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ORIGINAL - SURVEY OR EXPOSITION

Exploring the development trajectory of single-machine production scheduling

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Abstract
The Single-Machine Scheduling Problem (SMSP) serves as the cornerstone of scheduling theory. Almost all developments in production planning and control were initially introduced and tested within single-machine production settings. Exploring the literature on SMSPs illuminates the entire development trajectory of scheduling theory. This study employs the Main Path Analysis (MPA) for a systematic review that is the first of its kind in the literature on SMSPs. By analyzing 2904 articles, the main path and key branches that highlight the most significant documents in the development trajectory as well as the specifics in different development stages of the field are identified. The literature is analyzed in four stages of development, following a chronological order. Within this framework, the focus of the studies, key technologies, and characteristics of each period are discussed. Cluster and keyword analysis are employed alongside MPA to identify and explore the predominant themes in the field, including SMSPs with deterioration and/or learning effects, solution methods for SMSPs with various constraints, solving SMSPs integrated with maintenance-related variables and constraints, advanced solution methods and constructive heuristics for minimizing delays, as well as agent-based methods for single-machine scheduling. The review outcomes consist of directions for future research based on the development trajectories of SMSPs.

Keywords Production planning and control · Optimization · Systematic review · Main path analysis (MPA) · Cluster analysis (CA)

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1 Introduction

Scheduling is *"the process of allocating limited resources to perform certain tasks or activities within a given timeframe, including all the constraints imposed"* (Baker, 1974). Production scheduling helps to plan and control the operating parameters of a shop floor to improve margins and reduce costs while maximizing efficiency without necessarily upgrading the system. The Single-Machine Scheduling Problem (SMSP) has been employed for planning tasks in various operating systems to optimize their performance. In this context, an operator, a production line, a process, a factory, or an entire manufacturing shop floor with a bottleneck can all be modeled as a 'single-machine' system. As a classic engineering optimization problem, SMSP constitutes the backbone of scheduling theory with nearly all developments in the field having been introduced and/or tested in a single-machine environment.

Scheduling problems have been extensively investigated in industrial engineering, production engineering, operations management, operations research, management science, and data science, with a primary emphasis on mathematical formulations and solution methodologies. Given the nearly 17,000 published articles, researchers may find it challenging to find significant research gaps and contribute to the further development of various scheduling domains. A big data analysis-based overview of the literature on SMSP can aid in identifying unexplored research directions, expanding the existing body of knowledge, or introducing new research ideas across various scheduling domains.

Few comprehensive reviews exist regarding the literature on SMSPs. Koulamas (2010) surveyed single-machine *total tardiness* scheduling problems. Martinelli et al. (2022) conducted a literature review on single-machine scheduling in *make-to-order* production. Most recently, Koulamas and Kyparisis (2023) presented a review that examined dynamic programming algorithms for offline deterministic SMSPs, addressing their classifications, enhancements, and applications in broader scheduling contexts over the past 50 years. These reviews have limited scope and cannot provide a comprehensive overview. Besides, they have used traditional review approaches. To the best of the authors' knowledge, the only comprehensive reviews of the literature on SMSPs were conducted by Gupta and Kyparisis (1987) and Maxwell (1964), and the most recent review dates back more than 37 years. Providing a fresh and insightful perspective on the development of SMSPs is timely and bridges a gap.

The present study conducts a systematic review of the literature on SMSPs to address the following question. What are the knowledge diffusion patterns in the literature on SMSPs and how could this impact the future development trajectory of the field? To answer this, the Main Path Analysis (MPA; Hummon & Dereian, 1989) method is employed to identify the important research themes, problem features, and solution algorithms in SMSPs. The relevant literature is analyzed at various development stages to synthesize the research results in each period and identify the main topics of interest and key technologies. MPA operates within citation networks, commonly referred to as a network 'directed acyclic graph' within graph theory. The network can be constructed from datasets comprising diverse document types, including patents, papers, and court decisions, where objects demonstrate referencing or inheritance relationships. Patent citation networks predominantly serve to investigate technological development trajectories, while paper citation networks form the foundation for literature reviews and the examination of knowledge dissemination pathways. Hummon and Doreian (1989) propose a two-step methodology to analyze the development of DNA theory. First, citation links, initially considered to be of equal importance, are assigned traversal weights. Subsequently, the links with the highest traversal weights are traversed to identify the most significant citation chain, termed the "main path," which represents a pivotal pathway within the citation network. Additionally, in this study, Cluster Analysis (CA; Girvan & Newman, 2002) is employed to categorize the published literature into groups. Based on this categorization, in-depth discussions on the development of each subfield are provided as references for future research. MPA and CA are conducted using the *MainPath 465* software. *Pajek* (http://mrvar.fdv.uni-lj.si/pajek/) is utilized to visualize the main trajectories of the citation network. Finally, the citation maps are generated using *VOSviewer* (https://www.vosviewer.com/) to depict the primary keywords and their correlations within each cluster.

The rest of this review article is organized into three sections. Research methods and materials are briefly explained in Sect. 2. The results and discussions are presented in Sect. 3. Lastly, the review is concluded in Sect. 4 followed by the suggestion of some directions for future research.

2 Research methodology

The methodology comprises three phases: 'data collection', 'data processing', and 'data analysis'. The 'data collection' phase begins by confirming the keywords and search protocols. For the systematic review and citation network analysis, the necessary data is collected from the Web of Science (WoS) database. The database includes documents indexed in the Science Citation Index Expanded (SCIE), Social Sciences Citation Index (SSCI), Arts & Humanities Citation Index (AHCI), and Emerging Sources Citation Index (ESCI). The search field was filled by ((TS = ("single machine") OR TS = ("single-machine") OR TS = ("one machine") OR TS = ("one-machine")) AND TS = ("scheduling")). Taking into account the studies published from and after 1987, a total of 4,527 related research papers were retrieved.

The 'data processing' phase involves cleaning up the original database and screening articles. The articles are screened by considering their titles and abstracts, following the PRISMA method (Liberati, 2009). Data processing revealed that 210 articles were disconnected from the citation network, 51 articles were perceived as irrelevant to scheduling, and 1308 articles did not directly study SMSPs, but only mentioned this scheduling variant in their content. A total of 2,904 articles passed the screening and cleaning processes and were organized in a unified file format. The compiled documents were subsequently imported into the software to confirm whether the quality measures were reasonable.

In the 'data analysis' phase, a citation network is first established. Using this network, MPA discovers all possible paths from the origin to the end of dissemination to identify the main path based on the most frequent occurrences. In the citation network, nodes represent documents; these are characterized as source nodes (origin of dissemination), intermediate nodes, and sink nodes (end of dissemination). A connection arrow represents the citations, with the tail indicating the cited article.

The MPA literature uses the following methods to identify the main path (Liu et al., 2019): [1] 'Search Path Count' (SPC), which counts all traversals between all sources and sinks; [2] 'Search Path Link Count' (SPLC), which counts the number of traversals when considering paths from all ancestors of a specific tail node to all sink nodes in the knowledge dissemination network; and [3] 'Search Path Node Pair' (SPNP), which counts how many times a citation link is traversed when running through all paths from its ancestors to descendants. SPLC is considered the most suitable method for identifying the path of knowledge dissemination in the literature (Liu et al., 2019). This is because SPLC reflects the intermediate nodes as both carriers of the previous knowledge and developers of new knowledge, as opposed to SPC, which recognizes the intermediate nodes as mere knowledge carriers.

When dealing with a weighted network representing the number of SPLC-based traversals, various methods can be employed to identify the main path. The 'Global Main Path' (GMP) is utilized to find the path(s) with the highest cumulative weight, offering insights into the development trajectory of the literature on SMSPs. Additionally, the 'Key-route Main Path' (KMP) method is employed to incorporate the top ten most cited references from the network. The path analysis commences separately from the tail and tip nodes of the highly weighted arrows and proceeds toward the origin and sink nodes of the network, respectively.

As a complementary step, CA is employed for the hierarchical clustering of articles. CA uses the article citation network as input and generates a tree-like diagram based on the similarities among the articles. The computational procedure consists of three steps: (A) finding the shortest paths from each node to every other node in the network; (B) calculating the number of steps taken from the network's source node to a given node using Eq. (1) for all nodes, starting at the bottom of the tree; (C) removing the edge(s) with the highest score. Completing this three-step procedure transforms the network into distinct groups of nodes.

$$\text{Edge Credit} = \left(1 + \sum \text{Incoming Edge Credit}\right) \times \frac{\text{Score of Destination}}{\text{Score of Start}} \tag{1}$$

In the data analysis phase, calibrating the MPA algorithm includes setting parameters such as the number of key routes and the search method. Overall, the statistical analysis provides an overview. MPA, when performed using the *MainPath* software, uncovers the knowledge development trajectory of SMSPs. The outputs of the *MainPath* software are then imported into the *Pajek* software for visualization. CA categorizes the documents into primary clusters to identify the subfields of SMSPs. *VOSviewer* compiles the results of the CA and keyword analysis while considering the author's keywords to analyze the inherent correlations. The outcomes of these phases are utilized to explore the development trajectories of SMSPs and to propose future research directions in the field.

3 Results analysis

3.1 Preliminaries

Before analyzing the literature, the database must be checked to ensure that the collected data represents the entire literature. For this purpose, the precision rate (Eq. 2) and the percentage of digital object identification (DOI, Eq. 3) are calculated. The precision rate and DOI percentage exceeding 70% confirm the adequacy of the database (Liu et al., 2019).

$$\text{Precision Rate} = \frac{\text{Network Size}}{\text{Number of papers in the original database}} \times 100\% \tag{2}$$

$$\text{DOI Percentage} = \frac{\text{DOI Total}}{\text{CR Total}} \times 100\% \tag{3}$$

In terms of the precision rate, 2904 out of the 2958 articles in the original database are included for further analysis after excluding isolated points, resulting in a precision of 98%. For evaluating the database's DOI percentage, 2904 documents were imported into the analysis software to examine their citation relationships. From 73,855 citations, 58,368 records are present in the database, which represents approximately 79.03% of the existing records.

Table 1 Notations representing the SMSPs studied in the literature

Field	Notation	Definition	Field	Notation	Definition
β	p_t	Processing time	γ	C_{max}	Maximum completion time
	r_t	Release time		E_{max}	Maximum lead time
	d_t	Due date		L_{max}	Maximum lateness
	C_t	Completion time		T_{max}	Maximum tardiness
	F_t	Flow time		$\sum T_j$	Total latency
	W_t	Waiting time		$\sum F_j$	Total flow time
	L_t	Lateness		$\sum C_j$	Total completion time
	E_t	Earliness		$\sum E_j$	Total lead time
	T_t	Tardiness		$\sum U_j$	Total number of tardy jobs
	U_t	Number of tardy completions		$\sum U_j$	Weighted number of tardy jobs
	ST_{si}	Sequence-independent setups		TSC	Total cost
	ST_{sd}	Sequence-dependent setup time		TST, TNS	Total number of setups/changeovers, Total number of spare parts

Finally, key-route values of 10, 20, 30, 40, 50, and 60 are tested to determine the best setting for KMP. This setting determines how many top-SPLC links should be included to identify a critical extended main path that is neither divergent nor limited. When the key-route value is set at ten, there are 37 resulting articles, only one more than that in the main path. Setting the key route at 30 or more causes the number of articles to begin to converge to 60, representing the overall developments in the SMSP's literature.

The notations characterizing the SMSPs in the literature are listed in Table 1. In this table, 'field' refers to the three-field $\langle\alpha|\beta|\gamma\rangle$ symbolization method introduced by Graham et al. (1979). The first, second, and third fields of the notation system refer to the operating environment (single-machine: $\alpha = 1$), technical constraints (β), and optimization objective (γ), respectively.

3.2 Descriptive statistics

The statistics regarding the number of published articles from 1987 to date show a consistent upward trend. We used the Logistic growth model from the Loglet Lab4 software to project the future trend of SMSPs in Fig. 1. The dotted section of the curve represents the actual cumulative number of SMSP articles over the years, whereas the dashed line represents the projected trend. The literature on SMSPs entered the growth stage in 2004 and is expected to reach saturation and enter the maturity phase in the early 2030s.

Figure 2 displays the top 20 journals considering the number of published articles; the European Journal of Operational Research is leading by a significant margin, followed by

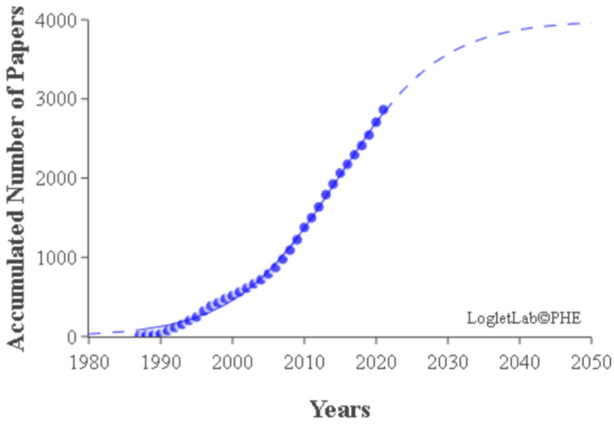


Fig. 1 Growth trend of the literature on SMSPs

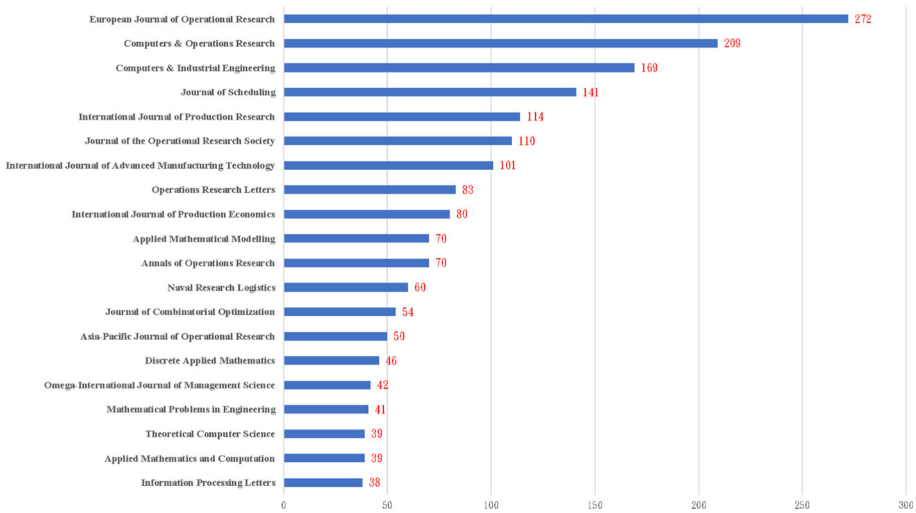


Fig. 2 Top journals in the literature on SMSPs

Computers & Operations Research, Computers & Industrial Engineering, Journal of Scheduling, and the International Journal of Production Research. In general, the top contributing journals mainly belong to the fields of manufacturing, operations research, and industrial engineering, and secondary contributions come from computer science, mathematics, and applied science. Figure 3 illustrates the top ten contributing countries based on the number of publications and citation counts with China, the United States, and Taiwan leading the competition.

To assess the quality of published works, the average traversal weight SPLC is used to highlight the relative significance of publications. Considering a network of 2904 publication nodes and 23,774 citation links, the top 10 countries according to the average SPLC value are listed in Table 2. It is observed that Japan has a lower-than-average SPLC value compared

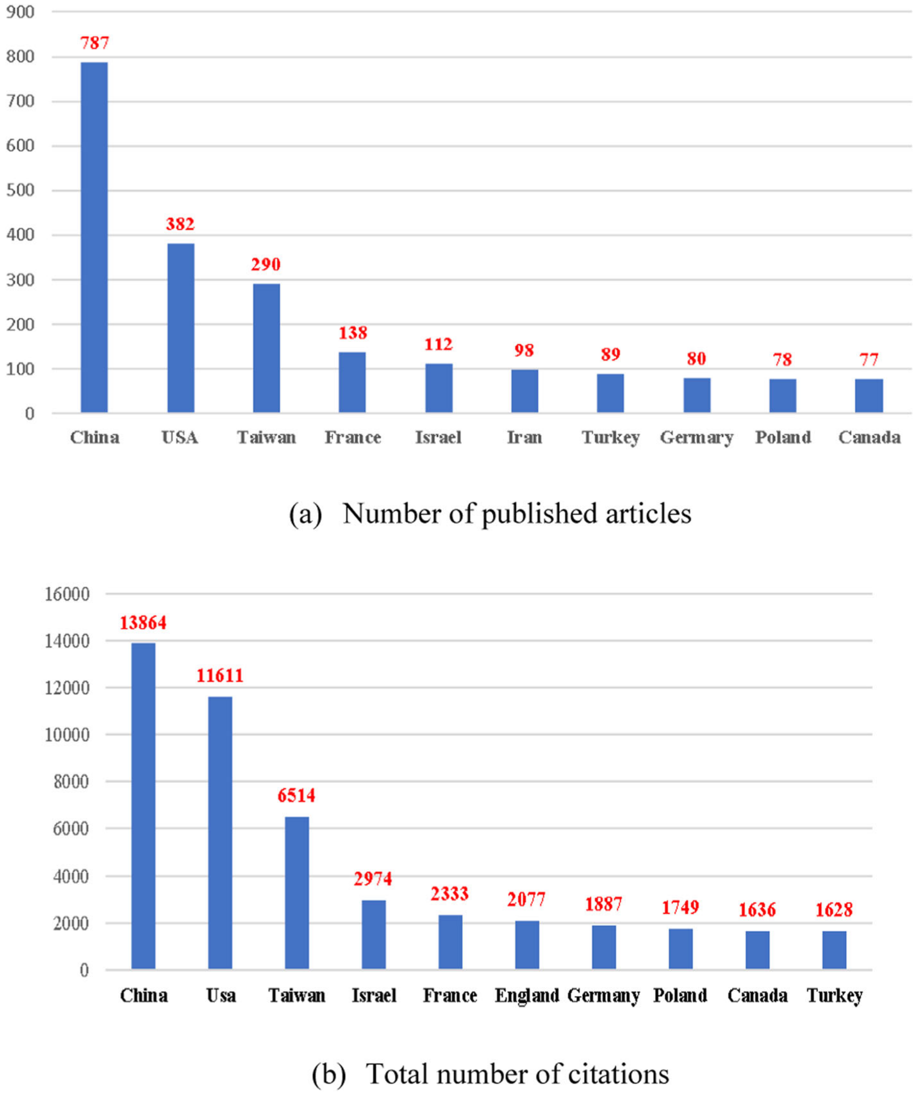


Fig. 3 Top contributing countries in the literature on SMSPs

to the overall average, while the other regions/countries exhibit above-average performance. Publications from Hong Kong universities are considered highly influential, comprising 18 seminal articles.

3.3 Main path analysis

This section reviews the 36 articles that shape the main path, categorizing them into three development stages with the main path links shown in Fig. 4.

Table 2 Ranking of influential regions/countries

Country	Documents	Percentage (%)	Average SPLC	Country	Documents	Percentage (%)	Average SPLC
Hong kong	18	0.62	53.80	Tunisia	3	0.10	22.19
Israel	112	3.86	40.33	Germany	80	2.75	18.08
Poland	78	2.69	37.45	USA	382	13.15	10.83
China	787	27.10	36.53	Türkiye	89	3.06	9.92
Taiwan	290	9.99	32.17	Japan	35	1.21	8.69

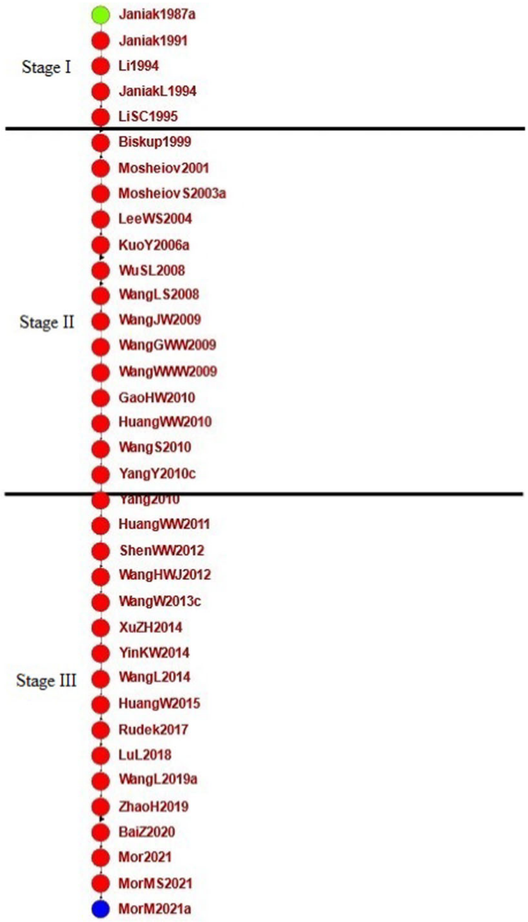


Fig. 4 The main path on the development trajectory of SMSPs

- Stage I (1987–1999): SMSPs focusing on resource allocation;
- Stage II (1999–2010): SMSPs focusing on the learning effect and group technology; and
- Stage III (2010 to date): SMSPs considering both learning and degradation effects.

3.3.1 Development stage I

The initial development stage of SMSPs was centered on resource allocation, taking into account various constraints. The source of the main path can be traced to Janiak, (1987), who proposed the unconstrained SMSP, where a single machine was considered the sole production resource, and the processing time depended on the number of allocated resources. Several linear resource consumption models were proposed, taking into account various optimization criteria, including maximum completion time, maximum tardiness, maximum cost, and the weighted sum of completion time. Several algorithms were developed to solve the SMSPs in polynomial time.

Janiak, (1991) explored SMSPs with common due dates and resource-dependent release times in the context of steelmaking. The release time was defined by considering a linearly decreasing function to minimize the total resource consumption while meeting a specific due date. He provided several approximation algorithms to solve this problem. Furthermore, he proved that the problem is '*NP*-hard' in a strong sense but becomes *NP*-complete when the release time reduction rate is assumed to be equal.

Li, (1994) extended Janiak's research by comparing job orders while considering two different resource consumption functions under various constraints. Using the linear function proposed by Janiak, (1991), Li, (1994) compared two resource consumption functions for SMSPs, considering release time and due date constraints across four scenarios: (i) minimize the total resource consumption within the common due date of all work orders, (ii) minimize the total resource consumption while constraining the total work order completion time, (iii) minimize the weighted total resource consumption and maximum work order completion time, and (iv) minimize the weighted total resource consumption and total job completion time. He compared the resource consumption function with the same and different effects on the processing time of all job sequences, concluding that different functions result in different computational complexities. Finally, he discussed optimal solutions for SMSPs with maximum completion time, total completion time constraints, and the bi-objective minimization of resource consumption and completion time.

JaniakL (1994) further discussed the problem of minimizing the total weighted completion time under resource consumption constraints. They proved that SMSP with release date and resource consumption constraints is generally *NP*-hard and attempted to exactly solve the problem under a special case where a job's weight equals its resource consumption rate, with the resource consumption function being linear. (Li et al., 1995) attempted to exactly solve the SMSP to minimize total resource consumption and total delays, considering a non-increasing convex function of resource consumption, and job orders with a common due date. The authors investigated two special cases and identified properties to develop efficient pseudo-polynomial-time algorithms for solving them.

In general, the research at this stage analyzes the relationship between resource consumption and single-machine scheduling and finds feasible solutions according to various optimization criteria.

3.3.2 Development stage II

Biskup, (1999) introduced the concept of a learning effect to SMSPs. Before this seminal work, several studies had explored workers' capacity to learn and their potential impact on enhancing production efficiency. In essence, it is not recommended to assume that workers maintain a constant level of efficiency over time. In addition to the learning effect, this study was the first to investigate the relationship between job processing time and job sequence position.

Mosheiov (2001) continued Biskup's research and found that under the learning effect model, some classical scheduling rules are ineffective. However, under specific conditions, they can be solved in polynomial time. Mosheiov developed the Earliest Due Date (EDD) rule to minimize maximum tardiness, and the Weighted Shortest Processing Time (WSPT) rule to minimize the sum of weighted makespans, thereby minimizing the number of tardy jobs. He argued that the problem can be solved in $O(n^3)$ time complexity when adding a job does not affect the learning effect. He also showed that the SMSP with makespan can be 'exactly' solved through pairwise exchanges if the job sequence lacks a learning effect index, and the best sequence is determined using the Smallest Processing Time (SPT) rule. Finally, the article extended the study of SMSPs to homogeneous dual machines.

Mosheiov (2003) extended the learning curve setting to allow differential learning rates among jobs. They proposed a general polynomial-time algorithm capable of solving a wide variety of problems, including those related to maximum completion time and total flow time minimization, due date assignment, and minimization of total flow time on unrelated parallel machines. Building on these findings, LeeWS (2004) explored the dual-criteria SMSP with learning effects and proposed a branch-and-bound algorithm along with a heuristic for approximating the optimal solution.

The variable speed learning model was introduced by (Kuo & Yang, 2006). They developed a polynomial-time algorithm and extended the group scheduling problem to include time-varying learning effects. The study also provided two polynomial-time algorithms to solve the problem, considering the minimization of the makespan and total completion time, respectively. Finally, they showed that a single-machine group scheduling problem with a time-dependent learning effect is 'polynomial time solvable'.

Wu (2008) discussed degeneracy in the single-machine group scheduling problem. The study assumes that both the actual processing time and the group setup time are simple linear functions of the starting time. The objective is to determine a sequence that minimizes the completion time of all jobs and the maximum completion time. The paper provides theorems regarding optimal assignments and group sequences. The theorems assert that in the single-machine group scheduling problem aiming to minimize the makespan, the optimal schedule satisfies the property that the group sequence and the work order sequence in each group can be arranged in any order. Additionally, in the single-machine group scheduling problem that aims to minimize the total completion time of all jobs, the optimal approach involves sorting the work orders within each group according to the Smallest Deteriorating Rate (SDR) rule and sorting the work orders by rank groups in non-decreasing order.

Wang (2008) extended the research of Wu (2008) to discuss SMSPs with degenerate jobs and Group Technology (GT). They assumed that both group setup times and job processing times are increasing functions of their starting time; that is, an SMSP variant with a time-changing learning effect. They showed that the problem can still be solved in polynomial time when aiming to minimize the makespan and total weighted completion time.

Wang (2009b) introduced the SMSP with past-sequence-dependent setup times. They investigated the impact of degradation and learning effects on the setup time associated with past sequencing, where the processing time of a job is a function of its start time and position in the sequence, and the setup time is proportional to the length of the processed job. They showed that the problem of minimizing the completion time, the total completion time, and the sum of δ powers of the completion time can be solved in polynomial time. They also showed that under certain conditions, the problems of minimizing the total weighted completion time, minimizing the maximum delay problem, and minimizing the number of tardy jobs can be solved in polynomial time.

Wang (2009a) studied the deterioration effect, considering that the SMSP has a degradation effect that increases with time, within the GT assumption. A new learning effect model was developed, and it was shown that the problem can be solved in polynomial time when general linear degradation and GT are considered simultaneously. Wang (2009c) studied time-dependent deterioration in the SMSP. They showed that the single-machine makespan minimization problem is polynomially solvable even when time-dependent deterioration is introduced into the job processing times. They also showed that the optimal schedule for the total completion time minimization problem is V-shaped according to the normal work processing times.

Gao et al., (2010) extended the research of WANGWWW2009 for a special case where the processing time is dependent on the start time and decreases, subject to precedence constraints. The authors conclusively proved that the SMSP can be solved in polynomial time to minimize the total weighted makespan. (Huang et al., 2010) explored the SMSP with deteriorating jobs by extending it to the early penalty cost, where the deterioration of work is modeled as a linear function of time. The authors showed that two special cases of this problem can be solved in polynomial time: the problem with an equal-weighted monotonic penalty objective function and the problem with a weighted linear penalty objective function.

Wang (2010) addressed a variant of the SMSP with the group setup times and processing times depending on the start time and following a decreasing function. They proved that the problem of minimizing the makespan is polynomially solvable under the assumption of decreasing linear deterioration and that the problem of minimizing the total weighted completion time becomes polynomial-time solvable under the scaling assumption that the decrement in working time is a linear function of time. Finally, (Yang & Yang, 2010) considered both deterioration and learning effects and proved that the optimal job sequence and group sequence can be found in polynomial time; this article initiated the third stage of development of the SMSP's literature.

Overall, the research during the second stage of development was mainly concerned with incorporating the learning effect, exploring its impact, and developing solution strategies.

3.3.3 Development stage III

The main path during the third development stage shows that scholars proposed different models and solutions for SMSPs with both deterioration and learning effects. The most dominant considerations were resource allocation, starting time, location-related effects, and resource consumption.

Yang and colleagues suggested that deterioration and learning effects can exist simultaneously and should be considered together in SMSPs. (Yang & Yang, 2010) proved that the optimal job sequence and group sequence under this extension of SMSPs can be found in polynomial time. YangY2010c focused on minimizing the makespan and total completion time and showed that the former is polynomially solvable while the latter is solvable under special conditions. Yang (2010) introduced a new SMSP extension that takes into account the onset time-related learning effect and the position-related deteriorating effect to minimize the makespan, total completion time, and total absolute deviation of completion times (TADC) and solved them in polynomial time.

Huang (2011) proposed a polynomial time solution algorithm under resource constraints. They explored two single-machine group scheduling problems with deterioration and learning effects, considering two scenarios: minimizing the makespan and minimizing the total resource consumption problem under the maximum completion time below a given limit. Shen (2012) extended the study of (Yin et al., 2010), ensuring that the work order processing time changes due to deterioration and learning effects. They provided a counterexample to support the importance of this inclusion.

Wang & Wang, 2013a, b; Wang et al., 2012; Xu et al., 2014 all introduced new deterioration functions and proved that the problem of makespan minimization can be solved in polynomial time under certain conditions. WangHWJ2012 proposed a new SMSP where jobs deteriorate over time and have independent setup times; they also incorporated group technology to include a simple linear degradation function. WangHWJ2012 finally proved that the problem of minimizing the makespan problem in a special case can be solved in polynomial time when deterioration and GT are considered simultaneously. WangW2013c explored SMSPs with a convex resource-dependent allocation of processing times and deteriorating jobs and proposed a mathematical model where job processing time is related to the starting time and resource allocation. A polynomial-time algorithm was proposed to solve this problem, considering the makespan, total completion time, and absolute standard deviation of total completion time and total resource consumption cost. It is worth mentioning that WangW2013c's study opened a relatively popular branch line for the development of SMSPs. XuZH2014 considered the SMSP with GT and release time as a linearly proportional deterioration function, where the actual processing time of jobs is an increasing function of their start time. They developed a highly competitive heuristic algorithm and also proved that the problem is solvable in polynomial time to minimize the makespan.

Wang & Liu (2014), Yin et al., (2014) combined resource allocation and learning effects and proposed polynomial-time algorithms to solve the problem. Yin studied the single-machine group scheduling problem, combining resource allocation, learning effects, and deteriorating jobs, where the processing time depends on location, start time, and allocated resources. They proposed two resource allocation functions to minimize the weighted sum of completion time and total resource cost and demonstrated that these problems have polynomial solutions under the condition that the number of jobs in each group is the same. WangL2014 explored a dual-objective extension (minimizing the total weighted completion time and the maximum cost) of group scheduling with a single machine, in which setup and processing times are increasing functions of their starting times. They also proposed a polynomial-time algorithm to solve the problem and demonstrated that the bi-objective group scheduling problem can be solved in polynomial time.

Huang (2015) introduced the position-dependent deterioration model and proved that a polynomial-time algorithm can be developed to solve several special cases. In their model, the processing time of a job is defined by an increasing function of the total weighted normal processing time of the previous jobs, where the weight is determined based on the position of the job. (Rudek, 2017)'s article aims to minimize the total weighted completion time while taking into account learning and deterioration effects based on the sum of processing times. The authors showed that this SMPS variant is *NP*-hard in the strongest sense. LuL2018 (Lu & Liu, 2018) studied four dual-criteria SMSPs with resource allocation and location-dependent workloads. The first criterion is minimizing scheduling time (i.e., manufacturing time, the makespan, the total absolute difference in makespan, and the total absolute difference in waiting time), and the second criterion is minimizing the total resource consumption cost. Wang & Liang (2019) considers a single-machine group scheduling problem with deterioration effects and resource allocation with controllable processing times. The goal is to limit resource availability to a given range and minimize the makespan. They showed that the problem can be solved under two special cases, and heuristic algorithms were developed to solve the general problem.

Bai & Zhao (2020); Mor (2021); Mor & Mosheiov (2021); Zhao & Hsu, 2019) all considered the dependence of the jobs' processing times on the starting times, along with other factors, and proposed polynomial-time and pseudo-polynomial-time algorithms to solve the problems. ZhaoH2019 studied the scenario where both processing times and start times deteriorate, and the processing time of a job is a linear increasing function of its start time. The goal was to minimize the weighted number of late jobs. Ultimately, they developed a pseudo-polynomial solution algorithm and a fully polynomial-time approximation scheme (FPTAS) to solve this problem. BaiZ2020 studied SMSPs with DeJong's learning effect and machine availability constraints. Initially, a polynomial-time approximation scheme (FPTAS) was developed to solve the problem. Subsequently, the FPTAS was extended to solve SMSPs involving deteriorating jobs, DeJong's learning effect, and machine availability constraints.

Mor (2021) explored a single-machine batch scheduling problem. The article examined three performance measures: maximum completion time, total completion time, and total weighted completion time. They assumed that batches have the same capacity, each batch may contain multiple orders of different sizes, each order size being less than or equal to the same capacity, and orders must be processed in a maximum of two consecutive batches. The problem is subject to an upper bound on the total rejection cost. They showed that all three problems can be solved in polynomial time. Later, (Mor & Mosheiov, 2021) extended the problem to account for common due dates and common due windows problems in batch scheduling. They developed polynomial-time dynamic algorithms to solve the two extensions. Numerical experiments showed that both algorithms can easily solve instances of medium size.

Finally, (Mor et al., 2021) studied a single-machine batch scheduling problem with rejectable jobs. They considered three objective functions: minimizing the makespan, minimizing the total makespan, and minimizing the total weighted makespan while ensuring that the jobs exceeding the cost upper limit can be rejected. It was confirmed that the three problems are *NP*-hard, and the author developed a pseudo-polynomial dynamic programming algorithm to find near-optimum solutions.

In general, the research during this period was focused on finding efficient solutions for SMSPs. Different methods including new mathematical models, polynomial-time algorithms, and heuristic algorithms were developed to address problems, such as scheduling problems that consider complex factors such as deterioration effects, learning effects, resource allocation, starting time, position-related effects, and resource consumption. The main contribution of these studies is that they not only solve theoretical problems but also provide solutions to challenges that may be encountered in practical operations.

3.4 Key-extensions path analysis

This section elaborates on the development branches that have *emerged from the main path*. As depicted in Fig. 5, the key branches are developed in three major streams.

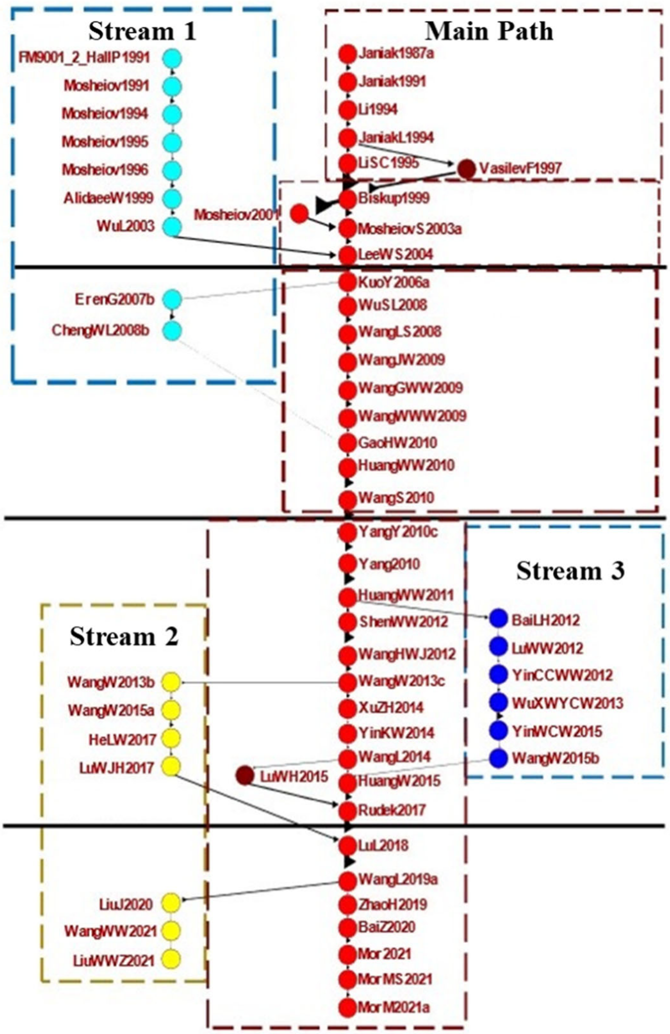


Fig. 5 Important development branches from the main path

In Fig. 5, the nodes on the overall main path are marked in red, while the branches are differentiated in various colors according to their research themes. The detailed discussion of the three development branches is as follows:

Stream 1 (light blue) The nodes in the first stream are represented by light blue, where resource allocation subject to various constraints is the main focus. This branch began with the studies of Hall et al., (1991), both focusing on earliness and tardiness penalty costs. The first study showed that when jobs have different weights while considering a soft due date, the problem becomes *NP*-complete. The second study aimed to minimize the Total Weighted Earliness and Tardiness (TWET) subject to a restrictive common due date, proving that the problem is *NP*-hard.

Mosheiov (1991) focused on deteriorating jobs and proved that the best scheduling method for minimizing the makespan is to use a V-shape policy. In this method, the first job is sorted in descending order of the rate of deterioration, while the subsequent jobs are sorted in ascending order of the rate of deterioration. Mosheiov (1994) proposed a simplified linear deterioration model and explored different optimal scheduling strategies. He suggested that as the number of jobs increases, the start times of the jobs can be delayed, rendering the basic processing time irrelevant. Mosheiov (1995) explored various industry scenarios in which step-deterioration occurs; he proposed several heuristic algorithms to solve the problem considering the makespan. Mosheiov (1996) extended his earlier study the research to encompass nonlinear job deterioration, in which the processing time of the job increases with the start time; this study addressed the problem of minimizing the total weighted makespan and proved that the optimal scheduling method is of Lambda type and can be solved in polynomial time.

Alidaee & Womer (1999) conducted a literature survey to examine methods for handling time-dependent processing times. They discussed the issues related to linear, step-linear, and nonlinear processing time functions. Wu (2003) extended the study on linear deteriorating jobs to include availability constraints and proposed a simplified model. They showed that the problem of minimizing makespan can be solved by transforming it into a 0–1 integer programming problem and simplifying it under certain conditions.

During the same period, another branch emerged from the main path. Vasilev (1997) extended the studies of Li and Janiak (1994), where resource consumption is related to the release time of the jobs, aiming to minimize the total consumption with constrained makespan and total completion time, and extended the consumption function to the case of convex decreasing. However, despite its promise, the new development branch was quickly overshadowed by Biskup (1999).

The next two studies belong to a more recent phase of development. Continuing the study of (Eren & Güner, 2007) explored the bicriteria SMSP with a learning effect and proposed an integer programming model to minimize the weighted sum of total completion time and total tardiness time. Cheng (2008) proposed a new learning model in which, the actual processing time of a job depends on its scheduled position and the processing time of the processed jobs. They showed that, under certain assumptions, the problems of maximum and total completion times, as well as the total weighted completion time and maximum tardiness, can be solved in polynomial time.

Overall, the studies in this branch have laid the essential theoretical foundations and optimization tools for subsequent research on exploring the real-world use cases of SMSPs. More recent developments in the field have diverged into two distinct development branches; one in 2011 and the other in 2013. While the main path (represented by red nodes) considered both deterioration and learning effects, the yellow nodes (i.e., the left branch) lie at the intersection of learning effects and resource allocation, and the dark blue nodes (i.e., the right branch) are commonly used for solving SMSPs considering the makespan. The two streams are discussed in the following sections.

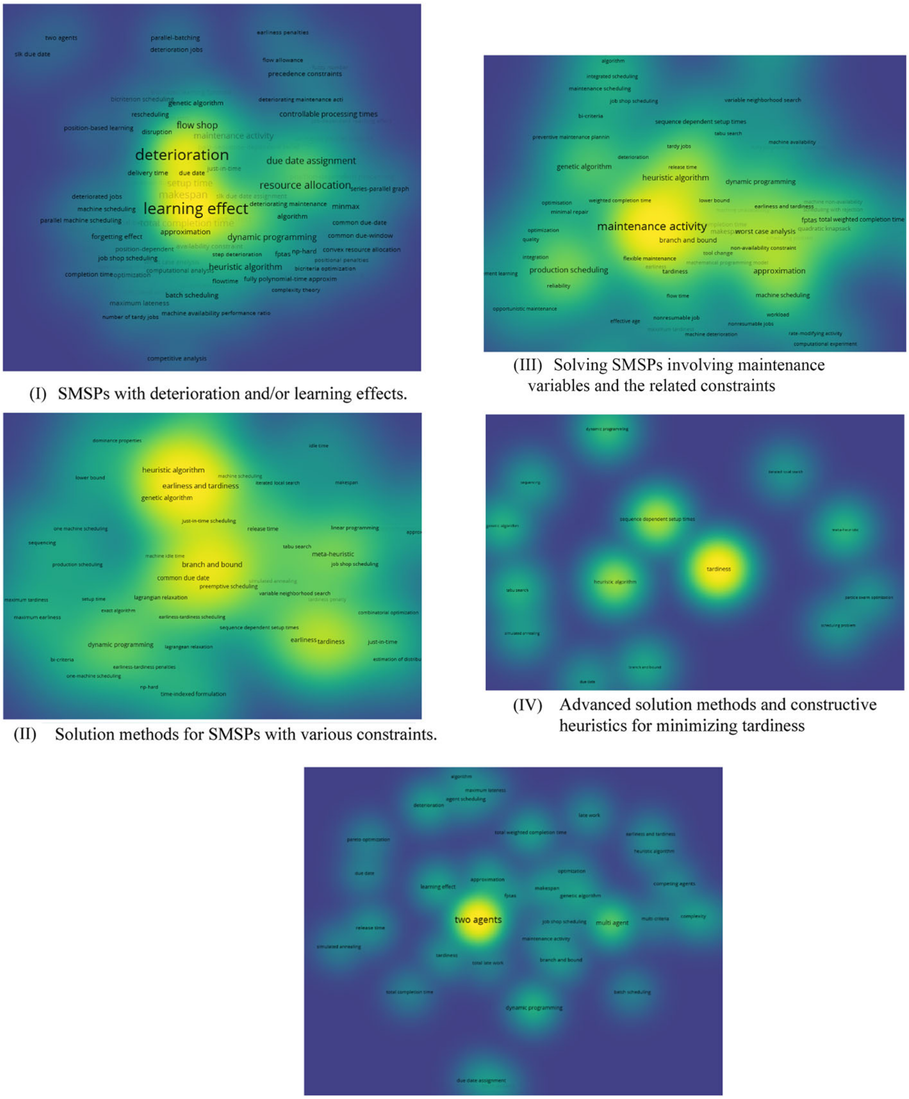
Stream 2 (yellow) (Wang & Wang, 2013a, 2013b) extended the main path research of WangW2013c to establish the new branch. They investigated the SMSP with due date allocation, aiming to minimize earliness, lateness, and total resource consumption costs simultaneously. Two different processing time functions and three different deadline allocation methods were analyzed. They found polynomial-time solutions to each combination. Wang (2015a) focused on job-dependent learning effects and the SMSP with convex resource-dependent processing times. They found a polynomial-time algorithm for all the studied objective functions. He et al.(2017) examined resource-constrained SMSPs with general truncated job-dependent learning effects. They assumed that the processing time of a job is a function of the number of resources allocated to the job, general job-related learning effects, and job-related control parameters. They developed polynomial-time algorithms to minimize various objective functions and processing time functions under both linear and convex resource allocation. Lu et al., (2017) studied the SMSP with setup times and convex resource allocation, where the learning effect is influenced by the position between groups and within the group. The objective was to minimize the makespan under limited available resources. They showed that the best result was obtained by sorting each group's work according to the SPT rule.

Liu (2020) explored an SMSP that simultaneously considers due dates, learning effects, and convex resource allocation while minimizing the total weighted resource consumption cost. In their model, the learning effect is location-based and job-related. The objective was to simultaneously determine the optimal job sequence and resource allocation such that the total weighted resource consumption cost is minimized, subject to a bounded total weighted resource consumption. Wang et al., (2021) studied an SMSP with learning effects and convex resource allocation. Under relaxed deadline assignment, the goal was to determine the optimal order of jobs and groups, the optimal deadline assignment, and the optimal resource allocation to minimize the weighted sum of early and late penalties, general traffic allowance, and resource consumption costs. They showed that the problem can be solved in polynomial time in three special cases and developed heuristics and Tabu search algorithms to solve the general problem. Finally, (Liu et al., 2021) studied the single-machine group scheduling problem with due window assignment and position-dependent weights. The goal was to minimize the weighted sum of the delay and due window allocation costs. The authors showed that the problem can be solved in polynomial time using different methods of assigning due windows.

Overall, in the early development period of this stream, resource allocation SMSPs were examined from different perspectives, such as due date allocation and learning effects. During the second development period, the studies focused on solving SMSPs by incorporating learning effects and resource allocation. Both groups developed new methods to solve the problems studied in polynomial time.

Stream 3 (dark blue) Inspired by the work of (Bai et al., 2012) studied the single-machine group scheduling problem with deterioration and learning effects. They assumed that assignments in the same group have general position-dependent and time-dependent learning effects. The authors considered the problem of minimizing the total completion time in two special cases and showed that it can still be solved in polynomial time. Lu (2012) studied single-machine scheduling with learning effect and proposed two new learning models that combine existing models through addition and multiplication. They proposed polynomial-time algorithms for different objective functions, including minimizing the total completion time, the makespan, and the h^{th} power of completion times.

Yin (2012). studied a dual agent SMSP with assignable due dates. In their model, each agent tries to minimize the completion time of its assigned job. The objective was to assign each job a due date and a position in the sequence such that the weighted sum of the conflicting goals of the agents is minimized. They conducted complexity analysis and solved the proposed problems for different combinations of maximum lateness time, total (weighted) lateness time, and total (weighted) tardy jobs (Wu et al., 2013) also studied a two-agent SMSP with job deterioration. The objective of this model was to minimize the weighted makespan of the first agent's jobs while ensuring that the maximum lateness rate of the second agent adhered to within an upper limit. The authors proved the model to be *NP*-hard and proposed several sorting rules and a lower bound to develop a branch-and-bound algorithm. Furthermore, they devised a Tabu algorithm to approximate the optimal solution.



(I) SMSPs with deterioration and/or learning effects.

(II) Solution methods for SMSPs with various constraints.

(III) Solving SMSPs involving maintenance variables and the related constraints

(IV) Advanced solution methods and constructive heuristics for minimizing tardiness

(V) Agent-based methods for single-machine scheduling.

Fig. 6 Keyword clouds for the clusters

Yin (2015) introduced a new deterioration model considering time and position dependence. They confirmed that the problem of minimizing makespan can be solved in polynomial time. The study further showed that an optimal schedule can be found with minimum, maximum, or V-shaped processing times, depending on the problem parameters. Finally, (Wang & Wang, 2015b) explored SMSPs with precedence constraints while considering simple linear deterioration. The goal was to minimize the weighted sum of waiting times, where the processing time of a job follows a simple linear function of its execution start time. They proved that the general variant of the problem, as well as extensions for jobs with weak

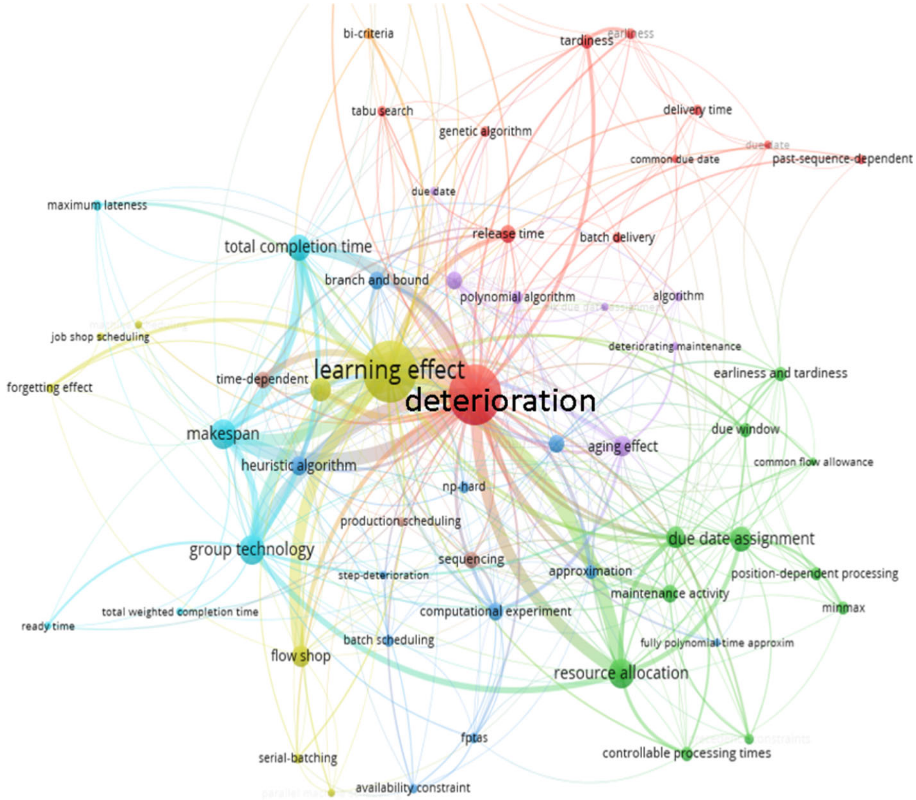


Fig. 7 Keywords correlation diagram of Cluster I

(strong) parallel chains and series–parallel digraph precedence constraints, can all be solved in polynomial time.

Overall, the aforementioned research covers problems such as deterioration and learning effects, as well as dual agent SMSPs, and developed polynomial-time solution methods to facilitate their industry applications. Given the advances in computational hardware, the SMSP literature's development in exact optimization may soon be used as a basis for exactly solving more complex scheduling problems. In addition to the above streams, Lu (2015) branched off from the main path by studying SMSPs with variable job processing times. To minimize the makespan and the total completion time, they showed that the optimal schedule exhibits a V-shaped property in terms of processing time for special cases. The study attracted considerable attention, although most criticized its findings.

3.5 Prevailing themes

The CA approach identified a total of 23 major themes. The word clouds in Fig. 6 highlight the keywords that appeared five times or more in each cluster, with yellow color (larger circle sizes) indicating a higher frequency. The cluster (theme) name is determined by considering the edge-betweenness measure for clustering the articles, the most frequently used keywords, and the respective correlations within each cluster. The number of articles in the following top clusters is 674, 300, 211, 168, and 140, respectively.

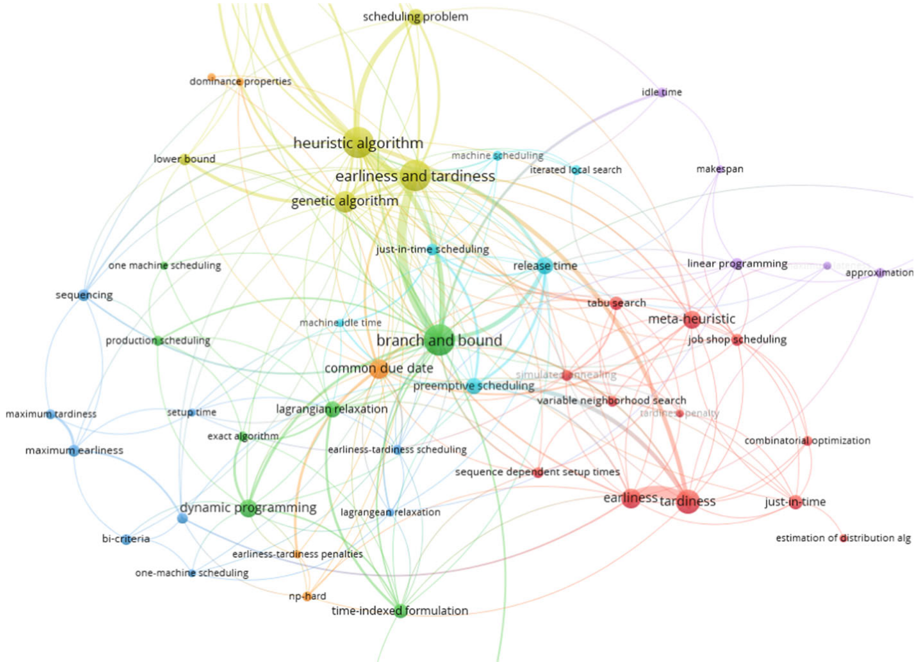


Fig. 8 Keywords correlation diagram of Cluster II

(I) SMSPs with deterioration and/or learning effects.
(II) Solution methods for SMSPs with various constraints.
(III) Solving SMSPs involving maintenance variables and related constraints.
(IV) Advanced solution methods and constructive heuristics for minimizing tardiness.
(V) Agent-based methods for single-machine scheduling.

The correlation among the keywords within each cluster is analyzed to identify the most practical relationships and meaningful academic contributions. Figure 7 shows the keyword correlations within the first cluster that have two cores, deterioration, and the learning effect. In this figure, the thickness of the paths indicates the strength of the link between the two keywords. Important links in the first cluster are observed from three points of view: the problem of interest, the solution method, and the optimization objectives. Resource allocation and due-date allocation exhibit the strongest link in case of the desired problem. Among the methods, batching techniques and heuristic algorithms represent the strongest linkage. When considering the objective function, the makespan and the total completion time stand out more than others. The primary observation is that the first cluster contains the main path, and the keywords within this cluster are consistent with those from the MPA results.

The keyword correlation diagram of the second cluster in Fig. 8 shows that it possesses several cores; no group of keywords exceeded 15 percent of the overall repetitions, implying that the cluster cannot be adequately represented using a few keyword groups. The main themes encompass branch and bound solution methods for solving SMSPs with different constraints, like release date constraints, and optimization objectives such as earliness and tardiness penalty cost, as well as delay penalty cost optimization. The second cluster contains some of the articles located on the key-extension paths.

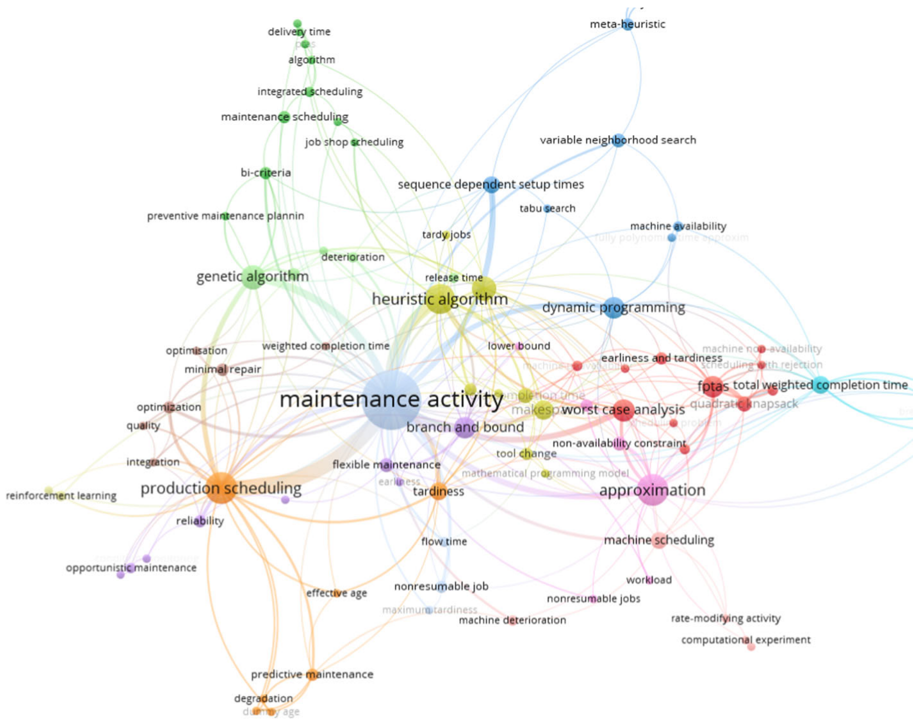


Fig. 9 Keywords correlation diagram of Cluster III

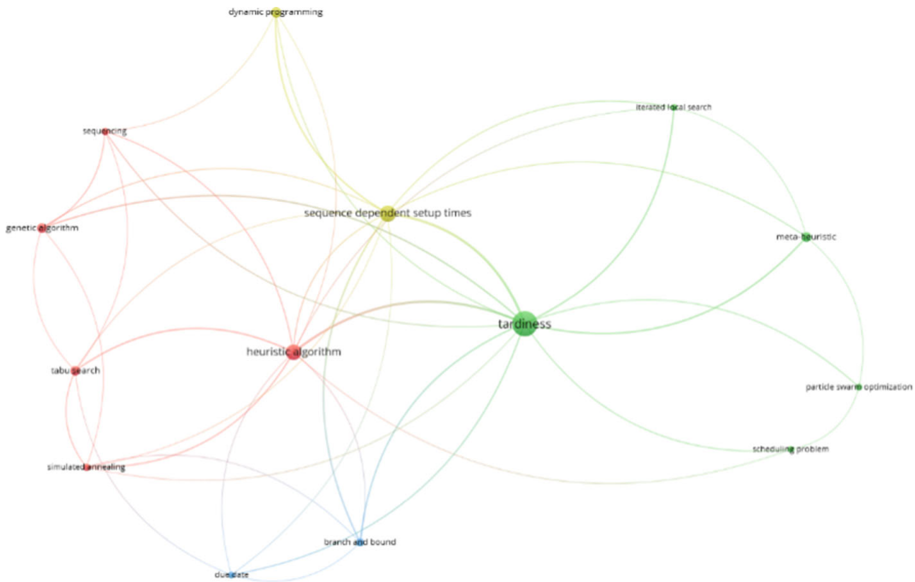


Fig. 10 Keywords correlation diagram of Cluster IV

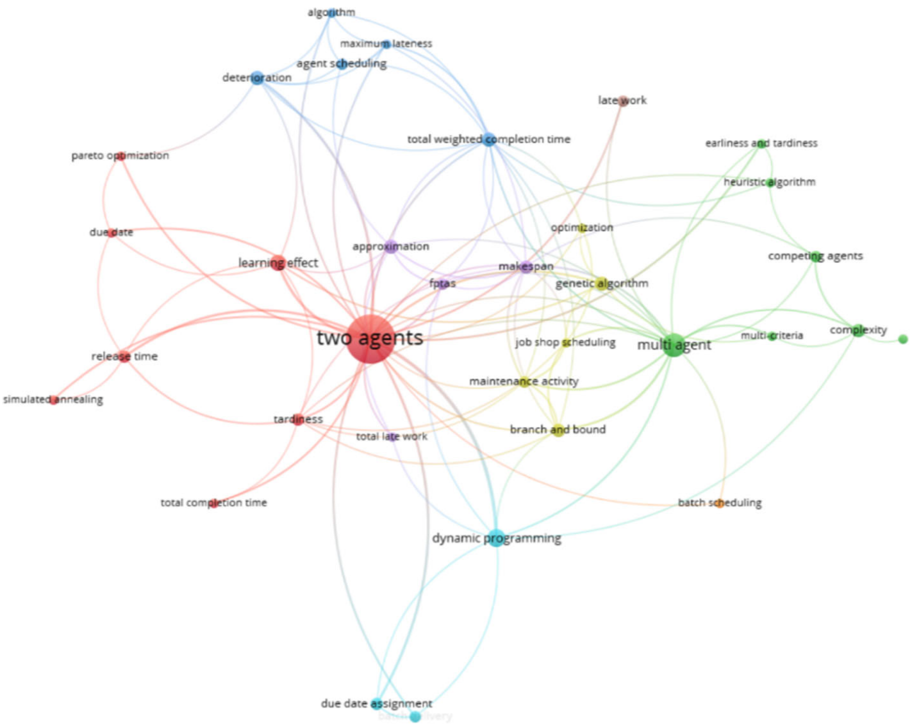


Fig. 11 Keywords correlation diagram of Cluster V

Figure 9 shows the keyword correlations within the third cluster. This cluster differs from the first two clusters by demonstrating a distinct focus on one keyword: "maintenance activity" is identified as the main core of the third cluster, accounting for more than 40% of the total relationships. The main keywords correlated with "maintenance activity" are method-related, including heuristic algorithms, dynamic programming methods, branch and bound methods, and genetic algorithms; these showed the most significant interactions with SMSPs that include maintenance variables.

The correlation diagrams of the last two clusters are presented in Figs. 10 and 11, respectively. Cluster IV is formed around correlations between time-related factors, like tardiness, and solution methods for solving the problem of interest. In contrast, the last cluster exhibits correlations between agent-based methods and a wider range of scheduling features both in terms of optimization objectives and operational constraints. Overall, the last cluster reflects a more diverse literature compared to the other four clusters. An analysis of the growth trends in these clusters shows that the third cluster is expected to continue its growth, while the development of the other clusters is expected to reach the saturation stage in the coming years.

4 Concluding remarks

This systematic review analyzed the main path of knowledge dissemination and seminal development branches within the literature on SMSPs. By analyzing articles published after

1987, the study categorized the literature into different stages and themes of development. Based on these categorizations, the review article discussed trajectories, growth trends, key technologies, and practical features incorporated into SMSPs. The findings provided insight and directions for future research.

When considering current research trends, it becomes clear that the aspects of learning and deterioration effects, particularly in their relationships to group technology, have received limited attention in the existing SMSP studies. These features should be thoroughly explored in the context of SMSPs, taking into account various setup times, resource allocation, due data assignments, and maintenance requirements. Total weighted tardiness (Bożejko et al., 2006; Wodecki, 2008) and the number of tardy jobs (Bożejko et al., 2024) have been well studied in the SMSPs literature. Developing parallel and quantum algorithms for SMSPs considering novel optimization objectives, like energy consumption and workload balance is another suggestion for future research. Additionally, there is room for further development in the integration of uncertainty into SMSPs as well as multi-objective optimization. Furthermore, it is essential to recognize the ongoing industrial transformation and the impact of disruptive technologies in shaping new research directions within the SMSP literature. To name a few, future studies may investigate the Internet of Things (IoT), digital twins, 5G, cyber security, blockchain, as well as Generative AI and their possible implications for scheduling, particularly in the context of SMSPs. Although additive manufacturing-based scheduling problems have received recent recognition among production research scholars (Ying et al., 2022), this new scheduling domain remains underdeveloped, both in terms of problem complexity and solution methodologies.

Our literature review is dedicated to SMSPs; future research may consider expanding the scope of the review to draw a broader picture of scheduling theory and its milestones. In addition, exploring the cross-research between scheduling theory and related fields, such as supply chain and operations management, may shed light on new research ideas. Reviewing real-world use cases and patents based on scheduling theory and its variants is another review topic that will help bridge the gap between scheduling theories and practice. Finally, using alternative big data analysis and learning methods may complement MPA in uncovering less tangible aspects within the scheduling literature.

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Data availability The raw/processed data required to reproduce these findings can be provided upon reasonable request.

Declarations

Conflict of interest The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Consent to participate Not applicable.

Consent to publish Not applicable.

Ethical approval. Not applicable.

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