# Analysis of Civil Engineering Infrastructure in Norway with Solutions Based on Structural Health Monitoring and Artificial Intelligence

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**Abstract**

Ageing civil infrastructures such as bridges and building causes many consequences from practical and economical point of view. Especially in northern Norway, the impact of extreme arctic conditions is intense on civil engineering infrastructures (see [39]). With the increased loading due to the prosperous seafood industry and increased cargo activity is putting additional pressure on the aged infrastructures. Research and development of new methods is needed for the damage detection in these structures. In this paper we present, discuss and analyze the situation concerning bridges in Norway with a special focus on northern Norway. Moreover, based on the research in [40], [41] and [42] we describe and emphasise the importance of structural health monitoring methods, artificial intelligence and machine learning when trying to solve these serious problems of structural damage detection especially in arctic regions.

**Keywords:** Structural health monitoring, Vibration analysis, Operational modal analysis, Damage detection, Bridges, Arctic conditions, Finite element model, Signal processing, Wavelet transform, Artificial intelligence, Machine learning, Neural network, Wavelet network and Statistical methods.

**AMS Classification (2010):** 35A22, 45A05, 44A15, 65T50, 65T60

## 1 Introduction

This paper is based on the recent Ph.D. thesis [40], especially the papers [39], [41] and [42], where we highlighted the importance of using artificial intelligence and machine learning for damage detection in structures such as bridges and high-rise buildings. In this paper, we investigate the scale of ageing bridge infrastructure in Norway with focus on the special problems appearing in arctic regions.

Signal processing plays a very important role in extracting the features from the data generated by the sensors to find damages. Therefore, signal processing is like the heart of structural damage detection methods. With the continuous advancement in technology new techniques are under development for structural health monitoring (SHM)

Some new artificial intelligence (AI) and machine learning (ML) algorithms that are of importance for structural health monitoring (SHM), operation modal analysis (OMA) and finite element (FE) model updating are discussed in this paper. In fact, AI algorithms/techniques such as Deep Learning, Long Short-Term Memory and Ant Colony Optimization were briefly discussed in [9], [45], [46] and [47] for various smart city applications involving time series analysis and flow distribution. These algorithms can be crucial for further development of smart SHM solutions in the future.

This paper is organised as follows: In Section 2 we present and briefly discuss the huge problems caused by the ageing infrastructures in Norway with a special focus on the situation of all bridges in northern Norway. Section 3 covers the topics of SHM, OMA, FE model updating and damage detection. In Section 4 an overview of the current state-of-the-art of SHM with AI are presented. Machine learning, neural network and recurrent neural network are presented with a focus on structural damage detection. Finally, in Section 5 we present some concluding remarks including some suggestions of future research in this important area.

## 2 Aged civil engineering infrastructure

Damages in structures occur during its operational lifetime due to various environmental or human factors. Lack of maintenance and monitoring can lead to accumulation of damages with time that can significantly decrease the performance of the structures, change in natural symmetry or even destruction. In general, civil engineering structures are designed with a lifetime of 50 to 100 years. In this lifetime, structures are assumed to meet the expected structural integrity. But in general, the structures are prone to unpredictable and unexpected damages arising due to various factors in the lifetime of a structure.

Ageing of civil engineering infrastructures such as bridges, tunnels and buildings cause many problems with great consequences, both from practical and economical points of view. Governments and municipalities around the world have to spend more time and budget for maintenance, repairs or construction of new structures in place of deteriorated or damaged ones, so the citizens can have a decent service.

Infrastructure maintenance costs for the governments around the world are on the rise, as a lot of infrastructures around the globe are approaching towards the end of its life cycle. Moreover, due to scarcity of expert work force to analyze such challenges, it is adding up to the problem. Analysis of such problems is important e.g. in northern Scandinavia, since such problems are even more serious due to the fact that the impact of extreme arctic conditions is quite intense (see e.g. [40]).

A Norwegian newspaper Verdens Gang (VG) got access to a report published by Statens Vegvesen (The Norwegian Public Roads Administration) in 2017. According to this report there are approximately 16,791 bridges in Norway and Statens Vegvesen have been violating inspection rules for many of them. It was discovered that for one of every two bridges, proper inspection is lacking. Moreover, approximately 1087 bridges in Norway have damages that are described as serious or critical according the internal classification system of Statens Vegvesen (see [48]). In April 2022 it was revealed by the government-

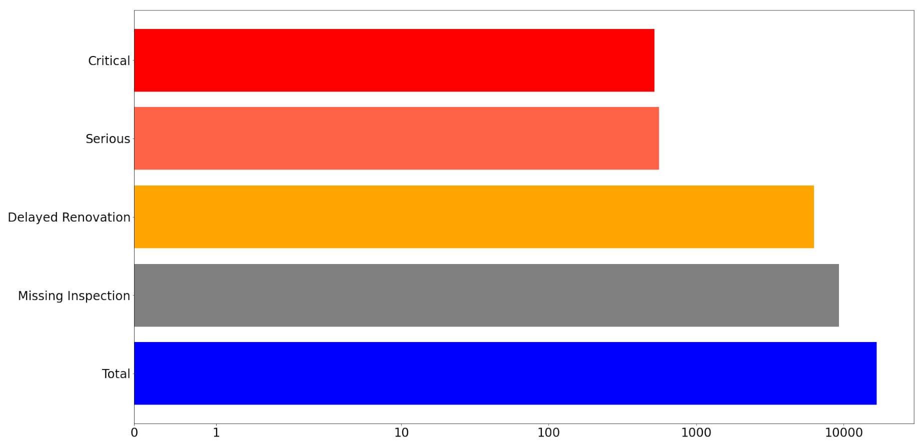


Figure 1: Histogram of state of Norwegian bridges in logarithmic scale.

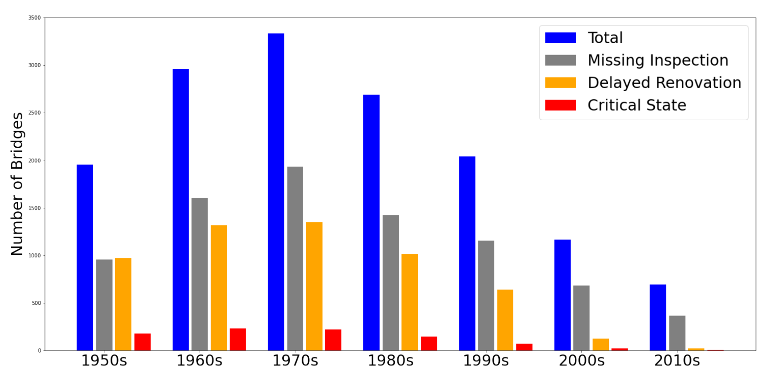


Figure 2: Histogram of Norwegian bridges, grouped by build decade.

owned broadcasting company NRK that around 1000 bridges in Norway were still not up to standard (see [31]).

The newspaper VG also created a map of bridges in Norway, using data acquired from the internal database Brutus used by Statens Vegvesen, which uses classifications such as seriously injured, delayed renovation and lacking inspection (see [49]). From the analysis of this data that is available on the VG website, an analysis of all the bridges in Norway is done and presented in logarithmic scale in Figure 1. Classification is made with respect to bridges that are missing inspection, delayed maintenance action, serious and critical that need action. Moreover, in Figure 2 a histogram of Norwegian bridges classified over different decades, is presented.

In northern Norway, large amounts of seafood cargo is exported along the public roads. The seafood industry of Norway, as of 2021, exported for 12 billion euros and contributed to around 10 percents of Norwegian export earnings. The seafood industry has seen 7 percent year on year growth since the year 2000, essentially doubling every ten years (see [13] and [29]). The seafood industry is expected to keep growing at the same rate, and has already in the first part of 2022 seen record growths of 20 percent year on year (see [34]). Especially in the sparsely populated northern Norway this is expected to put ever increasing loads on already struggling infrastructure. Thus the ageing infrastructure has to be tested and maintained with respect to the increased loading. A down time or failure of any such infrastructure can lead to substantial economic losses and even human lives.

A detailed study is conducted in this paper that maps the clusters of bridges in Nordland, Troms and Finnmark county where the large parts of the seafood industry is concentrated. A graph of all the bridges in northern Norway is presented in Figure 3 and Figure 4. As we know from geographical constraints,

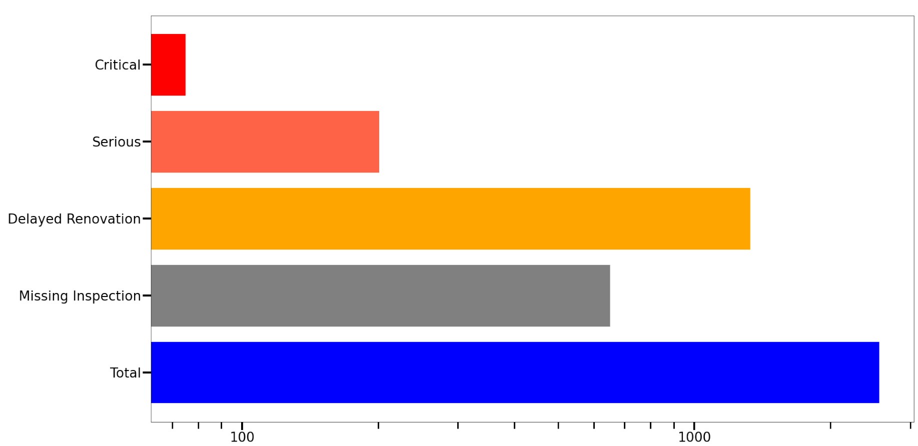


Figure 3: The current state of bridges in the two northernmost Norwegian counties – Nordland, as well as Troms and Finnmark.

Norway is very long, so in order to have better visualization the area of focus that is from Nordland to Finnmark is split into 3 regions. These figures illustrate the extent of problem in northern Norway. Figure 5 is from latitude 57 degree (border of Nordland and Trøndelag) till latitude 66 degree passing Bodø. Figure 6 covers the geographical area from from latitude 66 degree from Bodø till latitude 68 degree. Finally, Figure 7 cover Finnmark from latitude 68 degree Tromsø till 69 degree that is Nordkapp. The hexagon blocks indicate the density of bridges that lack inspection while red circles represents the bridges in critical or serious state.

With this in focus Statens Vegvesen has put a higher priority to investigate and do maintenance of bridges that are critical or are seriously damaged. In the year 2019, a major damage was found in the construction of the Herøysund bridge located on the west-coast in Nordland county in Norway (see Figure 8). As a result concerned authorities decided that the special transport was no longer allowed to drive over the bridge (see [3]).

Later in 2020, Nordland county and the Norwegian public road administration decided to work on building a new bridge that would be located just south of the current bridge. The new Herøysund bridge is expected to cost about 270 million NOK and is expected to be finished in the summer of 2024. Moreover, it was decided that the maintenance and reinforcements will be carried out on the Herøysund bridge so it is safe to use until the new Herøysund bridge opens.

**Remark 2.1** A detailed study of the Herøysund bridge will be presented in our forthcoming article. This is possible because we have the concrete data for this case. By doing this we can do a similar analysis for all other bridges in the region of northern Norway. For a more detailed description see Remark 5.1.

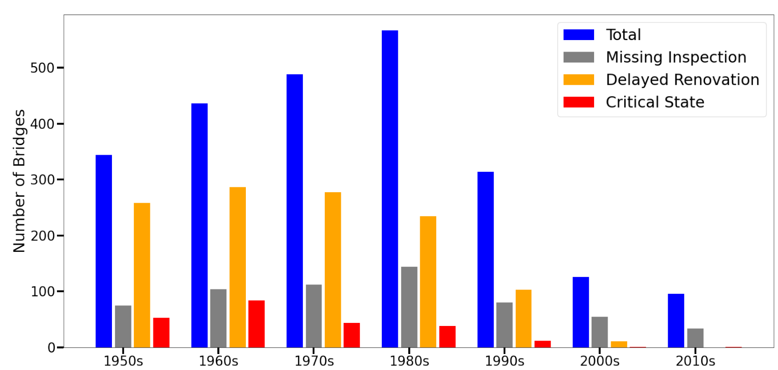


Figure 4: The same data as in Figure 3, ordered by the decade the bridges were built during.

The remaining part of this paper is inspired by the recent research presented in the PhD thesis [40] where some methods presented can be used to overcome these serious problems in northern Norway. In the next section we present structural health monitoring (SHM) along with the importance of artificial intelligence in SHM.

# 3 Structural health monitoring

Operators/owners of civil engineering infrastructure such as bridges, dams and tunnels are mostly municipalities or government owned enterprises in Norway. As for now, infrastructure assets management decisions are based on visual inspections, which could be aided by localized diagnosis techniques such as the

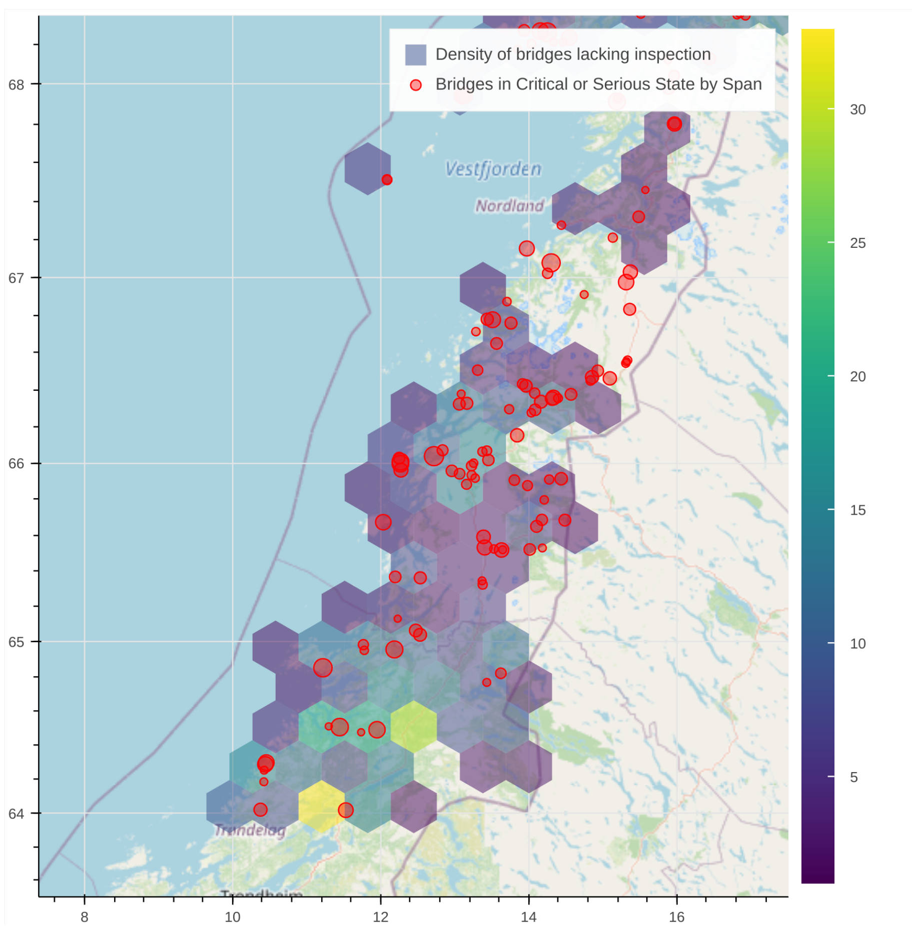


Figure 5: The bridges in southern and middle part of Nordland county.

use of acoustic, ultrasonic or magnetic field non-destructive testing methodologies. Nevertheless, these testing methodologies have several limitations such as, inaccessibility to some parts of the structure, inability to detect internal damage, localization of the damage, and it is challenging to carry out continuous monitoring with such techniques.

With the advancement in technology, new techniques are under continuous development for the monitoring of structures. These techniques are commonly called structural health monitoring (SHM) techniques. SHM refers to the process of systematizing, implementing and characterizing a damage detection strategy in civil, mechanical and aerospace engineering structures (see [12]). The process involves the observation of structure over the course of time with periodically spaced static and dynamic response measurements, extraction of

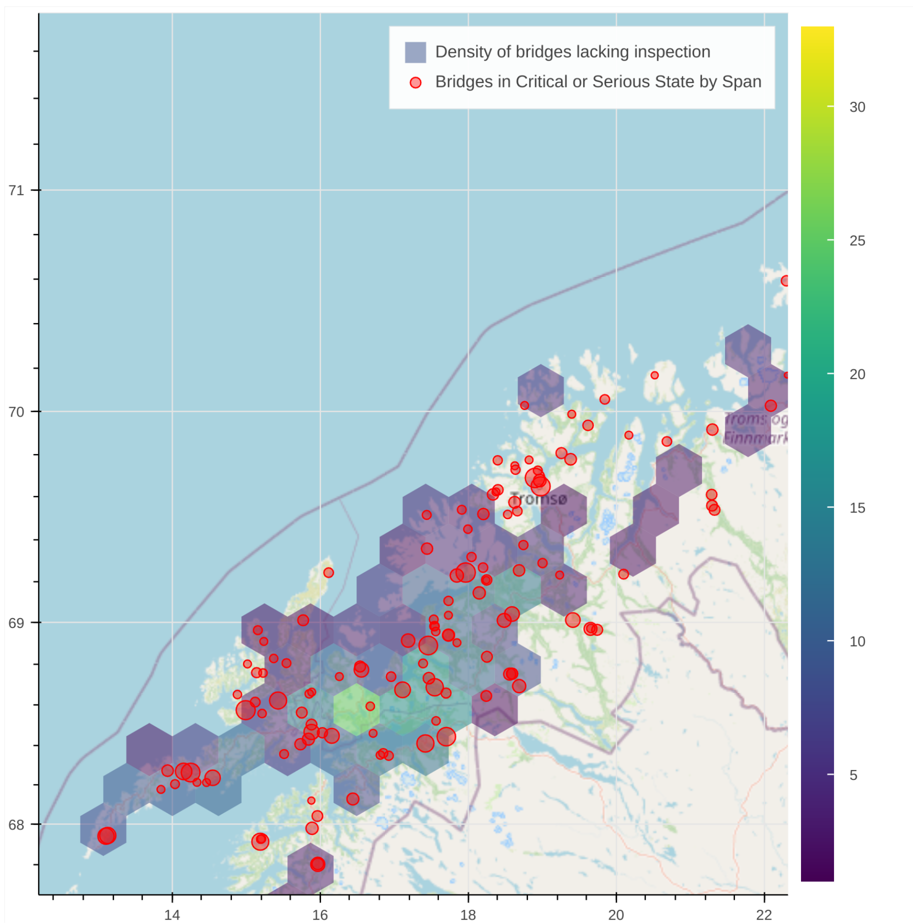


Figure 6: The bridges in the northern Nordland county and southern Troms and Finnmark county.

damage sensitive features from measurements and finally statistical analysis of these features to estimate the current state or health of the structure.

In a typical SHM system sensors are distributed throughout the structure, that are used to estimate the condition of the structure. A damage is defined as an intentional or unintentional change to the material or geometric properties of the structures, including the changes in the boundary conditions or system connectivity which adversely affect current or future performance of the structures (see [12]).

In order to do damage detection and localization, the raw data generated by sensors is processed for extraction of damage sensitive features. For example in a vibration based SHM system, accelerometers are used to find the key parameters: mode shapes, mode frequencies and mode damping. Once these parameters have been estimated, damage detection algorithms can be utilized

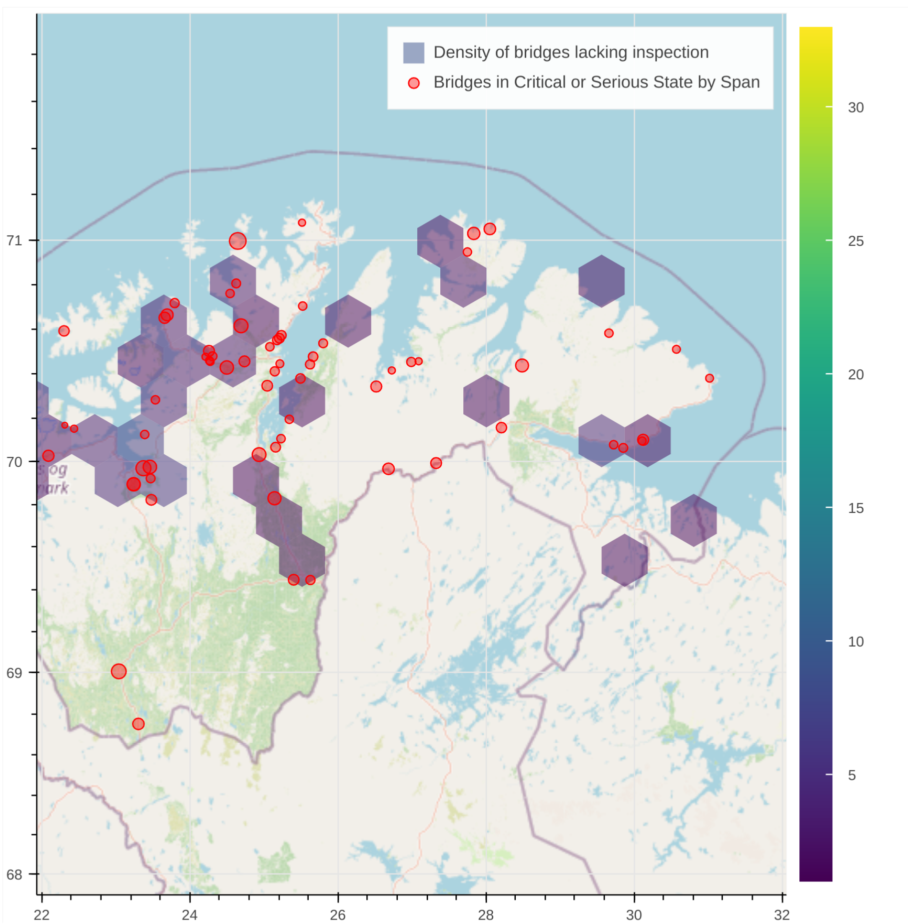


Figure 7: The bridges in the northern Troms and Finnmark county.

to figure out the magnitude of damage occurred, if any (see [17] and [21]).

The ordinary differential equation that represents a linear dynamical model of a vibrating system is as below (see [33] and [42]):

$$M\frac{d^2u(t)}{dt^2} + C_2\frac{du}{dt} + Ku(t) = B_2f(t). \tag{1}$$

The different terms in the equation are described as: $M$ is the mass matrix, $C_2$ is the damping matrix, $K$ is the stiffness matrix, $B_2$ is the selection matrix (input matrix), $f(t)$ is a vector with nodal forces and the solution $u(t)$ of this differential equation is the vector with nodal displacements (see [33]). A mathematical model to compute the modal parameters is described in detail in [33] and [42].

In general traditional forced vibration tests with artificial excitation forces can be performed on large structures, but such tests are costly and complicated. Moreover, other vibration sources such as wind and traffic are treated as noise.

Finite element (FE) model updating is one of the most popular methods nowadays to improvise the numerical models for various civil engineering structures such as bridges, high-rise buildings, and mechanical structures such as steel bridges, wind mills, off-shore structures, etc. Moreover, the FE model is updated and improved by updating the numerical response with respect to the

Figure 8: Herøysund bridge.

observed experimental behaviour of the structure (see [18]). FE updated model is also known as a digital twin. It is very crucial to have correct information about the structure. In this context OMA and FE updated model both play very important roles.

Operational modal analysis (OMA) is gaining popularity where ambient vibrations from the wind and traffic are considered as unknown input, and output-only analysis is done to determine the resulting vibration modes. OMA technique has been tested on a steel truss structure bridge in Luleå (see [41]), a high-rise fire tower building in Luleå (see [42]) and various other structures around the world, see also the PhD thesis [40]. OMA are Multiple Input and Multiple Output (MIMO) techniques, so these techniques can estimate the closely shape modes and repeated modes for a high degree of accuracy. Single Input Multiple Output (SIMO), Multiple Input Single Output (MISO) and Single Input and Single Output (SISO) are traditional testing procedures that are not able to find repeated poles due to lack of mode separation.

**Remark 3.1** In the new upcoming project the plan is to test this OMA technology on the old Herøysund bridge that is going to be demolished in 2024, and the new Herøysund bridge that is going to become operational in 2023 in the county of Nordland in Norway.

Statistical pattern recognition paradigm for SHM is a model where a comparison between two different states of the structure, one being initial/normal or undamaged state and the other being the damaged or a state with defects is made. For example in case of a bridge, a label of critical damage, need of inspection or need of maintenance can be assigned by comparing the bridge to a database of healthy bridge. This database can be accumulated over the time and be used for training a mathematical framework of machine learning algorithms. This will further be discussed and described in the forthcoming paper.

The sensors used for SHM generate lots of data, thus signal processing techniques makes the heart of SHM. Various signal processing techniques that are of great importance for SHM and OMA have been discussed and compared in our previous article (see [42]). Wavelet analysis is an effective mathematical and signal processing tool that is based on time frequency analysis and overcomes some of the limitations of conventional Fourier analysis based methods. In our forthcoming article we will focus our discussion on wavelets in the context of damage detection and artificial intelligence. However, for the readers convenience already here in Appendix A we give some historical remarks and newest development of wavelets from the first Haar wavelets till the remarkable development described in more the 50 books.

**Remark 3.2** SHM systems have very many sensors installed, so the challenge of synchronization of data due to various sampling rates appears naturally. Further, the issue of missing data can appear due to various factors such as sensitivity of sensors or other environmental factors such as low wind or failure to record data. In a SHM system problem of data synchronization and missing data appears naturally due to various types of sensors that are involved (see [18]). Recent work in machine learning aims to alleviate such issues, which will be addressed in a forthcoming article.

# 4 Artificial Intelligence and Machine Learning

While the terms are often used interchangeably, Artificial Intelligence (AI) and Machine Learning(ML) have different meanings, and an important relation between each other. The term AI refers to any algorithm that can be used to make a machine (or computer) perform a task. This range of tasks is enormous, and encompasses anything from simple path-finding algorithms to advanced autonomous drones. As such, the term ML is included under the AI umbrella. Where ML differs from other types of AI can be suggested from the name. ML algorithms are able to learn from data. This data can be collected from many sources, including, but not limited to, real-world sensors or images (see [26]), simulated worlds (see [38]) or data created by humans such as text (see [6]). Typically, machine learning is divided into the three paradigms supervised, unsupervised and reinforcement learning. For the purposes of this paper, we present supervised and unsupervised learning.

## 4.1 Supervised learning

Supervised learning is a machine learning paradigm which aims to learn a function that maps a set of input data points to a set of target data points. This function should generalize well and also should be able to make good predictions for unseen data points. The problem of supervised learning is often solved by finding the closest points in the input space to the target points using a distance function, i.e., by finding the nearest neighbor of each data point. The intuition is that the predicted target point will be the nearest neighbor of the data point, which is closest to the data point.

## 4.2 Unsupervised learning

Contrary to supervised learning, unsupervised learning algorithms do not require labeled data. The goal of unsupervised learning is to learn the structure in the data, such as grouping some examples together, or finding similar examples. For instance, an unsupervised clustering algorithm, such as K-means, can automatically partition a dataset into different groups. It is important to note that the quality of the results produced by unsupervised learning algorithms are typically lower than those of supervised learning algorithms.

Next, we introduce neural networks, which is a very important and common machine learning model. Countless variations and improvements exist, but we explain the basic version which is the foundation for more advanced models.

## 4.3 Neural networks

Neural networks are models of computation inspired by the structure and function of biological nervous systems, including the brain. They are a paradigm of machine learning and are used in the development of artificial intelligence. A common choice for the structure of a neural network is the layered feed-forward network, in which a set of input nodes receives data from a set of previous nodes, which receive data from another set of nodes, and so on; the last layer is called the output layer. In the layered feed-forward network, information flows only in one direction, which is called the "feed-forward" direction. Information can spread out to the output layers by activating all the nodes in the layers between the input and output layers. Each of the nodes in the network has a strength and all the nodes are connected with each other. The networks are trained by calculating the error of the network, which is the difference between the desired output and the output of the network, using back propagation (see [35]).

Consider a network $N$ with $w$ connections, $x$ inputs and $y$ outputs. NNs are function approximators, and as such can be expressed as a function $y = f_N(w, x)$. The weights $w$ maps the inputs $x$ to the outputs $y$. The weights $w$ are usually provided from a random distribution, while the inputs $x$ are the data the network is trying to learn from, such as sensor data, images or signals. The outputs $y$ is then the variable or classification the network is optimizing towards.

Given an input sample $p_j(t)$, for each neuron $j$ in the network, its contribution to to the outputs $o_i(t)$ can be described as:

$$p_j(t) = \sum_i o_i(t) w_{ij}. \tag{2}$$

where the elements $w_{ij}$ in the matrix $[w_{ij}]$ represents the intermediate product between each layer.

Next, we describe some variations and improvements on neural networks that are of importance for upcoming section describing AI in the SHM field.

### 4.3.1 Recurrent neural networks

Recurrent Neural Networks (RNNs) are a variation of neural networks that include a sequential – or looping property. As such, they can be used to predict the next element of a sequence. They are especially useful for modelling sequences. The most common and famous RNN variation is called the Long Short-Term Memory (LSTM) (see [20]), which was designed to address the vanishing gradient problem found in earlier versions of the RNN. The vanishing gradient problem appears in cases where the weights of the network become so small that they effectively will not change. As such, the network will not train. This problem is improved by improving learnable gating mechanisms in the network, which allows better control over the information flow.

### 4.3.2 Residual Networks

Inspired by the architecture of the LSTM, one of the motivating factors for the Residual Network (ResNet) was to avoid the vanishing gradient problem. However, it is not the preservation of sequentiality that is the main goal, but rather to make it easier to train and optimize very deep neural networks. This is achieved by utilizing skip connections – basically meaning that the neurons do not only have to communicate between directly neighbouring layers, but can also communicate between distant layers. Like with the LSTM, there are also gating mechanisms to control information flow (see [19] and [44]).

### 4.3.3 Generative Adversarial Networks

The Generative Adversarial Network( (GAN) is in fact not a network architecture, but rather a protocol that two neural networks use to generate new data. The training data is then used as a statistical baseline for the generation of new data. Indeed, there are two networks, the *generator* (generative network) and the *discriminator* (discriminative network), both which having different, competing goals. The generative network creates data samples (by guessing from a sample distribution) that is then evaluated by the discriminative network (which knows the full distribution). As such, the generator will incrementally get better at emulating the true data distribution. This will lead to the generated samples becoming more and more like the true distribution. This technique can be especially effective for generating synthetic data (see [15]).

## 4.4 AI techniques for SHM and vibration analysis

With the recent technological advances in computer vision, artificial intelligence (AI), and machine learning (ML), we are now witnessing a new era of computer-based automated systems in the damage detection of facilities, infrastructure, and vehicles. In this subsection, we review the use of AI and ML for automated detection of damage in the SHM and OMA spaces.

Combinations of wavelets and ML approaches such as NNs have been explored in the SHM space, with the most basic approach being to first transform the vibration signal with DWT, and then using the NN to train on the transformed signal (see [37]).

A recent study presents a novelty-classification framework applicable to SHM problems. LSTMs are utilized to perform the classification. Then, a GAN and its generated data objects are used to improve the low-sampled data class classification (see [43]). Similarly, various deep learning approaches are explored for automated Structural Damage Detection (SDD) during extreme events. Among the approaches are ResNet for classification. ResNet is also combined with a segmentation network for categorizing and locating structural damage (see [4]). The applicability of Transfer Learning (pre-trained image models) to SHM problems shows both promise and concern (see [7]). Unmanned Aerial Vehicles combined with computer vision and deep learning has been shown to be a fast, cheap and effective means of SHM for civil infrastructures (see [32]). Data sets from the construction industry is also used to benchmark machine learning architectures, such as in the paper [25] where a large amount (56,000) of images of cracks in concrete were used for training a novel algorithm for crack detection. A Deep Belief Network (a NN that only has connections between layers, but not between neurons) was used, and the configuration of neurons and layers in the network was self-organized using Adaptive Restricted Boltzmann Machine.

Clearly, recent years has seen much work in the SHM space with regards to techniques involving AI and computer vision. However, less work can be found involving AI and vibration analysis from measured sensor data. The work in this field has been more concerned with traditional statistical and mathematical models.

However, some work combining ML with vibration analysis in the SHM space has appeared, recently. An ensemble deep learning technique that combines a Convolutional NN with Dempster-Shafer theory (DST) is proposed, and called CNN-DST. The framework shows robust performance compared to other state-of-the-art classification methods (see [50]). GANs have also been used for synthetic data generation in the context of vibration analysis for SHM (see [27]).

# 5 Concluding Remarks

**Remark 5.1** In the new upcoming project the plan is to test this OMA technology on the old Herøysund bridge that is going to be demolished in 2024, and the new Herøysund bridge that is going to become operational in 2023 in Nordland county in Norway. The main aim is that this can essentially help us for the better understanding of bridges with similar issues and take precautionary steps before the damage in bridges can become serious. In this forthcoming article we describe more details concerning this important motivation.

**Remark 5.2** In the recent PhD thesis [40] some new statistical and mathematical results were stated and proved, which hopefully can be useful in the required improvements of the traditional methods in this area of structural health monitoring and artificial intelligence. So far a lot of development has been done in the bounded systems for Fourier analysis and inequalities. Further development of new Fourier analysis techniques (see [5]) and inequalities also in unbounded orthogonal systems (see [1] and [2]) and signal processing problems in nonseparable function spaces (see [36]) can provide or help in the improvement of the signal processing techniques used for the damage detection in suspension bridges and related structures.

**Remark 5.3** In our new paper we aim to analyze the bridge by using the methods above and also the new theoretical findings in the Ph.D. Thesis [40]. It is especially important to note that the wavelet system (see Appendix A) is unbounded and the traditional theory of Fourier inequalities do not cover this case. In the recent Ph.D. thesis also some new statistical methods were stated and applied. In particular, non-separable function spaces were used (see [36]), which can further help to tackle similar problems for bridges, especially in northern Norway.

**Remark 5.4** The literature search reveals that the intersection of AI/ML and SHM has a long history and many important studies has been conducted in this field. However, the use of AI/ML combined with vibration analysis applied to SHM still needs further research.

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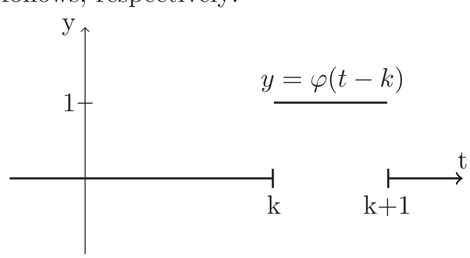
# A On Wavelet Theory

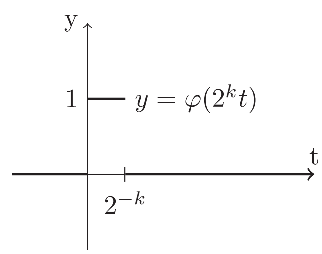
There exists both a discrete version (comparable with Fourier series) and a continuous version (comparable with Fourier transforms) of Wavelets Theory. The discrete version can be described as follows:

The Classical Haar Mother Wavelet $\varphi$ and first Haar Wavelet $\psi$ are defined as follows:

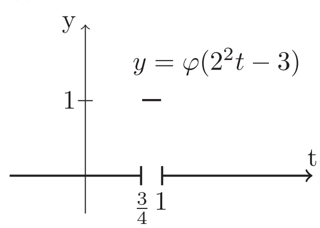
$$\varphi(t) = \begin{cases} 1, & 0 \leq t \leq 1 \\ 0, & \text{elsewhere} \end{cases} \qquad \psi(t) = \begin{cases} 1, & 0 \leq t \leq \frac{1}{2} \\ -1, & \frac{1}{2} < t \leq 1 \\ 0, & \text{elsewhere} \end{cases}$$

The translations of $\varphi(t-k), k \in \mathbb{R}$ and $k : t$ dilations of $\varphi$ can be represented as follows, respectively:




We can also combine Dilation and Translation, as follows:




The series $\sum_0^\infty \alpha_k \varphi_k(t)$ is called the Haar series of $f(t)$. And since the system $\{\varphi_n\}$ is orthonormal, from the general Fourier theory it follows that $f(t)$ can be reconstructed exactly as follows:

$$f(t) = \sum_0^\infty \alpha_k \varphi_k(t)$$

from its "basis functions"

$$\varphi_k(t) = 2^{\frac{n}{2}} \varphi(2^n t - k)$$

and the corresponding Haar(-Fourier) coefficients

$$\alpha_k = \int_0^1 f(s) 2^{\frac{n}{2}} \varphi(2^n s - k) ds \ .$$

**Remark 1** Wavelets are functions that slice data into differing frequency components. As such, the scale and resolution will match for each component. This means that wavelets can accommodate both large and small features, depending on the scale and resolution. Wavelets are better at handling signals containing discontinuities and sharp spikes compared to traditional Fourier methods.

**Remark 2** These original Haar wavelets can be variated in various ways e.g. involving different mother wavelets. The different wavelet families make different trade-offs between how compactly the basis functions are localized in space and how smooth they are. The *Daubechies wavelet* family is one such example. Usually, each wavelet in a family is named after the number of vanishing moments it contains. A vanishing moment is a rigorous mathematical term that relates to the number of coefficients a wavelet has. The more vanishing moments, the higher complexity can be represented by the scaling function. For applications even more general discrete wavelets are used in different programs. In the basic cases the function space $L^2$ is used. But for applications it is sometimes important to consider more general function spaces like Besov spaces.

**Remark 3** In recent decades, wavelet methods have shown themselves to be of considerable use in Fourier analysis and related applications. The strength of wavelet methods lies in their ability to describe local phenomena more accurately than the traditional expansions in sinus and cosinus can. This is because wavelet functions are localized in space. Thus, wavelets are ideal in many fields where an approach to transient behavior is required, for example, in considering acoustic or seismic signals, image processing, damage detection in bridges (see [40]). For applications that are even more general, discrete wavelets are used in even more different applied fields such as astronomy, nuclear engineering, subband coding, signal and image processing, neurophysiology, music, magnetic resonance imaging, speech discrimination, optics, fractals, turbulence, radar, human vision, and pure mathematics applications such as solving partial differential equations ( see e.g. [16]).

**Remark 4** As mentioned above, there exists also the continuous wavelet transform, which is an integral transform, comparable with the Fourier transform. Both of these transforms are very important for various types of applications. For more information, see also the books referred to in the next remark.

**Remark 5** From the first discoveries of Alfred Haar (1885–1933) it has been an almost unbelievable development of the wavelet theory. The reasons are both the interest from the mathematical point of view and the applications described above. In particular, more than 50 books on the subject has been written. Here we just mention [8],[14],[22],[23],[24],[28] and [30], as well as the papers [10] and [11], which illustrates various aspects of this broad science and also how many well-known authors have been involved.