# Reliability Assessment of Computer in Design Phase Under High Censored Setting

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*Abstract* - **Assessment of reliability of personal computer is a challenge for the developer as the lack of sufficient data. Ordinary statistical approach depending on large dataset has less convincible result for the developer to make decision. Prior to massive production, the computer manufacturer runs a life test by picking up a certain number of new computers to run to failure to enlarge the data set. Nevertheless, as the defect rate of the modern computer at this stage is very low, the life data are right-censored with high censoring rate that up to 90%. This paper adopts a moment method to analyze the life data to accommodate the highly censored problem, and a case study is presented to access the reliability.**

*Keywords* - **Reliability assessment, Computer, High Censored Setting, Moment method**

## I. INTRODUCTION

The significant reliability improvement of VLSI and the error-tolerance mechanism in the computer hardware ensures the high reliability of the modern computer. The essential constituent components of a modern computer such as CPU, memory, hard driver, flat screen, mainboard etc. has very low failure rate. The Hard Driver Disk (HDD), e.g. considered as a vulnerable part of the computer, the failure rate can be as lower as below 2% per year [1]. The motherboard can be as low as 0.0026 per month [2]. The high reliability of computer also owns to the great efforts of manufacturers, who assemble these components into a computer system. Customer therefore benefits from the high reliability of computer.

The architecture of a consumed computer, used in our daily life, does not differ from each other significantly. Except for military purpose or for special applications where reliability requirement is extremely high, some manufacturers are not willing to put efforts to redesign the architecture and the reliability issues is not significant [3]. However, the intensive competition in the market result to the decreasing of computer price, as the manufacturer choose to reduce the cost to survive from the intensive competition and maintain their market share. The lower price implies the lower reliability of the components. Manufacturer will balance the reliability and the cost.

For this purpose, during the design of computer, reliability engineers play a key role to balance them. They choose proper suppliers with acceptable price. For non-critical system, e.g. fans, manufacturer just requests supplier to conduct accelerated life test and to provide estimated life. For critical component, e.g. the mainboard, manufacturer could conduct thermal test and accelerated life test by themselves. Later on, at system level, the manufacturer employs other strictly reliability test such as thermal test, falling test, vibration test, mean time between failure (MTBF) tests etc. This paper discusses the reliability assessment of the computer when it is in its design phase. The section II discusses the life test in the design phase. Section III reviews the mathematical methods for reliability assessment. Section IV presents a case study. Section V presents discussion.

## II. LIFE TEST IN DESIGN PHASE

When the design of the computer is completed, if the reliability is predicted to be able to reach the required level, manufacture will produce a small amount of computers. For these new computers, manufactures will conduct a life test to evaluate the life. As massive production has not started, the available number of manufactured computers is a small number. If the new computers can meet the requirement, massive production could proceed. This test is thus significant.

In this test, all the new computers are stored in the big walk-in chamber, similar to accelerated life test. However, the temperature and humidity level applied are just a little bit higher than the normal using condition. During this stage, most computers should be able to function without any failure during the test period, as the reliability have grown to certain level in this phase. However, few of them could still fail. If reliability level could not meet the requirement, manufacturer has to improve the design. If they deliver them to market with low reliability, the manufacturer will bear the heavy repair cost and the brand will be devalued, the manufacturer could lose market share.

The test duration prefers longer period. However, for normal consumed computer, longer period of test is impossible due to cost and the intensive competition in the market. The computer could be outdated even during its test period if the test period is too long. In mathematical term, the reliability test for this situation is right-censored, and it is highly censored problem, as most computers have not failed. It is a small sample size problem as number of computers on test is small. Analyzing reliability for such highly censored with small sample size is challenging. Data analysis method based on large number theory would lead to unreasonable results, and consequently it would lead to wrong decision later on.

From the failure mechanism perspective and from our previous experiences, ignoring the software fault, computer is composed of CPU, HDD, mainboard, power fan, CPU fan, screen, etc. The failure pattern of this complex electronic and mechanic mixture follows bath-tub curve. Initially, the failure rate will be high due to the improper installation, poor connection, or the damage of some component during transportation. Thereafter, the failure rate tends to be constant. The failure in that period is relatively rare and random. After the random failure phase, failure rate tends to be high again. It is due to the wear of some electro-mechanic units, for example, fan and HDD, etc. This paper neglects the wear out phase, as running the life test to wear out phase is costly, and it is not necessary. In our case, the censor rate could be as high as to 90%. For example, in our case, if the computer's failure rate takes 10% per year, the reliability after 1 year is still as high as approximate 90%. Truncating the test at end of one year, the censored rate is around 0.9. The problem of this case will be obviously high censored problem.

## III. REVIEW OF PARAMETER ESTIMATION FOR HIGH CENSORING RATE

As the number of failures during the production stage is low, most of the tests are stopped without finding any failure. This problem complicates the parameter estimation. Weibull distribution is the most widely used distribution for reliability and this distribution has been intensively investigated [4]. This paper uses Weibull distribution to fit the data. The CDF (Cumulative Distribution Function) of the Weibull distribution is written as follows:

$$F(t) = 1 - \exp\left(-(t/\alpha)^{\beta}\right) \tag{1}$$

Suppose the observed failure times are $t_1, t_2, \ldots, t_{r-1}, t_r$. The time terminating the experiment is $T$. The maximum estimator of the Weibull distribution considering the time-truncating situation is [5, 6]

$$\hat{\alpha} = \left(\frac{\sum t_i^{\hat{\beta}} + (n-r)T^{\hat{\beta}}}{r}\right)^{1/\hat{\beta}}; \tag{2}$$

$$\frac{\sum_{i=1}^{r} t_i^{\hat{\beta}} \ln t_i + (n-r)T^{\hat{\beta}} \ln \mathrm{T}}{\sum_{i=1}^{r} t_i^{\hat{\beta}} + (n-r)T^{\hat{\beta}}} - \frac{1}{\hat{\beta}} = \frac{1}{r}\sum_{i=1}^{r} \ln t_i; \tag{3}$$

For highly censored situation, the (2)(3) is still applicable. The advantage of this estimation method is it has a well-developed mathematical foundation. The variance of the estimators can be evaluated from fisher information matrix readily [5]. The goodness fit test method based on maximum likelihood estimation is also well developed. However, the performance of the estimation method for small and highly censored situation should be investigated.

Theoretically, maximum likelihood estimators in general are biased. The biasness could be significant for small data size, i.e., the estimator tends to deviate from true value far. From this perspective, the classical likelihood estimation method is not a good approach for small sample size [7-9]. The biasness is worsened for high censored situation, even the sample size is large [7].

Bayesian method is another popular method to address the small sample size problem. This method is advantageous when prior information of the unknown parameters is well known. This prior information is the prior distribution of the parameters of interest. The Bayesian approach is shown in (4)

$$\pi(\alpha, \beta/t) \propto L(t; \alpha, \beta)\pi(\alpha, \beta) \tag{4}$$

where the $\pi(\alpha, \beta)$ is the prior distribution, the $L(t; \alpha, \beta)$ is the likelihood function. A simple solution to solve the prior information problem is to use the non-informative prior distribution. When using non-informative prior distribution, the Bayesian approach is essentially a maximum likelihood estimation method. The Bayesian approach requires high computational cost to obtain the posterior distribution $\pi(\alpha, \beta/t)$ by using simulation, for which the Markov Chain Monte Carlo (MCMC) is the common method [7]. The MCMC has much lower computational efficiency than the maximum likelihood method.

Moment method is another method used in state of art. One moment method is to use the classical first moment and second moment in the Weibull distribution to estimate the unknown parameters. The k[th] moment of the Weibull distribution is given by

$$m_k = \alpha^k \Gamma(1 + \tfrac{k}{\beta}) \tag{5}$$

The moment estimator can be obtained from the equation

$$\begin{cases} \alpha\Gamma\left(1+\frac{1}{\beta}\right) = \frac{\sum t_i}{n} \\ \alpha^2\Gamma(1+\frac{2}{\beta}) = \frac{\sum t_i^2}{n} \end{cases} \tag{6}$$

where n is the sample size. For censored situation, the $t_i$ is the truncating time T. The solution to (6) requires numerical method. This method is simple, but rarely used in reliability engineering, and the discussion of performance of this moment method is rarely found in state of art.

Another method is proposed by the Yu and Peng [10]. This method is based on the Peaks-over-threshold (POT). This method using the generalized Pareto distribution to approximate the extreme distribution. The paper claims the proposed method is not worse than the ML method. The limitation of the method is it can only estimate the quantile of the reliability. It cannot be able to use to estimate the Weibull distribution parameters. Moreover, the method is applicable to Type II censoring situation [10]. Other methods such as least square method is also applicable

[11]. For complete data, least square method is not biased. However, for highly censored situation where most products under test do not fail, the least square method could have high error.

Yang and Sirvanci proposed another parameter estimation method by considering the Weibull distribution as extreme distribution [12]. This method is claimed it can outperform the maximum likelihood estimator and has good performance for the small sample size problem, and it is suitable for the type I censoring. Yuan elaborated this method and concluded the moment method can outperform the maximum likelihood estimation by simulation study[13]. This paper uses this moment method to evaluate the data obtained from life test in the lab.

## IV. EVALUATION ON COMPUTER RELIABILITY

In order to assess the reliability of the newly developed computer. Manufacturer conducted a life test which dated as June, 16th, 20XX. As these new computers are newly manufactured and the available number of computers is only 18. Large scale producing will proceed if the reliability can satisfy the reliability requirement. The test duration last around 1 years. It terminates at 309th day. The failure is defined as: if any error shows in the operation system, the computer is considered failed. The test results found that total 4 out of 18 computers failed during the test. Table I shows part of the original data.

Table I
TIME TO FAILURE OF COMPUTERS

| No. | Start Date | Failure Date | Duration in Days |
|---|---|---|---|
| -- | -- | -- | -- |
| 2 | 16.06.20xx | / | 309 |
| 3 | 16.06.20xx | / | 309 |
| 4 | 16.06.20xx | / | 309 |
| 5 | 16.06.20xx | / | 309 |
| 6 | 16.06.20xx | 20xx-06-16 Down | 1(Failed) |
| -- | -- | -- | -- |
| 10 | 16.06.20xx | 20xx-12-14 down and unable to start up | 180(Failed) |
| -- | -- | -- | -- |

The sample size for this case is 18. The censoring rate is $\frac{14}{18} \approx 78\%$, which is above 50%. It is highly censored setting. The moment estimator method is applied to estimate the parameters. Estimators are calculated from (7) and (8) for shape and scale parameter respectively [5].

$$\hat{\beta} = \frac{nh(\hat{p})}{\sum_{i=1}^{n} I(\ln T - \ln t_i)} \quad (7)$$

and

$$\hat{\alpha} = \frac{T}{[-\ln(1-\hat{p})]^{1/\hat{\beta}}} \quad (8)$$

The estimated parameter values are shown in Table II. The confidence intervals are obtained at 98% significance level that is calculated from (9) and (10) [5].

$$[\alpha_L, \alpha_U] = \left[\exp\left(\hat{\mu} - z_{\left(1-\frac{\alpha}{2}\right)}\sqrt{Var(\hat{\mu})}\right), \exp\left(\hat{\mu} + z_{\left(1-\frac{\alpha}{2}\right)}\sqrt{Var(\hat{\mu})}\right)\right] \quad (9)$$

$$[\beta_L, \beta_U] = \left[\frac{1}{\hat{\sigma} \times w}, w/\hat{\sigma},\right] \quad (10)$$

The CI is loose due to the small sample size.

Table II
ME ESTIMATOR FOR WEIBULL DISTRIBUTION

| | Estimator | Upper bound (Significance Level 98%) | Lower bound (Significance Level 98%) |
|---|---|---|---|
| $\hat{\alpha}$ | 15,948 | 6,497,833 | 39 |
| $\hat{\beta}$ | 0.35 | 1.6 | 0.08 |

Results show the shape parameter $\hat{\beta}$ is less than 1. The hazard function is deceasing against time. As shown in the Fig. 1, the computer's failure rate is high in the initial days and then approach to constant. This failure rate represents the first two stage of the bath-tub curve. Hazard function is high in the first stage due to improper installation, connection and some cases are due to the improper transportation. The failure rate of second stage is constant. The occurrence of failure in that stage is seldom. In this case, for these computers, the hazard rate in the second stage is approximately 0.006 per month. The third stage is not considered in this paper as the life test is terminated before the computer reach to its wear out stage.

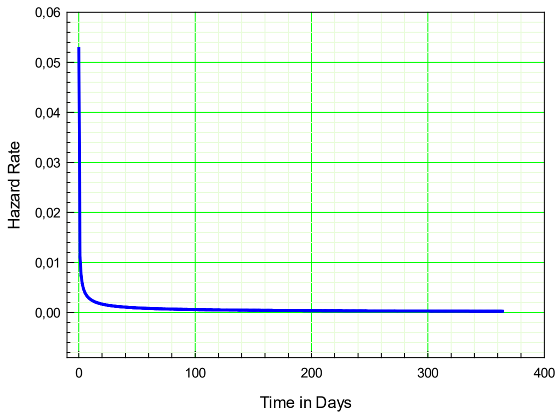


Fig. 1. Hazard Rate Function of the new Designed Computers

The probability density function is plotted in Fig 2. It can see that roughly the PDF is almost zero after 100days. The failure rate plot shows the same pattern. The results are consistent with the previous experience. Computers are a complex system. The most failure will show in the beginning of its service life. Once the computer can survive the initial service life, it is more likely to survive for a long time, normally 2-3 years without any dysfunction. However, at the end of its service time, the number of failures is more likely to increase. The common causes of computer failure in wear out state are due to failures of fan from the power supply system, keyboard and hark driver disk. As most customers replace its computers within 3 years, some manufacturers mainly concern the problems within the first three years to save cost.

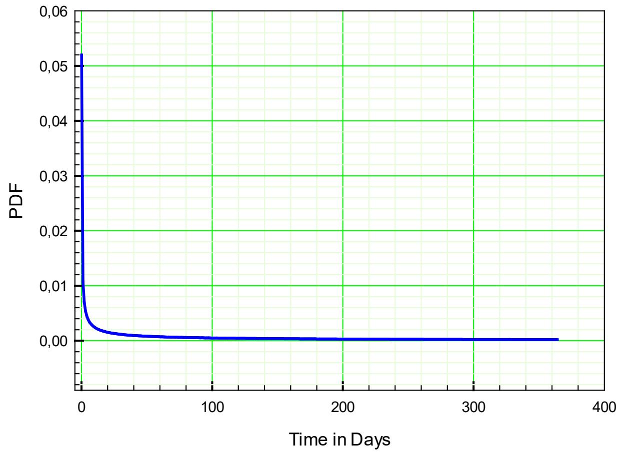

Fig. 2. PDF of the new batch computers

The predicated reliability is another issue the manufacture concerns. After obtaining the estimator for the model, the reliability function against time is

$$R(t) = e^{-(t/15948)^{0.35}} \quad (11)$$

The value for the discrete five years is tabulated in Table III. The reliability for the first year is very important, as for common customer, most manufacturer nowadays provides one-year free guaranty. For these computers, after one year, the reliability reduced to 77%. Manufacturer can evaluate if the reliability 77% is acceptable, by considering other factors such as cost. The upper and lower bound of the reliability is calculated from [13].

$$[R_L(t), R_U(t)] = [\exp(-expU_U), \exp(-expU_L)] \quad (12)$$

The expected reliability and its confidence interval are plotted in Fig. 3. As shown in Fig. 3, the confidence interval become large with time. The duration of the test life is truncated within one year, so the larger the time deviated from the truncating time, the less of the life data contains information of this time. The confidence interval will become bigger as well. However, predicting reliability in long term, for example 10 years is not necessary. One reason is the computer is outdated after 10 years, and most customers replaced their computer before 10 years. Another reason after 10 years, the failure rate of the computer could reach the wear-out stage, where the Weibull distribution is not applicable.

TABLE III
RELIABILITY PREDICTION

| Year | R | R Lower | R Upper |
|---|---|---|---|
| 1 | 0.766 | 0.733 | 0.775 |
| 2 | 0.712 | 0.475 | 0.761 |
| 3 | 0.676 | 0.288 | 0.753 |
| 4 | 0.649 | 0.167 | 0.747 |
| 5 | 0.626 | 0.09 | 0.741 |

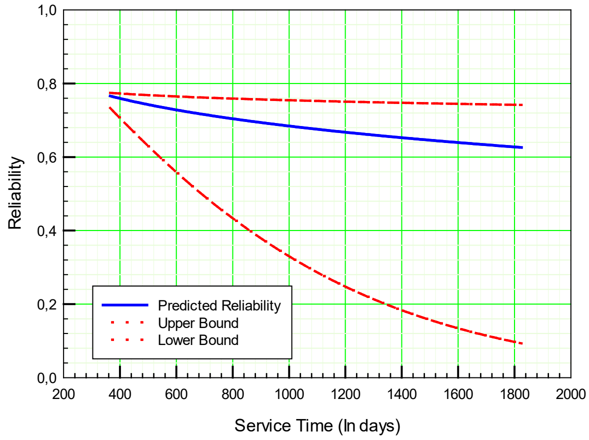

Fig 3. Estimated Computer Reliability against Time

Another desired application of reliability is that one can apply it to estimate the expected number of survived computers at given time point. Suppose number of N of computers sold into market. The probability of a computer survived at time t is the $R(t)$ discussed above. Based on the Bernoulli distribution, the mean of survived number of computers is then $\widehat{N} = N \times R(t)$. The lower bound at $100(1-\alpha)\%$ confidence interval is $\widehat{N} = N \times R(t) - z_{(1-\frac{\alpha}{2})} \times \sqrt{N \times R(t) \times (1-R(t))}$. If initially total 1000 computers are sold out and the reliability function used is (11), Fig 4 shows the expected survived computers against time in days.

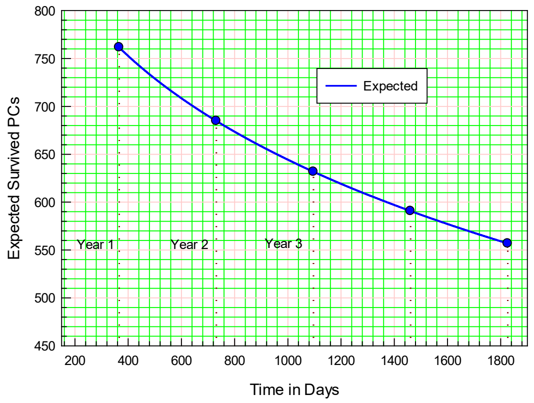

Fig 4. Estimated number of survived Computers against Time

Numerically, after 3 years, the estimated number of survived computers is around$\widehat{N} = 632$. The lower bound of survived number of computers is 596. Almost half of the computers are estimated to fail within 3 years. Decision maker can consider conducting screen test to filter the defected computer to pass the initial stage in the bath-tub curve. But one has to be aware that the failure data do not differentiate the failure from the operation system and the failure induced by hardware. If excluding the failure from operation system, the results could be much better.

## V. CONCLUSION

The analysis results demonstrate the failure rate of the desktop computer exhibits a deceasing failure rate in its early stage. The failure rate tends to constant and is estimated as low as 0.6% per month at end of the first year. The reliability is as around 77% at the end of first year, which is not high, while the failure takes account of the software failure induced by the operation system. The reliability of sole hardware should be much higher than that number. Our future work will focus on to develop a method to narrow the confidence interval to improve the credibility of the analysis results.

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