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Abstract

The efficient operation of heavy machinery is crucial to the success of mining and civil construction operations. To guarantee this performance, equipment performance is assessed using the Reliability Index, which analyzes failures to study the ability of a system to carry out its intended functions under predetermined conditions. On the other hand, the failure rate and operational environmental condition (such as management decisions, maintenance performance, etc., that are defined as "risk factors") over the life cycle of industrial systems pose a significant challenge to reliability analysis. This paper proposes an approach to address these challenges by extending Weibull family functions with regression models. Then, the effectiveness of this approach using 12-month failure data from the Komatsu 785-5 dump-truck engine refueling system is demonstrated. The most appropriate reliability function for two scenarios with different environmental conditions over the performance interval is fitted. The results indicate that both scenarios exhibit lower reliability than the baseline, highlighting the influence of environmental conditions on equipment reliability.

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RESEARCH ARTICLE

Enhancing fuel injection system reliability through Weibull family functions analysis

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Abstract

The efficient operation of heavy machinery is crucial to the success of mining and civil construction operations. To guarantee this performance, equipment performance is assessed using the Reliability Index, which analyzes failures to study the ability of a system to carry out its intended functions under predetermined conditions. On the other hand, the failure rate and operational environmental condition (such as management decisions, maintenance performance, etc., that are defined as "risk factors") over the life cycle of industrial systems pose a significant challenge to reliability analysis. This paper proposes an approach to address these challenges by extending Weibull family functions with regression models. Then, the effectiveness of this approach using 12-month failure data from the Komatsu 785-5 dump-truck engine refueling system is demonstrated. The most appropriate reliability function for two scenarios with different environmental conditions over the performance interval is fitted. The results indicate that both scenarios exhibit lower reliability than the baseline, highlighting the influence of environmental conditions on equipment reliability.

Keywords: Mining truck, reliability, weibull distribution family, proportional hazard model, fuel injection system

1. Introduction

In the 21st century, the world has become more complicated in terms of social, political, economic, technological aspects, etc. This complexity is reflected in many man-made products and systems. The expectations of customers to receive a product or service (a specific output from a specific system) are defined according to the ability of that product or system to perform the desired activity at a specific time and under specific environmental conditions, which is called reliability in engineering sciences [1]. Every year, billions of dollars are spent on producing various types of equipment for use in different mines worldwide, and this cost is increasing rapidly. On the other hand, the competitive economy has required mining companies to update their operations through technology upgrades, optimization, or changes in the existing system [2]. With new technological changes, mines have become increasingly mechanized and somewhat automated in various sectors. Heavy machines were introduced to the mining industry in the early 1950s, and the second big wave of change was achieved in the early 1990s with the introduction of computers. After the introduction of heavy machines and complex mining equipment, mining systems have also become more complicated, and therefore, the management of these systems requires new approaches. Reliability engineering is a new management method that improves system performance through behavior measurement. This science examines the possibility of failure operation and its performance by examining the failures that occurred in a system. Failure in reliability analysis is defined as the inability to perform the expected activity due to

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various factors such as high stress (like fracture, buckling, deformation, and adhesion) and depreciation (such as corrosion, rubbing, penetration, crack initiation, diffusion, and, radiation) can cause it. In addition, they may be classified as mechanical (deformation, buckling, fracture, cracking, creep and creep cracks), electrical (electrostatic discharge, dielectric hazards, connection hazards), thermal (heating, thermal expansion and contraction), radiation (radioactivity, secondary cosmic rays), chemical (corrosion, oxidation, etc.) or a combination of these [1]. From the mid-1960s to the late 1980s, this index was introduced by researchers such as Levkovich & Chalenko [3], Al'tshuler [4], Ivko and co-workers [5], Freidina and co-workers [6], these years can be known as the period when the mining community became familiar with the concept of reliability. These articles are mostly short, and due to the weakness of the data bank and the lack of development of statistical modeling software, they are not very strong in terms of content. In the late 1980s, Goodman investigated the reliability of emergency escape routes in room and base coal mines [7]. Kumar and his co-workers (1989) implemented a coherent process for analyzing the reliability of loading fleets [8]. In the next two decades, similar to the process proposed by Kumar again [9], Javad Barabady et al. [10,11]. In the latest research on the approach mentioned earlier, Gustafson et al. compared two conventional and automatic underground loaders [12]. Hashemi and his colleague, Rakhshanimehr, have also had software reliability and simulation activities in recent years [13,14]. The common feature of the research reviewed up to this point about reliability is ignoring the environmental conditions considered "risk factors or covariates" in the analysis. In these studies, classical statistical methods such as distribution functions (normal, exponential, Weibull, etc.) and simple models based on time data, such as the power law process, carry the major share of analysis. However, as it is clear from the title of these factors (environmental conditions), they are considered environmental factors such as the type of rock, road conditions, weather conditions, etc., and it is necessary to enter them to achieve more accurate analysis and close to the real conditions. Regression approaches, such as the proportional hazard model (PHM), are among the approaches that include the effects of risk factors in the analysis and modeling of the system's reliability. In 1993, in an article, Kumar and Klefsjö, for the first time, gave a relatively complete review of the proportional hazard model (PHM) to use to include the effect of environmental conditions in the reliability analysis [15]. A year later, the researchers mentioned above used the PHM to analyze the reliability of electricity transmission cables [16]. In 1995, Kumar also extended the PHM for repairable systems [17]. Kumar and Westberg's research in 1996 also continued on the loading machine, and during it, the assumption of constant risk factors over time for the PHM was evaluated [18]. Prasad and Rao (2002) examined three study samples [19]. From 2005 to 2012, Ghodrati et al., for the first time, used the combination of the effects of environmental conditions, such as weather, humidity, operator skills, etc., on reliability in managing spare parts such as washers and brake systems and hydraulic jacks. In this research, in addition to estimating the number of spare parts needed, they also determined the time to renew the stock (purchase again to fill the stock) [20]. In 2011, Abbas Barabadi used the stratified Cox regression model (SCRM) in reliability analysis [21]. In 2012, Wijaya presented his Ph.D. dissertation on improving the accessibility of tunnel-boring devices [22]. In 2014, Rahimdel and co-workers analyzed the reliability of the drilling rig fleet [23]. In the same years, Sinha and Mukhopadhyay used FMEA and TTT-plot methods to check the stone crushing system [24]. In recent years, Nouri et al. analyzed the reliability of the stone crusher system in the cement factory in different subsystems such as stone crusher, conveyor, stacker, and reclaimer [25,26]. He has also studied loading, transportation, and storage machinery [25,27]. In 2018, Hotma also studied the reliability of loading subsystems with the same approach [28]. In the same year, Moniri and colleagues used PHM to analyze the loading system [29]. In 2019, Shakhatreh and his friends introduced a new lifetime distribution to describe and analyze the failure rate. The main part of their attention was on the fundamental relationship between the failure rate and the average remaining life according to their change points [30]. In 2020, Zhang addressed an old and fundamental problem in estimating the reliability of Weibull parameters and reliability with zero-failure data [31]. In 2021, Amirzadi and his friends focused on the inverse generalized Weibull distribution's scale parameter and reliability estimates [32]. Kayid et al., in 2022, also presented a flexible inverse modified Weibull model with a concave Weibull probability diagram that describes various reliability phenomena [33]. In reliability engineering, it is known that complex systems usually have more than one failure mode or cause. Between reviewed literature, it has not been used appropriate distribution in the PHM models. On the other hand, some studies have ignored the influence of environmental conditions (risk factors) and fixable distribution (as a baseline function). However, most researchers use Weibull distributions for data analysis due to their strong and flexible nature. Consequently, this approach is highly effective in adapting to various failure data analysis. Therefore, this article attempts to provide a coherent framework for reliability analysis with a suitable function of various Weibull distribution functions under the title "Weibull distribution family" and to consider the effects of risk factors. The article is divided into three general parts, including research theory, which provides general information about reliability and the proposed methodology. In the next part, the breakdowns of the engine fueling system of a mining dump truck are analyzed using the proposed methodology, and finally, the results are analyzed.

2. Research theory and methodology

Having a correct estimate of the behavior of each system (subsystem, component, or part) plays a fundamental role in its planning and function. It is one of the most significant challenges for engineers. As mentioned, using different modes of the Weibull function, or in better words, the "Weibull distribution family", is one of the ways to face this challenge. Despite the existence of different modes for this model or its family, one of the key issues in applying the Weibull function is the lack of practical guidance for choosing the most appropriate model from this family. However, this model alone cannot cover all aspects of a function. To address this weakness, this article also considers environmental conditions as risk factors (or covariates) in addition to this family. Figure 1 illustrates the proposed algorithm for entering the effect of environmental conditions on performance indicators. In this algorithm:

- **First stage:** The system's identification and differentiation rely on the researcher's diverse perspectives. It is essential to clearly distinguish between system levels and various failure modes, ensuring no overlap.
- **Second stage:** Identifying the mechanism of failures in the system and its components can be achieved through condition monitoring, inspection, pre-existing failure data, and information from sensors.
- **Third stage:** Collecting the required information from different sources and integrating this information to form a general data bank that can respond to analyses based on time data and the impact of environmental conditions.
- **Fourth stage:** Determining the impact of risk factors.
- **Fifth stage:** Determining the reliability function is done in parallel, ignoring the effects of environmental conditions.
- **Sixth stage:** The basic function will be integrated with the influence of environmental conditions, and the behavioral function will be obtained for any desired state in different scenarios.

Due to the scope of the statistical issues, more details about each stage are discussed based on the actual data on the fueling system failure of the Komatsu 785-5 dump truck engine from the Sungun copper mine.

2.1. Identifying and restricting the system (first stage)

According to the meeting and expert interviews, a Komatsu 785-5 dump truck from the Sungun copper mine was selected as the machine to be analyzed. The Sungun copper mine complex is 85 km northwest of Ahar City and 35 km north of Varzeqan. It has geographical coordinates of 46° 42 min 20 s of longitude and 38° 41 min 30 s of latitude. Cold and frosty winters and mild summers are the climatic features of the Sungun region. The average maximum temperature is 33°C in summer and 20°C in winter. This machine is critical because the number of these machines is limited due to the large volume of loading devices. Significant repair work is focused on this device, and many management decisions are based on it.

The dump truck system in the mine, based on the studies conducted and the opinions of the specialists of the repair shop (shift manager, mechanical engineers, repairmen, operators), is divided into five subsystems: 1. Machine: including the body, operator's cabin, container, and tires, 2. Power transmission: spring and shock absorbers, wheels, and appendages, 3. Hydraulics: all pumps, jacks, hydraulic motors, hydraulic equipment, and hoses, 4. Gearbox: all parts and components of gearboxes and turbines, and 5. Engine: mechanical-diesel engine, which, according to failure statistics and costs, this subsystem was considered a critical subsystem.

2.2. Identifying the mechanism of the occurred failures (second stage)

In the next step, based on the type of failures that occurred and the configuration of the system, which was examined according to experts' opinions, the

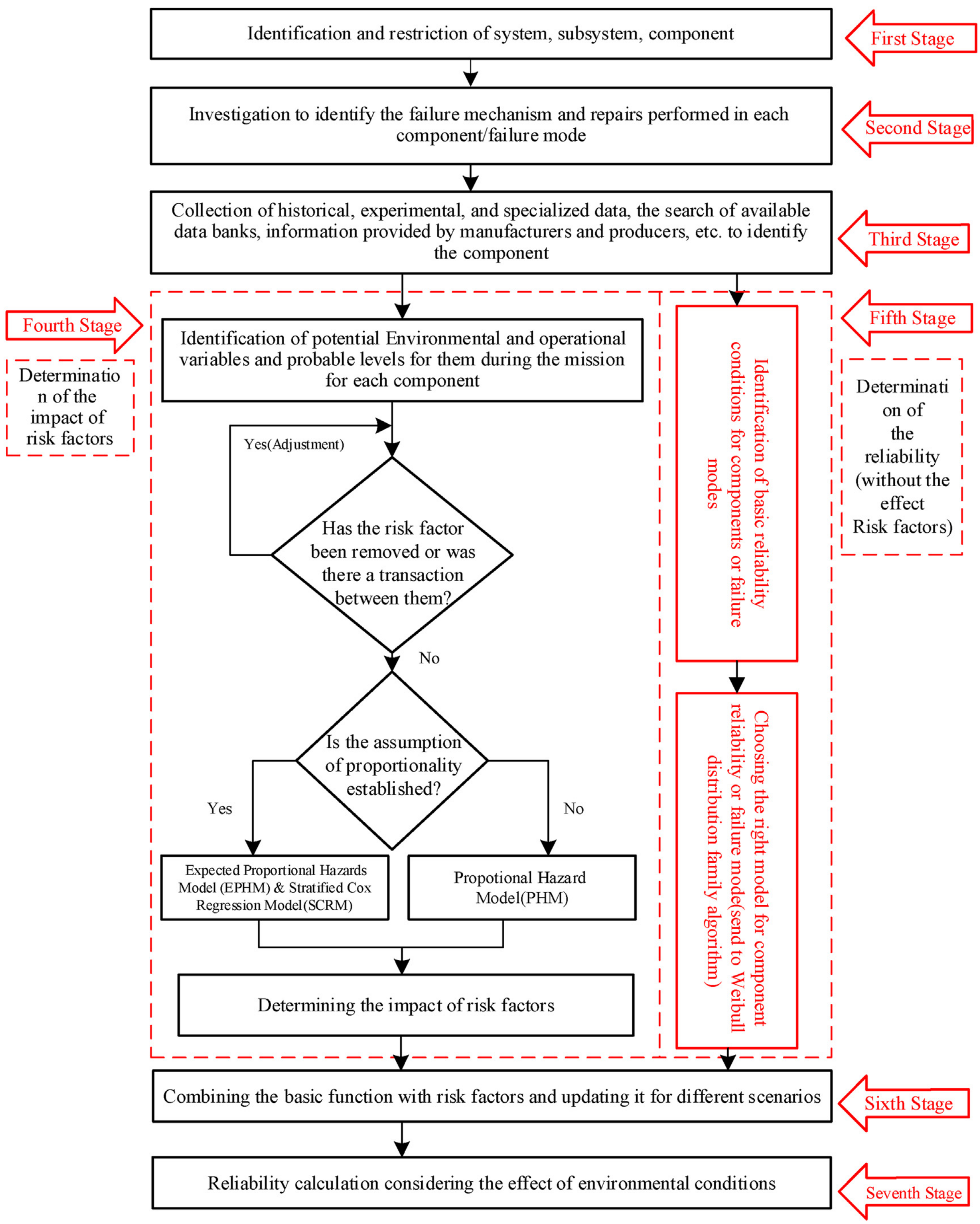

Fig. 1. Reliability analysis by considering the impact of environmental conditions.

fault tree for the five main failures considering the engine structure was determined in Figure 2. As can be seen, four types of failure have occurred for the engine as follows:

- Mechanical: failure of the apron, turbocharger, radiator, etc.
- Fuel injection (fuel circulation and transfer), such as puncture of the pipe, wear of diesel pump needles, etc.
- Pneumatic: failures related to the transfer of wind energy in the system, such as compressor failure, hoses, etc.
- Electrical: burning out of the lamp, horn failure, starter: all failures related to the starter system.

Figure 3 shows the Pareto chart for various failure modes. In this chart, the columns show the number and frequency percentage of breakdowns related to each mode, the upper curve displays the number and

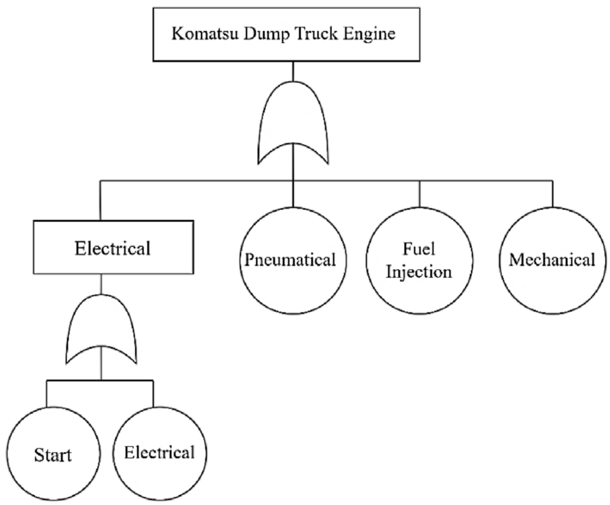

Fig. 2. FTA diagram of dump truck engine failure.

cumulative percentage of breakdowns, the vertical axis on the left shows the number of accumulations, and the right axis represents the percentage of frequency and cumulative frequency. At the bottom of the chart, the failure mode and the numerical values of the number, frequency percentage, and cumulative frequency percentage of each failure mode are included. Based on this chart, it should be noted that 101 failures were recorded for the engine, of which 36 failures were related to the fuel injection system. This includes about 40% of the total failures, so this failure mode was chosen for wider analysis.

2.3. Collecting and extracting the required data (third stage)

The data required in this research are time data in the form of "time between failures (TBFs)" and the effects of environmental conditions in the form of risk factors. The desired data were collected for 12 months from various sources, including daily reports, repair shops' meteorology, interviews and meetings, warehousing, etc. Table 1 shows an example of system data along with risk factors. In this table, the first column is the failure number, the second column is the time between failures in hours, the third column is the failure status (Status) in terms of complete failure (f) or censored (s), and the next column shows the risk factors including a discrete risk factor which consists of weather conditions (z_1), which is divided into four parts; clear and sunny (4), partly cloudy (3), cloudy (2) and heavy fog (1). Continuous risk factors include slope (z_2), hourly capacity (z_3), precipitation (z_4), and temperature (z_5).

2.4. Determining the effects of risk factors (fourth stage)

As mentioned, the system's reliability is a function of the system's time and operating environmental conditions. Hence, the reliability study requires a framework that includes technical, operational,

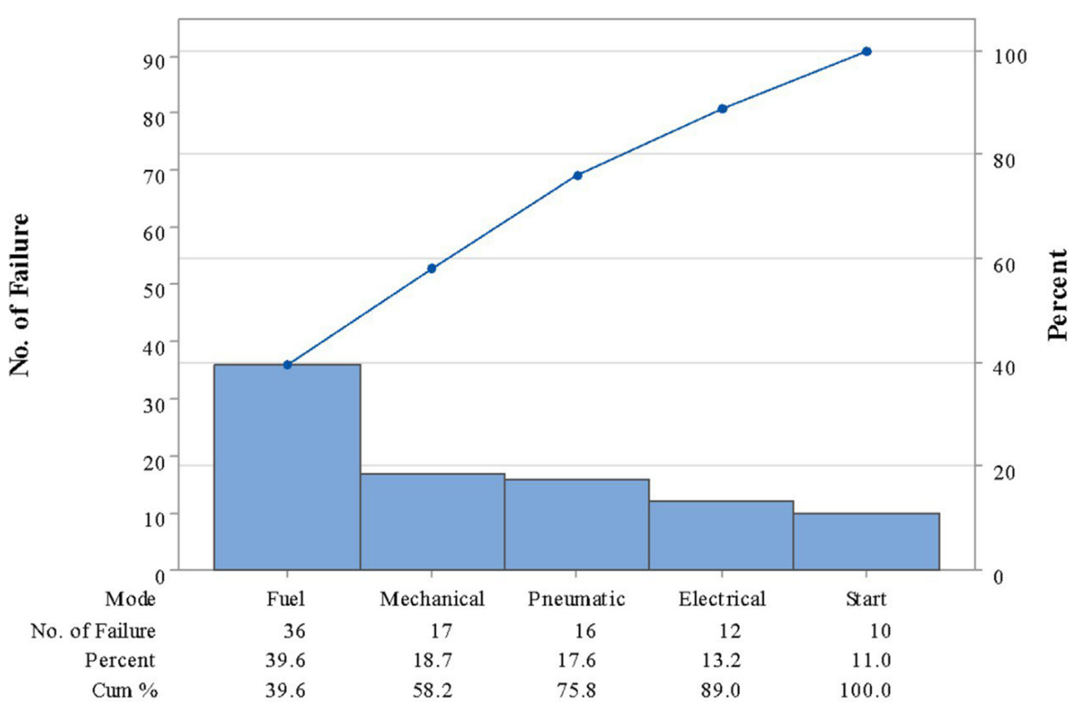

Fig. 3. Pareto diagram of dump truck engine failure modes.

Table 1. Example of TBF data of dump truck engine system along with failure risk factors.

Failure data	Failure status	Risk factors				
		slope (z_2)	hourly capacity (z_3)	precipitation (z_4)	temperature (z_5)	weather (z_1)
23.75	*f*	0	3.07	0.02	0.47	1
26.75	*f*	0.03	1.45	0.02	0.56	1
10	*f*	0	2.9	0	1.39	4

commercial, and managerial issues and risk factors in general [34]. This article uses the proportional hazard model and extensions to estimate reliability features better. These models were used in the 1970s due to their ability to calculate the reliability of a system [15,35]. Risk factors change randomly and may also change failure times [36]. As mentioned, PHM and its various extensions, such as SCRM and EPHM, are the most widely used regression models for introducing risk factor effects. Models based on risk factors in reliability analysis are mainly based on the proportional hazard model. PHM is a non-parametric or semi-parametric approach first developed by Cox (1972) for survival data in the medical field [37]. This model is a valuable statistical process for estimating failure risk according to the system conditions and environment. This model assumes that the risk function is a component or subsystem. It combines its basic risk rate function and an expression including the effects of risk factors [38]. The hazard rate function of this model is expressed as equation (1):

$$\lambda(t,z) = \lambda_0(t)\psi(z,\alpha) \tag{1}$$

In this equation, $\lambda(t, z)$ is the hazard rate function (response variable). z: risk factor (a linear vector containing risk factor parameters) that includes the degree of effect of each risk factor on the risk rate, and t represents the time until failure in a device or the time it is working. $\lambda_0(t)$: baseline hazard rate, $\psi(z, \alpha)$: link function [39], it is a function for which different states can be considered. Exponential mode $\exp(z\alpha)$, logistic mode $\log(1 + \exp(z\alpha))$, inverse linear $1/(1 + (z\alpha))$, and linear $1 + (z\alpha)$ are some of these modes, of which the exponential mode is the most widely used [15]. Assuming an exponential function for the function $\psi(z, \alpha)$, the hazard rate becomes (2):

$$\lambda(t,z) = \lambda_0(t)\exp(z\alpha) = \lambda_0(t)\exp\left(\sum_{i=1}^{n} z_i\alpha_i\right) \tag{2}$$

The multiplication factor $\exp(z\alpha)$ can indicate the risk of failure due to the presence of the risk factor. In $z\alpha = \sum_{i=1}^{n} z_i\alpha_i$ equation, α is a column vector of unknown parameters of the model or regression coefficients related to the risk factor [15]. The reliability function for PHM is also in the form of equation (3) [40]:

$$R(t,z) = (R_0(t))^{\exp\left(\sum_{i=1}^{n} z_i\alpha_i\right)} \tag{3}$$

In this equation, $R_0(t)$ is the baseline reliability, which is only based on time. This model is based on the time independence of risk factors, the proportional ratio between two risk rates over time, known as the proportionality assumption (PH). This assumption is statistically expressed as equation (4) [41]:

$$\frac{\mu_i(t,z_1)}{\mu_j(t,z_2)} = \theta, \text{constant overt} \tag{4}$$

In this equation, μ_i is risk rate i and μ_j is risk rate j. This assumption is rejected if the risk rate graphs are crossed for two risk factors, z_1 and z_2. Of course, various methods exist to evaluate this assumption, including graphical methods, a goodness-of-fit test process, and a method based on time-dependent variables [41]. If the risk factors are time-dependent, the accelerated failure time model (AFT) or some extensions of PHM, such as the stratified Cox regression model (SCRM), can be used. For more information, refer to Refs. [19,21,41]. According to the algorithm, all obvious risk factors are extracted from various sources, interviews, and meetings. The possibility of merging some of these factors or even removing them is examined. According to the above algorithm, the first step is to check the presence or absence of dependency between risk factors. The correlation test between the risk factors and the time data was performed for this purpose. The analysis results for the risk factors and system operation time can be seen in Table 2, which shows the dependency of the risk factors. As it is clear from the dependency test result, there is no dependency between the risk factors.

In the next step, the assumption of the proportionality of the risk rate (PH) should be evaluated to ensure the non-dependency of risk factors on time. For this purpose, using the PH assumption evaluation test provided for each risk factor is possible. There are different methods for this test. Graphic methods are one of the most widely used. This

Table 2. Dependence of risk factors on working periods.

Statistic	Dependence of risk factors				
	z_1	z_2	z_3	z_4	z_5
Pearson correlation	−0.093	−0.033	−0.167	−0.147	−0.214
p-value	0.59	0.848	0.329	0.391	0.21

method provides a visual image of the parallelism of risk rates, from which proportionality should be assumed. In this method, they are visually compared to determine their parallelism after drawing the risk rate for different values of the risk factors. However, the problem is, what kind of parallelism is meant by "parallelism?" Because this issue will lead to separate conclusions based on different logic. The next problem will be layering continuous risk factors such as rainfall or temperature. Because more layering will make the layer thinner and the data inside it less.

The third problem is evaluating the PH assumption for several risk factors simultaneously. For this purpose, an analytical GOF test is used to evaluate this assumption. In this treatise, Harrell and Lee's test, a modified form of Schoenfeld's test (1982) known as the "Schoenfeld's residuals" test, is applied. This method is more attractive because it presents results based on p-values and provides easier evaluation using mathematical logic. A significant p-value of more than 0.1 indicates the acceptability of the PH assumption [42].

In contrast, a p-value smaller than 0.05 for a risk factor indicates that the PH assumption is not satisfied. In other words, the null hypothesis test states that if the PH hypothesis is established, the Schoenfeld residuals will not correlate with time (Ho: $\rho = 0$). If the null hypothesis is rejected, the PH hypothesis will also be rejected. The results of the test for risk factors in outpatient data are presented in Table 3. As can be seen, the p(PH)-value for risk factors is not significant at 5%. Therefore, PH is valid for all risk factors. So, the assumption of proportionality or time-independent risk factors for system failure data can be accepted. According to the algorithm, the PHM model estimates the effects of risk factors. In Table 4, the model fitted to failure data is entered. In this table, the coefficient values of the risk factors are indicated by Alpha, and the hazard ratio of the coefficients is specified by Exp (Alfa). The risk rate indicates that if one unit of the factor increases, the overall risk rate will increase by that amount. For example, the temperature risk factor, the risk factor coefficient is 1.39, and the risk rate is 4.017. This means that a one-unit increase in the risk factor will increase the risk rate capacity by almost 400%.

Table 3. p-values to evaluate the assumption of the PH of system failure risk factors.

Statistic	Dependence of risk factors				
	z_1	z_2	z_3	z_4	z_5
Pearson correlation	–	–	–	0.072	0.029
p-value	–	–	–	0.877	0.869

Table 4. Fitted the model to estimate the effects of risk factors.

Model		Proportional hazard model (PHM)				
Statistic		α	S.E.	Wald	p-value	exp(α)
Risk factors	Z_4	4.2	1.5	7.5	0.01	68.85
	Z_5	1.4	0.5	6.9	0.01	4.02

It should be mentioned that the backward step-wise model is completely used to analyze models based on the risk factor. The results are related to the last step of the analysis.

Table 4 fitted the model to estimate the effects of risk factors (α) and also shows the standard error (S.E.), Wald coefficient (Wald), significance value (p-value), and the effective hazard rate value of each risk factor (hazard rate) in an exponential form.

2.5. Determining the reliability function of the Weibull family (fifth stage)

As mentioned, a system's functional behavior should be determined before making any decision. By considering the system's downtime and sleep time, reliability and repairability are two critical behavioral indicators of overall performance. In the following, the accessibility index will combine these two to establish a maintenance strategy. In any case, a detailed analysis of the above two indicators is the main pillar of the subsequent analysis. In this regard, the Weibull distribution function is one of the most versatile and flexible statistical functions. It can cover wide changes or coincidences in the data of these two indicators. Weibull family applications and extensions can be found in the following references [43,44]. In 2004, Murphy reviewed the types of Weibull models and discussed 40 types of this function and their relationship to the two-parameter Weibull. Most Weibull family functions have specific shapes in the Weibull probability plot (WPP). For example, the double Weibull model has an S shape, and the two-parameter Weibull model can be seen as a straight line in this diagram [45]. The axes of the WPP diagram will be based on the Weibull transformation in the form of equations (5) and (6):

$$y = \ln(-\ln(1 - F(t))) \tag{5}$$

$$x = \ln(t) \tag{6}$$

Where t represents a lifetime of the component and $F(t)$ is the probability of failure before time t and y is called Weibull probability plot versus x. This diagram provides a systematic process to determine the most suitable model for a data set. In this section, using exactly this, a guide according to Figure 4 is proposed for choosing the appropriate model from the Weibull family. This algorithm is proposed in six general steps as follows:

2.5.1. First step: collecting data

In the first step, the required data is collected after checking the system and identifying different levels. At this stage, to analyze the failure data, the risk factor of the weather condition ($z_1 = 3$) and the average value of the risk factors of rainfall and slope (z_2) were considered basic values.

2.5.2. Second step: preliminary analysis

At this stage, preliminary analyses, including calculating the minimum, maximum, average, variance, mode, correlation, and quartiles, are performed to gain an initial insight into the data. At this stage, bootstrapping was used to verify the results. Bootstrapping is a computational-statistical-computer method to determine estimators' accuracy from sample data. The term bootstrap refers to a self-starting process without external input. This method accurately calculates standard errors and confidence intervals for estimators such as mean, mode, median, percentiles, correlation, and regression coefficients. Bootstrapping refers to estimating an estimator's properties (such as variance) by utilizing measures of these properties in an approximate distribution of the entire sample data. With this technique, it is possible to estimate almost any statistic of the sample data distribution with only a very simple method. Generally, this method is considered one of the resampling methods. When a set of observations can be assumed from an independent and equally distributed population, bootstrapping can be implemented by creating several replicates. Each replicate is a random sample taken from the original data set. Also, this method can be employed to test statistical hypotheses. This method is usually used as an alternative to inferential methods based on parametric assumptions when doubt exists about these assumptions. Also, we use bootstrapping in parametric inference when calculating the standard error of the calculation formula is complicated [47].

2.5.3. Third step: determining the reliability model

In this step, a sample of data is selected to determine the analysis model. The most significant parameter in this step is the number of observations (failures), which is indicated by n and is divided into three categories: small ($n < 20$), medium ($20 \le n \le 50$), and large ($n > 50$). If the number of failures is small, all the data are used to draw the WPP chart. The average data will be selected from the bootstrapping or jack-knife approach to generate new samples, and the WPP chart will be used for each of them. The bootstrap approach, as previously discussed, considers the initial data as input and produces updated data samples by removing or replacing the data. The jack-knife approach is similar to this method, removing one data in each iteration [43,48].

Finally, for failures with a large number, the data is divided into two groups of 20% (S1) and 80% (S2) of the total data. The model selection is done for $S1$, and $S2$ validates the selected model. This will be done k times, depending on the data type and sensitivity. According to the extracted data bank and the diagram illustrated in Figure 3, the number of fuel injection failures is 36. This is part of the medium category, which requires data generation. This article used a bootstrapping approach with 20 production iterations. Figure 4 shows the WPP chart that must be drawn for all production data. The TBF diagram and batch of fuel injection failure data are shown in Figure 5. As can be seen, these graphs have a slight S shape (Dogleg) and do not have asymptotes when the data are connected, drawn roughly with dashed lines.

This position happens when there is not enough data available, the process of collecting and recording data is facing an error, or the collected data has more than one state and is the so-called "multi-modal data set", which should be fitted by several different models. As a result, according to the Weibull mixture distribution model algorithm, it will be suitable for describing these data. Weibull mixture distribution is one of these models that plays a fundamental role in operational applications. Bokar believes that when you know nothing about the system structure, you can estimate its reliability with this function [49].

Composite functions are discrete or finite mixtures obtained from linearly integrating two or more functions. Functions can have a normal, exponential, Weibull, etc. distribution. A simple description of this function can be expressed as follows: the population under analysis consists of $2 \ge n$ subpopulations, and the contribution of each sub-population to the total function and its value for the whole society is in the form of equation (7) [45]:

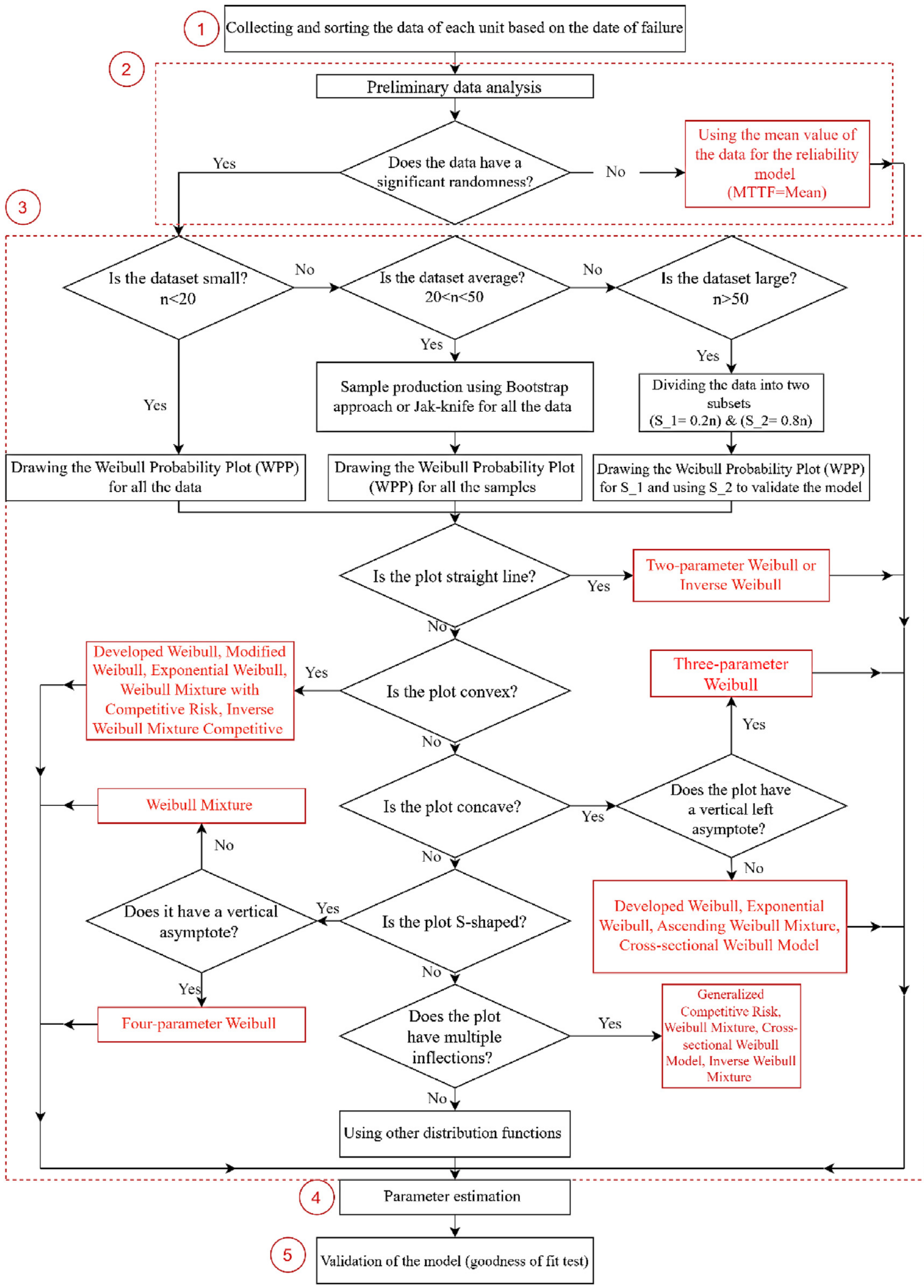

Fig. 4. Algorithm for selecting the appropriate model for reliability from the Weibull family (adapted from (Barabadi, 2013)).

$$\sum_{i=1}^{n} \omega_i = 1 \quad i = 1, 2, \ldots, n; 0 < \omega_i < 1 \tag{7}$$

In this equation ω_i is a contribution of each sub-population (layer). Therefore, for a random variable t from the population, the density function of the composite distribution can be expressed as equation (8):

$$f_m(t) = \sum_{i=1}^{n} \omega_i f_i(t) \quad i = 1, 2, \ldots, n; 0 < \omega_i < 1 \tag{8}$$

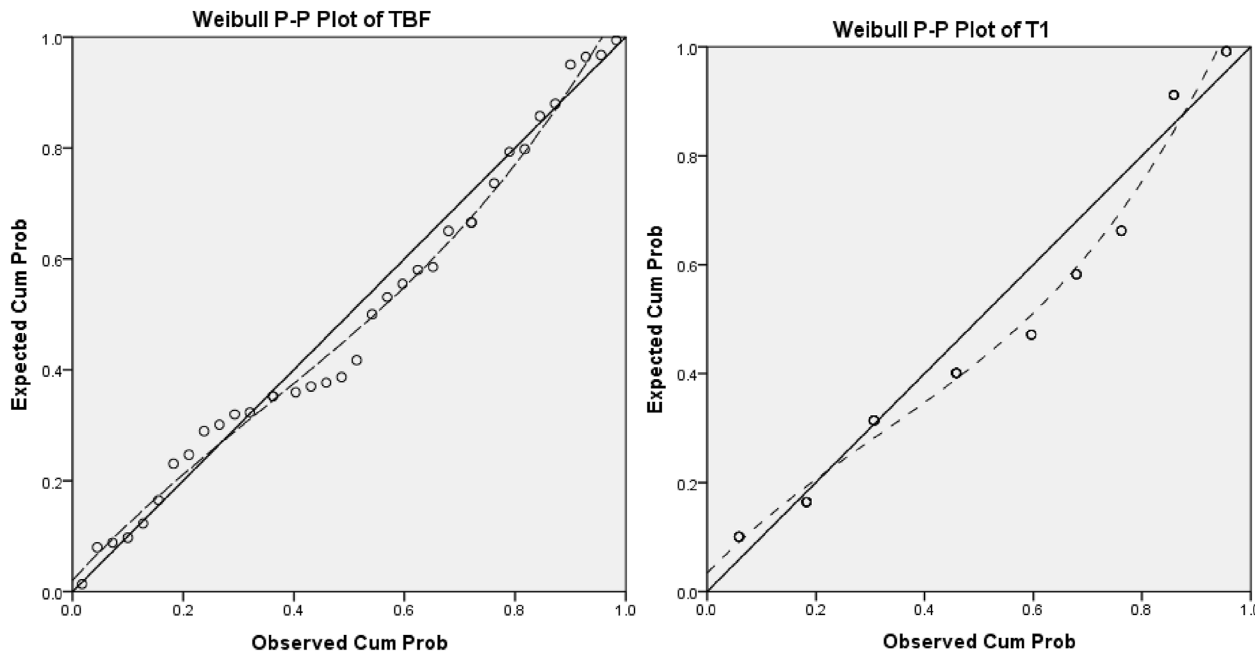

Fig. 5. WPP diagram of fuel injection failure mode.

In this equation $f_m(t)$ is the density function of the composite distribution and $f_i(t)$ is the density function of each sub-population. The reliability function is represented by equation (9):

$$R_m(t) = \sum_{i=1}^{n} \omega_i R_i(t) \quad i = 1, 2, \ldots, n;\ 0 < \omega_i < 1 \tag{9}$$

Also in this equation $R_m(t)$ is the reliability function of the composite distribution and $R_i(t)$ is the reliability function of each sub-population. If we assume a two-parameter Weibull (shape parameter (β) and scale (η)) of all the distribution functions of the sub-populations, the shape of the reliability function will be as shown in equation (10):

$$R_m(t) = \sum_{i=1}^{n} \omega_i \exp\left[-\left(\frac{t}{\eta_i}\right)^{\beta_i}\right] \quad i = 1, 2, \ldots, n; 0 < \omega_i < 1 \tag{10}$$

In this regard, if $n = 2$, the distribution function is called a “Two-fold Weibull mixture” [50–52].

2.5.4. Fourth & fifth step – parameter estimation and goodness-of-fit test of functions

According to the results of the WPP diagram, the mixture distribution function should be used for fuel failure. The four-fold Weibull mixture function was fitted based on the goodness of fit test. Figure 6 shows the WPP diagram of fuel injection failure data. As seen in this graph, the line fitted to the data covers the real state of the data. In a way, it provides the most accurate fit for the data.

Table 5 shows the results of the adapted functions for the failure mode. The function fitted to this state is the four-fold Weibull mixture, and the PHM function defines risk factors. In the next columns, the contribution of each layer (ω_i) and the corresponding shape and scale parameters are estimated. In the last two columns, the goodness-of-fit coefficient of the Kolmogorov-Smirnov test is compared to the critical value, and the test’s p-value, which is 100%, is determined.

The failure reliability function is in the form of equation (11):

$$\begin{aligned} R_{(\text{Fuel system})}(t) = {} & 0.2 \exp\left[-\left(\frac{t}{8.38}\right)^{1.63}\right] \\ & + 0.31 \exp\left[-\left(\frac{t}{22.5}\right)^{7.99}\right] \\ & \times \exp\left[-\left(\frac{t}{44.63}\right)^{6.77}\right] \\ & + 0.31 \exp\left[-\left(\frac{t}{113.79}\right)^{2.13}\right] \end{aligned} \tag{11}$$

2.6. Entering the effects of risk factors on system performance indicators (sixth step)

As discussed, the fitted function in the previous step is considered a “baseline function”. In this step,

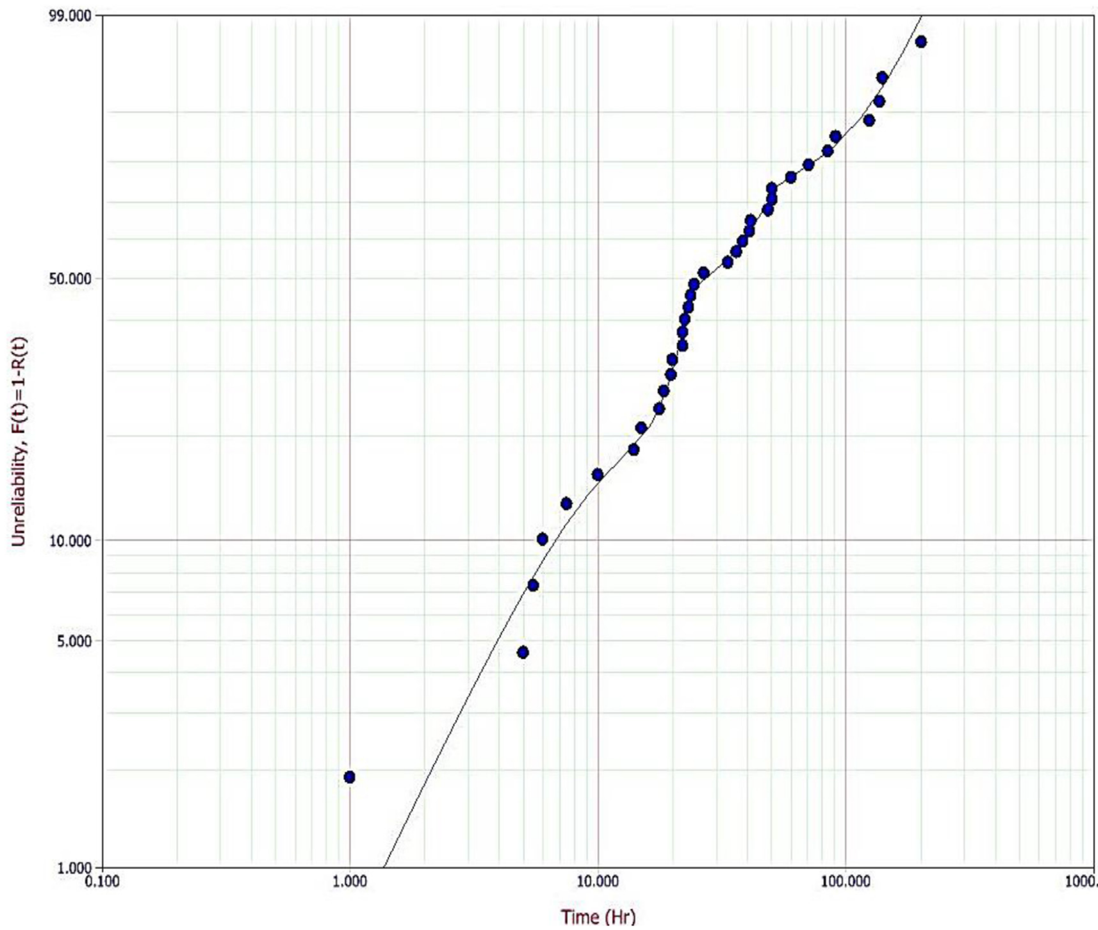

Fig. 6. WPP diagram of failure data of failure of fuel injection.

Table 5. Fitted function and goodness of fit test (GOF).

Distribution or model	Risk factors model	Estimation of parameters and contribution of fitted functions						Goodness-of-fit	
		subpopulation 1			subpopulation 2			K-S test	p-value
		portion	beta	eta	portion	beta	eta		
Weibull-mixed (4-fold)	PHM	0.2	1.63	8.38	0.31	7.99	22.5	0%	100%
		subpopulation 3			subpopulation 4				
		portion	beta	eta	portion	beta	eta		
		0.19	6.77	44.63	0.31	2.13	113.79		

these functions will be updated based on environmental conditions. Because the baseline functions are Weibull, the extension of the approach presented by Dr. Ghodrati can be used to include the effects of environmental conditions in the composite function [40]. He proved that the risk factors only affect the scale parameter value (η) and do not change the shape parameter (β). These changes can be defined with new parameters of shape (β_{si}) and scale (η_{si}) to introduce the influence of environmental conditions. If β_{0i} and η_{0i} are respectively fitted to the shape and scale parameters of each fold in the basic functions, then (β_{si}) and (η_{si}) are respectively as equation (12) [40]:

$$\beta_{si} = \beta_{0i} \eta_{si} = \eta_{0i} \left[\exp\left(\sum_{i=1}^{n} z_i \alpha_i \right) \right]^{-\frac{1}{\beta_{0i}}} \tag{12}$$

As a result, the reliability function (R_{sm}) by entering the effects of risk factors is presented as follows:

$$R_{sm}(t) = \sum_{i=1}^{n} \omega_i \exp\left[-\left(\frac{t}{\eta_{si}}\right)^{\beta_{si}}\right] \quad i=1,2,\ldots,n; 0<\omega_i<1 \tag{13}$$

Accordingly, the function of reliability and repairability for mechanical failure modes is equation (14). According to Table 6, two scenarios are considered to check the impact of risk factors. In these scenarios, two risk factors affecting reliability, namely rainfall and temperature, were considered. If relations (14) are used, the diagram of reliability and risk rate of the failure mode for 50 h of system operation in the basic mode, scenarios 1 and 2, is shown in Figure 7.

As can be seen, both scenarios exhibit lower reliability than the basic state. This highlights the impact of environmental conditions on reliability. The chaotic state of the hazard rate function in three different scenarios also demonstrates the impact of risk factors. Furthermore, the changes in this graph during the performance period confirm the use of the composite function in describing industrial systems' failure behavior.

Table 6. Defined scenarios for reliability.

Row	Scenarios		z_4	z_5
1	First semester	cheap maintenance	0.039	0.485
2	Second semester	expensive maintenance	0.040	0.117

3. Conclusion

In the literature review, one of the shortcomings in analyzing the analysis system's functional behavior is primarily associated with time data. This type of study focuses on time data (failure times) and ignores the impact of environmental factors (risk factors) on reliability. The issue appears to be rooted in two places. Firstly, researchers usually ignore the mathematical analysis process and infer the involvement of effects from the same time data. Second, apart from the science of analyzing risk factors simultaneously with time data, and much more significant than the first

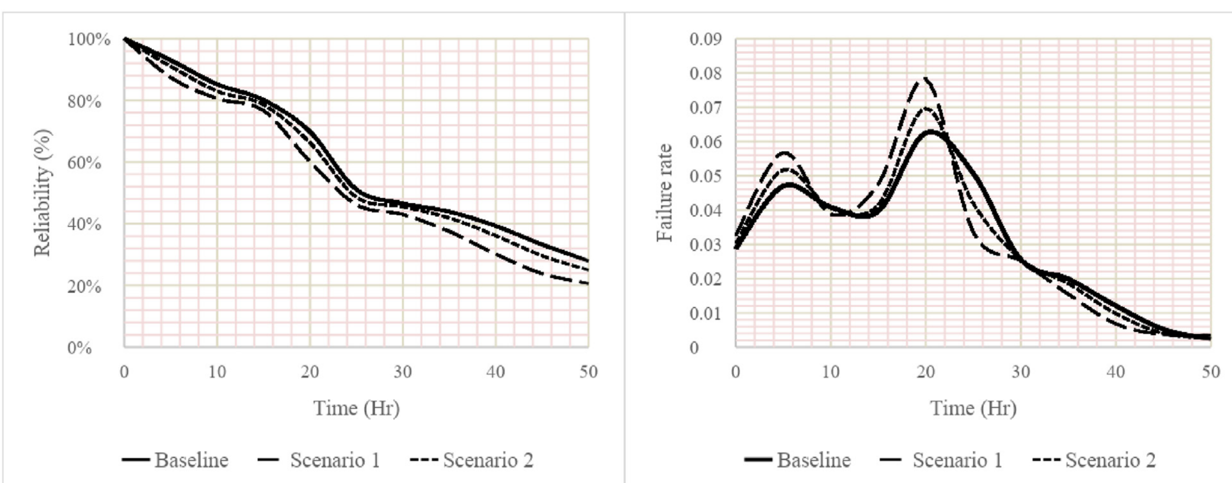

Fig. 7. Figure reliability function and mixture failure rate function for fuel injection system failure mode.

$$\begin{aligned} R_{sFuel}(t) = {} & 0.2\exp\left[-\left(\frac{t}{8.38\left[\exp(4.232z_4+1.39z_5)\right]^{-\frac{1}{1.63}}}\right)^{1.63}\right] + 0.31\exp\left[-\left(\frac{t}{22.5\left[\exp(4.232z_4+1.39z_5)\right]^{-\frac{1}{7.99}}}\right)^{7.99}\right] \\ & + 0.19\exp\left[-\left(\frac{t}{44.63\left[\exp(4.232z_4+1.39z_5)\right]^{-\frac{1}{6.77}}}\right)^{6.77}\right] + 0.31\exp\left[-\left(\frac{t}{113.79\left[\exp(4.232z_4+1.39z_5)\right]^{-\frac{1}{2.13}}}\right)^{2.13}\right] \end{aligned} \tag{14}$$

reason, is the lack of access or the inability to form a data bank for this type of analysis. Because in industrial environments, especially in mining operations, data registration has often not been done or recorded correctly. In addition, registration sources are so extensive that integration is regrettable. This is even though regardless of the effects of risk factors on performance behavior, especially in harsh environments such as mining, it will lead to analyses far from reality. Therefore, accurate and realistic analysis of system behavior requires considering these effects. The next point is that according to the behavior of industrial systems in the form of a bath-tub diagram during their lifetime, both classical methods and methods based on risk factors, such as the proportional hazard model, cannot cover this type of system behavior in the form of an integrated function. One of the main limitations of this model is the lack of compliance with reliability process changes during the research period. Therefore, this paper presents a coherent framework for integrating temporal data and risk factors with the Weibull family functions. In addition to integrating the system's statistical behavior with environmental effects, this approach also covers the reliability graph's variable shape (more realistic mode) during the performance interval. In the analysis of the case study of the Songun copper mine with the proposed approach, in the first step of the algorithm of failure tree analysis (FTA) and the drawn Pareto diagram of the engine of a mining dump truck system, the fuel injection subsystem was identified as a critical subsystem.

The appropriate database was extracted from the collected information in the next step. About 80% of this information is quantitative and includes various reports. The remaining 20% is qualitative, and experts' opinions in different departments must be considered. The reliability performance index should be determined to understand failure behavior accurately. The following estimates the proposed algorithm for integrating environmental conditions into reliability. First, the number of risk factors' effects was estimated using regression methods, and second, the basic state of reliability was estimated using the Weibull family functions. In the first step, Pearson's coefficient was used to examine the relationship between the risk factors of the fuel supply system, including weather conditions, and four continuous risk factors: slope, hourly capacity, precipitation, and temperature were examined. The results showed no specific dependence between them; therefore, the selected risk factors are independent. After determining the risk factors, the assumption of proportionality (PH) was evaluated using an analytical method to select the appropriate regression function in the next step. The test result indicated that at a significance level of 5%, no dependence on time was observed for the risk factors. Therefore, the PH assumption is valid. Therefore, PHM was chosen to determine the impact of the risk factors. Secondly, to solve the problem of trend changes during the working interval, an algorithm was proposed based on the Weibull family distribution functions. This was the first time. In this algorithm, the Weibull Mixture Distribution is utilized for data with trend changes during the working interval. In this function, after determining the baseline function from the Weibull family based on the presented algorithm, the parameters of this function were updated based on the effects of risk factors to integrate the regression function from the Weibull family functions. Next, to evaluate the effects of environmental conditions on performance, two different scenarios were used to check the system's performance in different situations. The results obtained for these two scenarios in the reliability diagram clearly show the impact of risk factors and the variability of the distribution function during the performance.

Ethical statement

The authors state that the research was conducted according to ethical standards.

Funding body

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Conflict of interest

The authors declare no conflict of interest.

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