



Article Gifted Students' Actualization of a Rich Task's Mathematical Potential When Working in Small Groups

Anita Movik Simensen * and Mirjam Harkestad Olsen

Department of Education, Faculty of Humanities, Social Sciences and Education, UiT The Arctic University of Norway, 9037 Tromsø, Norway; mirjam.h.olsen@uit.no

* Correspondence: anita.m.simensen@uit.no

Abstract: This article examines gifted students' (ages 13–16) groupwork on a rich task in mathematics. This study was conducted in Norway, which has an inclusive education system that does not allow fixed-ability grouping. The purpose of this study was to better understand how to cultivate mathematical learning opportunities for gifted learners in inclusive education systems. The analysis was conducted from a multimodal perspective, in which students' coordination of speech, gestures, and artifact use was viewed as part of their learning process. The findings contribute to discussions on gifted students as a heterogeneous group. Moreover, our analysis illustrates how giftedness can be invisible, leading to unrealized potential and low achievement. We suggest that more attention be paid to teaching by adapting to gifted students' individual needs, particularly if the intention is to provide high-quality learning opportunities for gifted students in inclusive settings.

Keywords: giftedness; inclusive education; mathematics education; rich tasks

1. Introduction

Among the many challenges in mathematics classrooms, the one that has historically been almost invisible in the Norwegian education system is how to cultivate mathematical learning opportunities for gifted students. Norwegian teachers lack deeper knowledge about giftedness, as it is not being addressed during their teacher education [1]. These teachers' experiences align with conclusions in the official Norwegian report "More to gain—Better learning for students with higher learning potential" [2]. Resources for identifying and cultivating learning opportunities for gifted students are nearly nonexistent (p. 33). In mathematics, the lack of these resources has resulted in few Norwegian students achieving an advanced or high level in mathematics (p. 33). In a study of pre-service teachers' thinking about teaching gifted students in inclusive classrooms, Lassila et al. [3] reported a "need to equip future teachers with research-based knowledge about how different groups, particularly the gifted, experience varying pedagogical actions" (p. 319). Accordingly, our aim is to remind readers that gifted students are heterogeneous; that is, they have different strengths and diverse interests [4].

In Norway, inclusion has resulted in heterogeneity and a greater range of mathematical talent within the same class, with the gifted students in mathematics having a lower likelihood to meet more likeminded students. As such, these gifted students could hardly flourish in their area of mathematical talent. Gifted students in Norway have reported that their teachers often position them as assistant teachers in mathematics classrooms [5], thereby placing them outside the learning processes taking place among their peers and giving them more opportunities for repetition than developing mathematical knowledge [6]. The paradox is that these challenges are reported in an education system based on inclusive principles. Inclusive education systems aim to cultivate classrooms in which interaction among students at different levels can create a learning space that embraces

Citation: Simensen, A.M.; Olsen, M.H. Gifted Students' Actualization of a Rich Task's Mathematical Potential When Working in Small Groups. *Educ. Sci.* **2024**, *14*, 151. https:// doi.org/10.3390/educsci14020151

Academic Editor: Chua Tee Teo

Received: 27 November 2023 Revised: 26 January 2024 Accepted: 30 January 2024 Published: 31 January 2024



Copyright: © 2024 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https://creativecommons.org/license s/by/4.0/). diversity. Olsen [7] addressed some complexities of inclusion from a Norwegian perspective: "Norway has high ambitions when it comes to offering an inclusive school, ambitions that are clarified both ideologically and formally. Nevertheless, there appears to be a gap between ideology and how the pupils experience inclusion" (p. 109). Gifted students are reporting that they are not experiencing academic inclusion and have asked for opportunities to discuss mathematical problems with other gifted students [8]. They seek learning situations in which they have opportunities to work together with peers on a higher level, that is, situations in which they can discuss mathematical problems with other gifted students [8]. This study is an attempt to overcome this problem by using tasks that can be described as rich to challenge the students in diverse groups. Olsen [8] reported on gifted students' experiences of not being offered adapted teaching, which is in line with research reporting that students with extraordinary learning potential tend to find regular classrooms boring, leading to frustration and low motivation [9], which may lead to underachievement and dropout [10,11].

Skovsmose [12] discussed how different discourses or narratives can influence tensions between ideology and practice, portraying inclusion as inclusive landscapes in which "children with different cultural backgrounds can be brought together, and new perspectives are established" (p. 82). This conceptualization of inclusion shares similarities with Roos' [13] description of inclusive practices:

The discourse of teaching for maximising opportunities in mathematics for all. This discourse construes inclusion by focusing on teaching to enable all students to participate. Even though their points of departure are different and the studies discuss different subjects, the common theme is teaching to maximise opportunities to learn. (p. 33)

In this article, we have chosen to build on the understanding of inclusive practices as described by Roos [13] and Skovsmose [12]. We view inclusiveness as "processes of participation in mathematics education that every student can access" [13] (p. 231). Overall, we view inclusive mathematics education as a way to achieve social justice.

This understanding of inclusive practices affects both our and the national understanding of giftedness. The Norwegian Directorate for Education and Training emphasizes that giftedness "includes not only students who perform at a high and advanced level, but also students who have the potential to do so" [2], a point that Olsen [4] has also made (see Figure 1).

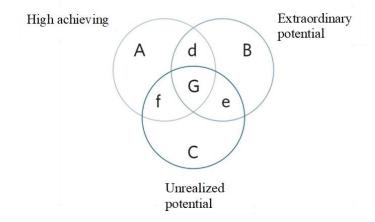


Figure 1. Variability in gifted students and achievement [4] (p. 17).

Figure 1 illustrates some of the complexities of giftedness, namely that not all gifted students are high achievers. For example, it is possible to have extraordinary and unrealized learning potential (e). Gifted students who feel that their school is not meeting their needs are at risk of low achievement and developing negative emotions toward education. One consequence of this might be to drop out from school. Furthermore, students with extraordinary learning potential (B) can be high achievers (A) and still have unrealized learning potential (C), as presented in Section G. The group of students who are high achievers (A) comprise students who are succeeding at school and getting good grades, possibly due to extraordinary learning potential (d) or well-structured and effective work habits. Students with extraordinary learning potential (B) are not always high achievers (d) and might have unrealized potential, which may lead to lower performance (e). The students in Section C have unrealized learning potential; that is, they might be high achievers (f), might have extraordinary potential (e), might be both high achievers and have extraordinary potential (G), or might have "only" unrealized learning potential.

We view gifted students as a heterogeneous group; that is, we advocate for an understanding of gifted students as diverse, with some being high achievers and others not. However, we argue that gifted students should also be offered learning opportunities that meet their abilities and aptitudes, which we refer to as "adapted teaching." In this article, we seek to better understand how to cultivate mathematical learning opportunities for all gifted learners. We believe that gifted students' request for more opportunities to work together with other gifted students should be taken seriously. More precisely, we investigate two different small groups of gifted learners while they work on a rich task. The research question guiding our work is as follows: *How do gifted students actualize a rich task's mathematical potential when working in small groups*?

2. Theory

The study presented in this article viewed learning as an activity strongly influenced by learners' cultural–historical context and background; that is, learning cannot be viewed as something that occurs individually as a cause–effect process. Instead, it is about potential mathematical knowledge being actualized by learners within the activity systems in which they participate [14]. Following this view, mathematical concepts are viewed as historically and culturally developed phenomena that can be actualized through learners' mediating actions. These actions open space for learners to take up mathematical learning opportunities. Our understanding of mathematical learning opportunities as a sociocultural phenomenon aligns with how Foyn et al. [15] described students' access to mathematics:

This study shows that students' access to mathematics is connected to participation, which makes participation and access inevitably interconnected by the idea of inclusion. Inclusion in mathematics education is not easily described or attained. The analyses indicate that inclusion is a complex process of participation where both ideological and societal issues, as well as individual and subject-specific issues, must be considered in the educational endeavour. (p. 244)

One consequence of our conceptualization of mathematical learning opportunities is that learning can be observed through "our participation in the activity that makes this way of thinking present in the singular" [14] (p. 139). The use of the word "singular" here refers to actualization, that is, actions taking place during learning processes. When knowledge is actualized, learners have opportunities to participate in mathematical learning processes. Through the fine-tuned coordination of speech, the body, gestures, symbols, and tools, mathematical learning opportunities can be mediated and contribute to new knowledge [16]:

Thinking, hence, does not occur solely in the head but in and through language, body and tools. As a result, from this perspective, gestures, as a type of bodily action, are not considered as a kind of window that illuminates the events occurring in a "black box" they are not clues for interpreting mental stages. They are rather genuine constituents of thinking. (p. 113) This is not to say that all thinking and mathematical learning are observable, but when mathematical objects are actualized, they can be observed. Ontologically, as Radford [14] explained, objects of knowledge are mediated by activity and can be illustrated through three elements (see Figure 2).

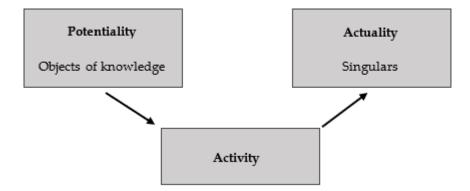


Figure 2. The objects of knowledge, singulars, and mediating activity [14] (p. 137).

One central point in Radford's reasoning is that mathematical objects of knowledge are abstract and general; that is, they are pure possibilities and can be actualized using singular examples. For example, an equation is not a mathematical object of knowledge, but a singular action or product exemplifying a mathematical object. This is why language, the body, and tools are important in gaining insight into mathematical learning opportunities: Mathematical objects of knowledge are a potentiality and become an actuality when mediated through different modalities (e.g., bodily actions, gestures, speech, and use of artifacts).

Gagné [17] pointed out that gifted students need adaptations to perform at the level of their abilities. Many gifted students find regular teaching methods frustrating [9], and they have expressed having low motivation to perform, as they believe they have already demonstrated their knowledge [11]. Shernoff et al. [18] investigated different groups of students and found that those who face challenges in line with their abilities and prerequisites will be in a learning flow. However, students who do not receive enough challenges based on their abilities will experience boredom and may drop out. If we supplement this with the higher proportion of behavioral problems that exist among gifted students compared with the total population [19], underachievement [20], and perfectionism [21], the importance of facilitating academic inclusion for these students is emphasized.

A high degree of creativity is viewed as a central trait of gifted students [22]. Idsøe [23] pointed out that gifted students can overwhelm their surroundings because of the insight and originality they demonstrate in their work. The teacher plays an important role in cultivating students' mathematical learning opportunities. Another important resource is the task. To meet our understanding of learning processes as a sociocultural phenomenon and our interest in the principle of inclusive education, we chose to use a mathematics task that can be described as a rich task. Boaler and Dweck [24] reported that rich tasks are appropriate for cultivating creative and inspiring opportunities to learn mathematics. They presented the following characteristics of such tasks: It is challenging, but accessible; students can view it as a puzzle; students are empowered to use visual thinking; multiple ideas can be proposed in the classroom; students respect both their own and their peers' mathematical ideas; and students encourage collaborative work (pp. 62–63). We argue that rich tasks have the potential to cultivate and encourage learning situations that can be observed within the frames of potential knowledge, actualization, and activity systems [14]. Moreover, extant research has indicated that rich mathematics tasks provide

opportunities for all students to contribute mathematically, regardless of previous achievement and gender [5,25].

3. Methods

In this study, we qualitatively examined how gifted students actualized a rich task's mathematical potential when working in small groups. This study was conducted in Norway on students aged 13–16. In Norway, fixed-ability grouping is not allowed [5]; however, a national program for students viewed as gifted in mathematics and natural sciences began as a pilot program in 2016 and is part of the government's strategy for supporting gifted and high-achieving students. As Nissen et al. [26] found, gifted students are not always high achievers. In this national program, gifted students were invited to participate based on the following criteria: they were very motivated to participate in the program; their teachers recommended them; and they had a history of high achievement in mathematics and natural sciences. The national program aims for students to experience being socially and academically included in mathematics and science and to be challenged at their academic level.

3.1. Informants

The national program was a starting point for our study, and some students participating in this program were invited to take part in our research. Those agreeing to participate were followed at the case level, in which we interviewed them and video-recorded their work in small groups in both regular classroom and talent center settings. In this article, we reported findings after analyzing the video recordings and students' (written) products from the observed sessions.

For gifted students, participating in the national program was an opportunity to both meet other gifted students at one of the national STEM learning centers four times during the school year (two days each) and receive teaching adapted to their mathematical level. When they were not at the national learning center, they attended their regular mixed-ability classes. To pursue our interest in mathematical learning opportunities for gifted students, we video-recorded them in two small groups: one in their regular class, in which group participants had different academic levels, and one at the learning center, in which all group participants were viewed as high achievers. We repeated this methodological approach with students from three more classes. These students' teachers viewed them as gifted. Altogether, we video-recorded six small groups in regular classrooms at four different schools and eight small groups comprising only gifted students. That is, six gifted students were studied in detail (motivation letter for the national program, interviews, observations in both heterogeneous and homogeneous groups). The observed sessions varied in length between 45 and 90 min each.

The video recording allowed us to examine their mathematical reasoning within the theoretical frame of Radford [14], that is, from a multimodal approach to mathematics learning in collective solution spaces. This is in line with Levav-Waynberg and Leikin [27], who posited that collective solution spaces can be a helpful "tool for examining the mathematical knowledge and creativity of participating students" (p. 78). Following the complexity of giftedness presented in Figure 1, not all mathematical learning can be captured through test scores. For example, the students in Section d have both extraordinary and unrealized learning potential, and they are at risk of not being identified and, thus, not being offered adequate learning opportunities. Video recording and the theory of objectification allowed us to investigate their mathematical learning opportunities outside the limits of test scores.

3.2. Analytical Process

The data presented in this article comprised video observations and students' (written) products from working in small groups. The videos captured students' actual communication, and the written products have the potential to provide insight into some of the complexities of students' multimodal expressions. When a student says something, points to something, and writes/draws something simultaneously, these three modalities contribute complementary information that provides a more detailed picture than each modality taken separately.

The video observations were transcribed and coded using NVivo (a qualitative data analysis program), allowing us to synchronize the videos with associated transcriptions. Thus, we conducted coding in vivo [28], in which we emphasized actual multimodal interactions between students in our coding process. Based on our theoretical framework, the theory of objectification, we viewed students' coordination of speech, gestures, and artifact use while working in small groups to illustrate their actualization of mathematical objects. While coding, we focused on how the students actualized mathematical ideas. For example, if a student pointed to a figure on the worksheet and said, "this equals that," we viewed it as an actualization of an idea. Our aim was not to develop categories, but rather to better understand how gifted students actualized a particular rich task's mathematical potential.

3.3. The Rich Task Used in the Small Groups

Inclusive settings are characterized by a diversity of students who perform at different levels, with varying learning styles and interests. While we noted the ongoing discussion about ability grouping or inclusive settings, e.g., [29], the research presented in this paper sought to contribute to develop resources that can cultivate mathematical learning for all students, particularly gifted students. Because rich tasks have been found to promote mathematical learning opportunities for pupils performing at different levels [30,31], we used these kinds of tasks to gain insight into gifted students' learning opportunities in both heterogeneous and homogeneous settings. In this article, we reported findings related to students' work on one rich task, asking them to find the ratio of two squares (see Figure 3).

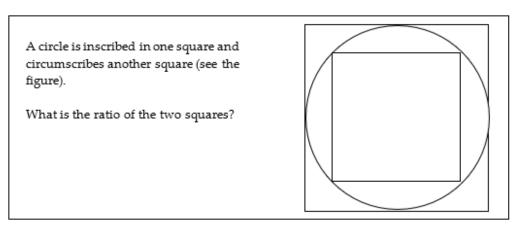


Figure 3. Task used in the data presented in this article.

The task can be solved in several ways, using multiple methods, thereby potentially offering mathematical opportunities to a variety of students. We piloted this task previously and reported preliminary findings that suggested that it meets the criteria for being defined as a rich task (see Figure 4).

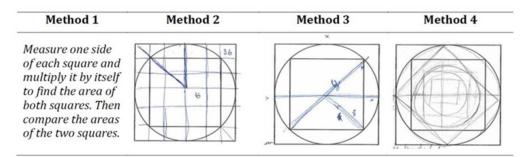


Figure 4. Four solution methods revealed in our study [5].

The first two methods are based on measurement and proportional reasoning. For both, the students began by calculating both squares' areas, and then comparing the two numbers to determine the ratio. Below, we focus on the next two solution methods, which involve dynamic aspects of students' mathematical generalizations. Both are based on a mental transformation in which the students rotated the inner square 45°. In our analysis, this rotation depicted the students' argument, justifying that the inner square's diagonal equals the outer square's sides. After this initial mental transformation, the two solutions took different paths. Method 3 was developed into a symbolic generalization based on the Pythagorean theorem, and Method 4 was developed into multimodal reasoning leading to a non-symbolic generalization. Below, we provide more details about our results from the students' work related to Method 4. While we have focused on written products thus far, we address speech, gestures, and the use of tools below.

3.4. Limitations and Ethical Considerations

One of this study's limitations is that we did not have access to information about whether our informants are high-achieving only, or whether they also have extraordinary learning potential. However, these students' teachers know them very well and have identified them as both high-achieving and gifted (see Section d in Figure 1).

Another limitation is the analytical approach we have taken. Coding in vivo provides opportunities to honor and spotlight participants' communication, with the potential to provide authentic insight into students' mathematical communication. However, it may also hinder the generalization and comparison of other studies' findings. Despite possible reliability problems, we ascribed the greatest importance to acknowledging students' voices when developing an understanding of giftedness. Therefore, we argue that these kinds of studies may help develop knowledge about gifted students and their mathematical learning opportunities.

This research was conducted based on Norway's national guidelines and evaluated by the national ethics committee (NSD, now SIKT). All participants and their parents provided voluntary consent to participate. The participants were informed that they had the right to know what data we collected about them. We also informed them that they had the right to withdraw from this study at any time and without citing any reason.

4. Results and Analyses

Based on our research question about how gifted students actualize a rich task's mathematical potential when working in small groups, we begin this section by exemplifying rich tasks' potential to promote inclusive learning spaces in mathematics. One of the characteristics of a rich task is its low entrance; that is, the question should be easy to understand so that all students can be invited to participate in solving the posted problem. This might seem like an important characteristic only for low-achieving students, but we remind the reader that gifted students constitute a heterogeneous group. Many gifted students struggle in some school situations; therefore, adapted tasks and teaching are crucial for this group of students' mathematical learning opportunities. Figure 5 illustrates how a group of gifted students used the task in Figure 3 as a starting point to develop a common understanding of the proposed problem.



Figure 5. Students using the figure on the worksheet to discuss how to solve the task.

The students' body language and gestures give the impression that all group members are on task, focusing on the worksheet; that is, the group had a common starting point that framed the upcoming problem-solving process. A frequent comment that we observed in several of the groups was, "It must have something to do with the circle. If not, it would not have been there" (Hanna, 10th grade). We viewed this as demonstrating how the figure on the worksheet cultivated students' reasoning and, thus, can be viewed as a basis for students' actualization of the task's mathematical potential.

Table 1 provides an excerpt from a conversation that exemplifies the task's low entrance, illustrating how the students started to focus on the relationship between the smaller square's diagonal and the larger square's sides. Tom and May, the students whose comments are cited, are in ninth grade (14 years old). They worked together with Ida, who was silent during the episode presented in Table 1. The three students are from the same school, but from different classes. We defined the group as homogeneous because the students' teachers viewed all three as high achievers in mathematics.

Line Name		Utterance		
1	Tom	Do you understand?		
2	May	I think I do understand the problem to solve, but not how to solve it.		
3	Tom	Yes, I would like to say, you know, the diameter of the circle. This is this much [pointing to the		
		diameter with his hand]. If this had been two triangles, the hypotenuse would have been the diagonal.		
4	May	Yes.		
5	Tom	It is, in a way, the same as the diameter of the circle.		
6	May	Yes, I see, but I don't understand how to find this.		
7	Tom	Do you think we need to calculate, or do you think we can just see it if we think [imagine]?		
8	May	I'm not sure; how do we actually calculate it? We have not really got any number, or anything		
		like that.		
9	Tom	No, that is true, but we can, in a way. We could have used numbers. I need to write it down		
		because the diameter, you know, if this had equaled five [points to the sides of the larger square],		
		the diameter would have also equaled five because the diameter has the same length as this		
		[points to one side of the larger square]. Then, this had been three, and this had been four. No, it		
		wasn't. Forget that; this is really challenging.		
10	May	Yes.		
11	Tom	Do you think we are answering the task if we write that the diagonal of the smaller square equals		
		the sides of the other square?		
12	May	I think, maybe; I'm not sure.		
13	Tom	We can perhaps write it down?		
14	May	Yes.		

Table 1. The task had a low entrance.

This excerpt revealed that May started by claiming that she understood the problem posed but did not know how to solve it. This illustrates the task's low entrance. The question is easy to understand, but not necessarily easy to answer. In Line 11, Tom actualizes the relationship between the smallest square's diagonal and the larger square's sides. The idea about this relationship can be viewed as a mathematical idea based on abstract thinking. Simultaneously, Tom used the actual length of five to actualize and argue that the diagonal and the sides have the same length. Our analysis found that most students used drawings to illustrate this relationship, rotating the smaller square 45° in the drawing (Figure 6a). However, some students demonstrated this relation by cutting pieces of paper and rotating the piece representing the smaller square by 45° (Figure 6b).

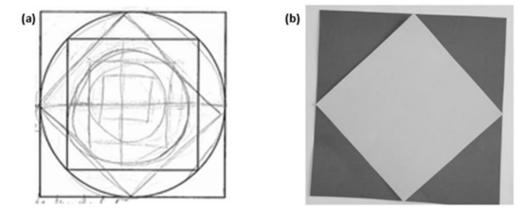


Figure 6. The drawings and figures used to illustrate the relationship between the diagonal of the smaller square and the sides of the larger square ((**a**,**b**), respectively).

The excerpt in Table 1 demonstrates that the task offers opportunities to discuss different methods for solving the task. The students moved between actualizations on a concrete level (using numbers) and actualizations on a more abstract level (comparing the smaller square's diagonal and the larger square's sides).

Students' understanding of the relationship between the smaller square's diagonal and the larger square's sides is crucial to solving the problem later using the Pythagorean theorem. Table 2 presents the communication between Anna (seventh grade) and Jon (ninth grade). Tina (eight grade) is also a member of this group, but she participated sparingly during this episode. The group could be viewed as homogeneous, comprising students participating in the national talent program. However, the group could also be viewed as heterogeneous because the students are from different grades (ages 13–15). In the excerpt, the students discussed the problem posed and how to solve it.

Line	Name	Utterance
15	Jon	Yes, because we are not supposed to measure anything.
16	Anna	But we have this square, but two of these [points to two of the corner triangles in Figure 6a] equal one-fourth.
17	Jon	What?
18	Anna	Two of these [points to one of the corner triangles in Figure 6a], these two [points to two of the corner triangles in Figure 6a]. (They) equal one-fourth [of the larger square].
19	Jon	That makes it half.
20	Tina	Yes, one-fourth.
21	Anna	Four of these. This [points to the smaller square] is made up of four of these [points to the corner triangles in Figure 6a].
22	Jon	Half as big? Or am I wrong? Let's see. We have these
23	Anna	They are too small.
24	Jon	Hmm? About. We do it like this, and then this fits here. Now we have used this part. Then, this is one and two. And then this one is half the size of that one.
25	Anna	Let's write it down How can we explain our thinking? This is right- angle triangles. Divide the figure. This is not a semicircle; it would have been a semicircle if we had cut the figure here. This is like a parabola thing. The whole figure is made up of right-angle triangles.
26	Jon	Made by the outer circle parts together with the sides.
27	Anna	And then we have a square inside the larger square that equals the smaller square.

Table 2. The problem posed is challenging for the students.

Jon and Anna discussed what was required to solve the task properly, and in Line 15, Jon said that they were not supposed to measure anything. Anna drew lines between the figure on the worksheet and the fractions: "But we have this square, but two of these [points to two of the corner triangles in Figure 6a] equal one-fourth" (Line 16). To connect the figure on the worksheet with her explanation, she pointed to the worksheet while explaining. This indicated that the artifacts were contributing to the students' actualization of the ratio. They continued their conversation about the task, pointed to the figure on the worksheet, and found that the smaller square was half the size of the larger square. The analysis revealed that several of the students actualized this ratio by using pieces of paper to model the problem and solved it by rotating the papers to examine and actualize how the two squares were related. Figure 7 is a snapshot of one of the groups' uses of pieces of paper to examine and actualize the ratio between the two squares dynamically.

The students' use of pieces of paper to actualize the ratio between the two squares exemplifies that concrete materials have the potential to cultivate students' opportunities to actualize mathematical objects' dynamic aspects.

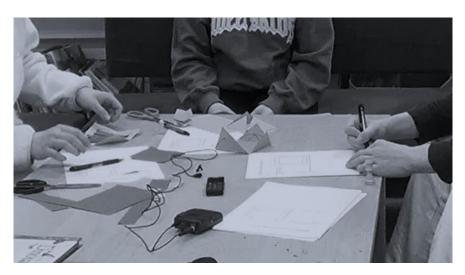


Figure 7. Students used pieces of paper to examine the relationship between the squares.

While Jon and Anna used the drawing illustrated in Figure 6a to discuss the ratio of the two squares, the group in Figure 7 used pieces of paper. Figure 8 demonstrates how this group examined the same relationship that Jon and Anna identified. The group justified their answer in two ways. First, they made the shape in Figure 8a, arguing that the light triangles can be moved to, and overlap exactly with, the dark square.

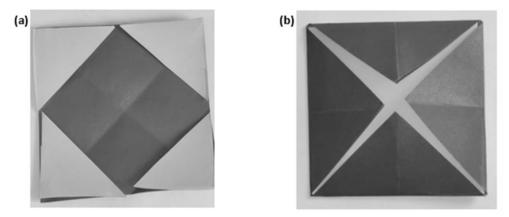


Figure 8. Two ways in which the students justified their answers with pieces of paper: (a) cutting the smaller square in four pieces and covering the corners of the larger square; (b) folding the corners of the larger square when the smaller square is inside the larger.

Both methods illustrated in Figure 8 started from the rotation presented in Figure 6. In Figure 8a, the students cut the smallest square into four congruent right-angle triangles and actualized that the triangles cover the difference between the smaller and larger squares; that is, the students argued that two of the smaller triangles cover exactly the same area as the larger square. They used the same justification when using the figure in Figure 8b. However, they chose to fold the difference, so the larger square enclosed the smaller square. The results were identical, but the students experienced the methods as two different ways to solve the problem.

Our analysis has illustrated that the gifted students in our study used a variety of approaches to actualize mathematical objects when working on the actual task. The students' discussions about what is viewed as appropriate mathematical actualizations provided insight into how the students coordinated others' multimodal communication, the figures on the worksheet, and their own mathematical knowledge to actualize mathematical ideas that are central when working on this task. Furthermore, their use of speech, gestures, and artifacts created opportunities to actualize mathematical objects as dynamic, that is, a potentially. Regarding motivation, the gifted students in our study expressed that working on mathematical problems in homogeneous groups provided them with interesting and challenging learning opportunities. In fact, some of the groups denied stopping working when the sessions ended. When we stopped the videorecording, the students themselves organized a plenary discussion in the classroom. That is, they spent the break discussing the multiple solution strategies found and used by different groups.

5. Discussion

Inspired by the research question guiding this article — *How do gifted students actualize a rich task's mathematical potential when working in small groups?* — and our findings, we decided to conduct our discussion around two points of interest: (1) gifted students' collaboration on a rich task and (2) the importance of adapted teaching for gifted learners.

5.1. Gifted Students' Collaboration on a Rich Task

The official Norwegian report "More to gain-Better learning for students with higher learning potential" [2] requested more research on how to cultivate learning situations for gifted students. We view the national program for students deemed gifted in mathematics and natural sciences as a response to this request. More precisely, we recognize this program as a resource intended to offer learning spaces in which gifted students can come together and work on inquiry-based problems. This is a way to achieve academic inclusion, as well as social inclusion, by allowing students to interact with others who are high achievers in these subjects. In this regard, we hope that our study will contribute to a deeper understanding of how to cultivate mathematical learning opportunities for gifted learners in inclusive settings. By "inclusive settings," we mean classrooms designed to be flexible, sometimes structured for heterogeneous groups and other times for homogeneous groups. What is important, in our opinion, is that gifted students are not readily identifiable in the classroom. This point has been underscored in research on low-achieving students, which found that these students contribute sophisticated and abstract mathematical ideas [25,30,32]. Therefore, flexible grouping is important, as it will motivate all gifted students, including those not visible to the teacher, because of low achievement, to participate in collaborative work that seeks to cultivate sophisticated mathematics reasoning. Olsen [7] reported that high-achieving mathematics students have a desire to learn and develop their understanding, preferably in interactions with other high-achieving students. In this sense, group work in a homogeneous group should stimulate a high level of commitment among all participants. Our study includes several examples of groups of three, in which one of the students did not participate actively in the mathematical conversation. We do not know why, but this finding suggests that even though content-rich tasks provide good opportunities for the collaborative actualization of mathematical ideas, this organization does not guarantee that all students are participating. This finding illustrates that not all students are demonstrating their mathematical potential when working in small groups. Moreover, if students do find mathematical problems to be too easy, they may not be motivated to participate in group work. Extant research has found that students who did not formerly appreciate group work in lower education changed their minds when they were challenged academically at the university level [6].

One consequence of students who do not participate actively in group work is that they might be viewed as less talented than their peers. For example, from the excerpt presented in Tables 1 and 2, Ida and Tina could be viewed as low achieving or unmotivated based on their lack of participation; however, their teachers viewed them as high-achieving and motivated when it comes to mathematics. Unfortunately, we do not have information about what working style they prefer, but we wonder whether group work might not be their favorite way of working. If this is the case, lots of group work could conceal their learning potential and further demotivate these students, leading to under-achievement and perhaps dropout. Considering that learning opportunities that are made available are of importance to students' achievement and motivation, we viewed the Norwegian understanding of giftedness as a strength. The Norwegian Directorate for Education and Training emphasizes that giftedness "includes not only students who perform at a high and advanced level, but also students who have the potential to do so" [2]. Olsen [4] has also made this point (see Figure 1).

Olsen's point aligns with the sociopolitical discourse behind the "Norwegian" interpretation of inclusive education, namely that ability grouping may harm some students. Criticisms of ability grouping are based on the tendency to limit mathematical learning opportunities for members of low-achieving groups [33]. How Olsen [4] portrayed giftedness, as presented in Figure 1, provides insight that might be helpful in understanding some of the reasons for this harm and the limitations elicited. Within a discourse similar to that of the Norwegian Directorate for Education and Training, Olsen [4] and Tirri and Laine [10] discussed ethical challenges related to gifted students in inclusive education. One of the issues addressed relates to the barriers to learning that may be created by misconceptions about giftedness. For example, they explain that gifted students tend to be viewed as gifted in everything, a misconception that can lead teachers to believe that gifted students can learn anything on their own (p. 243). Through our research, we contribute further to the discussions that Olsen [4] and Tirri and Laine [10] have already generated.

5.2. The Importance of Adapted Teaching for Gifted Learners

We acknowledge that communication about mathematical learning opportunities for gifted learners is not just about teachers, researchers, and news media. Gifted learners' voices and actions are of great importance in understanding how they experience and take advantage of learning opportunities in inclusive education. Several studies have problematized the roles available for gifted learners in collaborative work, e.g., [6,7,34]. Gifted learners reported that they spend time helping their lower-achieving peers instead of engaging in productive pursuits with challenging tasks [5,6,10]. Flexible grouping and rich tasks can be a counterweight to this.

Our data indicate that the rich task we used opened opportunities to use multiple methods, some more sophisticated than others. It has been argued that these kinds of tasks are beneficial for low-achieving students [5,26]. From our understanding of gifted learners as a diverse and heterogeneous group, we argue that gifted learners also need rich tasks' flexibility and openness. We previously reported on these kinds of tasks as being beneficial for cultivating gifted students' creative mathematical reasoning [5]. The study reported here has contributed more details on this picture. Considering that gifted students have different strengths and weaknesses, not all gifted students will solve all mathematical problems easily. Therefore, opportunities to use their own strengths and experiences when working on rich tasks are of great importance for gifted students. If inclusive mathematics education is to be viewed as opportunistic for all students to participate in the actualization of mathematical knowledge, the mathematics education field must acknowledge the heterogeneous nature of giftedness [4].

We suggest that rich tasks have the potential to cultivate and encourage learning situations that can be observed within the potential knowledge, actualization, and activity systems frames [14]. Simultaneously, our findings indicated that not all gifted students are active in this work, and they strengthen the view of teaching as adapted, particularly if the intention is to meet all students' needs. When we say "adapted" here, we mean flexible when it comes to organization (individual and group work), tasks (problem-based, procedure-based, etc.), and access to actualize mathematical ideas through a combination of speech, gestures, and artifacts. Although this view is recognized in the mathematics education field, extant research has found that other fields have differing views. For example, mathematics as language- and culture-neutral is one perspective visible in news media, and beginning teachers are frustrated because they have experienced method rigidity in mathematics classrooms [35,36]. Our findings demonstrated that some students were almost invisible during the observed communications (see Tables 1 and 2). Ida and Tina did not participate much; therefore, they could be viewed as outsiders from the homogeneous groups. These retrospective reflections on how some gifted students might be invisible reminded us of the complexity that Skovsmose [12] addressed, namely that one way to interpret inclusive mathematics education is that it entails the provision of spaces in which differences come together. An understanding of inclusiveness as learning situations in which all students feel academically, socially, and culturally included requires education systems that appreciate and celebrate diversity. We understand this in line with inclusive landscapes that aim to achieve social justice, as Skovsmose [12] described.

The gifted students in our study appreciated working on a rich task in small groups, although not all students were participating actively. We also acknowledge that gifted students are different, and we see the variety of participation in relation to how participation tends to differ in heterogeneous groups.

6. Conclusions

Overall, we view inclusive mathematics education as one way to achieve social justice, and we hope to contribute to communication aimed at cultivating mathematical learning opportunities for gifted learners in inclusive settings. Our findings indicate that gifted students may also benefit from coordinating speech, gestures, and artifact use to actualize mathematical objects. Therefore, we suggest that flexibility in inclusive education is an important aspect of supporting gifted learners in mathematics education. According to our data, the gifted learners examined in this article are a heterogeneous group that demonstrated diverse ways of participating in group work and solving mathematical problems. Some were very talkative, others were quieter, and some contributed mainly through gestures and the use of artifacts. This illustrates the variety of strengths and learning/communication styles that characterize gifted students' participation in group work in inclusive settings.

One of the limitations of our study is that we did not interview the students after observing their work in small groups. Interviewing them and asking them to share their retrospective experiences would provide more insight into what adapted teaching might look like in inclusive classroom settings.

Author Contributions: Conceptualization, A.M.S. and M.H.O.; Methodology, M.H.O.; Formal analysis, A.M.S.; Writing—original draft, A.M.S. and M.H.O.; Writing—review & editing, A.M.S. and M.H.O.; Project administration, A.M.S. and M.H.O. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding

Institutional Review Board Statement: The study was conducted in accordance with the guidelines given by The National Committee for Research Ethics in the Social Sciences and the Humanities (NESH), and approved by the Norwegian Centre for Research Data (protocol code 242336, date of approval: 7 September 2022).

Informed Consent Statement: Informed consent was obtained from all subjects involved in the study.

Data Availability Statement: Data for the larger study are unavailable due to privacy or ethical restrictions.

Conflicts of Interest: The authors declare no conflict of interest.

References

- 1. Lenvik, A.; Jones, L.Ø.; Hesjedal, E. Teachers' perspective on gifted students with extraordinary learning potential in Norway: А descriptive study with primary and secondary teachers. Gift. Talent. Int. 2022, 37. 163 - 172.https://doi.org/10.1080/15332276.2022.2138796.
- 2. NOU 2016:14. More to gain—Better learning for students with higher learning potential. 2016. [Official Norwegian Reports] Available online: https://www.regjeringen.no/en/dokumenter/nou-2016-14/id2511246/ (accessed on 31 January 2024).

- 3. Lassila, E.; Hyry-Beihammer, E.K.; Kizkapan, O.; Rocena, A.; Sumida, M. Giftedness in Inclusive Classrooms: A Cross-Cultural Examination of Pre-Service Teachers' Thinking in Finland, Austria, Turkey, the Philippines, and Japan. *Gift. Child Q.* **2023**, *67*, 306–324. https://doi.org/10.1177/00169862231183652.
- Olsen, M.H.; Skogen, K. (Eds.) Læringspotensial; Cappelen Damm Akademisk: Oslo, Norway, 2019; pp. 11–26. ISBN 978-82-02-59812-9.
- Simensen, A.M.; Olsen, M.H. Tasks that enhance creative reasoning: Supporting gifted pupils in inclusive education systems. Including the Highly Gifted and Creative Students–Current Ideas and Future Directions. In Proceedings of the 11th International Conference on Mathematical Creativity and Giftedness (MCG 11), Hamburg, Germany, 22–24 August 2019; pp. 202–208.
- Smedsrud, J.H.; Nordahl-Hansen, A.; Idsøe, E. Mathematically Gifted Students' Experience with Their Teachers' Mathematical Competence and Boredom in School: A Qualitative Interview Study. *Front. Psychol.* 2022, 13, 2954. https://doi.org/10.3389/fpsyg.2022.876350.
- Olsen, M.H. A practical-theoretical perspective on the inclusive school in Norway. In *Dialogues between Northern and Eastern Europe on the Development of Inclusion: Theoretical and Practical Perspectives*; Routledge, New York, NY, USA, 2021; pp. 99–114. ISBN 978-0-367-40988-3.
- 8. Olsen, M.H. Tilpasset opplæring når elever har stort læringspotensial. In *Tilpasset Opplæring*, 1st. ed.; Olsen, M.H., Haug, P., Eds.; Cappelen Damm Akademisk: Oslo, Norway, 2020; pp. 65–92. ISBN 978-82-02-66586-9.
- Neihart, M. Social and Emotional Development of Gifted Children: What Do We Know?; Routledge, Waco, Tex, USA: 2002. ISBN 978-1-59363-643-2.
- 10. Mathisen, A.R.P.; Olsen, M.H. Relasjoner i pedagogikken: i lys av Baumans teorier. In *Relasjoner i pedagogikken. I lys av Baumans teorier*; Olsen, M.H., Ed.; Universitetsforlaget: Oslo, Norway, 2016; pp. 98–112. ISBN 978-82-15-02644-2.
- Tirri, K.; Laine, S. Ethical Challenges in Inclusive Education: The Case of Gifted Students. In *Ethics, Equity, and Inclusive Education*; Forlin, C., Gajewski, A., Eds.; International Perspectives on Inclusive Education; Emerald Publishing Limited, Bingley, UK: 2017; Volume 9, pp. 239–257. ISBN 978-1-78714-153-7.
- Skovsmose, O. Inclusions, meetings and landscapes. In *Inclusive Mathematics Education: State-of-the-Art Research from Brazil and Germany*; Kollosche, D., Marcone, R., Knigge, M., Penteado, M.G., Skovsmose, O., Eds.; Springer International Publishing: Berlin/Heidelberg, Germany, 2019; pp. 71–84. ISBN 978-3-030-11517-3.
- 13. Roos, H. Inclusion in mathematics education: An ideology, a way of teaching, or both? *Educ. Stud. Math.* 2019, 100, 25–41. https://doi.org/10.1007/s10649-018-9854-z.
- Radford, L. The epistemological foundations of the theory of objectification. In *Teaching and Learning Mathematics. Some Past and Current Approaches to Mathematics Education*; Branchetti, L., Ed.; Isonomia, On-line Journal of Philosophy Epistemologica: 2015; Volume 7, pp. 127–149. ISSN 2037-4348. Available online: https://isonomia.uniurb.it/teaching-and-learning-mathematics-some-past-and-current-approaches-to-mathematics-education/ (accessed on 31 January 2024).
- 15. Foyn, T.; Solomon, Y.; Braathe, H.J. Clever girls' stories: the girl they call a nerd. *Educational Studies in Mathematics* **2018**, *98*, 77–93, https://doi:10.1007/s10649-017-9801-4.
- Radford, L. Why do gestures matter? Sensuous cognition and the palpability of mathematical meanings. *Educ. Stud. Math.* 2009, 70, 111–126. https://doi.org/10.1007/s10649-008-9127-3.
- 17. Gagné, F. Constructs and models pertaining to exceptional human abilities. In *International Handbook of Research and Development* of *Giftedness and Talent*; Pergamon Press: Oxford, UK, 1993; pp. 69–87.
- Shernoff, D.J.; Csikszentmihalyi, M.; Shneider, B.; Shernoff, E.S. Student engagement in high school classrooms from the perspective of flow theory. Sch. Psychol. Q. 2003, 18, 158–176. https://doi.org/10.1521/scpq.18.2.158.21860.
- Guénolé, F.; Louis, J.; Creveuil, C.; Baleyte, J.M.; Montlahuc, C.; Fourneret, P.; Revol, O. Behavioral profiles of clinically referred children with intellectual giftedness. *Biomed. Res. Int.* 2013, 2013, 540153. https://doi.org/10.1155/2013/540153.
- 20. Reis, S.M.; McCoach, D.B. The Underachievement of Gifted Students: What Do We Know and Where Do We Go? *Gift. Child Q.* **2000**, 44, 152–170. https://doi.org/10.1177/001698620004400302.
- 21. Guignard, J.H.; Jacquet, A.Y.; Lubart, T.I. Perfectionism and anxiety: A paradox in intellectual giftedness? *PLoS ONE* 2012, 7, e41043. https://doi.org/10.1371/journal.pone.0041043.
- 22. Reis, S.M.; Renzulli, S.J.; Renzulli, J.S. Enrichment and Gifted Education Pedagogy to Develop Talents, Gifts, and Creative Productivity. *Educ. Sci.* 2021, 11, 615.
- 23. Idsøe, E.C. Elever med Akademisk Talent i Skolen; Cappelen Damm akademisk: Oslo, Norway, 2014.
- 24. Boaler, J.; Dweck, C. Mathematical Mindsets: Unleashing Students' Potential through Creative Math, Inspiring Messages and Innovative Teaching; Chichester: John Wiley & Sons, Incorporated: Chichester, UK, 2015.
- Barclay, N. Valid and valuable: Lower attaining pupils' contributions to mixed attainment mathematics in primary schools. *Res. Math. Educ.* 2021, 23, 208–225. https://doi.org/10.1080/14794802.2021.1897035.
- Nissen, P.; Kyed, O.; Baltzer, K.; Skogen, K. Talent i Skolen: Identifisering, Undervisning og Utvikling; Pedagogisk psykologisk forlag, Namsos, Norway: 2012. ISBN 978-82-7767-092-8.
- Levav-Waynberg, A.; Leikin, R. The role of multiple solution tasks in developing knowledge and creativity in geometry. J. Math. Behav. 2012, 31, 73–90. https://doi.org/10.1016/j.jmathb.2011.11.001.
- Given, L. The SAGE Encyclopedia of Qualitative Research Methods; SAGE Publications, Incorporated, California, USA, 2008. https://doi.org/10.4135/9781412963909.

- 29. Francis, B.; Archer, L.; Hodgen, J.; Pepper, D.; Taylor, B.; Travers, M.-C. Exploring the relative lack of impact of research on grouping' in England: А discourse analytic account. Camb. I. Educ. 2017, 1–17. 'ability 47. https://doi.org/10.1080/0305764X.2015.1093095.
- 30. Boaler, J.; Sengupta-Irving, T. The many colors of algebra: The impact of equity focused teaching upon student learning and engagement. J. Math. Behav. 2016, 41, 179–190. https://doi.org/10.1016/j.jmathb.2015.10.007.
- Mellroth, E. The suitability of rich learning tasks from a pupil perspective. In *Proceedings of the Tenth Congress of the European* Society for Research in Mathematics Education (CERME10, Dublin, Ireland, 1–5 February 2017); Dooley, T., Gueudet, G., Eds.; DCU Institute of Education and ERME: Dublin, Ireland, 2017; pp. 1162–1169.
- Simensen, A.M. Matematiske Læringsmuligheter for alle: En Styrkebasert Flerkasusstudie om Elever Som Presterer lavt i Matematikk sin Deltakelse i Heterogene Smågrupper. Ph.D. Thesis, Universitetet i Agder, Kristiansand, Norway, 2022; Volume 365.
- 33. Straehler-Pohl, H.; Fernández, S.; Gellert, U.; Figueiras, L. School mathematics registers in a context of low academic expectations. *Educ. Stud. Math.* 2014, *85*, 175–199. https://doi.org/10.1007/s10649-013-9503-5.
- 34. Olsen, M.H.; Mathisen, A.R.P.; Sjøblom, E. Faglig Inkludert?: Fortellinger fra Elever med Ulik Måloppnåelse, 1st ed.; Cappelen Damm akademisk: Oslo, Norway, 2016.
- 35. Andersson, A.; Foyn, T.; Simensen, A.M.; Wagner, D. Storylines in Voices of Frustration: Implications for Mathematics Teacher Education in Changing Times. *Educ. Sci.* 2023, 13, 816.
- Andersson, A.; Ryan, U.; Herbel-Eisenmann, B.; Huru, H.L.; Wagner, D. Storylines in public news media about mathematics education and minoritized students. *Educ. Stud. Math.* 2022, 111, 323–343. https://doi.org/10.1007/s10649-022-10161-5.

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.