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Revisiting the development trajectory of parallel machine scheduling

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ABSTRACT

The research on the Parallel Machine Scheduling Problem (PMSP) has undergone significant development. The most recent comprehensive review of published studies dates back to early 2001. This article presents an algorithmic review of PMSPs, using Main Path Analysis (MPA) to identify seminal knowledge diffusion and development trajectories. This research also sheds light on the less tangible aspects of the PMSP's development by employing Cluster Analysis (CA). Our findings indicate that the scheduling of semiconductor production operations received recent and growing attention, which is mostly driven by the industry's strategic nature. Specifically, the research cluster relevant to setups —material preparation, tool changes, machine settings, testing, and adjustments— requires investigation to address case-specific operational needs. From a theoretical perspective, further development of batch scheduling is needed, particularly when conflicting objectives are considered. Additionally, developing approximation algorithms for multi-objective optimization to integrate non-financial considerations into production scheduling is expected to continue as a growing research topic.

1. Introduction

Manufacturing shop floors are primarily designed based on process-based (e.g., Single Machine and Flow Shop), product-based (e.g., Job Shop), or parallel-machine layouts. When comparing the Parallel Machine Scheduling Problems (PMSP) with other classical scheduling problems such as the Single Machine Scheduling Problem (SMSP), the Flow Shop Scheduling Problem (FSSP), the Hybrid Flow Shop Scheduling Problem (HFSSP), and the Job Shop Scheduling Problem (JSSP), distinct challenges arise in both modeling and solving. For example, unlike SMSPs, where jobs are processed by only one machine, and FSSP/HFSSPs, where jobs follow a predefined sequence on machines, PMSPs involve processing multiple jobs simultaneously on parallel machines. Efficiently managing concurrency presents a major modeling consideration. Capturing the interactions and dependencies among jobs and machines brings with it additional complexities. Besides, optimally allocating jobs among parallel machines to minimize machine idle times and maximize machine utilization is a critical challenge in PMSPs. In contrast to FSSPs and HFSSPs, where job routing is predefined, PMSPs necessitate the allocation of jobs to parallel machines considering job characteristics and machine capabilities and constraints. This dynamic nature of resource allocation adds complexity to both modeling and solving PMSPs. Overcoming these challenges requires innovative approaches in both problem formulation and solution methodologies.

Production with parallel machines is prevalent in strategic industries such as semiconductor manufacturing and foundries. Prime examples of parallel-machine processes include photoresist processes (coating, exposure, and development using parallel machines with similar performance) and thin film deposition (chemical vapor deposition with different parallel machines to complete wafer testing) in the semiconductor industry, as well as metal production (steelmaking-continuous casting, rolling, and heat treatment processes) in foundries. In the literature, parallel machines, hybrid flow shops, and re-entrant flow shops are all pertinent formulations for addressing scheduling challenges in semiconductor manufacturing. Each formulation has its own set of benefits and limitations, and the choice between them depends on various factors, such as the nature of the manufacturing process, equipment availability, production constraints, and objectives. Compared to the formulations of hybrid and re-entrant flow shops, PMSP enhances flexibility in allocating resources, mitigates bottlenecks in the production process, enables better quality control, simplifies complexity in formulation and difficulty in implementation, and avoids deadlocks and production halts, especially in systems with high interdependencies between tasks and resources. On the other hand, the

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limitations of PMSP are that they do not fully capture the complexity nor explicitly consider the reentrant nature of tasks in semiconductor manufacturing processes, potentially leading to suboptimal schedules. Beverage production, printing, pharmaceuticals, and forging industries are other examples of industries where manufacturing shop floors are designed with parallel-machine production configurations (Biskup et al., 2008; Cheng et al., 2021; Wu et al., 2022). Considering the growing demand and resource limitations in the mentioned industries, production scheduling of parallel machines is expected to receive relatively more attention. The trend graph in Fig. 1 —drawn using the Logistic Function in LogletLab— illustrates that the literature on PMSP is in the growth stage of development and is expected to continue its sharp incline.

The literature on PMSPs is rich with over 2,271 published articles. Some of the seminal works for optimizing the PMSP and its variants include duality-based solution algorithms (van de Velde, 1993); Column Generation (Chen and Powell, 1999a; van den Akker et al., 1999); greedy randomized adaptive search procedure (GRASP; Rojanasoonthon and Bard, 2005); linear programming models, heuristics, and exact methods (Chan et al., 1998; Dell'Amico et al., 2008); preemptive relaxation (Şen and Bülbül, 2015); and robust polynomial-time approximation schemes (Skutella and Verschae, 2016); decomposition methods (Tran et al., 2016); Branch-and-Price Algorithm (Kowalczyk and Leus, 2018), Branch-Cut-and-Price (Oliveira and Pessoa, 2020), and dispatching rules for PMSPs considering different machine efficiencies (Wu et al., 2024).

Cheng & Sin (Cheng and Sin, 1990) conducted the first major survey on PMSPs, which was published in 1990. More recent articles surveyed the PMSP literature with a limited scope. Fowler & Mönch (Fowler and Mönch, 2022) reviewed scheduling problems with a focus on parallel batch processing. Three articles reviewed parallel machines as part of broader reviews of several scheduling extensions while focusing on specific mathematical settings (Adamu and Adewumi, 2016; Alidaee et al., 2021; Zhu and Wilhelm, 2006). The latest comprehensive review of PMSPs dates back to 2001 (Mokotoff, 2001). Reviewing the PMSP literature in its entirety is therefore perceived as relevant, and it is timely to discuss the development trajectory and the existing gaps to provide a reference for future research.

The main contribution of the present study is to update an important scheduling domain that has not been comprehensively reviewed in the past 23 years. By understanding how ideas have evolved and diffused within the field, researchers can identify opportunities for new contributions, push the boundaries of knowledge in PMSP research, and inspire innovation. Main Path Analysis (MPA) was introduced by Hummon and Dereian (1989) to identify the main development trajectory in a field using a local main path search method based on the links between published references (such as academic articles, patents, and law cases). This initial application involved describing the citation network of the Deoxyribonucleic Acid (DNA) theory. Subsequently, the

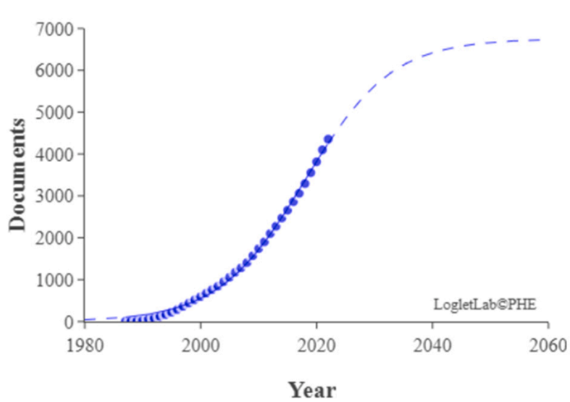


Fig. 1. The forecasted trend for the PMSP literature.

MPA method was expanded to accommodate large networks using global search algorithms by Mrvar and Batagelj (Mrvar and Batagelj, 2016). The key-path-based MPA was proposed by Liu and Lu (Liu and Lu, 2012), incorporating seminal works —based on the number of times an article is cited— into the network of main paths. MPA has been employed to explore the main development trajectory in various fields, including medical tourism (Chuang et al., 2014), text mining (Jung and Lee, 2020), data quality (Xiao et al., 2014), the Internet of Things (Rejeb et al., 2022), circular economy (Rejeb et al., 2023), carbon footprint (Yu and Chen, 2022), lean supply chain management (Yu and Ye, 2023), and data envelopment analysis application in sustainability (Zhou et al., 2018). Preliminary investigations showed that MPA has not been utilized to analyze any branches of the scheduling literature.

We apply an algorithmic review (Schryen and Sperling, 2023) with a broad research question, and comprehensive search strategy, using mathematical and computational basis for the literature analysis. The research question under consideration is:

- What are the knowledge diffusion patterns in the literature on PMSPs? How might this impact the future development trajectory of parallel-machine scheduling?

To answer these questions, the MPA and Cluster Analysis (CA) methods are employed to analyze the PMSP literature with a focus on the problem features and solution methodology. Pajek is used to visualize the knowledge development path, and VOSviewer software is employed to conduct keyword analysis and determine the themes of major research clusters. The seminal articles, the global main development path, and the observed patterns provide a basis for discussing the development trajectory of parallel-machine scheduling.

The rest of this review article is organized into three sections. Section 2 provides a summary of the materials and methods. Section 3 includes an overview of PMSPs, bibliometric analysis, discussions based on MPA and the key development paths, as well as classification based on CA and keyword analysis. Finally, Section 4 concludes the study and offers suggestions for future research directions based on the findings.

2. Materials and methods

This section outlines the data collection and analysis elements of the research process, as depicted in Fig. 2.

2.1. Keyword selection, data collection, and screening

The literature on PMSPs is gathered from the core database Web of Science (WoS), which includes the Emerging Sources Citation Index (ESCI), Science Citation Index Expanded (SCIE), Social Sciences Citation Index (SSCI), Arts & Humanities Citation Index (A&HCI), Book Citation Index (BCI), and Conference Proceedings Citation Index (CPCI). The following search protocol/keywords are considered: "parallel + machine* + scheduling*", "parallel machine* + scheduling*", and "parallel machine scheduling". The collection includes all papers published in and after 1987, the year when the literature was first collected in the WoS database, until the end of 2022. To include the retrospective literature from and before 1986, secondary data from Cheng & Sin (Cheng and Sin, 1990) are considered, which include all the published works after the seminal work of McNaughton (McNaughton, 1959). A total of 4463 items have been obtained. After the screening, which consisted of removing duplicates, isolated articles (documents that have neither cited others nor been cited by others), and those perceived as irrelevant, 2271 articles are left for further analysis.

2.2. Main path analysis

To construct the citation network, each document is represented by a node, which can be either a source node (the origin of the field), an

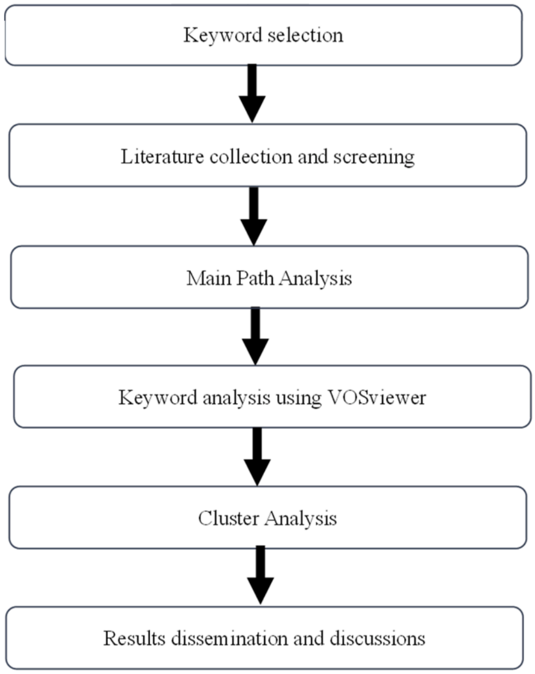


Fig. 2. The multi-method collection and analysis procedure.

intermediate note, or a sink node (the end of dissemination in the field), connected by directed arrows to the rest of the network. MPA finds all possible paths from the source point(s) to the sink point(s), and from these paths, the ones with the most frequent passages are called the main paths. A path constitutes many connections (citations), with the tail node representing the cited article and the head node representing the citing article. Three major methods are used to calculate the connection weights in the network for determining the main paths:

- The Search Path Count (SPC) method counts how often a connection is traversed when considering all possible paths from all the sources to all the sinks in the network.
- The Search Path Link Count (SPLC) method counts how often a connection is traversed when exhausting considering all possible paths from all the ancestors of the tail node, including itself, to all the network sinks.
- The Search Path Node Pair (SPNP) method counts how often a connection is traversed when running through all possible paths from all the ancestors of the tail node to all the descendants of the head node.

According to Liu et al. (Liu et al., 2019), the SPLC method best represents the knowledge diffusion scenario in literature development. Therefore, it is considered to calculate the connection weights in the network. In the weighted network, there are different alternatives for finding the main paths. In this study, two methods, namely the Global Main Path and the Key-route Main Path methods, are considered. The Global Main Path identifies the most important citation chain (i.e., the path with the greatest total weight) to explore the literature development track in PMSPs. The Key-route Main Path method starts the search from the most cited link(s) and progresses toward the source and sink nodes to identify ten major and interrelated development trajectories while ensuring that the seminal documents are included in the analysis.

2.3. Cluster analysis

CA (Girvan and Newman, 2002) is a hierarchical clustering approach that groups all sample points into a single group and classifies them into clusters through an iterative layer-by-layer process by removing specific links. This procedure takes the articles' citation network as input and produces a tree-like diagram representing similar groups of articles as output, respectively. The computational procedure of CA starts by determining the number of shortest paths from each node to every other node. Next, the edge credit score is calculated for each link using the following formulation: $\text{Edge Credit} = (1 + \sum \text{Incoming Edge Credit}) \times \frac{\text{Score of Destination}}{\text{Score of Start}}$. The calculations begin at the nodes at the bottom of the tree. In this context, the score of node 'a' (i.e., destination and start nodes in the formulation) corresponds to the number of steps required to reach node 'a' from the network's source node. This procedure continues until all remaining nodes are scored, and the edge scores of all nodes are summed up and then divided by two (due to the network being an undirected graph). As a final step following the calculating of edge scores, the edge(s) with the highest score are removed, resulting in the formation of multiple clusters.

2.4. Softwares

This study employs the *MainPath 465* software, developed by the Institute of Technology Management, National Taiwan University of Science and Technology. Upon importing the database txt file into the software, the first step involves removing isolated points based on which the global main path and the key-route main path are obtained. Subsequently, *Pajek* is employed for network analysis and visualization of the main path trajectory within the citation networks. Lastly, *VOSviewer* is utilized to generate citation maps for network visualization, label visualization, and density representation. For this purpose, after consolidating words with similar meanings into a singular, fixed noun through the use of this software, the keywords that exhibit higher frequencies within each cluster are compiled to signify the primary keyword of the respective cluster.

It is important to note that the documents displayed on the main path are evaluated based on the precision rate (Eq. (1)) and the digital object identification number (Eq. (2)) to ensure the significance of the articles in the field's development. The index is suggested to be higher than 70 percent (Liu et al., 2019).

$$\text{Precision} = \frac{\text{Network Size}}{\text{Number of papers in the original data}} \tag{1}$$

$$\text{DOI Percentage} = \frac{\text{DOI Total}}{\text{CR Total}} \times 100\% \tag{2}$$

The precision rate of our database is 98 percent (=2271/2326 × 100%), and the DOI percentage is 79 percent (=51612/65412 × 100%). These figures indicate the high accuracy of the analysis presented in the following section.

3. Results

3.1. Overview

PMSPs can be subdivided into three categories based on their similarities and processing patterns; these are identical, uniform, and unrelated parallel machines. These PMSP variants, along with the most relevant keywords, are considered to provide a visual overview. The network, label, and density views of the PMSP literature are shown in Figs. 3–5, respectively. The network and label views assign topics to clusters based on research keywords. Each topic is depicted by a circle, with its size representing its significance (weight) in relation to the frequency of keywords in the cluster. The connection distance between the circles reflects the correlation of common citations between the two topics. The closer the distance, the stronger the correlation of common citations between the two topics. In the network view shown in Fig. 3, the most frequently occurring keywords related to PMSPs include unrelated parallel machines, uniform parallel machines, learning effect, competitive analysis, and batch scheduling. The label view is distinct

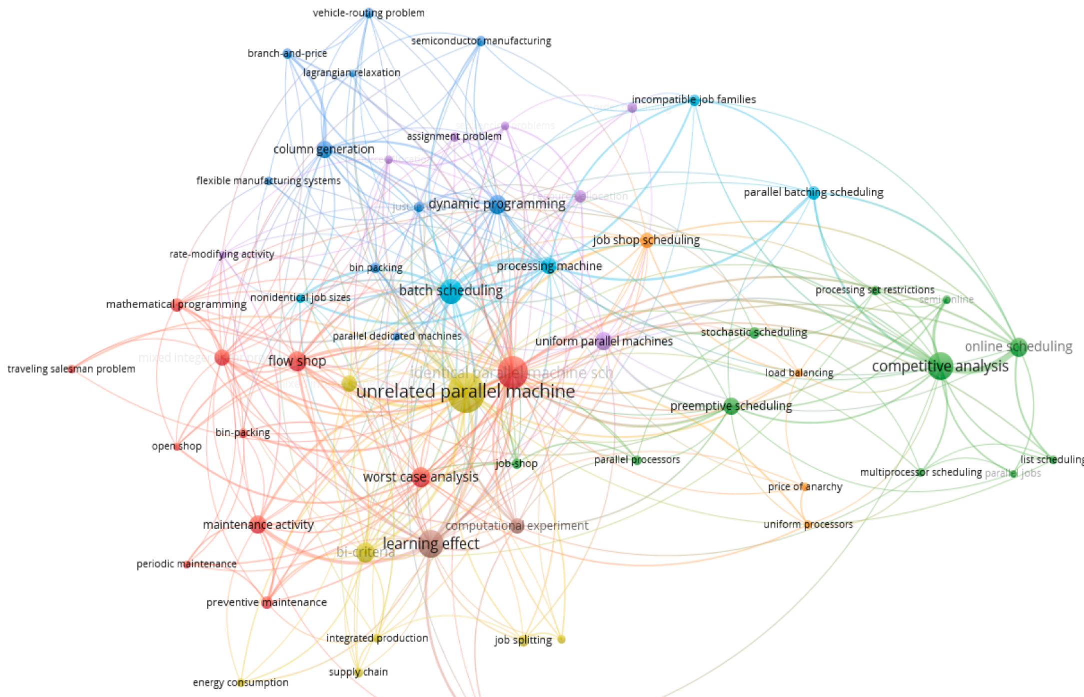


Fig. 3. Network view of the PMSP literature.

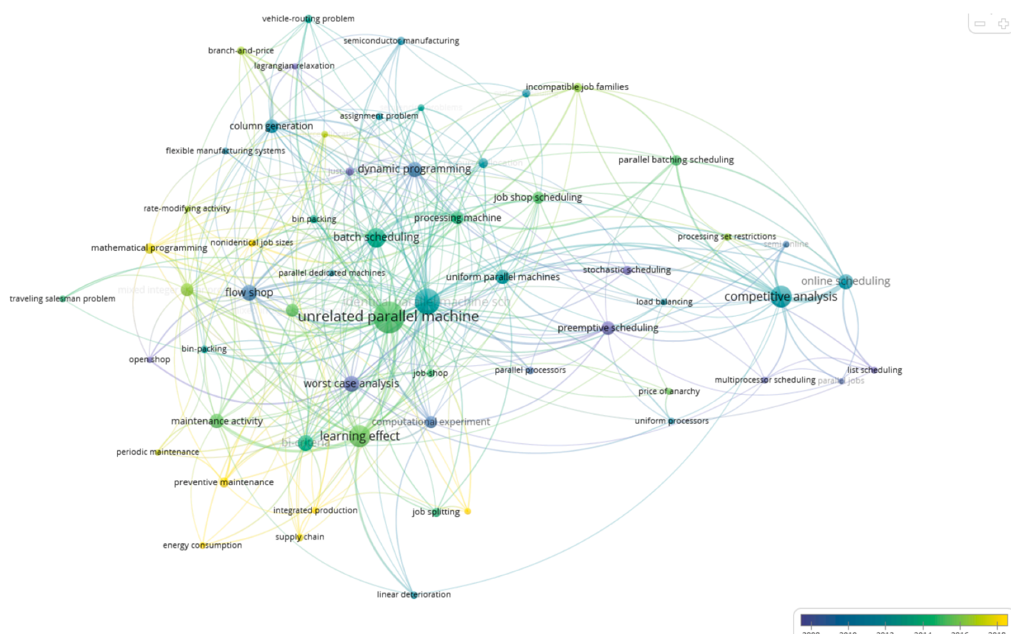


Fig. 4. Label view of the PMSP literature.

because it allows one to identify the main development topics in different years/periods. As depicted in Fig. 4, the primary topics from 2008 to the present include multiprocessor scheduling, worst-case analysis, uniform parallel machines, unrelated parallel machines, learning effects, preventive maintenance, energy consumption, and integrated production. The density view highlights areas by considering both the quantity and importance around each node, which helps understand the distribution of various topics. As shown in Fig. 5, the majority of studies on PMSPs focused on unrelated parallel machines, learning effects, competitive analysis, and batch scheduling.

3.2. Bibliometric analysis

For the bibliometric analysis of the PMSP literature, the *g*-index is used as an indicator for evaluating journals' influence, given that it is more sensitive to highly cited articles than the *h*-index. Furthermore, the *h*-index is considered a secondary criterion for prioritizing the documents with the same *g*-index. The results for the top twenty journals are presented in Table 1 to assess their influence on PMSP development.

Journal statistics show that the PMSP literature has been published in 280 journals (on Scopus). Among these, the European Journal of Operational Research (EJOR) has made the most significant

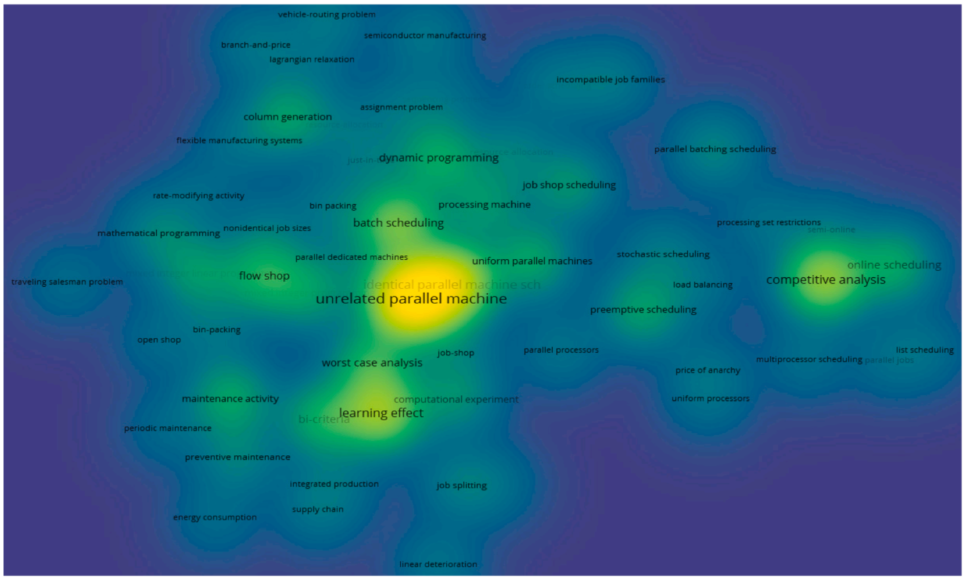


Fig. 5. Density view of the PMSP literature.

Table 1
Top contributing journals in PMSPs.

Ranking	Journal	*g*-index	*h*-index	Active years
1	European Journal of Operational Research	64	40	1990 ~ 2022
2	Computers & Operations Research	60	42	1991 ~ 2022
3	International Journal of Production Research	45	29	1988 ~ 2022
4	Computers & Industrial Engineering	41	30	1993 ~ 2022
5	International Journal of Production Economics	37	24	1991 ~ 2022
6	International Journal of Advanced Manu Technology	34	25	2002 ~ 2022
7	Journal of Scheduling	31	20	2001 ~ 2022
8	Naval Research Logistics	31	19	1994 ~ 2019
9	Operations Research Letters	31	18	1992 ~ 2020
10	Expert Systems with Applications	31	17	1992 ~ 2022
11	Discrete Applied Mathematics	31	16	1991 ~ 2022
12	Journal of Intelligent Manufacturing	29	19	1991 ~ 2022
13	Journal of the Operational Research Society	29	17	1989 ~ 2021
14	Applied Soft Computing	28	19	2011 ~ 2022
15	IIE Transactions	26	18	1994 ~ 2014
16	Applied Mathematical Modelling	26	18	2008 ~ 2021
17	Theoretical Computer Science	24	17	1994 ~ 2021
18	Operations Research	19	15	1989 ~ 2019
19	Information Processing Letters	18	12	1994 ~ 2022
20	Engineering Optimization	18	12	1996 ~ 2021
	Average	32	21	

contributions to the development of the field. The journal mainly focuses on the practical and innovative application of Operations Research, along with the theoretical basis and methodology that contribute to its advancement. Significantly, the most recent comprehensive review of the PMSP literature was published in this journal, and the published papers carry considerable influence. The second is Computers & Operations Research (CAOR), which focuses on optimization methods within Operations Research. The third journal, the International Journal of Production Research (IJPR), particularly focuses on decision-making, operations management, and logistics to tackle complex production systems. Next on the list is Computers & Industrial Engineering (CAIE), which compiles literature to address industrial engineering problems and emerging applications related to industrial engineering. Following that, the International Journal of Production Economics (IJPE) and the International Journal of Advanced Manufacturing Technology (IJAMT) are ranked next. IJPE is an interdisciplinary journal that focuses on engineering and management issues in manufacturing, aiming to disseminate knowledge with financial and economic implications. IJAMT encompasses research on advanced manufacturing technologies and innovation, topics related to mechanical engineering, material science, industrial engineering, automation engineering, production management, and quality control.

Bibliometric analysis indicates that a total of 3721 researchers have contributed to the development of the PMSP literature. For analyzing their contributions, the *g*-index is employed as the primary indicator to identify the top twenty researchers, while the *h*-index serves as a tie-breaker. In cases where both indicators are equal, the number of contributions as the first author will be used to resolve the tie. The statistical outcomes of the authors are presented in Table 2.

The top nine authors have been active from 1992 to the present, all of whom are affiliated with Asian universities. First, Edwin Tai Chiu Cheng, the author of the comprehensive review of PMSPs titled “A state of the art review of parallel machine scheduling research” (Cheng and Sin, 1990) which was published in 1990, has been cited 732 times in Google Scholar. Second, Joseph Yuk Tong Leung, who passed away in 2018, contributed the seminal PMSP article titled “Scheduling parallel machines with inclusive processing set restrictions” (Ou et al., 2008), published in 2008, which has received 88 citations so far. The third contributing author is Min Ji, who published “Minimizing resource consumption on uniform parallel machines with a bound on makespan” (Ji et al., 2013). The paper has received 45 citations. Jinjiang Yuan, Kai Li, and Ming Liu are the next contributing authors, with their most cited

Table 2
Top contributing authors in PMSPs.

Ranking	Author	g-index	h-index	1st Author	Active years	Articles
1	Cheng, Edwin Tai Chiu	29	20	7	2007 ~ 2022	43
2	Leung, Joseph Yuk Tong	22	15	5	2006 ~ 2017	22
3	Ji, Min	19	12	10	2007 ~ 2022	20
4	Yuan, Jinjiang	18	11	2	2008 ~ 2022	37
5	Li, Kai	18	10	12	2009 ~ 2022	25
6	Liu, Ming	17	9	10	2009 ~ 2022	23
7	Yang, Dar-Li	16	13	3	2008 ~ 2018	16
8	Ying, Kuo-Ching	16	12	4	2010 ~ 2021	16
9	Pardalos, Panos	16	10	0	2008 ~ 2021	16
10	Woeginger, Gerhard	15	12	3	1992 ~ 2001	15
11	Ng, Chi To	15	11	0	2007 ~ 2022	17
12	Jia, Zhao-Hong	15	9	12	2014 ~ 2022	20
13	Werner, Frank	15	7	0	2009 ~ 2021	15
14	Mosheiov, Gur	14	10	5	2007 ~ 2020	14
15	Shabtay, Dvir	13	9	7	2006 ~ 2022	13
16	Pei, Jun	13	9	3	2015 ~ 2021	13
17	Liu, Xinbao	13	9	1	2015 ~ 2022	13
18	Lei, Deming	13	7	7	2018 ~ 2022	13
19	He, Yong	13	7	5	1998 ~ 2006	15
20	Tan, Zhiyi	13	6	4	2007 ~ 2021	14

papers being "Bicriteria scheduling of equal length jobs on uniform parallel machines" (Zhao and Yuan, 2020) (published in 2020, cited 10 times), "Parallel machine scheduling problems in green manufacturing industry" (Li et al., 2016) (published in 2016, cited 92 times), and "Multi-objective optimization of parallel machine scheduling integrated with multi-resources preventive maintenance planning" (Wang and Liu, 2015) (published in 2015, cited 116 times), respectively. These authors are affiliated with universities in the top contributing countries listed in Table 3. The statistics reveal that 2271 documents were authored across 144 countries. China ranks first with a total of 591 articles (34.02 %), followed by the United States, France, Germany, and Taiwan.

Table 3
Top contributing countries in PMSPs.

Ranking	Journal	g-index	h-index
1	China	591	34.02
2	USA	319	18.36
3	France	180	10.36
4	Germany	109	6.28
5	Taiwan	99	5.70
6	Iran	99	5.70
7	Canada	93	5.35
8	Turkey	83	4.78
9	England	64	3.68
10	South Korea	50	2.88
10	Italy	50	2.88
	Total	1737	

3.3. Main path analysis

In this section, the SPLC method is employed to identify the global and key-route main paths of the PMSP's development trajectory. The three-field $\langle\alpha|\beta|\gamma\rangle$ notation introduced by Graham et al. (1979) is utilized to label the seminal literature within each cluster. This notation system forms the basis for reviewing the knowledge diffusion paths. This notation system delineates the operating environment (α), operating constraints (β), and the optimization objective (γ).

Field	Definition	Notation
α	Identical parallel machines	P/P_m
	Uniform parallel machines	Q/Q_m
	Unrelated parallel machines	R/R_m
γ	Maximum completion time	C_{max}
	Maximum delays	$L_{\max}$
	Maximum lateness	T_{max}
	Tardiness	T_j
	Earliness	E_j
	Completion time	C_j
	Weights	W_j
	Total cost	TC
	Minimum cost function	γ
	Total absolute difference in cost	$TADC$
	Total absolute difference in waiting time	$TADW$
	Total waiting time	TW
Practical features/ constraints		
β	Machine breakdown	*brkdwn*
	Job splitting	*split*
	Time-related multiple-rate modifying activities	RMA_{j-time}
	Buffer capacity (length – k)	$B(k)$
	No preemption, jobs arrive in decreasing order of size	*Decr*
	The processing time is unknown, and the order of job processing time is known	*ordinal*
	Online scheduling	*On-line*
	Predetermined sequence	*prec*
	Tree-like sequence	*tree*
	Preemptive scheduling	*pmtn*
	Deadline/due date	d/d_j
	Processing time	P_j
	Sequence-dependent setup	ST_{sd}
	Release/ready time	r_j
	Machine availability time	a_k
	Machine qualification	M_j
	Flow time	F_j
	Job size	s_j
	Machine capacity	Q_k
	Parallel batch processing	*p-batch*
	Batch processing	*batch*
	Machine maintenance	M_m
	Expired times	e_{ij}
	Sequence-independent setup	s_{ijm}
	Machine batch capacity	S_i
	Pollution cost	K_i
	Deterioration rate of machines	b_j
	Processing time of machines	a_j
	The maximum batch size of machines	B_i
	Incompatible jobs	*incompatible*

3.3.1. Global main path

The global main path within the PMSP's knowledge diffusion network comprises a single source, one sink, and 17 intermediate nodes (documents). The visualization of the analysis results is presented in Fig. 6. In the main path, the articles are labeled with the surname of the first author, the initials of the surnames of other authors (if any), and the publication year. The direction of the arrow indicates the citation relationship, with thicker arrows representing a higher citation weight.

The source node of the main path, i.e., Cheng, in which Cheng (1989) studied the PMSP with a common due date, $P| < d > |\sum(\alpha E_j + \beta T_j + \gamma d)$, has been acknowledged as the initial point of the knowledge development path. They initially demonstrated that the optimal common due date in single-machine scheduling aligns with the completion time of a

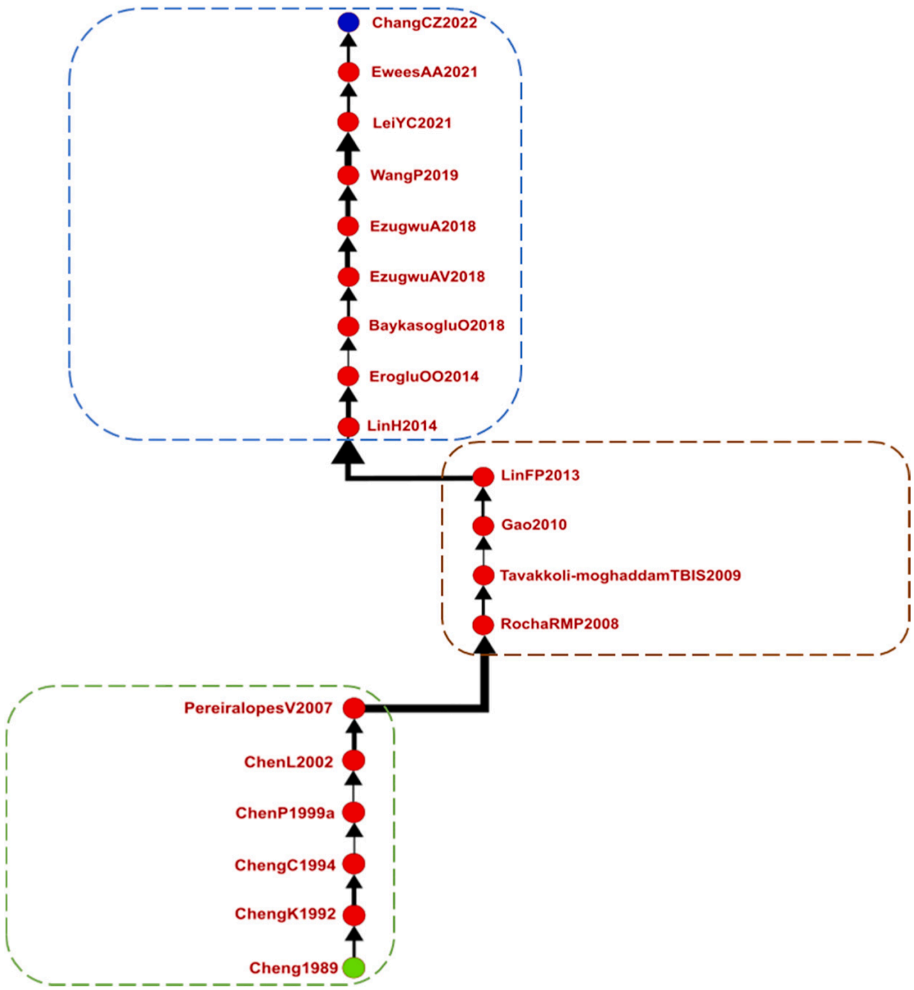


Fig. 6. The global main path in the development trajectory of PMSPs.

particular job, and the author established the extension of this discovery to the PMSP. Cheng was a pioneer in devising a heuristic algorithm to solve the PMSP with due dates and job assignments. By proving that $P| < d > |\sum(\alpha E_j + \beta T_j + \gamma d)$ is *NP*-hard, Cheng and Kahlbacher (1992) affirmed the feasibility of employing heuristics to address the problem. Subsequently, Cheng and Chen (1994) explored optimizing the problem by accounting for the total penalty function, $P| < d >, P_j = P|\sum(\alpha E_j + \beta T_j)$. They demonstrated that under the condition of equal processing times for all jobs, a polynomial computing time complexity algorithm can solve the problem using an exact approach.

Chen and Powell (1999b) introduced an integer programming model for the PMSP, aiming to minimize the total weighted earliness and tardiness, $P| < d > |\sum(\alpha_j E_j + \beta_j T_j)$. The proposed model was further decomposed using Dantzig-Wolfe decomposition, resulting in a main problem and two sub-problems. In the solution algorithm, both the last-in-first-out (LIFO) and the best lower limit rules were incorporated. Additionally, they developed an exact approach, i.e., a Branch-and-Bound Algorithm based on the Column Generation Procedure, to solve this Just-In-Time variant of PMSP. Chen and Lee (2002) extended the concepts introduced by (Chen and Powell, 1999b) and formulated a Branch-and-Bound algorithm to address the PMSP with a common due window, $P| < \hat{d}_l, \hat{d}_r > |\sum(\alpha_j E_j + \beta_j T_j)$. The authors approached the problem with set partitioning and utilized the column generation method presented by Janiak et al. (2015) to solve the linear relaxation of the model. Empirical experiments demonstrated the effectiveness of the proposed solution method for instances of up to size 45. Pereira Lopes and de Carvalho (2007) studied the PMSP while considering factors such as machine availability, job release dates, and sequence-dependent setup times, $R|a_k, r_j, ST_{sd}|\sum w_j T_j$. They proposed a branch-based pricing model inspired by the concept presented by Chen and Powell (1999b), and developed a Branch-and-Price Algorithm with an acceleration column generation method to solve the problem. Their findings demonstrated that the proposed solution method accelerates the optimization process by over seven times, yielding reasonable solutions for instances involving up to 50 machines and 150 jobs.

Rocha et al., 2008) proposed Mixed-Integer Programming (MIP) models and a Branch-and-Bound algorithm for addressing the unrelated PMSP with $R|s_{ijm}|C_{\max} + \sum w_i T_i$ settings. They developed a Greedy Randomized Adaptive Search Procedure (GRASP) and a metaheuristic that utilized the GRASP-generated solution as an upper bound for computations. Tavakkoli-Moghaddam et al. (2009) proposed a Mixed-Integer Linear Programming (MILP) formulation to address the $R|ST_{sd}|\sum w_j T_j$ problem. They developed an efficient metaheuristic based on Genetic Algorithm (GA) to address the bi-objective problem. The metaheuristic's performance was demonstrated by illustrating its superiority over the Branch-and-Bound method. Gao (2010) developed a vector Artificial Immune System (VAIS) to solve the $R|M_j|\sum w_j E + w_j T_j + C_{\max}$ problem. By utilizing instances from a textile manufacturing company, the authors conducted a comparison between the proposed solution algorithm and the Vector Immune Genetic Algorithm. The results indicated that VAIS exhibits higher competitiveness. In their study, Lin et al. (2013) developed two heuristic algorithms and a metaheuristic based on GA to solve three multi-objective variants of the PMSP, i.e., $R\|C_{\max}, \sum w_j T_j$, $R\|\sum w_j F_j, \sum w_j T_j$, and $R\|C_{\max}, \sum w_j F_j$,

$\sum w_j T_j$. Their conducted experiments validated the approach's substantial computational efficiency and its ability to generate superior non-dominated solutions.

Lin and Hsieh (2014) proposed a MILP model for PMSP that considers both ready times and setups, $R|ST_{sd}, r_j|\sum w_j T_j$ and developed a heuristic as well as an iterative hybrid metaheuristic to solve this problem. Their proposed heuristic approach extended the Apparent Tardiness Cost with Setups and Ready Times (ATCSR) algorithm (Pfund et al., 2008), which was originally designed to solve unrelated parallel machines. The authors found that their proposal for the heuristic approach is a breakthrough and their proposed metaheuristic outperforms the Tabu Search (TS) and the Ant Colony Optimization (ACO) algorithms.

Yilmaz Eroglu et al. (2014) addressed PMSPs with setup times, $R|ST_{sd}|C_{\max}$. They developed a GA hybridized with the local search proposed by Chang and Chen (2011), which demonstrated superior performance compared to the ACO algorithm developed by Arnaout et al. (2010). Baykasoğlu and Ozsoydan (2018a) introduced $P|ST_{sd}, r_j, M_j, brkdwn|C_{\max}, L_{\max}$, and proposed a multi-start constructive heuristic to solve the online and dynamic variant of the PMSP. The authors utilized real-world instances where the production procedure could be significantly shortened.

Ezugwu et al. (2018) proposed an improved version of the Symbiotic Organisms Search (SOS) algorithm to address the $P_m|ST_{sd}|C_{\max}$ problem with a focus on improved efficiency. Through comprehensive experiments that compared SOS with the Simulated Annealing (SA), ACO, hybrid GA with dominance properties, and hybrid SA with dominance properties, the authors demonstrated the substantial impact of their work. Ezugwu and Akutsah (2018) introduced an improved local search-based Firefly Algorithm (FA) for solving the $P_m|ST_{sd}|C_{\max}$ problem; They compared the proposed algorithm with eight improved metaheuristics based on SA, ACO, and ABC, in order to verify the competitive performance of the FA-based method.

Wang and Pan (2019) extended the Imperialist Competitive Algorithm (ICA) by incorporating two constructive heuristics and the competition doctrine from the Estimation of Distribution Algorithm (EDA) method to solve the $R|M_m|C_{\max}, \sum T_j$ problem. Their findings demonstrated that the multi-objective optimization achieved by their algorithm outperforms both the multi-objective Harmony Search (HS) and multi-objective multi-point SA algorithms. Lei et al. (2021) developed an improved Artificial Bee Colony (ABC) to optimize the bi-objective PMSPs considering unrelated parallel machines. The authors showed that their solution method is more effective than both the multi-objective multi-point SA algorithm (Wang and Pan, 2019) and the Tabu-enhanced Iterated Pareto Greed algorithms. Ewees et al. (2021) combined FA with the Salp Swarm Algorithm (SSA) to solve the $R|ST_{sd}|C_{\max}$ problem. Considering a total of 540 instances, their proposed algorithm demonstrated superior performance compared to the improved versions of GA, TS, and ABC algorithms, exhibiting significantly improved convergence toward the optimal solution.

Lastly, Chang et al. (2022) directed their attention toward the PMSP extensions tailored for scheduling Printed Circuit Board Assembly (PCBA) processes using Surface Mount Technology. Their study encompassed various practical aspects, including machine qualification restrictions, priority restrictions, unequal job release times, and shared resources, i.e., $R|M_j, ST_{sd}, prec|C_{\max}, \sum_{j=1}^{n} T_j$. The authors proposed a Non-dominated Sorting Genetic Algorithm (NSGA-II) to solve the dual-objective optimization problem and showed that their proposed optimization approach resulted in a reduction of up to 10 percent in makespan and 53 percent decrease in delays.

In general, the Branch-and-Bound algorithm based on column generation has emerged as a widely adopted solution approach for addressing PMSPs. Research published after 2008 has shifted its emphasis towards dual- and multi-objective optimization, leading to the extension of mainstream metaheuristics such as GA, FA, and ABC for the resolution of PMSPs.

3.3.2. Key extensions

The key-route analysis examines the interrelationships among various development paths to identify emerging issues that were previously studied but have been relatively underexplored in subsequent years. The key extensions identified for the main path consist of one source, two sinks, and a total of 24 intermediate nodes, as shown in Fig. 7. In this section, we review the documents present on the extension branches that did not appear on the global main path (Section 3.3.1).

The first extension, initiated by Xing and Zhang (2000), diverged from the main path that includes research by Cheng (1989), Cheng and Kahlbacher (1992), and Cheng and Chen (1994). Within this branch, the concept of preemptive PMSP was introduced, allowing for job splitting, i.e., $P|split|C_{\max}$. The authors proposed a heuristic algorithm enhanced with a machine learning module, introduced the Maximum Completion Time Estimation (MCTE), and demonstrated the achievability of a worst-case performance ratio of $7/4 - 1/m (m \geq 2)$. Logendran and Subur (2004) investigated a novel variant of PMSPs with unrelated machines $R|M_j, r_j, d_j|\sum w_j T_j$. They extended upon the Apparent Tardiness Cost (ATC) rule and developed several heuristics based on the maximum-frequency strategy, minimum-frequency strategy, fixed/variable size taboo list, and long/short-term memory of TS. Their experiments revealed that the heuristic based on long-term memory and maximum frequency excels at addressing small-scale problems, whereas the minimum frequency strategy proves to be more effective for handling medium and large. This study was subsequently extended by Logendran et al. (2007) to incorporate sequence-dependent setup times; they introduced four solution initialization methods to improve the optimization outcomes.

The second extension was initiated by Ezugwu (2019) exploring $R|ST_{sd}|C_{\max}$. The author extended the SOS and SA algorithms and also developed a hybrid approach combining SA and SOS to solve the problem. With a focus on enhancing local search capabilities, they showed that the hybrid algorithm performs significantly better. Jouhari et al. (2019) developed a hybrid approach incorporating both SA and Sine Cosine Algorithm (SCA) to solve $R|ST_{sd}|C_{\max}$, where SCA builds upon the solution generated by SA in pursuit of improved outcomes. Their experiments revealed that the proposed hybridization is a breakthrough in terms of convergence. SARAÇ and TUTUMLU (2021) and Saraç and Tutumlu (2022) are the sink nodes in the key-extension paths; both studies investigated the $R|ST_{sd}, M_j, split|C_{\max}$ problem, encompassing aspects such as setup times, machine qualification, and job split into unrelated parallel machines. The former study adopted GA by modifying its computational mechanisms to solve large-scale instances and demonstrated a significant reduction in completion time through the proposed approach. The latter study proposed two distinct mathematical formulations for PMSPs, considering both job-split and non-job-split scenarios. Through a case study, the authors showed that their PMSP-based optimization improves operations' makespan by around 85 percent.

The research conducted by Chen et al. (2022) is another confluence of the key-route main path. This research addresses the ion implantation process issue in wafer manufacturing, which is modeled as $R|r_j, e_{ij}, ST_{sd}|C_{\max}$. They introduced local search methods centered around job removal and insertion, resulting in a reduction of the total setup time. Both the main and key-extension paths culminated at the sink node, represented by Chang et al. (2022).

In summarizing the key findings, it becomes evident that PMSP with a common maturity constraint dominated the field between 1989 and 1994. By 2000, research had transitioned its focus toward the Branch-and-Bound algorithm rooted in column generation. Subsequently, Xing and Zhang (2000) expanded PMSPs to accommodate job splitting. During the middle phase of PMSP's development path, specifically from 2008 to 2013, the primary focus encompassed dual- and multi-objective

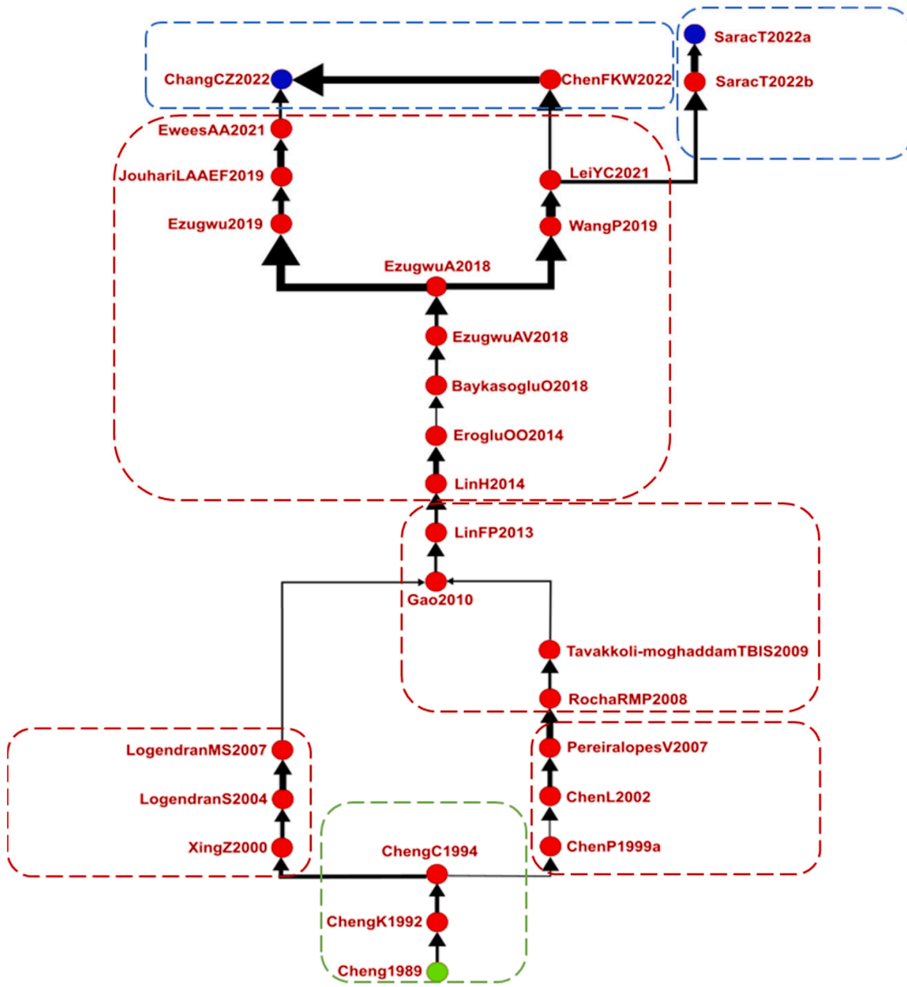


Fig. 7. The key-route main path in the development trajectory of PMSPs.

PMSPs, whereas subsequent studies leaned towards the advancement of sophisticated metaheuristics (Ezugwu, 2019; Ezugwu and Akutsah, 2018; Wang and Pan, 2019). These investigations transferred their insights to the recent study by Chang et al. (2022); this study also serves as a convergence point for the discourse on semiconductor-related issues within PMSPs. In summary, PMSP studies along the main trajectory, characterized by minimal constraints and a single-objective function, have evolved towards multi-objective optimization incorporating a broader range of practical constraints. This pivotal extension has notably broadened the application scenarios of PMSPs, especially within wafer manufacturing and PCBA contexts.

3.4. Cluster analysis

A cluster review based on CA is now provided. Categorized into 20 clusters, the top five clusters with 489, 144, 103, 88, 68, and 55 articles are considered for discussion. The result is summarized in Fig. 8, wherein the proportion of keywords indicates the number of times they appeared in the cluster divided by the total number of documents in the cluster. The keywords with the greatest proportion value are regarded as the research topic, which is indicated by an asterisk "*". Observing that "Minimizing Makespan" is present in all clusters indicates that the main objective function of PMSPs is the Makespan. Sequence-dependent setups in PMSPs are at the growth stage, and the trend is expected to flatten later than the growth stage of Batch Scheduling, Learning Effect & Deteriorating Operations, and Approximation Algorithms are also in the growth stage. However, online scheduling is expected to be saturated earlier than the first four clusters.

3.4.1. Cluster I: sequence-dependent setups in parallel machine settings

The global main path of the first cluster consists of one source node, seven intermediate points, and one sink node, making a total of nine nodes on the global main path, with the latest four also being on the key-route main path (see Fig. 9). We now provide a deeper review of these research works, considering their influence on the research and development of PMSPs.

The source node of this cluster is the study of Lin and Hsieh (2014) where they developed and solved the MILP model of unrelated parallel machine scheduling with sequence-dependent setup times and ready times, aiming at minimizing the weighted tardiness i.e., $R|ST_{sd}, r_j|\sum w_jT_j$. Yilmaz Eroglu et al. (2014) improved the local search ability as a benchmark and proposed an algorithm based on the Genetic Algorithm (GA) to solve the $R|ST_{sd}|C_{\max}$ problem. Baykasoğlu and Ozsoydan (2018b) developed a dynamic scheduling variant to address the PMSPs with consideration of setups, release, and machine maintenance times. i.e., $P|ST_{sd}, r_j, M_j, brkdwn|C_{\max}, L_{\max}$. Starting from the study of Ezugwu et al. (2018), where they proposed a symbiotic organisms search algorithm to solve the $P_m|ST_{sd}|C_{\max}$ problem, the dominant focus of the cluster shifted to developing effective metaheuristics. Ezugwu and Akutsah (2018) developed a local search-based FA algorithm to solve the same problem.

Ezugwu (2019)) developed a symbiotic organisms search algorithm to solve the $R|ST_{sd}|C_{\max}$ problem, and introduced non-preemptive constraints to improve the solution quality. Jouhari et al. (2019) proposed

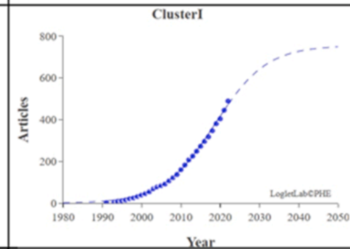
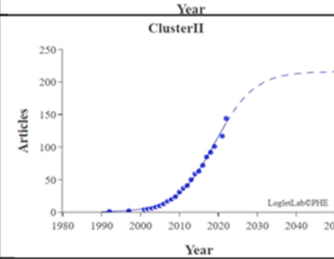
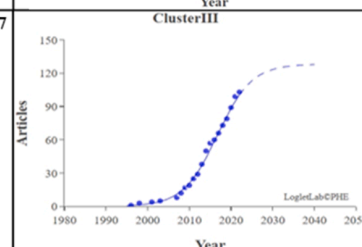
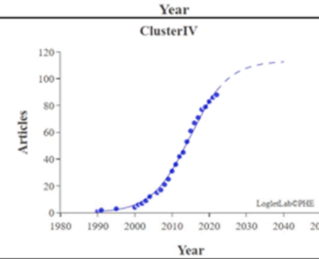
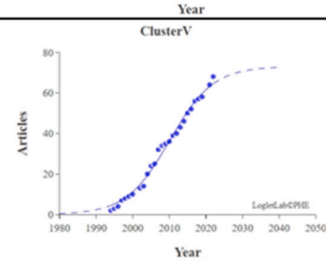
Cluster/Number of Articles	Ratio of Keyword	Growth Trend
Cluster I/489	*Sequence Dependent Setup Times 0.38 Minimizing Makespan 0.28 Algorithm 0.27 Unrelated Parallel Machine Scheduling 0.24 Heuristic Algorithms 0.24 Genetic Algorithm 0.21 Optimization 0.15 Total Tardiness 0.09 Total Weighted Tardiness 0.09 Tabu Search 0.11	
Cluster II/144	Minimizing Makespan 0.55 *Batch Scheduling 0.42 Genetic Algorithm 0.38 Processing Machine 0.29 Total Weighted Tardiness 0.19 Release Times 0.19 Non-identical 0.15 Job Families 0.14 Ant Colony Optimization 0.13 Branch and Bound 0.10	
Cluster III/103	*Learning Effect and Deteriorating Operations 0.67 Processing Times 0.38 Minimizing Makespan 0.34 Minimization 0.31 Maintenance Scheduling 0.22 Total Completion Time 0.15 Unrelated Parallel Machine Scheduling 0.14 Common Due-date 0.12 Availability Constraint 0.11 Completion-time 0.10	
Cluster IV/88	*Approximation Algorithm 0.35 Minimizing Makespan 0.24 Competitive Analysis 0.23 Online Algorithm 0.23 Eligibility Constraint 0.16 Service Provision 0.15 Unrelated Parallel Machine Scheduling 0.15 Multipurpose Machine Scheduling 0.10 Processing Set Restrictions 0.10 Restricted Assignment Problem 0.09	
Cluster V/68	*On-line Scheduling 0.60 Competitive Analysis 0.50 Approximation Algorithm 0.16 Identical Parallel Machines 0.15 Preemptive Scheduling 0.15 Minimizing Makespan 0.15 Semi-online Algorithms 0.13 Worst-case Analysis 0.12 Analysis of Algorithm 0.09 List Scheduling 0.06	

Fig. 8. Analysis of the top clusters in CA.

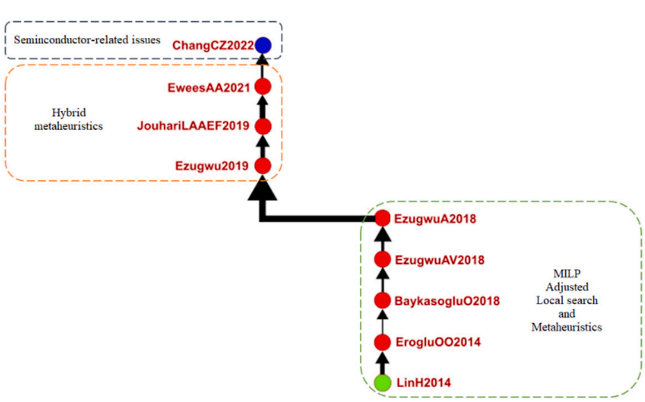


Fig. 9. The global main path of Cluster I.

the Sine-Cosine enhanced SA to solve the $R|ST_{sd}|C_{\max}$ problem. Ewees et al. (2021) developed the firefly-based SSA to solve the $R|ST_{sd}|C_{\max}$ problem. Serving as the sink node of the cluster, Chang et al. (2022) developed the non-dominated sorting genetic algorithm (NSGA) to solve the PMSP with machine eligibility restrictions, setups, and resource constraints, i.e., $R|M_j, ST_{sd}, prec|C_{\max}, \sum_{j=1}^{n} T_j$, aiming to minimize the maximum completion time and total operation delay time.

3.4.2. Cluster II: Batch processing

All articles on the global main path of the second cluster belong to the category of identical parallel batch processing machines, as shown in Fig. 10.

Being the knowledge source of the cluster, the study by Mönch et al. (2005) introduced two job assignment methods based on GA to solve the $P_m|r_j, batch, incompatible|\sum w_j T_j$ problem. Kashan et al. (2008) proposed a hybrid GA to solve the $P|p-batch, s_j, B_i|C_{\max}$ problem. In their study,

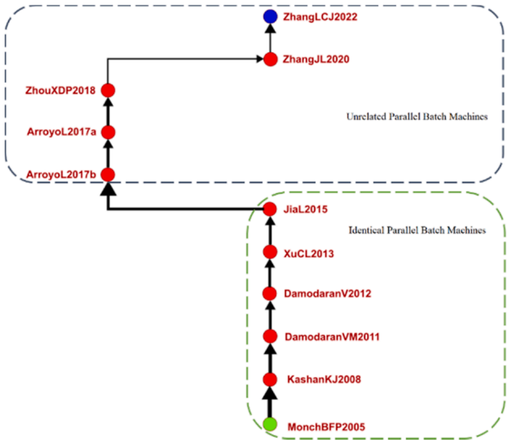


Fig. 10. The global main path of Cluster II.

the Random Batches Procedure was introduced, employing the Longer Processing Time (LPT) rule to assign the jobs to the same batch for each generated offspring, ensuring the feasibility of the solutions. The feasible batches are sorted based on the LPT rule before being assigned to the machine. Damodaran et al. (2011) considered the setup constraints and ready times and proposed the GRASP algorithm integrated with the Modified First Fit (MFF) stochastic heuristic to solve the $P|r_j, p-batch, s_j|C_{\max}$ problem, aiming at minimum completion time. Damodaran and Vélez-Gallego (2012) investigated the same problem and proposed an improved SA, which outperformed the GRASP algorithm. In continuation of this path, Xu et al. (2013) aimed at dual-objective optimization of the $P|r_j, p-batch, s_j|C_{\max}, T_{\max}$ problem, and proposed a Pareto-based Ant Colony System algorithm to solve this specific problem. Jia and Leung (2015) proposed an Ant Colony Optimization (ACO)-based metaheuristic to solve the $P|p-batch, s_j, B_i|C_{\max}$ problem using the Max–Min Ant System (Stützle and Hoos, 2000) to group jobs into batches and sequence them on-stage.

The study of Arroyo and Leung (2017a) is the first seminal work on the unrelated parallel batch machine problem. They proposed a MIP model to address the $R|r_j, p-batch, J_s, Q_k|C_{\max}$ problem, as well as a novel heuristic that divides operations into different batches. This heuristic uses the Earliest Completed Operation rule to assign each batch to the machine with the minimum completion time, and it completes the processing order of the batches on each machine simultaneously. They compared this heuristic with two other heuristics based on the modified best-fit method (Damodaran and Vélez-Gallego, 2012) and the Best-fit LPT (Arroyo and Leung, 2017a).

Arroyo and Leung (2017b) presented a MIP formulation and an Iterated Greedy (IG) Algorithm with Arroyo and Leung's (Arroyo and Leung, 2017a) heuristic as the initialization module to address the $R|r_j, p-batch, J_s, Q_k|C_{\max}$ problem. They demonstrated that their IG algorithm is superior to the GA, ACO, and SA algorithms. Zhou et al., (2018b) proposed a random-key-encoded GA to solve the $R|p-batch, J_s, Q_k|C_{\max}$ problem. In their optimization approach, the initial solution is generated in two ways: the first method adopts multiple scheduling rules, with the jobs being sorted according to the increasing sequence of job release time, and the machines are randomly selected. The other method involves randomly generating an initial solution and then using the First Fit Earliest Ready Time (FFERT) rule to form the batches. Zhang et al. (Zhang et al., 2020) considered the $P_m|p-batch, p, w_j, s_j, M_j|\sum w_j c_j$ problem, and proposed metaheuristics based on the Ant System (Dorigo et al., 1996) and Max-Min Ant System (Stützle and Hoos, 2000) to minimize the total weighted completion time.

The latest publication concerning the global main path of the second cluster, authored by Li et al. (2022a), also serves as its converging point. In their study, they developed the NSGA-CC algorithm to address the $P_m|p-batch, r_j, s_j, S_i, K_i, M_j|L_{\max}, TC$ problem of optimizing both the maximum delay and the total cost. They introduced the concept of hierarchical clustering to enhance the diversity of the parent solutions.

3.4.3. Cluster III: Learning effect and deteriorating operations

The study of Kang and Ng (2007) is recognized as the source node within the cluster of PMSPs with deteriorating and learning effects. In their study, time-dependent processing times $P_m|p_j = a_j + b_j t|C_{\max}$, confirming the problem's NP-hard nature. Huang et al. (2014) incorporated learning effects and degenerate operations into PMSPs to minimize the cost of total waiting time in $P_m|p_j|TCTADC$ and minimize the cost associated with the total absolute difference in waiting time in $P_m|p_j|TWTADW$. They showed that both models exhibit polynomial time complexity. Wang and Wang (2014) addressed the $R_m|p_{ij}|\gamma$ problem of minimizing the total costs. They demonstrated that, with a constant number of machines, the problem can be solved with polynomial time computational complexity.

Woo et al. (2017) explored the PMSP with time-related multiple-rate modifying activities $R|RMA_{j-time}|C_{\max}$. Due to the inability to solve the MILP model for large-scale instances, the authors proposed metaheuristics based on GA integrated with Triple-dimensional String and Rule-based Dispatching. Woo and Kim (2018) studied the $P|RMA_{j-time}|C_{\max}$ problem, proposed a MILP formulation, and solved it using exact methods for small-size instances. They also developed two math-heuristic algorithms: the GA and SA, embedded in the mathematical optimization, to solve large-scale instances. Ding et al. (2019) proposed the Ejection Chain Algorithm (ECA) for problems with sequence-dependent setup times and degeneration effects $R_m|ST_{sd}|C_{\max}, \sum_{j=1}^{n} w_j c_j$. They demonstrated that ECA is more efficient than SA. To regulate the balance between the exploration and exploitation power of the search algorithm, they adopted a random and greedy strategy to generate an initial solution and applied the Hill Climbing Algorithm as the local search method. Salehi Mir et al. (2020) considered the equivalent parallel machine scheduling problem with sequence-dependent setup time, degeneration and learning effects. They proposed a heuristic and two metaheuristics based on GA and ACO to solve the problem of minimizing the total completion time. They showed that ACO's performance is statistically superior to that of the GA-based algorithm for solving medium and large test instances.

In the latest research focusing on the main path, Kalaki Juybari et al. (2021) studied the PMSP with exponentially time-dependent degenerate jobs $P_m|P_{ij}|C_{\max}$. They proposed a MILP model and three metaheuristics based on GA, SA, and the Artificial Immune System (AIS) to solve the

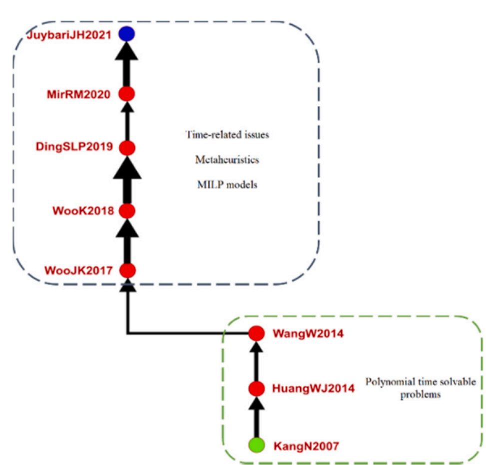


Fig. 11. The global main path of Cluster III.

problem where the SA-based algorithm performed better. Fig. 11 provides a visualization of the global main path for this cluster.

3.4.4. Cluster IV: Approximation algorithms

The fourth cluster, consisting of 88 documents, is formed by one source point, three intermediate nodes, and one sink node on the global main path of its development (see Fig. 12).

The knowledge source of this cluster is Glass and Kellerer (2007), who investigated the PMSP with job allocation constraints and developed a polynomial-time solution algorithm to solve the problem. They showed that when the processing time is limited to 1 and 2, the Absolute Worst-case Error Bound is 3/2. Despite its merits, the method is limited by the requirement that the processing time takes two values, and the difference between the two values cannot exceed 2.

Ou et al. (2008) considered parallel machines with the same processing speed $P|M_j|C_{\max}$ and presented an approximate solution using the Modified First Fit Decreasing mechanism with binary search and correction. They proved that the worst-case performance of the method is superior to that of the Worst-case Performance Ratio method, which is $\frac{4}{3} + \in$. Li and Wang (2010) studied the problem with different release times $P|r_j|C_{\max}$ and developed a polynomial-time approximation scheme to solve it. Mastrolilli (2003) considered machine indices when classifying and arranging jobs, and proposed a fully polynomial-time approximation scheme to solve the $P|M_j|C_{\max}$ problem. In the same production setting, Huo and Leung (2010a) introduced an advanced method with a worst-case performance bound of 7/4, comparing it with that of Glass and Kellerer (2007); they also investigated PMSPs with two and three machines and found that the worst-case performance of their solution method is 5/4 and 3/2, respectively.

The final document in the cluster, authored by Huo and Leung (2010b), explored the same problem, $P|M_j|C_{\max}$, and proposed an efficient approximation algorithm for the Tree-Hierarchical Processing Set, which demonstrated an enhancement in the bounds of the worst-case performance to 4/3. They also proposed the nested processing set algorithm that yielded a worst-case performance of 5/3. "In general, the literature within this cluster primarily focuses on addressing Set Restrictions in PMSPs.

3.4.5. Cluster V: Online scheduling

Out of a total of 68 documents, five appeared on the main development path of this cluster, with three of them serving as intermediate nodes (see Fig. 13).

The study of Kellerer et al. (1997) is identified as the knowledge source for this cluster. In their study, three variants of semi-online partition problems ($P_2|\mathrm{B(k)}|C_{\max}$) were proposed. The first variant assumed that items arrive one after another, and there is a buffer of length k partitions to maintain the items. The second variant considered the use of two parallel processors to simultaneously assign objects to a partitioned set. The last variant assumed that the total weight of the items was known, allowing the items to be assigned to a set upon arrival. The authors proposed several heuristics for solving these problems and proved that the worst-case performance limit of all heuristics is 4/3.

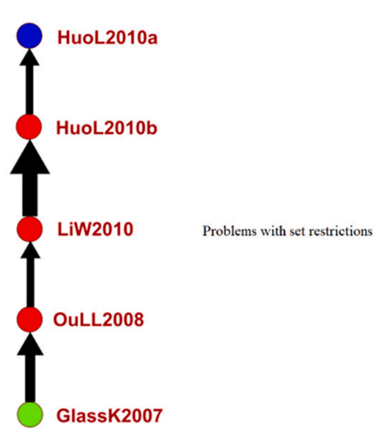


Fig. 12. The global main path of Cluster IV.

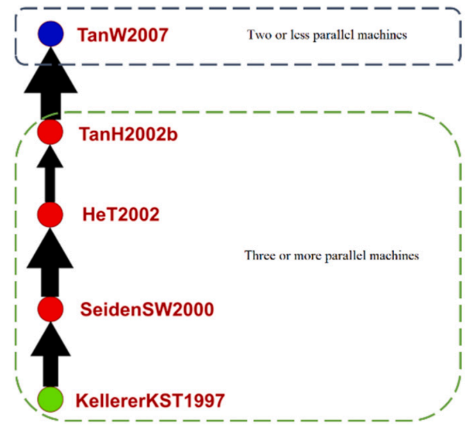


Fig. 13. The global main path of Cluster V.

Seiden et al. (2000) investigated the $P_{2,3}|Decr|C_{\max}$ problem. The authors established that when the number of machines is 2, the best competitive ratio has a value of 7/6. Similarly, for the number of machines is 3, the lower bound of the best competitive ratio is $\frac{(1+\sqrt{37})}{6}$. In the study by He and Tan (2002), the $P|ordinalon-line|C_{\max}$ problem was investigated, aiming at minimizing the maximum completion time of machines. The authors proposed an ordinal algorithm that utilizes ordinal data for the assignment decisions, and demonstrated that the method is capable of achieving the most competitive ratio of $\lceil\sum_{i=1}^{m} 1/i\rceil + 1$. Tan and He (2002) dealt with the $P_2||C_{\max}$ problem, and showed that their competitive ratio is 65 when considering the maximum processing time and the total processing time of all operations. They also demonstrated that when considering the total processing time of all operations, the competition ratio is 109 when the operations arrive in the order of non-increasing processing time.

The latest seminal work in the fifth cluster, conducted by Tan and Wu (2007), extended the research of Seiden et al. (2000) to a problem with semi-online machine coverage, considering $m \geq 3$ parallel machines and three optimization objectives: the total processing time of all jobs $P_m|sum|C_{\max}$, the maximum processing time of all jobs $P_m|\max|C_{\max}$, and the dual-objective problem, $P_m|sum\&\max|C_{\max}$. By developing a new algorithm, they demonstrated that when either the total or maximum processing times are considered, the competition ratio of the optimal algorithm is $m-1$, with a competitive ratio of 3/2. The competitive ratio becomes $m-2$ when four or more machines are included.

Overall, although the second and third clusters —batch processing and learning effect— are not part of the global and key-extension main paths, they have received recent recognition in the PMSP literature. From the main path clusters, approximation algorithms are gradually maturing, and recent studies on online scheduling are dominated by multi-objective optimization.

4. Conclusions

4.1. Concluding remarks

This study examined the development trajectory of parallel machine scheduling from 1959 up to the present day. The main development path and key extensions were analyzed to identify the most influential knowledge diffusion points in the literature on PMSPs. CA and keyword analysis were performed to discuss the top clusters and themes in the PMSP literature.

Comprehending the seminal works and the development trajectories in the research field of PMSPs provides valuable historical context. By analyzing seminal knowledge diffusion, researchers can identify the foundational contributions that have significantly influenced the development of PMSP research. Building upon these foundational contributions can provide a solid theoretical and conceptual basis for further research endeavors. Besides, studying the development trajectories of PMSP research can help visualize how different ideas, approaches, and methodologies have evolved and interacted over time. This mapping of trajectories reveals trends, patterns, and shifts in research focus, which can guide researchers in identifying current research gaps, emerging trends, and potential future directions for investigation. Finally, by understanding the diffusion of seminal knowledge, researchers can benchmark their work against established standards to evaluate its novelty, relevance, and contributions to advancing PMSP research. Overall, these insights can inform researchers about the possibilities for advancing knowledge in PMSPs by building upon existing ideas, addressing unresolved questions, or exploring new research.

A summary of the major observations of this study is as follows. The maximum completion time (Makespan) is the most commonly used scheduling criterion. The Branch-and-Bound algorithm based on column generation has been the most widely used method for solving optimization problems. In multi-objective optimization, which has attracted attention since 2014, the weighted tardiness $\sum w_j T_j$ and the makespan $C_{\max}$ are the most commonly used optimization objectives. The recent focus has been on introducing practical constraints, such as priority constraints and resource sharing. A comparison of the global path and key-path analysis results reveals that some nodes on the global main path overlap with the key extension main path, indicating that the recent development of PMSPs tends to focus on practical issues; examples include semiconductor wafer manufacturing and PCBA printing. Analysis of the keywords within CA identifies the following five major clusters, which can direct the researchers to find innovative research topics:

(1) **Sequence-Dependent Setups in Parallel Machine Settings**. During the initial phase (i.e., 2014 to 2018), the focus of this cluster was centered on improving local search capabilities and metaheuristics. However, the focus has shifted to practical issues, particularly within the semiconductor industry.

(2) **Batch Processing**. During the early stages of development, the main research direction focused on identical parallel batch processing; However, from 2017 to 2022, the emphasis shifted to non-identical parallel batch processing, which comprises the majority of research efforts. In recent times, dual objectives and hierarchical clusters have gained prominence.

(3) **Learning Effect and Deteriorating Operations**. During the initial phase of development for this cluster, the focus primarily revolved around studying the type of problem and derivation, while subsequent contributions addressed time limitations. The main focus of future research is expected to be on developing hybrid meta-heuristics.

(4) **Approximation Algorithms.** The majority of discussions within this cluster focused on addressing set constraints, with the makespan being the most extensively studied objective function.

(5) **Online Scheduling**. During the early development phase of this cluster (i.e., 1997 to 2002), studies focused on considering three or fewer machines. In more recent studies, PMSPs involving more machines have become prevalent.

To propel advancements in the field of PMSPs, researchers can delve into innovative research topics inspired by the five major research clusters identified in this study. More specific suggestions are provided below.

4.2. Future research directions

Drawing on current research trends, technological advancements, and practical challenges, the following future research directions are worthy of further exploration. *First*, future research may focus on integrating supply chain-related decision variables into multi-stage PMSPs to comprehensively consider the entire production and transportation processes. This might involve optimizing different stages in the supply chain for improved overall efficiency, for example, the integrated planning of the production and distribution activities (Wu et al., 2022).

Second, research on parallel additive machine scheduling problems requires new optimization strategies, given that this variant is different in that parts of various geometries should be processed at once on the machine's build platform; the study of (Ying et al., 2023) is a recent example that explored PMSP in metal 3D printing.

Third, additional studies could explore approaches for distributed PMSPs to tackle challenges in distributed manufacturing environments, for example, the study of (Wang et al., 2023). This involves efficiently coordinating and optimizing shops dispersed across different locations to achieve synergies in overall production.

Fourth, future research could invest in integrating modeling methods and artificial intelligence techniques to enhance the efficiency of solving PMSPs. Possible directions include intelligent optimization for improving scheduling approaches using machine learning technologies, like the recent article of (Li et al., 2024) that takes advantage of deep reinforcement learning.

Fifth, a significant research direction involves integrating Industry 4.0 considerations and technologies into PMSPs to achieve a more human-centered manufacturing environment. This may include the use of smart manufacturing, human-robot collaboration, and data analytics to enhance production performance (see Ghasemi et al., 2024).

Sixth, addressing the challenges of uncertain PMSPs requires more development. This involves developing new models to handle the uncertainty of the operational parameters (see Song and Leus, 2022); for example, Saraç et al. (2023) and Skutella et al. (2016) considered uncertain setup and processing times in PMSPs, respectively. Employing machine learning methods to adapt to such variations during actual operations remains a noteworthy development.

Seventh, a novel research direction involves applying inverse optimization to solve PMSPs, wherein decisions serve as input, determining objectives and/or constraints that render these decisions approximately or exactly optimal. This concerns developing algorithms and techniques to automatically adjust model parameters based on actual operational data, ensuring that schedules better align with real-world needs and conditions; the application of simheuristics for solving PMSPs by (Abu-Marrul et al., 2023) is a recent development of this type.

Eight, future research may focus on energy efficiency in PMSPs in light of increasing environmental awareness. The most recent contribution introduced the use of time-of-use tariffs for parallel machine scheduling (Feng and Peng, 2024); future research could test new scheduling strategies to reduce machine energy consumption, methods for modeling energy efficiency, and the integration of renewable energy sources to support net-zero manufacturing. Besides, new ways of formulating PMSPs will contribute to the further development of this research direction; the works of Li et al. (2022b), Naderi et al. (2023), and Kowalczyk et al. (2024) are the most recent examples.

The *ninth* direction stems from analyzing the impact of strategic or tactical shop-floor changes on PMSPs. Including product discontinuation decisions and/or the addition of extra machines to the shop floor (Rustogi and Strusevich, 2013) are good examples of this research direction. Game models for job-splitting, on the other hand, provide a different insight into the same research direction (Alon and Anily, 2023).

Finally, this paper is the first review of the scheduling literature based on MPA and CA. Therefore, other scheduling variants can benefit from the same sort of objective analysis in future studies. Besides, using other review methods that analyze the literature of PMSPs considering formulation and solution techniques, as well as the rapid development of solver engines and computing power's impact on modeling and solving

PMSPs is worthy of further exploration in future research.

One limitation of this study is related to the time-consuming process; the latest articles, especially those research works published in 2023, are not included in the MPA and CA results. To address this limitation, MPA can be periodically re-executed every 5 to 10 years to analyze the most recent trends in the relevant research field. Another limitation of this study is associated with the use of MPA to identify significant knowledge diffusion and development trajectories. Since this approach may not pay attention to less-cited but innovative literature, it could potentially create the impression that some articles related to PMSP were excluded from the review. However, the scope of this comprehensive review encompassed the entire PMSP literature. By leveraging the insights provided in this study, scholars working on PMSP-related topics can gain a deeper understanding of the state of the art, identify opportunities for innovation, and contribute to advancing the field.

CRediT authorship contribution statement

Kuo-Ching Ying: Validation, Supervision, Software, Resources, Project administration, Methodology, Conceptualization. **Pourya Pourhejazy:** Writing – review & editing, Writing – original draft, Investigation, Formal analysis. **Xin-Yi Huang:** Visualization, Methodology, Formal analysis, Data curation.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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