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An exploration of the concept of constrained improvement in data envelopment analysis

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ABSTRACT

Constrained improvement refers to regulating rivalry between companies in a particular industry by defining a framework or an evaluation mechanism. Such a mechanism results in a more equitable and healthy competitive environment. The primary motivation is that the best-performing players in a particular industry improve their performance such that the rest of the contenders remain competitive. This study investigates the concept of constrained improvement from a frontier analysis perspective, develops a systematic implementation framework, and explores a novel application of sensitivity analysis in Data Envelopment Analysis (DEA). Original programming approaches are developed to discover the stability region considering a variable returns to scale. The objective is to determine the extent to which the input and output of a decision-making unit (DMU) can be improved or worsened before the configuration of the efficient frontier changes. Furthermore, the permissible change radius for the decision-making unit is identified, considering all possible change directions. The applicability of the approach is demonstrated using numerical examples.

1. Introduction

Regulating rivalry is an effective way of creating a fair and healthy competition environment. This requires a union-like framework in which competition can be controlled. Such a framework measures the companies' competitiveness compared to their peers in the same industry while accounting for various performance indicators. Data Envelopment Analysis (DEA) is a cornerstone for efficiency analysis across various sectors, and the recent advancements continue to expand its application areas [1–5].

DEA applications range from healthcare to disaster response planning, environmental resource management, and beyond. For example, DEA is applied to evaluate Small and Medium Enterprises (SMEs) using smart, green, resilient, and lean practices to identify inefficient units and suggest improvements [6]. Innovative methods based on DEA have also been tested for designing and evaluating humanitarian supply chain networks under disruption using a weighted goal programming model and a two-stage network DEA to identify more efficient configurations [7]. As another example of DEA applications, a two-stage model was proposed to evaluate the utilization efficiency of urban water resources in Chinese cities, emphasizing the importance of improving water consumption efficiency through technological investments [8]. Moradi-Motlagh and Emrouznejad [9] developed statistical approaches to provide valid inferences in DEA estimations, addressing the deterministic nature of non-parametric models and offering confidence interval estimates for efficiency and productivity assessments. Peykani and Pishvaee [10] presented a DEA-based method for evaluating hospital performance under uncertainty.

Methodological advancements are another arena where DEA applications are extended; inverse DEA models for addressing resource overestimation issues are a prime example [11]. Ref. [12] introduced an inverse DEA method for efficiency analysis in mergers and acquisitions scenarios. Molla-Alizadeh-Zavardehi et al. [13] put forward several novel algorithms to enhance conventional DEA models, particularly addressing challenges associated with large datasets and maintaining input–output weights. Banking efficiency assessment was addressed by Shi et al. [14], where an enhanced Slacks-Based Measure (SBM) model was introduced. Banker et al. [15] investigated the impact of managerial ability on insurance company performance. Most recently, Santos-Arteaga et al. [16] assessed efficiency in European countries considering labor productivity and ICT value-added, showcasing the versatility of DEA in addressing diverse performance evaluation challenges.

DEA is a widely used mathematical model to estimate efficiency, but its accuracy relies on perfect data knowledge [17]. That is, DEA is data-oriented, and hence entry errors, statistical noise, and imprecise data affect its performance. In this situation, it is important to know

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how sensitive the DMUs are to possible changes in the input/output data. Sensitivity analysis in DEA initially emerged to investigate this issue.

Sensitivity analysis in DEA explores the impact of input and/or output data changes on the efficiency score of DMUs. This mathematical investigation of DEA can be employed to investigate a real-world situation where competitors in an industry union *change* their performance while ensuring that the contenders remain competitive despite the changes. To be specific, determining the maximum decrease in inputs (*e.g.*, downsizing) or increase in outputs (*e.g.*, increasing market share) of an industry's best-performing company so that the efficiency classification of others remains unchanged. The existing DEA literature explores the efficiency stability region when the performance of the efficient DMU *deteriorates* [18–25]. To our knowledge, DEA sensitivity analysis with an improving scenario has not been investigated. This study develops a novel programming approach to bridge this gap and extend the sensitivity analysis use cases of DEA. The objective is to determine the stability region and the permissible improvement radius for the members of the efficient frontiers considering Variable Returns to Scale (VRS). It is then extended to include the entire input–output space and changes in both improvement and deterioration directions all over the $\mathbb{R}^{m+s}$ space.

The remainder of this article unfolds in five sections. Section 2 provides a literature review and background to support the research gap. Section 3 introduces the basic concepts of the frontier analysis approach. Section 4 begins with the proposed improvement method and develops two DEA models to determine an improvement radius for the DMU under investigation. Another model is then developed to investigate the stability radius in the entire input–output space and all directions. Numerical analysis based on a case example is provided in Section 5. The article is concluded in Section 6 where major findings and directions for future research are presented.

2. Literature review

Efficiency in DEA can be analyzed through sensitivity and stability assessments to enhance accuracy and reliability, as highlighted in various research papers. The Shifted Geometric Mean (SGM) model is utilized for sensitivity analysis to identify indicators with the highest potential for sustainability improvement, aiding in decision-making for sustainability [26]. Inverse DEA, a post-DEA approach, focuses on determining optimal input and output quantities for each decision-making unit under perturbations to achieve efficiency targets, showcasing its versatility across sectors and applications [27]. Additionally, sensitivity analysis of random constraints and variational systems contributes to establishing well-posedness conditions for stochastic optimization problems, emphasizing robust stability properties in stochastic programming and variational inequalities [28].

Studies emphasize the challenges posed by uncertain data and non-unique optimal weights in traditional DEA models [29]. To address these issues, robust DEA models have been developed, incorporating robust optimization techniques to ensure stable and reliable performance evaluations, particularly in uncertain environments [29,30]. Also, the sensitivity of efficiency estimates in DEA is influenced by various factors such as distributional assumptions, choice of input variables, and functional forms, showcasing the importance of careful model selection and interpretation in efficiency analysis [31]. By integrating robust optimization theory with DEA, researchers have been able to enhance efficiency analysis, particularly in evaluating carbon emissions efficiency in the context of global warming mitigation strategies [32].

Dellnitz et al. [33] proposed a cross-efficiency method using strong defining hyperplanes to enhance efficiency scores, while Jiang et al. [34] presented an uncertain DEA model for scale efficiency evaluations, emphasizing the importance of considering scale efficiency alongside technical efficiency for decision-making processes. Furthermore, Sotiros et al. [35] discussed the dominance property in DEA, highlighting the necessity of non-dominated divisional efficiencies for accurate efficiency evaluations in Network DEA.

Jahanshahloo et al. [36] introduced the largest stability region for the BCC and Additive models with the support of hyperplanes. Liu et al. [25] explored sensitivity analysis for efficient DMU in the presence of data uncertainty; they deteriorated a class of DMU simultaneously and in the same directions while ensuring that the under-evaluation DMU remains on the efficient frontier. Boljunčić [37] used an iterative procedure so that changes do not impact the optimal basis matrix, and parametric programming was applied to obtain possible input or output perturbation.

Mozaffari et al. [38] adopted Multi-Objective Linear Programming (MOLP) based on weighted sums and the Step Method (STEM; interactive exploration procedure) to maintain a DMU's efficiency classification while applying simultaneous changes of all interval data. Abri et al. [39] employed a super-efficiency approach in DEA to determine the stability radius of efficient and quasi-efficient DMUs. Singh [40] proposed a multiparametric sensitivity analysis approach that classifies the perturbation parameters as *focal* and *nonfocal* and identifies critical regions for the efficient DMU under investigation. Wen et al. [41] and Khalili-Damghani & Taghavifard [42] extended the sensitivity analysis to work with fuzzy data. Hladik [43] proposed tolerance sensitivity analysis in linear programming and expanded the maximum tolerance region. Jahanshahloo et al. [19] developed a method for the sensitivity analysis of inefficient DMUs; they identified the 'exact necessary change region' where the efficiency score of specific inefficient DMUs changes to a defined efficiency score.

Abri [20] explored sensitivity analysis in returns to scale models using stability radius. Daneshvar et al. [44] proposed modifying the VRS model to improve the stability region of efficient units. Agarwal et al. [45] investigated the efficiency score's robustness by changing the reference set of the inefficient units using a new Slack Model (SM). Banker et al. [46] studied sensitivity and stability analysis in stochastic DEA and identified sufficient conditions to preserve the efficiency classification of all DMUs. Khodabakhshi et al. [47] extended sensitivity analysis in super-efficiency DEAs based on input relaxation super-efficiency measure. He et al. [48] determined the stability radius by simultaneously changing all interval data for all DMUs.

Zamani and Borzouei [21] presented a stability region by defining hyperplanes of the Production Possibility Set (PPS) while considering the addition of a new DMU to the set of observed DMUs. Dar et al. [49] employed a super-efficiency model based on input and output slacks. They analyzed the performance classification sensitivity and the returns to scale of DMUs, *i.e.*, Constant Returns to Scale (CRS), Increasing Returns to Scale (IRS), and Decreasing Returns to Scale (DRS). Ghazi et al. [50] investigated the improvement region for an inefficient DMU based on a value judgment using all the defining hyperplanes of PPS. Neralić and Wendell [22] demonstrated exploiting a priori information about admissible changes in the DMU to obtain a larger stability radius.

From the recent literature, Hladík [23] applied a tight linear programming approximation method to determine the stability region that preserves the efficiency classification of DMU under evaluation and maintains the ranking order. Khoveyni and Eslami [24] explored the internal structures of DMUs to determine their efficiency stability regions; their approach considers a two-stage production process when its inputs increase and intermediate products as well as final outputs decrease, while the data of the other two-stage production processes remain unchanged. Arabjazi et al. [51] identified the largest performance stability region for an 'extreme' efficient DMU whose input and output data can be changed in all directions. In contrast, the efficiency classification of the DMUs is preserved. They also identified the largest symmetric cell to the center of the 'extreme' efficient DMU under evaluation, which results in an efficiency stability radius. Sensitivity analysis, and stability radius determination in the presence of stochastic and fuzzy data have been studied by Arabjazi et al. [52,53].

Van Nguyen et al. [30] underscored the importance of model selection and distributional assumptions through their investigation of technical and scale efficiency estimates using stochastic frontier analysis (SFA). Similarly, Tian et al. [54] advocated for the application of the Dempster–Shafer theory (DST) in building energy research, highlighting the benefits of combining DST with machine learning methods to enhance the reliability of uncertainty and sensitivity assessments. Extending this theme, Zhu et al. [55] introduced a super-efficiency DEA model that incorporates spatial distance, thus improving stability against data perturbations and providing a robust method for evaluating decision-making units (DMUs). Mizuta et al. [56] contributed by examining the sensitivity of DEA models with different returns-to-scale (RTS) and orientations in the context of soil carbon sequestration (SCseq) efficiency across various Land Use/Land Cover (LULC) types in Florida. In another sector, Lou et al. [3] analyzed the efficiency of Scientific and Technological Innovation (STI) in universities, identifying an overall improvement in STI efficiency while also pinpointing specific areas for further enhancement. These diverse studies collectively illustrate that sensitivity and stability analyses are essential for gaining a comprehensive understanding of efficiency levels, identifying the key drivers of inefficiencies, and providing valuable insights to enhance performance across different fields.

Overall, the existing literature did not consider an improving scenario to find the efficiency stability radius or efficiency stability region for an 'extreme' efficient DMU; that is, these studies only assumed the case where changing the input/output data in a direction in which the 'extreme' DMU position deteriorates. This gap will be addressed to facilitate determining the extent to which the input and output of a DMU can be improved and/or deteriorated before efficiency classification changes.

3. Preliminaries

Efficient DMUs in DEA are used as references to evaluate the efficiency of other units. Therefore, it is of interest to investigate how changes in the inputs and outputs of efficient DMUs affect the overall efficiency analysis in the model. This analysis can show how sensitive the 'efficient frontier' is to the input and output data and whether small changes can cause changes in the identification of efficient DMUs. DEA results may not be stable when the analysis is sensitive to specific data changes. In this situation, exploring sensitivity helps analysts determine which efficient units are more resistant to data changes and act as a stronger reference to improve other units' performance or to make strategic decisions. Besides, sensitivity analysis allows managers to understand which inputs or outputs impose the greatest impact on unit efficiency; this informs managerial decisions about resource allocation and performance improvement.

Let us assume a set of n homogeneous DMUs, $\mathrm{DMU}_j; \mathrm{j} = 1, \dots, \mathrm{n}$, each of which converts m inputs into s outputs. Given $X_j = (x_{1j}, \dots, x_{mj})$ and $Y_j = (y_{1j}, \dots, y_{sj})$ as the observed input and output vectors of $\mathrm{DMU_j}$, respectively; also, $X_j \geq 0$, $X_j \neq 0$ and $Y_j \geq 0$, $Y_j \neq 0$. The PPS of VRS technology is shown in Eq. (1) considering a BCC model. On this basis, the envelopment form of the BCC model can be modeled in system (2).

$$T_v = \left\{ (X,Y) \;\middle|\; \begin{array}{l} X \geq \sum_{j=1}^{n} \lambda_j X_j \,,\, Y \leq \sum_{j=1}^{n} \lambda_j Y_j \,, \\ \sum_{j=1}^{n} \lambda_j = 1 \,,\, \lambda_j \geq 0, j = 1, \dots, n \end{array} \right\} \tag{1}$$

$$\begin{aligned} \theta^{o*} = \min \theta^{o} & \\ s.t. \quad & \sum_{j=1}^{n} \lambda_{\mathrm{j}} x_{ij} \leq \theta^{o} x_{io}, i = 1, \dots, m \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro}, r = 1, \dots, s \\ & \sum_{j=1}^{n} \lambda_j = 1, \\ & \lambda_j \geq 0, j = 1, \dots, n. \end{aligned} \tag{2}$$

In this formulation, x_{io} and y_{ro} are the ith input and rth output for the DMU_o under consideration ($o \in 1, \dots, n$), respectively. In this definition, DMU_o is called an efficient point if the BCC model's optimal value is equal to one ($\theta^{o^*} = 1$).

Based on Charnes et al. [57], a set of efficient DMUs can be classified into three groups: efficient points on the extremes (vertex) of the efficient frontier ($\boldsymbol{E}$), efficient but not an extreme point ($\boldsymbol{E'}$), and weak efficient points ($\boldsymbol{F}$). E consists of the points located at the vertices of the frontier, therefore it cannot be represented as a linear combination (with nonnegative coefficients) of the rest of the DMUs. In the case of VRS, we have a convex and not a linear combination. E' consists of efficient points that are efficient at both input and output orientations, but are not at the vertices. Finally, F consists of points that are efficient in the input orientation, and inefficient in the output orientation, or vice versa. The super-efficiency DEA model of Andersen and Petersen [58] is used to identify the classification of $\mathrm{DMU_o}$ (see Set (3)).

$$\begin{aligned} \theta_o^{\mathrm{super}*} = \min \theta_o^{\mathrm{super}} & \\ \text{s.t.} \quad & \sum_{j=1, j\neq o}^{n} \lambda_j x_{ij} \leq \theta_o^{\mathrm{super}} x_{io}, i = 1, \dots, m \\ & \sum_{j=1, j\neq o}^{n} \lambda_j y_{rj} \geq y_{ro}, r = 1, \dots, s \\ & \sum_{j=1, j\neq o}^{n} \lambda_j = 1, \\ & \lambda_j \geq 0, j \neq o, j = 1, \dots, n. \end{aligned} \tag{3}$$

On this basis, the optimal solution $\theta_o^{\sup er*}$ falls into one of the following categories.

- If $\theta_o^{\sup er*} > 1$ or (3) is infeasible, then $DMU_o \in E$.
- If $\theta_o^{\sup er*} = 1$, then $DMU_o \in E' \cup F$.
- If $\theta_o^{\sup er*} < 1$, then DMU_o is inefficient.

Definition 1. A region of permissible input/output changes is called a strong stability region if and only if DMU_o remains 'extreme' efficient after applying the changes. Furthermore, the efficiency classification of other 'extreme' DMUs must be preserved under these changes.

Definition 2. Given DMU_o, which is an 'extreme' efficient DMU, the largest radius of stability with the associated L_∞-norm is feasible if perturbations to DMU_o with L_∞-norm preserves the efficiency classification of other 'extreme' DMUs.

From a practical perspective, it is important to increase the productivity of a DMU without changing the efficiency classification of others. That is, obtaining a permissible region in which the inputs/outputs of the evaluated DMU are for possible improvement. These rates of change have important practical implications. The next section introduces a DEA-based method for sensitivity analysis and determination of the improvement radius of the 'extreme' efficient DMUs that define the efficient frontier. This follows by extending the model to allow data perturbations in all directions.

4. Proposed method

A decrease in the input value or increase in the output value of a DMU can impact the efficiency classification of the other extreme DMUs and result in the formation of a new efficient frontier. Taking DMU A – which is an 'extreme' point on the efficient frontier shown in Fig. 1 – as an example, decreasing the input takes A to A'; this, in turn, changes vertex B – which is also an 'extreme' efficient point – into a convex combination of A' and G. In this situation, point B becomes a new member of set E'. With a further decrease in the input value of A', point B becomes inefficient and will no longer be on the frontier. Similarly, when point A reaches point A'' as a result of increasing the output value, point C will be placed on the convex combination of A''

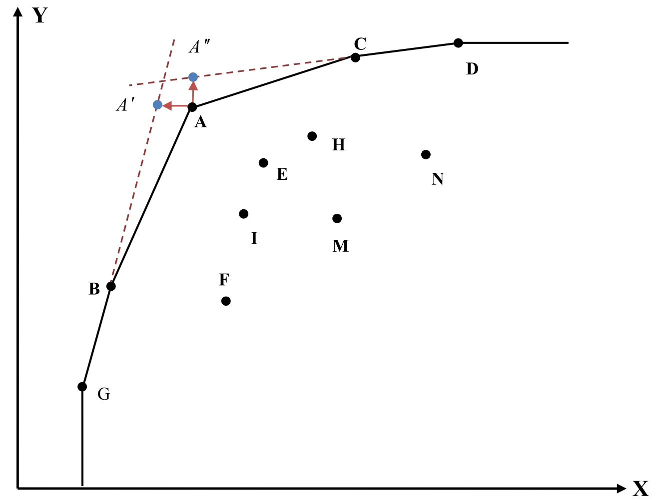

Fig. 1. An exemplary situation with one input and one output.

and D, and a further increase changes the efficiency classification of point C.

We are interested in determining how much the inputs (outputs) of the DMU under consideration can be decreased (increased) such that at least one of the other 'extreme' DMUs becomes a non-vertex efficient point in set E'. That is, the threshold within which improvements in DMU_o are possible before $\text{DMU}_{j_t \neq o}$ is displaced to the convex hull of the adjusted DMU_o (DMU_o^*) and its immediate neighbor (*e.g.*, point D).

Eqs. (4) and (5) are defined to represent, respectively, the absolute decrease of inputs, and absolute increase of outputs in the sensitivity analysis with an improving scenario.

$$x_{io}^* = x_{io} - \alpha_i, \quad \alpha_i \geq 0, \quad i = 1, \ldots, m \tag{4}$$

$$y_{ro}^* = y_{ro} + \beta_r, \quad \beta_r \geq 0, \quad r = 1, \ldots, s \tag{5}$$

In these equations, '*' represents the adjusted data for DMU_o; that is, $\text{DMU}_o^* = (X_o^*, Y_o^*)$. We extend the DEA model considering Equations (4) and (5) to explore the improvement region for $DMU_o \in E$ such that the efficiency classification of DMU_o and other extreme DMUs remains unchanged. Two separate cases of isolated input and output changes are first investigated to set the foundation for the integrated sensitivity analysis model that allows changes in all directions in the third subsection. It is worthwhile to note that each of the three models will be solved separately for every 'extreme' DMU while the data for the rest of the DMUs are constant.

4.1. Input model

The first mathematical formulation is developed to identify the exact improvement radius for DMU_o in the direction of *the* k^{th} *input*. This formulation is presented in the model (6).

$$\begin{aligned}
\min_{j_t \in E - \{o\}} \min \quad & \alpha_k^{j_t} \qquad \text{for each} \quad k = 1, \ldots, m \\
s.t. \quad & \sum_{\substack{j \in E \\ j \neq o \\ j \neq j_t}} \lambda_j x_{kj} + \lambda_o (x_{ko} - \alpha_k^{j_t}) = x_{kj_t} \\
& \sum_{\substack{j \in E \\ j \neq o \\ j \neq j_t}} \lambda_j x_{ij} + \lambda_o x_{io} = x_{ij_t}, \qquad i \neq k \\
& \sum_{\substack{j \in E \\ j \neq o \\ j \neq j_t}} \lambda_j y_{rj} + \lambda_o y_{ro} = y_{rj_t}, \qquad r = 1, \ldots, s \\
& \sum_{\substack{j \in E \\ j \neq j_t}} \lambda_j = 1 \\
& \lambda_{j \neq j_t} \geq 0, \alpha_k^{j_t} \geq 0.
\end{aligned} \tag{6}$$

The solution of this optimization problem, *i.e.*, $R_x = \alpha_k^* = \min \left\{ \alpha_k^{j_t *} \,|\, j_t = 1, \ldots, n, j_t \neq o \right\}$, represents the maximum possible decrease in the direction of the kth input of DMU_o; input changes less than α_k^* preserve the efficiency classification of $\text{DMU}_{j_t \neq o}$. In this definition, the kth input of DMU_o is reduced from x_{ko} to $x_{ko} - \alpha_k^*$ while keeping all other inputs and outputs constant. Therefore, if the kth input of the DMU_o is strictly reduced by α_k^*, all of the DMU_{j_t} will remain 'extreme' and on the efficient frontier.

Theorem. *In the case of a decrease in the* kth *input of* DMU_o*, the 'extreme' efficient* DMU_{j_t} *stays efficient if and only if* $\alpha_k^{j_t} \in \left\{ \alpha_k^{j_t} \middle| 0 \leq \alpha_k^{j_t} \leq \alpha_k^{j_t^*} \right\}$*, where* $\alpha_k^{j_t^*}$ *is an optimal solution to* (6).

Proof. Assume, for contradiction, that $0 \leq \alpha_k^{j_t^*} < \alpha_k^{j_t}$ then $x_{ko} - \alpha_k^{j_t} < x_{ko} - \alpha_k^{j_t^*}$. According to the first condition of the model (6), we have $\sum_{\substack{j \in E \\ j \neq o \\ j \neq j_t}} \lambda_j x_{kj} + \lambda(x_{ko} - \alpha_k^{j_t}) = x_{kj_t}$, which we will have from the $x_{ko} - \alpha_k^{j_t} < x_{ko} - \alpha_k^{j_t^*}$, and

$$\sum_{\substack{j \in E \\ j \neq o \\ j \neq j_t}} \lambda_j x_{kj} + \lambda \left(x_{ko} - \alpha_k^{j_t} \right) < \sum_{\substack{j \in E \\ j \neq o \\ j \neq j_t}} \lambda_j x_{kj} + \lambda \left(x_{ko} - \alpha_k^{j_t^*} \right)$$

$$x_{kj_t} < \sum_{\substack{j \in E \\ j \neq o \\ j \neq j_t}} \lambda_j x_{kj} + \lambda (x_{ko} - \alpha_k^{j_t^*}).$$

This contradicts that DMU_{j_t} is an 'extreme' efficient of *PPS*. A similar situation applies to models (7), (8), and (9) in the following section.

4.2. Output model

A stability analysis model is now developed to explore increased output data. The transformation based on Eq. (5) results in the model (7).

$$\begin{aligned}
\min_{j_t \in E - \{o\}} \max \quad & \beta_l^{j_t} \qquad \text{for each} \quad l \in \{1, \ldots, s\} \\
s.t. \quad & \sum_{\substack{j \in E \\ j \neq o \\ j \neq j_t}} \lambda_j x_{ij} + \lambda_o x_{io} = x_{ij_t}, \qquad i = 1, \ldots, m \\
& \sum_{\substack{j \in E \\ j \neq o \\ j \neq j_t}} \lambda_j y_{lj} + \lambda_o (y_{lo} + \beta_l^{j_t}) = y_{lj_t} \\
& \sum_{\substack{j \in E \\ j \neq o \\ j \neq j_t}} \lambda_j y_{rj} + \lambda_o y_{ro} = y_{rj_t}, \qquad r \neq l \\
& \sum_{\substack{j \in E \\ j \neq j_t}} \lambda_j = 1 \\
& \lambda_{j \neq j_t} \geq 0, \beta_l^{j_t} \geq 0.
\end{aligned} \tag{7}$$

In this model, the exact improvement radius for DMU_o in the direction of lth output can be described as $R_y = \beta_l^* = \min \left\{ \beta_l^{j_t *} \,|\, j_t = 1, \ldots, n, j_t \neq o \right\}$. Solving this optimization problem determines the maximum possible increase in the direction of the lth output of DMU_o, $R_y = \beta_l^*$ such that the efficiency classification of $\text{DMU}_{j_t \neq o}$ will be preserved for any values less than β_l^*. In this definition, output l of DMU_o increases from y_{lo} to $y_{lo} + \beta_l^*$ while other outputs and inputs are fixed. Other DMU_{j_t} are meant to remain 'extreme' and on the efficient frontier if the lth output of DMU_o is increased by $R_y = \beta_l^*$.

4.3. The integrated approach

The integrated mathematical model allows for simultaneous exploration of the inputs and output changes to identify the stability region of DMU_o while ensuring that the classification of other 'extreme' DMUs remains the same.

To find the stability region of the 'extreme' efficient unit under the evaluation of DMU_o, the inputs and outputs of DMU_o are shifted in the direction of $\vec{d} = \begin{pmatrix} dx \\ dy \end{pmatrix}$ by a step length of θ such that the 'extreme' efficient unit $DMU_{j_t \neq o}$ is placed in the convex combination of DMU_o and other 'extreme' efficient units. Moving more than θ is not possible because the efficiency status of extreme $DMU_{j_t \neq o}$ will be disturbed.

Model (8) is developed to identify the maximum value of θ so that the efficiency classification of the other vertical efficient units does not change. S_p represents the set of vertex (extreme) efficient points in this formulation.

$$
\begin{aligned}
&\min \theta \\
&s.t. \quad \sum_{\substack{j\in S_p \\ j\neq o \\ j\neq j_t}} \lambda_j x_{ij} + \lambda_o (x_{io} + \theta dx) = x_{ij_t} \quad i = 1,\dots,m \quad \forall j_t \in S_p - \{o\} \\
&\qquad \sum_{\substack{j\in S_p \\ j\neq o \\ j\neq j_t}} \lambda_j y_{rj} + \lambda_o (y_{ro} + \theta dy) = y_{rj_t} \quad r = 1,\dots,s \quad \forall j_t \in S_p - \{o\} \\
&\qquad 1dx + 1dy = 1 \\
&\qquad \lambda_j \geq 0, \theta \geq 0.
\end{aligned} \tag{8}
$$

Model (8) is the basis of the integrated programming approach with dx, dy, λ_j, θ representing the unknown variables. Moreover, $dx = \begin{pmatrix} dx_1 \\ \vdots \\ dx_m \end{pmatrix}$, $dy = \begin{pmatrix} dy_1 \\ \vdots \\ dy_s \end{pmatrix}$, and $\vec{d} = \begin{pmatrix} dx \\ dy \end{pmatrix}$ is a vector in space $\mathbb{R}^{m+s}$, which can be formulated as a linear combination of linearly independent vectors of e_j; $\vec{d} = \begin{pmatrix} dx \\ dy \end{pmatrix} = \sum_{\substack{j=1 \\ \alpha_j \in R}}^{m+s} \alpha_j e_j = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_{m+s} \end{pmatrix}$. Finally, $\theta d = \theta \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_{m+s} \end{pmatrix} = \begin{pmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_{m+s} \end{pmatrix}$, in which we have $\theta dx = \begin{pmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_m \end{pmatrix}$, $\theta dy = \begin{pmatrix} \theta_{m+1} \\ \theta_{m+2} \\ \vdots \\ \theta_{m+s} \end{pmatrix}$. On this basis, the base model can be transformed into a multiple-objective non-linear programming formulation using Model (9).

$$
\begin{aligned}
&\min |\theta_1| \\
&\vdots \\
&\min |\theta_{m+s}| \\
&s.t. \quad \sum_{\substack{j\in S_p \\ j\neq o \\ j\neq j_t}} \lambda_j x_{ij} + \lambda_o (x_{io} + \theta_i) = x_{ij_t} \quad i = 1,\dots,m \\
&\qquad \sum_{\substack{j\in S_p \\ j\neq o \\ j\neq j_t}} \lambda_j y_{rj} + \lambda_o (y_{ro} + \theta_{r+m}) = y_{rj_t} \quad r = 1,\dots,s \\
&\qquad \lambda_j \geq 0, \ \theta \text{ free}
\end{aligned} \tag{9}
$$

This approach pushes the target DMU_o's performance in all directions as far as it does not disturb other DMUs' classification. In this definition, values less than $|\theta_i|$ in the direction of the ith input and less than $|\theta_{r+m}|$ in the direction of the rth output will not alter the classification of DMU_{j_t}. That is, by increasing the input and output of DMU_o to a value greater than $|\theta_i|$, and $|\theta_{r+m}|$, DMU_{j_t} situates itself in the convex hull of the other 'extreme' DMUs. θ_i, θ_{r+m} are considered free-in-sign to allow movement in any direction. Besides, target functions are the absolute value of θ_i to ensure positive outcomes. The resulting model 9 is to be solved using the weighted sum method.

5. Numerical examples

This section begins with the small illustrative example in Table 1 to test the input and output models developed in Sections 4.1 and 4.2, respectively. We then present a case example from Ref. [59] to show the applicability of the integrated model developed in Section 4.3.

Table 1
Illustrative example with one input and one output.

Decision-making unit	Input	Output
DMU_1	3	2
DMU_2	4	5
DMU_3	5	6.5
DMU_4	6	8
DMU_5	8	3
DMU_6	9	9.5
DMU_7	10.5	10
DMU_8	12	10.5
DMU_9	13	7
DMU_{10}	14	11

Table 2
Results analysis considering simple input- and output-based models.

DMU_{j_t}	$\alpha_1^{j_t*}$	$\beta_1^{j_t*}$
DMU_1	3	$+\infty$
DMU_2	1	3
DMU_6	1.5	0.5
DMU_8	4	1
DMU_{10}	$+\infty$	3
R_∞	$R_x = min_{j_t} \alpha_1^{j_t*} = 1$	$R_y = min_{j_t} \beta_1^{j_t*} = 0.5$

Table 3
Case example with three inputs and three outputs.

Decision-making unit	Inputs			Outputs			CCR Efficiency
	X1	X2	X3	Y1	Y2	Y3	
DMU_1	12	400	20	60	35	17	1
DMU_2	19	750	70	139	41	40	1
DMU_3	42	1500	70	225	68	75	1
DMU_4	15	600	100	90	12	17	0.820
DMU_5	45	2000	250	253	145	130	1
DMU_6	19	730	50	132	45	45	1
DMU_7	41	2350	600	305	159	97	1

The initial evaluation of the illustrative example shows that DMU_1, DMU_2, DMU_6, DMU_6, DMU_8, and DMU_{10} can be categorized as set E, and entities DMU_5 and DMU_9 are perceived as inefficient. We also know that DMU_4 is an 'extreme' efficient unit. Table 2 reports the sensitivity analysis results of models (6) and (7).

In this table, '1.5' of DMU_6 indicates that the input of DMU_4 can be reduced by a maximum of 1.5 units while ensuring that DMU_6 remains on the efficient frontier. As another example, the output of DMU_4 can be increased by 3 units without deteriorating the efficiency of DMU_2. It is worthwhile to note that '$+\infty$' in the second column of DMU_{10} means that the input changes of DMU_4 do not impact the efficiency classification of DMU_{10}. To put the improving scenario in context, it is evident that DMU_4's output can be increased from 8 to 11 without changing the efficiency of DMU_{10}. The *R-value* in the last row shows that decreasing the input and increasing the output of the DMU_4 can be an upper limit of 1 and 0.5, respectively, such that the efficiency classification of all 'extreme' efficient DMUs remains stable.

It is more practical to account for the simultaneous input and output changes when analyzing the DMUs' performance. In the following case example, we consider three inputs and three outputs to investigate the permissible perturbations in a sector with seven entities. Data is summarized in Table 3.

Considering DMU_1 as the analysis target, Table 4 shows the permissible changes in inputs and outputs of this DMU considering its impact on the rest of the DMUs. The values under $|\theta_1|$ specify that the first input of DMU_1 can change at most 0.8 units; a perturbation greater than this will impact the classification of DMU_3. The results also show that changing the second input of DMU_1 is not permissible because any changes will impact the other four 'extreme' DMUs. It is

Table 4
Results analysis considering both inputs and outputs changes.

DMU_{j_t}	Inputs			Outputs		
	$\|\theta_1\|$	$\|\theta_2\|$	$\|\theta_3\|$	$\|\theta_4\|$	$\|\theta_5\|$	$\|\theta_6\|$
DMU_2	infeasible	infeasible	infeasible	infeasible	infeasible	infeasible
DMU_3	$\|0.8\|$	0	$\|-1.3\|$	0	$\|-16.86\|$	$\|3\|$
DMU_5	$\|-1.8\|$	0	0	$\|-10.15\|$	$\|-4.88\|$	$\|14.46\|$
DMU_6	$\|-1.38\|$	0	0	$\|10.98\|$	$\|-8.26\|$	$\|10.13\|$
DMU_7	$\|-5.02\|$	0	$\|82.12\|$	$\|-8.08\|$	$\|-7.93\|$	$\|-0.48\|$

important to note that the 'infeasible' in this table indicates that by moving DMU_1 in the direction of each of the inputs and outputs, DMU_2 will not placed in the convex hull of other 'extreme' efficient units and its efficiency classification does not change. The last three columns show the maximum change in the DMU_1's outputs, which, respectively, the first output remains unchanged, the second output can change up to 4.88 units, and the third output can change up to 0.48 units.

6. Concluding remarks

This study investigated the concept of constrained improvement, according to which, the best-performing players in a sector curb their progress on one major condition: that the rivals remain competitive. From a frontier analysis perspective, DEA sensitivity analysis is a basis for determining the stability region and the permissible improvement radius for the DMUs. The existing sensitivity analysis methods can merely investigate a worsening scenario. This gap was addressed by developing two programming models with improving scenarios where the 'extreme' efficient DMU under evaluation improves its performance without deteriorating the status of other 'extreme' efficient DMUs. An integrated model was then developed that allows for changes in all directions. Overall, the model addresses a critical gap in current DEA sensitivity analysis methodologies by offering the following features:

- Comprehensive sensitivity analysis with a multidirectional perspective within a single model: unlike existing models that often focus on unidirectional changes (either input or output changes independently), our model simultaneously considers all directions of change within one framework. Including all possible scenarios offers a more detailed and accurate sensitivity analysis.
- Comprehensive stability regions: by accounting for variations in all directions, our model identifies more extensive and detailed stability regions for efficiency. This ensures that decision-makers are aware of all potential efficiency scenarios, enhancing the reliability of the efficiency evaluation.
- Better insights for decision analysis: including a broader range of input and output variations helps decision-makers anticipate and prepare for a wider array of potential changes, leading to more strategic and robust operational adjustments.

The numerical examples confirmed that the developed approach enables a multifaceted frontier analysis. This can serve as a reliable tool for decision-makers and managers to adopt optimal strategies for improving the performance of the organizations or units under evaluation. The developed approach can also serve as a decision analysis tool for policymakers to regulate rivalry in an industry sector and for supply chain managers for supplier development programs.

Despite the novelty element, the developed programming approach is limited when it has to deal with anchor points in special cases. From the modeling perspective, the computational complexity of the developed non-linear approach is rather high and hence it may not be possible to use commercial solvers for large-scale problems. Future research may address these limitations to help extend the real-world applications of the concept of constrained improvement in DEA. Besides, we believe that employing the developed method for a real-world case study would interest decision analysis researchers in extending the use cases of DEA sensitivity analysis.

This study may inspire researchers to explore new research directions; here are some suggestions for future research. First, the concept of constrained improvement can be investigated considering other DEA variants to test and extend its use cases. The second suggestion comes from improving the proposed sensitivity analysis models to account for uncertainty and allow for using fuzzy, stochastic, and interval data. As a third suggestion, one may extend our method to work with network DEA; such a development will be useful for regulating rivalry across sectors or geographical regions. The concept of constrained improvement in network DEA could also be useful as a standardization framework for environmental monitoring and regulating carbon credit across sectors. The final suggestion relates to considering the improvement possibility in the performance of the smaller DMUs (*i.e.*, small and medium-sized enterprises-SMEs in a particular industry) in regulating rivalry. This may include special business cases like mergers and acquisitions.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The raw data required to reproduce the findings are available in Tables 1 and 3.

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