispatched power of the generators is exploited in the second stage to solve the SCUCED optimisation problem, in which the goal is to minimise the re-dispatching, operating, and start-up costs considering generators and network constraints. In particular, in this stage, an AC load flow is carried out to evaluate the overall operating condition of the system. The network constraints are included in the optimisation problem utilising linearised sensitivity factors to consider both active and reactive power balance, as well as the network losses. The approach is applied to a modified version of the IEEE 39-bus test system [8]-[9].

## II. PROPOSED METHOD

Fig. 1 shows the framework of the proposed bi-stage optimisation model. In the first stage, the ZDAM is solved by providing the generation bids, the required load and the interzonal flow bounds. It is a merit-order criterium market in which the UC constraints are neglected, and only the unit's rated power is considered. This formulation is based on the Pan European Single DAM, which the cross-border constraints must fulfil [10]. In the second stage, the dispatched power obtained from the ZDAM

is used to develop a SCUCED optimisation problem in order to fulfil generators and network constraints. The main advantage in subdividing the methodology into two stages is represented by the UC and ED re-dispatch involving the AC network constraints in order to define generation scheduling fulfilling the network requirements. In the European framework, these operations are usually developed in the Intraday-Market keeping a zonal detail of the transmission network [11].

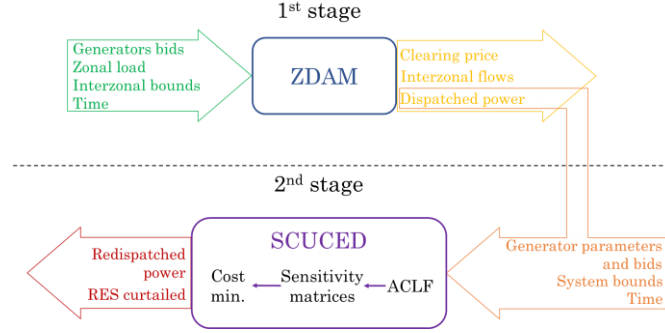


Fig. 1. Flow diagram showing the proposed SCUCED method.

The SCUCED is carried out by solving AC load flow (ACLF) routines, and sensitivity factors are evaluated in order to embed the linearised ACLF constraints in the problem constraints. Then, the re-dispatching cost minimisation is solved considering proper UC constraints and the network ones. In the following two subsections, the two-stage of the proposed method are described.

#### A. 1<sup>st</sup> stage: ZDAM model

The ZDAM optimisation problem is the same as proposed in [8], and in the following, it is briefly explained below. Consider an electrical power system made up of  $N_Z$  market zones with  $N_G$  generators installed among them and  $N_L$  interzonal connections. The ZDAM optimisation problem seeks to minimise the generation costs ( $C_T$ ) at a single time period ( $t_k$ ) over a time window composed of  $N_T$  time steps:

$$\min_{\mathbf{P}_G(t_k)} [C_T(t_k, \mathbf{P}_G)] \quad (1)$$

where the vector of the total active power dispatched ( $\mathbf{P}_G$ ) at the moment  $t_k$  is:

$$\mathbf{P}_G(t_k) = [P_{G_1}(t_k) \quad P_{G_2}(t_k) \quad \dots \quad P_{G_{N_G}}(t_k)]^T \quad (2)$$

and the total cost of generation ( $C_T$ ) at the moment  $t_k$  is:

$$C_T(t_k, \mathbf{P}_G) = \sum_{g=1}^{N_G} \sum_{s=1}^{N_S} C_g^s P_g^s(t_k) \quad (3)$$

where  $P_g^s(t_k)$  is the accepted active power of the  $s$ -th step (considering a set of  $N_S$  stepwise bids) of the  $g$ -th generator, and  $C_g^s$  is the marginal cost of the  $s$ -th bid step of the  $g$ -th generator. The objective function is subject to the following constraints:

$$\sum_{g=1}^{N_G} \sum_{s=1}^{N_S} \alpha_g^z P_g^s(t_k) - \sum_{l=1}^{N_L} \beta_l^z P_l^{tie}(t_k) = P_z^d(t_k) \quad (4)$$

$$\sum_{g=1}^{N_G} \sum_{s=1}^{N_S} P_g^s(t_k) - \sum_{l=1}^{N_L} P_l^{tie}(t_k) = \sum_{z=1}^{N_Z} P_z^d(t_k) \quad (5)$$

$$0 \leq P_g^s(t_k) \leq P_g^{s,max} \quad \forall s \in N_S, \forall g \in N_G \quad (6)$$

$$0 \leq \sum_{s=1}^{N_S} P_g^s(t_k) \leq P_g^{max} \quad \forall g \in N_G \quad (7)$$

$$P_l^{lb} \leq P_l^{tie}(t_k) \leq P_l^{ub} \quad \forall l \in N_L \quad (8)$$

in which (4) denotes the active power zonal balance ( $z = 1, 2, \dots, N_Z$ ), (5) is the total active power balance, (6) the maximum power of the  $s$ -th step of the  $g$ -th generator, (7) the maximum limit of the  $g$ -th generator and, finally, (8) the interzonal flow bounds of the  $l$ -th interconnection.

In particular, the variable  $P_l^{tie}$  represents the active power flow on the  $l$ -th interzonal connection. The parameter  $P_z^d$  is the load required in the  $z$ -th zone. Moreover,  $\alpha_g^z$  is a binary parameter that indicates in which zone the generator is installed, therefore,

it is equal to 1 if the  $g$ -th generator is installed in the  $z$ -th zone and 0 otherwise. The parameter  $\beta_l^z$  corresponds to the direction of the  $l$ -th interzonal power flows with the  $z$ -th zone, and it equals to 1 if it is entering the zone, -1 if it is exiting and 0 otherwise. Finally,  $P_l^{lb}$  and  $P_l^{ub}$  are the lower and the upper bounds of each zonal available transfer capacity (ATC), respectively. Their values are chosen with the  $N-1$  security criterion, as described in [8].

### B. 2<sup>nd</sup> stage: SCUCED method

The objective function of the SCUCED goal is to minimise thermal generator re-dispatching, operating, and start-up costs and the cost of the RES curtailment. The optimisation problem embeds, therefore: (i) minimum up- (MUT) and down-time (MDT), (ii) generators' active and reactive power limits, (iii) maximum branch power flow, and (iv) bus voltage constraints. The thermal unit operating costs are the unit marginal ones to perform in a perfect competition market. In contrast, a penalty fee is imposed on RES to avoid their curtailment (downward re-dispatch). Generators' limits involve the compliance of the minimum and the maximum power of both active and reactive power and the MUT and MDT. Moreover, these generators' parameters, as well as the marginal costs, depending on the power plant's technology and fuel. The ED is based on stepwise bids, in order to define a merit order criterium as well as in ZDAM. The problem is solved considering the generator's active power dispatch and the RES curtailment as control variables.

The proposed SCUCED problem is solved in three consecutive steps, the first is an AC load flow simulation according to the ZDAM results in order to define the initial operating point condition of the system. Downstream, the sensitivity matrices are evaluated in order to relate the dispatched active power variation to the line power flows, the bus voltages, and system power losses ones. The sensitivity factors, in particular, correlate one unit of redispatched active power of each generator to variations in each branch power flow and loss, as well as voltage of each node. As a result, the sum of the products of the redispatched power of the generators by the sensitivity factors of the respective variable and element yields the total variation on each network element (i.e., branch and node). Finally, the optimisation problem is solved exploiting the sensitivity matrix to model the linearised ACLF constraints of the voltages and power flows, as well as the network balance and losses.

## III. MERIT-ORDER CRITERIUM ZDAM RESULTS

In this stage, the authors formulated the problem and solved it as presented in [8]-[9]. The modified IEEE 39-bus test system has an installed capacity of 52% of RES, and several thermal generation units (TGU), both installed among three market zones called Z1, Z2 and Z3. The TGUs have a piecewise marginal price varying according to the technology and the fuel. The system's RES comprises 14 solar power plants (SPPs) and ten wind power plants (WPP) of different sizes, with a total installed capacity of 3,600 MVA. The TGU technologies are combined cycle (CC), combustion turbines (CT) and steam turbines (ST), supplied by Natural Gas (NG), Coal or Oil with a total capacity of 3,300 MVA among ten units. An equivalent 10,000 MVA generator represents the interconnection exchange with the rest of the transmission network; for the sake of simplicity, it will be called "Exchange". In the following, the results will be showed gathered according to technology and fuel.

Table 1 shows the active power limits ( $P_{MIN}$  and  $P_{MAX}$ ), the start-up costs ( $C_{SU}$ ), and the MUT and MDT of the TGUs. All the parameters, except the maximum power, have been obtained considering the available data of [12]-[13]. In particular, they are evaluated concerning each power plant's technology, fuel, and rated power. Therefore, the Exchange is the only generator devoid of proper technical parameters and start-up costs as it represents an equivalent interconnection exchange.

TABLE I. GENERATORS PARAMETERS AND START-UP COSTS.

Generator	$C_{SU}$ [\$]	$P_{MIN}$ [MW]	$P_{MAX}$ [MW]	MUT [h]	MDT [h]
CC NG 01	31703.82	114.75	382.50	2	2
CC NG 02	28181.70	102.00	340.00	2	2
CC NG 03	28181.70	102.00	340.00	2	2
CC NG 04	28181.70	102.00	340.00	2	2
CT NG 01	27843.52	114.75	255.00	1	1
CT Oil 01	9461.38	89.25	297.50	2	2
CT Oil 02	8109.75	76.50	255.00	2	2
Exchange	0.00	0.00	8500.00	0	0
ST Coal 01	39737.8	114.75	255.00	24	48
ST NG 01	20274.21	25.50	255.00	8	12
ST NG 02	20274.21	25.50	255.00	8	12

The ZDAM simulations are carried out during the yearly peak load day, and its hourly profile is shown in Fig. 2. The daily required energy is 72.85 GWh. The resulting dispatched generation is shown in Fig. 3. At 4:00 it occurs the minimum load, the wind production is sufficient to balance the load, with a curtailment of 16 MW, and the energy produced during the day is 13.38 GWh. The solar output subsists between 10:00 and 17:00, with a maximum output of approximately 833 MW at 14:00 and a daily production of 5.004 GWh. Fig. 4 shows the RES penetration percentage of the required load. The RES covers a daily mean of 37% of the total load, above the 32% of the European 2030 target [14]. Additionally, Fig. 3 shows that the ST Coal is the cheapest unit, followed by the Exchange, CC NG, CT NG, ST NG, and CT Oil units, as reported in [8]. In particular, the last is never cleared because the dispatching of RES and more affordable TGUs can supply the hourly load during the time horizon. The interzonal flow bounds are respectively  $\pm 1600$ ,  $\pm 1000$  and  $\pm 1000$  MW for Z1-Z2, Z2-Z3 and Z3-Z1. From 10:00 to 14:00 and from 16:00 to 22:00, Z2-Z3 reaches the lower bound, causing a market splitting of Z3.

#### IV. SCUCED RESULTS ANALYSES

The SCUCED optimisation problem is solved in DIgSILENT PowerFactory environment, taking advantage of the module Unit Commitment and Dispatch Optimisation. The AC load flow simulations are performed setting as reference machine the Exchange and the voltage of the busbar generators as in the original version of the IEEE 39-bus test system [15]. Considering that the desired voltage of the generator connected to bus 36 is 1.0635 pu, in the optimisation, the voltage bounds are set  $\pm 7\%$  of the rated voltage for all the busses. The maximum acceptable branch loading set in the problem, in percentage, is 100%. The marginal generation costs and the line ratings are also provided in [8]. The RES penalty costs for curtailment are set 150 \$/MWh. Considering the MUT and MDT of the generator ST Coal 01 and the time window of simulation, the optimisation is carried out contemporary, whereas the network sensitivity matrices are updated at each time step.

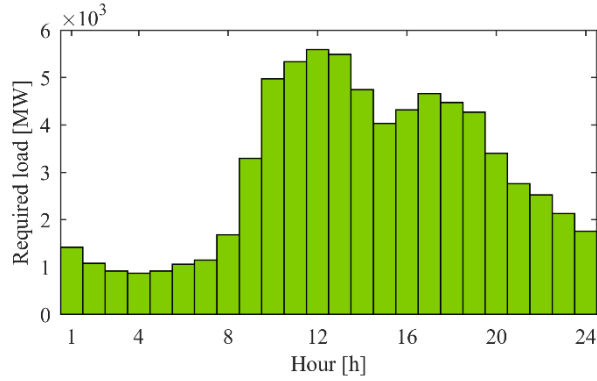


Fig. 2. Hourly load profile of the yearly peak load.

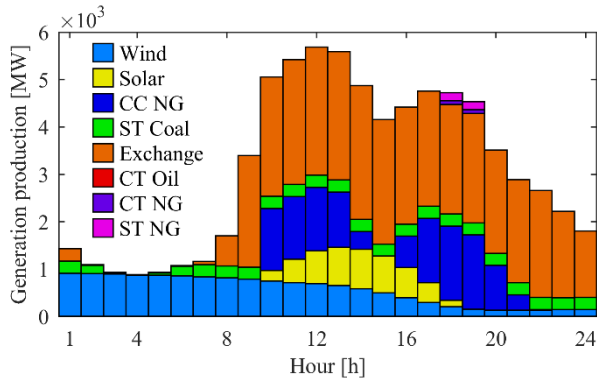


Fig. 3. Dispatched generation after the ZDAM solution.

Fig. 5 shows the net re-dispatched power after the SCUCED solving. Compared with the results of Fig. 3, it can be seen during 3:00-5:00 that the Coal generator is kept active at minimum power for the MUT constraint. During those hours, being a lack of production, only the reference machine downward re-dispatching and wind curtailment can allow the power balance. In the market splitting hours, the Exchange is the most exploited generator for upward movement re-dispatching. It is the second cheaper unit, and due to the  $N-1$  security criterium of the ZDAM boundaries, its dispatching was limited in the previous stage. Therefore, the total branch limits included in the SCUCED allow the increase in Exchange production, reducing the NG dispatched power, which is more expensive during those hours. At 18:00 and 19:00 the CT and ST NG units are dispatched in the ZDAM, but downstream the SCUCED solution both are shut down. The ST NG units have a MUT of 8:00, but both the technologies have a marginal price higher than the Exchange.

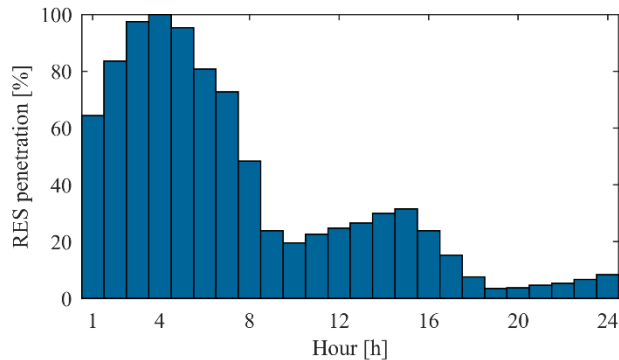


Fig. 4. Hourly RES percentage penetration.

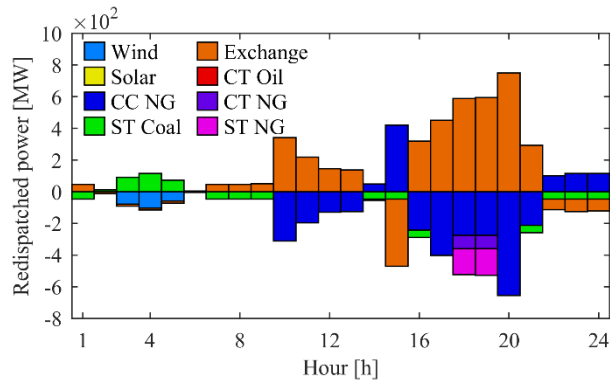


Fig. 5. Hourly net re-dispatched power.

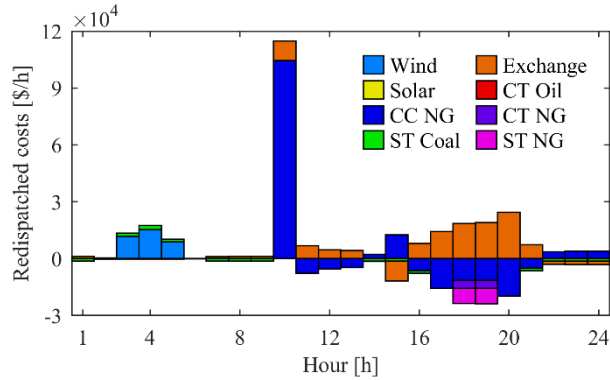


Fig. 6. Hourly total re-dispatching costs.

As it can be seen from Fig. 3, at 10:00, all the NG generators are started-up, and even if they are scheduled in the ZDAM, the software includes the start-up costs in the total dispatching cost reported in Fig. 6. For this reason, at 14:00 and 15:00, the NG is subject to an upper re-dispatch to avoid the start-up costs at 16:00. Moreover, this is a further cause of the CT and ST NG units being shut down. From 22:00 to 24:00, the algorithm prefers to keep one NG generator active rather than turn it off in the remaining hours, even if it is more expensive. This occurs because the optimisation minimises the operating costs, and the CC NG is slightly lower, with 4.02 k\$/h than the ST Coal one, which is 4.10 k\$/h from 22:00 to 24:00. The daily net re-dispatched energy, considering both upward and downward movement, is approximately 10.65 GWh.

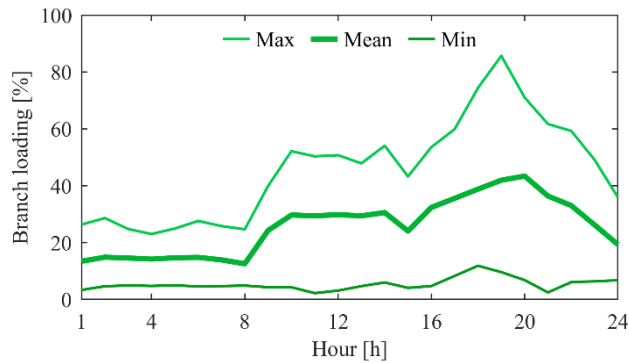


Fig. 7. Hourly maximum, mean and minimum line loadings.

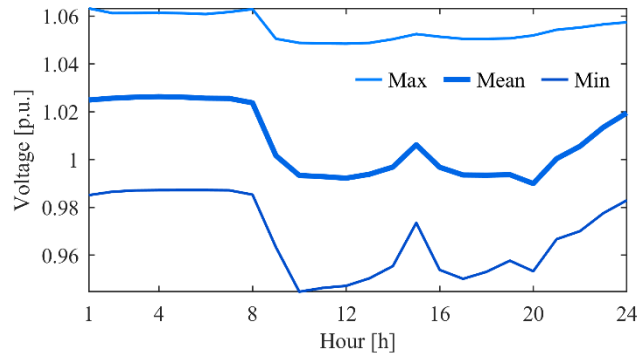


Fig. 8. Hourly maximum, mean and minimum nodal voltages.

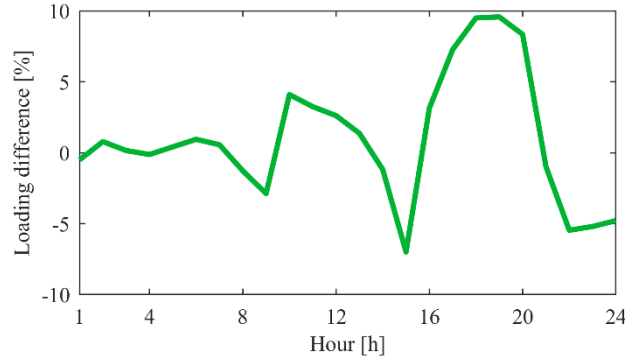


Fig. 9. Mean branch loading difference after and before the SCUCED.

Regarding the total re-dispatched costs reported in Fig. 6, the RES curtailment cost equals 36.08 k\$ for the three hours. At 10:00, the start-up costs of the CC NG units are included in the costs, with a total of 116.2 k\$. From 11:00 to 13:00 and from 17:00 to 19:00, the total revenues are higher than the expenses for the TSO, with a total profit of 13.58 k\$. In the remaining hours, the expenses exceed the revenues with a loss of 164.47 k\$. Fig. 7 and Fig. 8 show, respectively, the maximum, mean and minimum values of the branch loadings and nodal voltages after the SCUCED solution. For the nodal voltage, only the PQ busses are shown in Fig. 8 (i.e., from Bus 1 to Bus 29). Both the results respect the constraint limits set in the optimisation. To compare the results with those provided before the optimisation, in Fig. 9 and Fig. 10, there is a respective difference in the mean values.

As shown in [8]-[9], the line loading and nodal voltages do not exceed their limits before the optimisation as well. After the re-dispatching, the mean hourly branch loading difference varies from -7.0 % to +9.6 %. Considering the results of the re-dispatched power (Fig. 5), the greater loading difference follows the respective increase and the decrease of the Exchange power production. This occurs because the re-dispatched power of more TGUs spread in the system is balanced by one source located at one busbar. Similar to the voltages, the mean difference is neglectable in the first hours; on the contrary, in the remaining hours, the mean voltage varies from  $-1.43 \times 10^{-2}$  pu to  $+0.81 \times 10^{-2}$  pu.

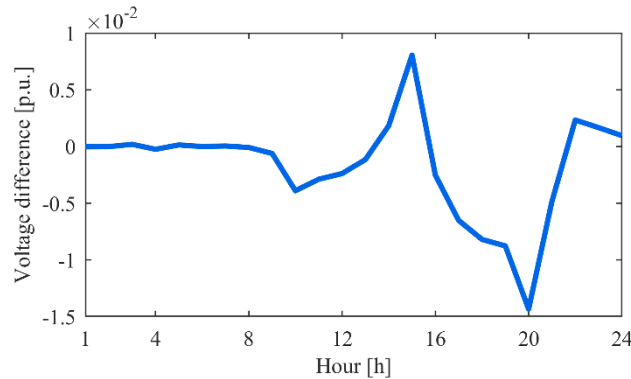


Fig. 10. Mean nodal voltage difference after and before the SCUCED.

Finally, a comparison of the system losses is presented in Fig. 11. Before the optimisation, the minimum and maximum losses were 12.2 MW and 139.8 MW, respectively, whereas after the optimisation, they were 11.9 MW and 176.3 MW. The variation of the active power losses is in line with the mean branch loading variation, causing a loss increase of 177.3 MWh during the day. It is worth to underling that the loss minimisation is beyond the purpose of this work.

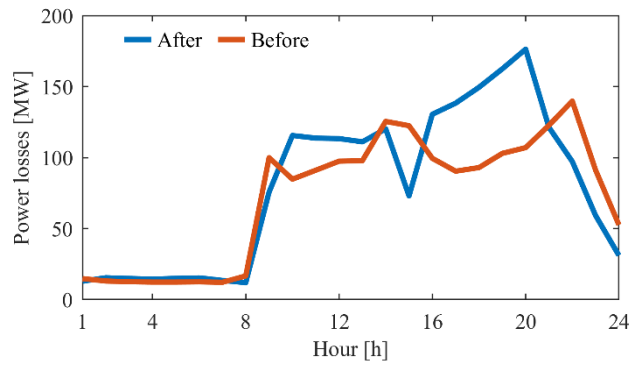


Fig. 11. Line losses after and before the optimisation.

## V. CONCLUSIONS

In this paper, a bi-stage SCUED is proposed. The main advantage of this approach is the simulation of SCUED problems considering linearised sensitivity matrices deriving from AC load flow equations in the optimisation problem. Therefore, power flow and voltage, as well as the UC, constraints are embedded in the proposed method. The method has been applied to a modified version of the IEEE 39-bus test system with 37% of RES penetration during the yearly peak hour day. The results show that the generation is re-dispatched, accomplishing generation and network constraints set in the optimisation problem. The RES has been curtailed only in the hours with a low load required, in which only wind power plants are dispatched in order to satisfy the MUT constraint of the ST Coal TUG. Moreover, the tool minimises the costs in each hour, and in six hours, the revenues are more significant than the expenses. The main drawback of this tool is the addition of start-up costs for the dispatched power scheduled in the previous market; in this work, the ZDAM schedule. This behaviour affects the start-up or shut-down of the involved generators to avoid paying further costs. From the network perspective, the optimal solution does not exceed the system constraints even in the hours when the most energy is re-dispatched. Further works will be developed, including additional UC constraints, extending the time window to one year of simulations, and considering RES and load uncertainties.

## VI. REFERENCES

- [1] H. Ma and M. Shahidehpour, "Transmission constrained unit commitment based on benders decomposition," *Electr. Power Energy Syst.*, vol. 20, no. 4, pp. 287–294, April 1998.
- [2] W. van Ackooij, I. Danti Lopez, A. Frangioni, F. Lacalandra, and M. Tahanan, "Large-scale unit commitment under uncertainty: an updated literature survey," *Ann. Oper. Res.*, vol. 271, no. 1, pp. 11–85, 2018, doi: 10.1007/s10479-018-3003-z.
- [3] C. Yonghong, P. Feng, Q. Feng, X. Alinson, Z. Tongxin, et al. "Security-Constrained Unit Commitment for Electricity Market: Modeling, Solution Methods, and Future Challenges", TechRxiv, Preprint, 2022, doi: 10.36227/techrxiv.19500710.v1.
- [4] D. Villanueva, A. E. Feijóo, and J. L. Pazos, "An analytical method to solve the probabilistic load flow considering load demand correlation using the DC load flow," *Electr. Power Syst. Res.*, vol. 110, pp. 1–8, 2014, doi: 10.1016/j.epr.2014.01.003.
- [5] Q. Ploussard, L. Olmos, and A. Ramos, "A Search Space Reduction Method for Transmission Expansion Planning Using an Iterative Refinement of the DC Load Flow Model," *IEEE Trans. Power Syst.*, vol. 35, no. 1, pp. 152–162, 2020, doi: 10.1109/TPWRS.2019.2930719.
- [6] J. WANG, H. ZHONG, Q. XIA, and C. KANG, "Transmission network expansion planning with embedded constraints of short circuit currents and N-1 security," *J. Mod. Power Syst. Clean Energy*, vol. 3, no. 3, pp. 312–320, 2015, doi: 10.1007/s40565-015-0137-8.
- [7] K. Purchala, L. Meeus, D. Van Dommelen, and R. Belmans, "Usefulness of DC power flow for active power flow analysis," *2005 IEEE Power Eng. Soc. Gen. Meet.*, vol. 1, pp. 454–459, 2005, doi: 10.1109/pes.2005.1489581.
- [8] G. Tricarico, R. Wagle, M. Dicorato, G. Forte, F. Gonzalez-Longatt, J. L. Rueda, "Zonal Day-Ahead Energy Market: A Modified Version of the IEEE 39-bus Test System", submitted to IEEE ISGT Asia 2022.
- [9] G. Tricarico, R. Wagle, M. Dicorato, G. Forte, F. Gonzalez-Longatt, J. L. Rueda, "A Modified Version of the IEEE 39-bus Test System for the Day-Ahead Market", submitted to IEEE ISGT Europe 2022.
- [10] ENTSO-E, "Single Day-Ahead Coupling (SDAC)". Available online: [https://www.entsoe.eu/network\\_codes/cacm/implementation/sdac/#:~:text=SDAC%20is%20an%20initiative%20between,power%20for%20the%20following%20day](https://www.entsoe.eu/network_codes/cacm/implementation/sdac/#:~:text=SDAC%20is%20an%20initiative%20between,power%20for%20the%20following%20day) (accessed on May 15, 2022).
- [11] ENTSO-E, "Single Intraday Coupling (SIDC)". Available online: [https://www.entsoe.eu/network\\_codes/cacm/implementation/sidc/](https://www.entsoe.eu/network_codes/cacm/implementation/sidc/) (accessed on May 15, 2022).
- [12] An Extended IEEE 118-Bus Test System With High Renewable Penetration. Available online: <https://item.bettergrids.org/handle/1001/120> (accessed on May 15, 2022).
- [13] I. Peña, C. B. Martinez-Anido and B. Hodge, "An Extended IEEE 118-Bus Test System With High Renewable Penetration," in *IEEE Transactions on Power Systems*, vol. 33, no. 1, pp. 281–289, Jan. 2018.
- [14] Renewable Energy, Moving Towards a Low Carbon Economy, Eur. Commission, Brussels, Belgium, 2018.
- [15] Anantha Pai, *Energy Function Analysis for Power System Stability*. Springer, 19.