Exploring student explanations: What types can be observed, and how do teachers initiate and respond to them?

OVE GUNNAR DRAGESET

This article presents different types of student explanations that can were observed, and how these were initiated and responded to. The research is based on the practice of five teachers, with all interactions having been analysed and categorized to develop the concepts. First, three distinct types of student explanation were found: explaining actions, explaining reasons, and explaining concepts. Secondly, the teachers' initiations were inspected, by studying the turn before each student explanation. Strong connections were found between the initiation and each type of student explanation. Thirdly, teachers' responses to the students' explanations were inspected, with three main types of response being found to all three types of student explanation: pointing out what to notice, requesting further detail, and confirming and moving on. The main contribution of this article is the conceptualization of students' explanations and the explanation of how these are initiated and responded to.

During classroom conversation, students contribute with different types of interaction. Drageset (2014) suggests that these interactions can be separated into five types: explanations, initiatives, teacher-led responses, unexplained answers, and partial answers. Of these, students' explanations might be of the greatest interest for further exploration. It is, of course, possible to focus on who explains and how frequently they do so, but instead, this article presents the development of concepts describing student explanations, as part of the classroom conversation, related to the following research question: What types of student explanation can be observed, and how are these initiated and responded to?

The types of student explanation have been reported in shorter versions of this paper, as part of an article about student interactions in general and as a conference paper (see Drageset, 2014, 2015). This paper gives a more in-depth and detailed presentation about these types of

Ove Gunnar Drageset
UiT – The Arctic University of Norway

Drageset, O. G. (2021). Exploring student explanations: What types can be observed, and how do teachers initiate and respond to them? *Nordic Studies in Mathematics Education*, 26 (1), 53–72.

student explanation, and new knowledge about how teachers initiate and respond to them.

Explanations in general

According to Dreyfus (1999), mathematics teaching at all levels "include[s] attempts to make learning more cooperative, more conceptual and more connected" (p. 85); a consequence of this is that students are more frequently asked to explain their reasoning. In summing up several correlational studies, Fuchs et al. (1996) state that just giving answers is not associated with learning for the provider or the recipient, while constructing explanations that clarify processes to help classmates arrive at their own solutions results in greater achievements for the provider. This indicates that providing explanations results in more learning than receiving explanations. However, explanations are not only a tool for learning. Reinholz (2016) argues that explanations should be an outcome of interest in their own right, as explanations of different types are fundamental to mathematics. This is evident in different standards and competency models. Niss and Højgaard Jensen (2002) suggest a model in which communication is one of eight competencies. The communication competency is about formulating one's own mathematical thoughts in different ways, and being able to understand and interpret others' mathematical utterances. Communication is also one of the five mathematical process standards of the *Principles and standards for school mathematics* (NCTM, 2000), which states that students should share ideas, clarify their understanding, and give explanations to each other. It is also emphasized that explanations should not only describe procedures but should include mathematical arguments and rationales.

However, what separates explanations from other utterances, and what types have been suggested in mathematics? According to Yackel (2001), explanations are communications made by students "in order to clarify aspects of their mathematical thinking that they think might not be readily apparent to others" (p. 5). Similarly, Balacheff (1988) argues that an explanation is "the discourse of an individual intending to establish for somebody else the validity of a statement" (p. 217). Levenson (2013) also sees explanations as communications that answer a question given explicitly or implicitly, and both Levenson (2013) and Johansson et al. (2014) suggest that explanations can be given to oneself.

Explanations of reasoning and explanations of process

Perry (2000) examined classrooms in the USA, China and Japan, and found differences in both the frequency and the type of mathematical explanation. One main difference was that the explanations in the Asian classrooms involved more complex topics than those in the USA. Also, the explanations in the Asian classrooms were more generalizable across problems, while the explanations given in the American classrooms were more often related to individual problems. Perry (2000) found that all students heard explanations of how to solve individual problems, but not all heard the more generalizable explanations of mathematical principles and functions (like how a procedure works and what a concept means). According to Perry (2000), "if a student can know why a procedure works and when to use it, that student will be better equipped to handle novel problems [...] than a student who does not know these things" (p. 204). With this, Perry (2000) points out that there are two types of explanation, one focused on how a problem is solved and the other on why methods work and when to use them. This distinction is similar to the difference between calculational and conceptual explanations, where calculational explanations describe a procedure to solve a problem while conceptual explanations describe the reasons for the steps (Fuchs et al., 1997). Dreyfus (1999) mentions the related difference between providing chronological accounts of actions carried out and pointing out connections and implications. Cobb et al. (2003) and Bowers and Doerr (2001) also make a similar distinction between an explanation of a process without reason and an explanation of the reasons related to a concept. This also relates to the work of Popper (1934/2002), who argues for a sharp distinction between the context of discovery (how did you find this out?) and the context of justification (how can we decide if this is true?).

The above concepts are summed up in table 1, which illustrates the distinction between the two types of explanation. In the right hand column,

Table 1. *Two types of explanation*

Explaining reasons and concepts	Explaining processes
General across problems	Specific to a problem
Generalizable explanations	Individual problems
Why methods work and when to use them	How a problem is solved
Conceptual explanations	Calculational explanations
Pointing out connections and implications	Chronological accounts of actions carried out
Explanations of reasons related to the concept	Explanations of process without reasons
Context of justification (how can we decide if this is true?)	Context of discovery (how did you find this out?)

there are keywords such as specific, individual, how, calculation, chronological, and process. Together, these concepts describe chronological explanations about how to solve specific problems. These are typically explanations of the steps taken in the actual process, from task to answer, and nothing else.

In the left hand column there are keywords such as general, why, when, concepts, connections, reasons, and justification. Together, these concepts describe explanations that are general and focused on reason, justification, concept, and connections. These are typically explanations of reasons beyond specific examples.

It is easy to think that explaining reasons is superior to explaining processes. Moreover, it is possible to argue that explaining reasons is more complicated than explaining processes. However, what would a lesson look like if no process was explained? Instead, one should look at these as two general types of explanation that serve different purposes.

Concepts such as arguments, justifications, and proofs are also closely related to explanations. Scholars use these concepts differently, and Dreyfus (1999) sums this up by saying that ”for mathematics educators there appears to be a continuum reaching from explanation via argument and justification to proof, and the distinctions between the categories are not sharp” (p. 102). It is clear that most authors treat arguments, justifications, and proofs as being related to explanations of reasons and concepts, while process explanations typically lack these. Figure 1 is a recap and comparison of the different types of explanation described in the literature. On the one hand, explaining processes is related to chronological explanations. On the other hand, explaining reasons and concepts seems to be related to arguments, justifications, or proofs. This does not mean that the similarity goes both ways, as some proofs are not explanatory (Stylianides et al., 2016).

These two types of explanation (process vs reason and concept) should not be seen as fundamentally different, but instead as belonging to each other in a dialectical relationship. This is illustrated in table 1. The relationship between the specific and the general relates to generalization

<table>
<tr><td>Argue</td><td>Justify</td><td>Proof</td><td>Chronological</td></tr>
<tr><td colspan="3">Explain reason and concept</td><td>Explain process</td></tr>
<tr><td colspan="4">Types of explanation</td></tr>
</table>

Figure 1. *Types of explanation*

from one or a few to all, and the discovery of a solution relates to the justification of the same. It is possible to work on one type of explanation at a time, but they might sometimes be looked upon as two sides of a coin.

Methodology

Identifying and initiating explanations

According to Sacks et al. (1974), turns are the most fundamental feature of conversation. People take turns talking, sequentially and one at a time. The different types of student explanation presented above typically come as a result of a question in the turn immediately before. According to Ingram et al. (2016), explanations that describe procedures generally follow from "how" questions, while responses to "why" questions include explanations of both procedures and reasons. This might lead to the idea that one can identify explanations by looking at the linguistic features in the turn just before, in the same way as Bailey et al. (2015) define explanations as turns given in response to questions that include the words "how" or "why", or as turns that include these responses as part of the explanation (such as turns that include "because" and "therefore"). However, Pimm (2014) questions whether linguistic features are necessary or sufficient for a turn to be identified as an explanation, and Leinhardt (2001) claims that explanations often do not have such linguistic markers. Of course, identifying explanations depends on their definition, and by defining explanations as turns including certain words (such as "because" and "therefore") following turns that also include certain words (such as "how" and "why"), one makes identification both precise and easy. However, this would not be meaningful if the consequence was that one overlooked other turns that have a similar function in the discourse. For example, Ingram et al. (2019) illustrate how a teacher can generate the need for student explanations without explicitly asking for them.

Conversation analysis

However, looking at single interactions, or turns, has a very limited scope. Even though people take turns in speaking, sequentially and one at a time, it is not possible to characterize a conversation as a series of individual actions. Instead, conversations are social practices in which each turn is thoroughly dependent on previous turns, and individual turns cannot be understood in isolation from each other (Linell, 1998). This means that categories describing different types of single turn are insufficient if one wants to study the discourse. Instead, one needs to study how different types of turn affect one another. Also, when responding, one or more responses are usually preferred to others. Sidnell (2010) exemplifies this by saying that the preferred response to a dinner invitation is to accept. If the invitation is accepted, there is no need for an explanation, but if it is rejected this then requires an accompanying explanation. Linell (1998) explains a similar concept called relevance, stating that some responses are more relevant than others. This means that there might be more relevant responses to specific types of turn that do not require an explanation, while other responses do. The concept of appropriation (Newman, 1990) highlights the need to look at both sequences and single interventions. Appropriation describes the process of teachers' interactive support for students, where feedback given on students' work helps students to learn the overall structure and purpose of the activities assigned to them. One effect of such appropriation might just be the development of relevant or preferred responses to certain types of turn (for example, to specific questions or answers). This means that there is a need to develop concepts that describe single interventions in order to study how they act together on a turn-by-turn basis. It might then become possible to obtain a better understanding of processes such as appropriation.

Using concepts from different frameworks enables us to identify specific types of turn and see how they relate to one another during a discourse. This article presents different types of student explanation that were identified, and which types of turns promoted each type of explanation and how teachers responded to them. In this way, short series of turns related to student explanations were investigated. By doing this, it was the ambition of this research to understand more of the work of teaching explanations.

Data collection and analysis

This article is based on a study of five teachers' practices. For each teacher, all mathematics teaching during one week was videotaped from the start of the topic of fractions: this amounted to four or five lessons of 45 minutes for each teacher. The camera followed the teacher, and a microphone attached to the teacher managed to catch everything the teacher said and almost everything that was said to the teacher or the entire class. All teachers, students and parents were informed about the project, and according to ethical guidelines in Norway gave free and informed consent, save for a few students who did not consent and were given the same lessons without being filmed. The filming was done by the author, who was positioned in a corner at the back of the classrooms. To avoid disturbing the students, the author did not move from the camera, and very rarely did any student turn around to look at the camera or try to make contact.

The five teachers were selected on the basis of a test and questionnaire, with the deliberate aim of selecting teachers with different levels of knowledge and different types of beliefs related to how students learn mathematics (mainly along an axis from emphasis on procedures to emphasis on concepts). At the same time, all five met the criterion that they were teaching students aged between 11 and 13. The teachers all had the typical Norwegian four-year generalist teacher education, which includes some education in mathematics, and they were all experienced, with between ten and thirty years of teaching practice each. Each classroom consisted of approximately 20 students. Some of the teachers were only four months into their first year with the class, while others had been teaching the same class for two or three years.

After the filming, the author transcribed all the videotapes. The analysis was a conversation analysis based on the transcriptions, in three phases. In the first phase, all student interactions were marked, and similar looking interactions were grouped together in a process with constant comparison and several cycles often described as a grounded approach, resulting in a gradual arrival of the five types of student interaction reported in Drageset (2014); Student explanations, Student initiatives, Partial answers, Teacher-led responses, Unexplained answers. Then, focusing on the student explanations, these were grouped into different types of explanation in a similar process. Quite soon, it emerged that certain keywords, such as ”why”, ”what”, ”how” and ”meaning”, could be used to group the explanations. However, other words or formulations could mean similar things, such as requests for reasons being grouped with questions asking why. Also, the keyword could sometimes be confusing: for example, there is a difference between ”how do you know” and ”how did you find out”, with the first set being grouped with ”why” and ”reason” and the latter being more about procedures and being grouped with questions asking ”what” and ”how”. In this way, the groups were gradually developed into distinct categories, using constant comparison and regrouping. This also illustrates that the development of the categories was not limited to using linguistic markers but instead characterized each turn according to its function in the discourse. While the focus was on single turns, they were characterized based on their role in the dialogue and not as isolated turns.

In the second phase, all teacher turns immediately before the student explanations were grouped and categorized in a similar process, and in the third phase, all responses to student explanations (the turns immediately after each student explanation) were grouped and categorized in the same way. As described above for the first phase, this was not a linear process, but instead used constant comparison and regrouping to develop each category further until all relevant data (all student explanations, and all turns immediately before and after each) were coded. It is worth noting that almost all the turns immediately before and after student explanations were teacher turns, in a typical IRE (Initiation–Response–Evaluation) pattern (Cazden, 1988; Mehan, 1979).

Findings

Three types of student explanation

The most frequent type of explanation found is illustrated in this excerpt (*explaining an action*).

Student: Then he gives one fourth of the remainder to his sister.

Teacher: Okay, what do you have to do now then?

Student: Then I have to take one fourth of one hundred which is twenty-five because twenty-five multiplied by four are ... [impossible to hear]. And then ... one hundred minus twenty-five, that is seventy-five.

The teacher asks what the student needs to do to find the answer, and then the student tries to explain what needs to be done, step by step. The first step is to find one fourth, and then the student needs to deduct this from one hundred. Such *explanations of action* were found quite frequently in all five classrooms. They are typically related to explaining how or what and to a standard method. Even though there is some variation between explaining standard methods and non-standard solutions, and some variation between explaining what has been done and what needs to be done, all these explanations naturally belong to the idea of explaining one's actions.

The following excerpt illustrates another type of explanation (*explaining a reason*).

Student: One sixth of eighteen equals three.

Teacher: Why?

Student: Because one ... three times six are eighteen.

Here, the student tries to explain why he knows that one sixth of eighteen equals three. Arguably, the explanation is incomplete, but it is also on the right track. *Explaining a reason* was common but not frequent, and the variations between the classrooms were noteworthy (see figure 3 for details). Within the category of explaining reasons, there is also variation, with more or less complete and more or less mathematically-founded explanations; but, together, they are all about explaining why an answer or method is correct. The difference between *explaining an action* and *explaining a reason* is similar to the distinction that Perry (2000) draws between explanations of how problems are solved and explanations of why methods work. *Explaining a reason* is similar to what Yackel (2001) calls justification and involves both arguments and justifications; Dreyfus (1999) ranks these higher than chronological explanations. A proof would be a formal mathematical way of explaining a reason. This means that while *explaining a reason*, students argue in order to justify, and might even give a proof. While Dreyfus (1999) differentiates between explaining (chronologically), arguing, justifying, and proving, he also talks of this as a continuum. In the data studied here, there is a clear distinction between explaining chronologically (*explaining actions*) on the one hand, and giving arguments and justifications (*explaining reasons*) on the other.

Even though the distinction between *explaining an action* and *explaining a reason* seems clear as presented above, there are, of course, borderline issues. For example, an explanation could include both an explanation of the steps to take and some general principles.

While the literature on explanations related to what is explained mainly finds only two distinct types – one being explaining a method or action, and one being explaining a reason – a third type emerges from these data. This is illustrated by an excerpt in which a student tries to explain what equivalent fractions means (*explaining a concept*).

Stud. A: It is fractions with the same value.

Teacher: Fractions with the same value.

Stud. B: If you have one half, that is a half, and then you have for example four eighths, and that is exactly the same only that the numerator is ....

Teacher: Yes. That the whole is divided into more and smaller pieces than in the first one. Yes.

Stud. C: Yes, that it is ... it is ... it is two different fractions that have the same value.

These three students all try to explain equivalent fractions, first with a statement that the fractions have the same value, then by an example, and then by saying precisely that it is two different fractions with the same value. All three students' explanations are about explaining the concept of equivalent fractions. Such *explanations of concept* emerged as a distinct type of explanation, different from *explanations of action* and *explanations of reason*. As with *explanations of reason*, *explanations* *of concept* often were incomplete and partial. *Explaining a concept* is itself distinct from *explaining an action*, as no action is explained, and from *explaining a reason*, as no solution or method is argued for.

Overall, in these five practices, one in eight student interactions was an explanation. More than half of those explanations were *explaining an action* (*what* and *how*), while *explaining a reason* (*why*) and *explaining a concept* (*meaning*) made up just over and under a quarter, respectively. Typically, one would find linguistic markers (”how”, ”what”, ”why”, ”meaning” when requesting explanations, and ”because” during the explanations), but this was not always the case (as described in the methods section).

As the literature review illustrates, scholars often define explanations using a dichotomy between two fundamentally different types of explanation, or a dialectical relationship between the two. One of these types focuses on how to calculate, offering a chronological account of actions without reasons; the other focuses on the reasons why, on connections, justifications, proofs, arguments, implications, and concepts. The review also illustrates that this second type of explanation is more diverse than the first. This diversity is illustrated by the findings above, where the data suggest a division into three main types of explanation, where concepts are separated from the second type. But would such a division be meaningful?

There are clear relationships between reasons (arguments, justifications, proofs) and concepts, and Fuchs et al. (1997) state that an explanation is conceptual if it describes the reasons for the steps of a solution. Concepts such as a square are defined by specific characteristics. These characteristics are often concepts themselves. Arguably, concepts are the basis on which reasoning rests. In formal mathematical proofs, one starts with definitions of concepts and uses these as tools in the reasoning. Using their characteristics, one could argue that a rhombus is always a parallelogram while the reverse is not always true. In this way, concepts are explicitly related to the process of reasoning. Consequently, working with explanations and the understanding of concepts per se can be seen as foundational work for reasoning. Arguably, an essential part of this would be to connect the concepts, as reasoning often involves the use of several concepts.

At the same time, one can explain a concept without reasoning – for example, by using a definition or illustration. It is also possible to explain the reason why half of 6/7 is 3/7 without applying the concept of fractions explicitly. One could explain that we have six pieces of a cake that was divided into seven equal pieces, and that sharing these pieces between two of us would give us three of the seven pieces each. It is clear that an understanding of the concept of fractions is the foundation of this solution, but it is not the concept that is explained.

While there is a clear connection between reasons and concepts, there is also a principal difference that arises from these data. The act of explaining a reason relates to a solution, a method or an action, while the act of explaining a concept does not. Instead, explaining concepts is about meanings, such as what ”equivalence” means and what ”denominator” means.

This means that separating explanations of concepts per se from explanations of reasons can give an insight into how classrooms work with the foundations needed for explaining reasons. As a result of this, a new figure is suggested in which explanations of concepts, reasons, and actions are related to the concepts from the literature review (figure 2). No separation was found in these data between the three types of explanations of reasons: proving, justifying, and arguing.

per se	Argue	Justify	Proof	Chronological
Concept	Explain reason and concept			Action (explain process)
Types of explanation				

Figure 2. *Types of student explanation, adjusted from figure 1 according to findings*

For illustration, the numbers of each type of student explanation in each classroom are included in figure 3 (as percentages of all student interactions). This illustrates how the five classrooms varied in the number of student explanations in total, with classroom E having more than three times as many explanations as classroom A. It also illustrates the variation in the amount of each type of explanation.

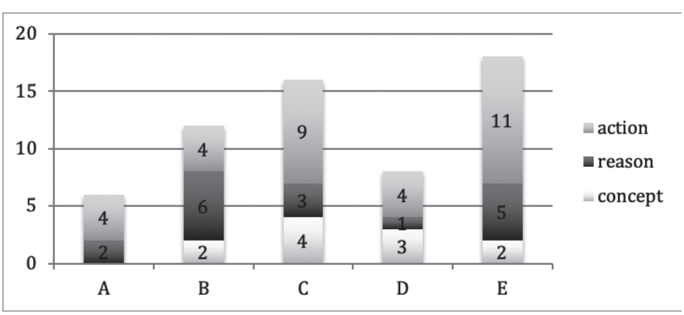

Figure 3. *The part of each type of student explanation in each classroom*
Note. The *y*-axis shows the percentage of all student interactions. The *x*-axis shows each classroom.

What initiated these explanations?

Investigating the turns immediately before each type of explanation resulted in quite clear findings. *Explaining an action* typically came in response to a teacher asking the student to relate how a solution was found, what was done to find the solution, or what needed to be done. The linguistic markers of ”how” and ”what” were found in almost all turns immediately prior to explaining an action. This seemed to be used both as a control of students' understanding and as a way to make the details explicit for others.

The initiation of *explaining a reason* had the most explicit linguistic marker, as such explanations almost always came as a response to a teacher asking why, and only a few times in response to equivalent expressions (can you explain the reason, what is the reason).

Explaining a concept was typically also closely related to a linguistic marker, as these explanations typically came in response to a teacher asking what a specific concept (like ”denominator”, ”numerator”, or ”equivalent fractions”) meant.

Even though these three categories were initiated in unusually distinct ways, there were some deviations. The most frequent of these was when the teacher asked why, or requested a reason in some other way, and instead got an explanation of the steps taken to arrive at the answer (explaining an action). As this was also sometimes accepted, it might indicate that this was part of the classroom's socio-mathematical norms, or that these norms did not explicitly or always differentiate between explaining actions and explaining reasons.

These distinct differences in how the three types of explanation were initiated also illustrate how strongly a turn (explanation) is dependent on the previous turn (initiation). This confirms the fundamental idea stated by Linell (1998), that each turn is thoroughly dependent on previous turns, and that individual turns cannot be understood in isolation from one another. It also gives a strong illustration of what it looks like when some responses are more relevant than others (Linell, 1998) or are preferred to others (Sidnell, 2010).

Teacher responses to student explanations

The teachers responded to the students' explanations in many different ways, but only a few responses were frequent. There were no noteworthy differences between the responses to the different types of explanation. This is somewhat surprising, given the strong evidence of turns depending on the type of explanation (Linell, 1998; Sidnell, 2010). As a consequence, this section will present the way in which the teachers responded to explanations in general, not to the three types of student explanation (*action*, *reason*, *concept*).

The most frequent type of response is illustrated by this excerpt from a discussion about fractions equal to one half. The teacher asks the students to find the denominator when the numerator is 34, and the fraction has to be equal to one half (*pointing out to notice*).

Student: Sixty-eight.

Teacher: Bravo. Sixty-eight [writes the fraction on the blackboard]. Because … what was the reason for this?

Student: Because three plus three is six and four plus four is eight.

Teacher: Yes. Double. Yes. Double the denominator related to the numerator.

The student explains the reason in a somewhat algorithmic way, in reality explaining the action of doubling. The teacher responds to this by pointing out and clarifying the general idea before the process continues with similar tasks, using the idea pointed out by the teacher. A similar response came in this excerpt (*pointing out to notice*).

Teacher: Three tenths and twenty-nine hundredths. Can you manage that one? Which one is the largest?

Stud. 1: Three tenths.

Teacher: You think that it is three tenths. How did you manage … It is entirely correct, but how did you think, then? How did you manage to solve it?

[Student 1 does not answer the question, so the irrelevant responses are omitted]

Stud. 2: Because twenty-nine hundredths become twenty-nine parts of a hundred, while thirty, no, three tenths becomes thirty hundredths.

Teacher: Precisely. Three tenths are the same as thirty hundredths, and that is larger than twenty-nine hundredths.

Here, the student explains the action needed to solve the task. In the last turn of the excerpt, the teacher repeats the student's explanation but also changes it a little, to emphasize that three tenths are the same as thirty hundredths and that is the larger amount.

The two excerpts above illustrate the response of *pointing out what to notice*: to tell the students what to notice, and sometimes reformulating the explanation to make it more general. *Pointing out what to notice* came in different forms, sometimes with only some words or parts of the explanation being pointed out, and at other times with the explanation being adapted. Within this variation one can see that, in this type of response, the teacher builds on the explanation in order to help other students understand or catch on to the idea. This variation also illustrates how the teacher establishes socio-mathematical norms, deliberately or not, by repeating acceptable explanations and adjusting explanations that are not complete, in an appropriation process. Such *pointing out to notice* was the dominant response to all three types of explanation in all five practices.

At other times, the teacher's response to a student's explanation looked like this (*confirm and move on*).

Student: One sixth of eighteen is three.

Teacher: Mmm [confirming]. What is three sixths of eighteen?

Student: Um ... I don't know.

Teacher: But if one sixth is three...

[Other students' comments omitted]

Student: Nine.

Teacher: Yes, but why?

Student: Because it becomes more. Three, six, nine.

Teacher: Three, six, nine, yes. One sixth of thirty?

Midway through this excerpt, the student arrives at the answer of nine. When the teacher asks why, the student responds by counting three steps of three (which is more of an explanation of an action). The teacher repeats the exact words of the student, accepts it and goes on. This illustrates the response of *confirm and move on*: that teachers sometimes just accept and confirm an explanation, with or without repeating it, and go on.

The difference between *point out to notice* and *confirm and move on* is at the core of orchestrating – when to go into detail and when to move on. While pointing out, emphasizing, and clarification are significant teacher actions in order to help other students understand, it also seems evident that a teacher cannot go into detail about every explanation. Then there would be little progress. Also, some student explanations might be assessed as sufficient, and then the teacher probably sees no need for an intervention.

At other times, the teacher could reply like this (*request for further detail*).

Teacher: What does it mean to find equivalent fractions? Anyone that can say something about it? What do you do then?

Student: There are many different fractions that have different denominators, but ... means the same anyway.

Teacher: Different denominators but means the same anyway. How would you clarify that?

The student's *explanation of the concept* of equivalent fractions is clearly insufficient, and the teacher responds by requesting further detail. This is also at the core of understanding fractions: how can they be equal and different at the same time? Such *requests for further details* were about addressing the critical details, potentially both to check the particular student's understanding and to make the details accessible to other students. These responses often asked for details about how the students arrived at an answer.

The dominant type of teacher response to student explanations was to *point out what to notice*. This could have at least two types of impact, one as part of the process of establishing socio-mathematical norms, and the other as part of helping both the actual student and the others to understand what is the most important thing to remember from this explanation. In order to *point out what to notice*, the teachers sometimes repeated students' explanations accurately, but more often altered them to make their point clearer.

Two other types of response occurred regularly. One was to *confirm and move on*, the other to *request further details*. Requesting further details signals that it might be necessary for the teacher to gain insight into the student's thinking and knowledge, and there is the potential to share details of the thinking that might help other students to understand.

Discussion

This article reports from at study aimed to explore students' explanations in the classroom and the way in which teachers initiate and respond to them. The main contribution of the article is the conceptualization of student explanations and of how these were initiated and responded to. First, three types of student explanation were developed from the data; *explaining an action*, *explaining a reason*, and *explaining a concept*. The teachers' initiation of, and responses to, each type of explanation were then inspected.

The three distinct types of student explanation refer to different types of mathematical work. *Explaining an action* is about sharing the way a solution was found sequentially, and such explanations are essential in order to help teachers to assess, and fellow students to follow, the line of thought. *Explaining an action* might be quite procedural, using standard methods or rules and explaining each step in the particular case. *Explaining a reason* is distinctly different, as it goes into the reason why a rule or a method is a mathematically justified choice in this case, or why an answer is correct. *Explaining a reason* is a type of mathematical work that goes to the core of mathematical understanding. While *explaining an action* and *explaining a reason* are typically related to solving tasks, *explaining a concept* is not. It is about explaining a concept per se. *Explaining a reason* is arguably closely related to conceptual understanding, but it might still be beneficial to study explanations of reasons and explanations of concepts as separate types. Explaining a concept per se can be seen as establishing the foundation for explaining a reason, and consequently of value in itself.

The study of how teachers initiated the three types of explanation gave rise to no surprises. *Explaining an action* came mainly in response to questions about how and what, *explaining a reason* was almost always in response to questions about why, and explaining a concept came as a response to a request to explain the meaning of a concept. This illustrates how strongly one turn affects the next turn, and is in line with established knowledge in conversation analysis (Linell, 1998). It is therefore surprising that no such pattern was found in the teachers' responses to the different types of explanation. Three main types of response were found, and these were the dominant teacher responses to all three types of student explanation. The most frequent type of response was to *point out what to notice*, and sometimes to reformulate the explanation to make it more general. A second regularly observed response was to *request further details*. By doing this, the teacher typically asked for an explanation of how the student arrived at the given answer or suggestion, or of what was done. This is essentially the same as initiating an explanation of an action. A third regularly observed response was to *confirm and move on*. This illustrated that there is not always a need or wish to point out, emphasize, or further detail all student explanations.

The connection to linguistic markers is also interesting. In these practices, it was almost always possible to identify initiations (requests) for student explanations by looking for linguistic markers ("what", "how", "why", "meaning"). This means that the teachers asked direct questions, and did not generate the need for student explanations without questions in the way Ingram et al. (2019) describe as an alternative method. Also, each linguistic marker led to a particular type of explanation, with few exceptions. However, while the student explanations frequently included one linguistic marker ("because"), this occurrence was far from being as consistent as in the initiations. Looking at the turn immediately after a student explanation, no consistent linguistic markers were found. Further research is needed to understand how practices that create the need for students' explanations are different from the practices of always asking explicitly, which is possibly related to how this affects student participation.

All three types of response can be seen as part of the teacher's work of establishing socio-mathematical norms, consciously or not. By pointing out particular explanations (or emphasizing or reformulating parts of them), by requesting clarification and further details, and by confirming and moving on, a teacher signals what is acceptable and what is not. Over time, and during an appropriation process, socio-mathematical norms might be formed. However, one would expect to find differences in how this works, possibly related to how consistent the teacher is, and this needs further research.

Why are the students so clearly affected by the prior turn when giving explanations, while the teacher seems not to be affected when responding to these explanations? As always, there might be several reasons. Could it be that the types of response developed are general types, and that one needs to go deeper to observe the turn-by-turn dependency? Alternatively, might the teachers have agendas that affect their responses more deeply than the different types of student explanation? There is a need to study whether the turn-by-turn dependency observed in a general conversation is also valid for a conversation between a teacher and student, which might be seen as a conversation between an authority in the field and a less knowledgeable person, or an authority and a subordinate. The authority might then not be as strongly affected by the previous turn as the subordinate. Alternatively, could it be a characteristic of teaching that the (teacher) agenda affects responses more strongly than the content (of the student turn)? These are questions that it would be interesting to study further.

Conclusion

There are three main contributions from this study. The first contribution is the concepts developed to distinguish between the different types of student explanation. The findings from this article suggest that it is meaningful to divide student explanations into three types (concept, reason, action, see figure 2) instead of just two types (reason and concept as one, and action as the other, see figure 1). The second contribution is the different types of teacher initiation and response to the student explanations. The explanations were typically initiated by asking what and how (actions), why (reason) and what something meant (concept), while the responses to the student explanations were mainly to *point out to notice*, *confirm and move on*, and *request further detail*. The third contribution is to note the way in which the teacher initiations followed easily observable linguistic markers, while the teacher responses did not. The lack of difference in teacher responses to different types of student explanation is a clear deviation from the rules of normal conversation, and might indicate that a teacher–student conversation is something that is really different from other types of conversation. Altogether, the concepts developed to describe the three types of student explanation and how teachers initiate and respond to these give us tools to study classroom conversations on a more detailed level than we were previously able to do.

References

Bailey, A. L., Blackstock-Bernstein, A. & Heritage, M. (2015). At the intersection of mathematics and language: examining mathematical strategies and explanations by grade and English learner status. *The Journal of Mathematical Behavior*, 40, 6–28.

Balacheff, N. (1988). *A study of students' proving processes at the junior high school level*. Paper presented at the Second UCSMP international conference on mathematics education.

Bowers, J. & Doerr, H. M. (2001). An analysis of prospective teachers' dual roles in understanding the mathematics of change: eliciting growth with technology. *Journal of Mathematics Teacher Education*, 4 (2), 115–137. doi: 10.1023/A:1011488100551

Cazden, C. B. (1988). *Classroom discourse: the language of teaching and learning*. Heinemann.

Cobb, P., McClain, K. & Gravemeijer, K. (2003). Learning about statistical covariation. *Cognition and Instruction*, 21 (1), 1–78.

Drageset, O. G. (2014). How students explain and teachers respond. In J. Anderson, M., Cavanaugh & A. Prescott (Eds.), *Curriculum in focus: research guided practice. Proceedings of the 37th annual conference of MERGA* (pp. 191–198). MERGA.

Drageset, O. G. (2015). Different types of student comments in the mathematics classroom. *Mathematical Behavior*, 38 (June 2015), 29–40. doi: org/10.1016/j.jmathb.2015.01.003

Dreyfus, T. (1999). Why Johnny can't prove. *Educational Studies in Mathematics*, 38 (1), 85–109. doi: 10.1023/A:1003660018579

Fuchs, L. S., Fuchs, D., Hamlett, C. L., Phillips, N. B., Karns, K. & Dutka, S. (1997). Enhancing students' helping behavior during peer-mediated instruction with conceptual mathematical explanations. *The Elementary School Journal*, 97 (3), 223–249.

Fuchs, L. S., Fuchs, D., Karns, K., Hamlett, C. L., Dutka, S. & Katzaroff, M. (1996). The relation between student ability and the quality and effectiveness of explanations. *American Educational Research Journal*, 33 (3), 631–664. doi: 10.3102/00028312033003631

Ingram, J., Andrews, N. & Pitt, A. (2016). Patterns of interaction that encourage student explanations in mathematics lessons. In G. Adams (Ed.), *Proceedings of the British Society for Research into Learning Mathematics*, 36 (1), 37–41.

Ingram, J, Andrews, N. & Pitt, A. (2019). When students offer explanations without the teacher explicitly asking them to. *Educational Studies in Mathematics*, 101, 51–66.

Johansson, M., Lange, T., Meaney, T., Riesbeck, E. & Wernberg, A. (2014). Young children's multimodal mathematical explanations. *ZDM*, 46 (6), 895–909. doi: 10.1007/s11858-014-0614-y

Leinhardt, G. (2001). Instructional explanations: a commonplace for teaching and location for contrast. In V. Richardson (Ed.), *Handbook of research on teaching* (4th ed., pp. 333–357). American Educational Research Association.

Levenson, E. (2013). Exploring one student's explanations at different ages: the case of Sharon. *Educational Studies in Mathematics*, 83, 181–203.

Linell, P. (1998). *Approaching dialogue. Talk, interaction and contexts in dialogical perspectives*. John Benjamins.

Mehan, H. (1979). *Learning lessons: social organization in the classroom*. Harvard University Press.

NCTM (2000). *Principles and standards for school mathematics*. NCTM.

Newman, D. (1990). Cognition, computing, and cooperation. In S. P. Robertson, W. Zachary & J. B. Black (Eds.), *Cognition, computing, and cooperation* (pp. 84–94). Ablex Publishing.

Niss, M. & Højgaard Jensen, T. (2002). *Kompetencer og matematiklæring: ideer og inspiration til udvikling af matematikundervisning i Danmark*. Undervisningsministeriet.

Perry, M. (2000). Explanations of mathematical concepts in Japanese, Chinese, and US first- and fifth-grade classrooms. *Cognition and Instruction*, 18 (2), 181–207.

Pimm, D. (2014). Authority, explanation, contention and register: language data and the surface search for essence. *ZDM*, 46 (6), 967–976. doi: 10.1007/s11858-014-0633-8

Popper, K. (2002). *The logic of scientific discovery*. Routledge Classics. (Original work published in German, 1934; first English translation 1959).

Reinholz, D. (2016). Improving calculus explanations through peer review. *Journal of Mathematical Behavior*, 44, 34–49.

Sacks, H., Schegloff, E. A. & Jefferson, G. (1974). A simplest systematics for the organization of turn-taking for conversation. *Language*, 50 (4), 696–735.

Sidnell, J. (2010). *Conversation analysis. An introduction*. Wiley-Blackwell.

Stylianides, G., Sandefur, J. & Watson, A. (2016). Conditions for proving by mathematical induction to be explanatory. *The Journal of Mathematical Behavior*, 43, 20–34.

Yackel, E. (2001). Explanation, justification, and argumentation in mathematics classrooms. In M. van den Heuvel-Panhuizen (Ed.), *Proceedings of PME 25* (pp. 1–9). Utrecht University.

Ove Gunnar Drageset

Ove Gunnar Drageset is a professor in mathematics education and assistant head of department for teacher education and pedagogy at UIT The Arctic University of Norway. His research interest is focused on classroom communication in general and particularly related to conceptualizing classroom interactions. Other interests include mathematical knowledge for teaching, beliefs related to mathematics teaching and learning, using drama to develop student-centered classroom communication, and developing research as an integral part of teacher education.

ove.gunnar.drageset@uit.no