

A Bond Trading Strategy Based on the Markowitz Model

The Norwegian Bond Market

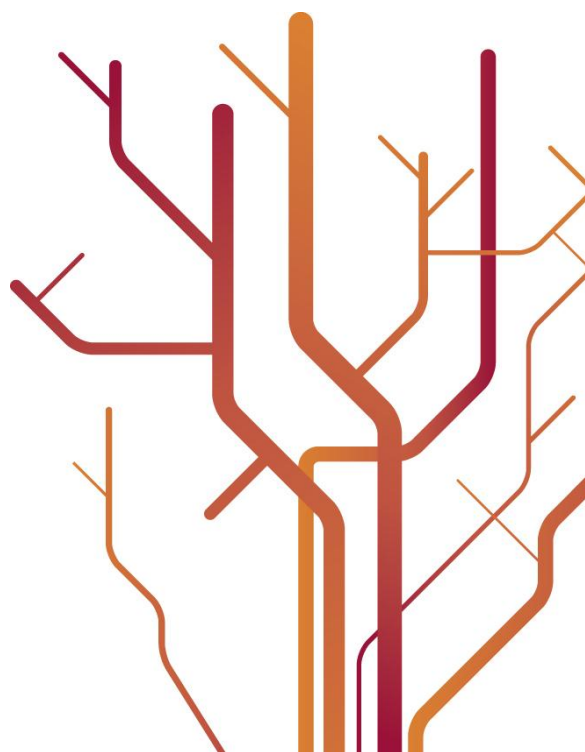


Walter Birkelund and Trond Jonas Skjærgård

Master thesis in Business Administration

(30 stp)

June 2011



Acknowledgements

First of all we would like to thank our supervisor, Ass. Prof. Espen Sirnes for his valuable comments and input during the course of this thesis, and especially for his programming help which enabled us to complete our master thesis. We also want to thank Sverre Braathen Thyholdt for his valuable comments. In addition, we are very grateful for the proofreading by Eirik Birkelund and Stig Rune Jensen.

Finally, we would like to thank our families, our fellow students and the University of Tromsø for five fantastic years.

Table of contents

Acknowledgements.....	ii
Table of contents.....	iii
Table of figures.....	vi
Table of tables.....	vii
Abstract.....	viii
1 Introduction.....	1
1.1 Background.....	1
1.2 Research problem.....	1
1.3 Problem description.....	2
1.4 Purpose.....	2
1.5 Benefit.....	2
1.6 Data and methodology.....	2
1.7 Previous reaserch.....	3
1.8 Outline and reader’s guide.....	3
2 From Theory to reality.....	4
2.1 The rational theory and the heuristic reality.....	4
2.2 Investor uses a portfolio to maximize the utility under uncertainty.....	4
2.3 Induction problem.....	5
2.4 Modelling and properties of time-series.....	5
2.5 Summary.....	6
3 Bond theory.....	7
3.1 Definition.....	7
3.2 Characteristics of bonds.....	7
3.3 Bond pricing.....	8
3.4 Interest rates theory.....	8
3.4.1 Current yield.....	8

3.4.2	Yield to maturity	9
3.5	Market structure.....	9
3.5.1	The U.S Bonds market	9
3.5.2	The European market	10
3.5.3	The Norwegian market	10
3.6	Bond rating.....	10
3.7	Bond portfolio management strategies	11
3.7.1	Passive strategies	11
3.7.2	Active bond management	11
3.8	The strategy in this paper	12
3.9	Bond Index/Reference Index	13
4	Modern portfolio theory (MPT).....	13
4.1	Mean and variance calculation.....	15
4.2	Diversification	16
4.2.1	Diversification in the Markowitz model.....	17
4.3	The efficient frontier and market portfolio.....	18
4.3.1	The Sharpe ratio.....	20
4.3.2	The risk free asset.....	21
5	Methodology	21
5.1	The Markowitz model and its mathematics	21
5.2	Optimum portfolio.....	22
5.2.1	Optimization with Riskless lending and borrowing with shortsale not allowed	25
5.2.2	Quadratic programming and Kuhn-Tucker conditions.....	26
5.3	Model weakness and state of the art	27
5.3.1	Model weakness	27
5.3.2	State of the art	28

5.4	Data	29
5.5	Programming and calculations.....	29
5.5.1	Sorting the raw data.....	29
5.5.2	Processing the data	32
5.5.3	VBA programming.....	36
5.5.4	Calculation of expected return, return, variance and standard deviation and the Sharpe ratio.....	42
5.6	Testing the level of significance	43
6	Summary of results and empirical investigation	44
6.1	Model 1	45
6.1.1	Shortsale not allowed all bonds.....	45
6.1.2	Shortsale not allowed Corporate Bonds.....	46
6.2	Model 2	47
6.2.1	Shortsale allowed all bonds	47
6.2.2	Shortsale allowed corporate bonds.....	48
6.3	Model 3	49
6.3.1	Shortsale not allowed on all bonds with risk-free asset when the expected return is less than the risk free rate.....	49
6.3.2	Shortsale not allowed corporate bonds with risk-free asset when the expected return is less than the risk free rate.....	50
6.4	Model 4	51
6.4.1	Shortsale allowed all bonds with risk-free asset when the expected return is less than the risk free rate.....	51
6.4.2	Shortsale allowed corporate bonds with risk-free asset when the expected return is less than the risk free rate.....	52
6.5	Summery of results	52
7	Conclusion.....	53
8	LIST OF REFERENCES	55

Table of figures

Figure 1 How diversification affect risk(Elton, et al., 2007, p. 60).	17
Figure 2 The efficient frontier. The optimal portfolios plotted along the curve have the highest expected return possible for the given amount of risk (Investopedia, 2003).	19
Figure 3 Efficient frontier and capital market line	20
Figure 4 Combinations of the riskless asset in a risky portfolio (Elton, et., 2007, p. 100).	22
Figure 5 Closing prices raw data from Excel (Oslo Stock Exchange, 2011).	30
Figure 6 PivotTable in Excel	30
Figure 7 The sheet "Traded".	32
Figure 8 The sheet "Last Traded"	33
Figure 9 The sheet "Return without coupons"	34
Figure 10 The sheet "Traded1"	35
Figure 11 The sheet "Return with coupon"	36
Figure 12 VBA snapshot of the main function OptPort(Sirnes, 2011)	37
Figure 13 Function Argument window in Excel	38
Figure 14 VBA snapshot of the Filter function (Sirnes, 2011)	39
Figure 15 VBA snapshot of the CalcMoments function (Sirnes, 2011)	39
Figure 16 VBA snapshot of the OptPortShortRF function (Sirnes, 2011)	40
Figure 17 VBA snapshot of the OptPortNoShortRF function (Sirnes, 2011)	41
Figure 18 VBA snapshot of the CalcCovar function (Sirnes, 2011)	41

Table of tables

Table 1 Testing the level of significance. 44

Table 2 Shortsale Not allowed all bonds 45

Table 3 Shortsale not allowed Corporate Bonds 46

Table 4 Shortsale allowed all bonds..... 47

Table 5 Shortsale allowed corporate bonds 48

Table 6 Shortsale not allowed on all bonds with risk-free asset when the expected return is less than the risk free rate. 49

Table 7 Shortsale not allowed corporate bonds with risk-free asset when the expected return is less than the risk free rate. 50

Table 8 Shortsale allowed all bonds with risk-free asset when the expected return is less than the risk free rate..... 51

Table 9 Shortsale allowed corporate bonds with risk-free asset when the expected return is less than the risk free rate. 52

Abstract

This thesis evaluates the possibility of using a trading strategy based the Markowitz portfolio optimization algorithm. The strategy is tested on the Norwegian bond market. As a main feature of our analysis, we use historical prices to estimate expected returns, return variances, and covariance of the different bonds.

Microsoft Excel 2010 and Microsoft Visual Basic for Applications 7.0 were applied to process the data and calculate the optimal portfolios.

The results from this paper are inconsistent. This implies that no reliable conclusions regarding the potential abilities of the trading strategy can be made. However, eight of the 32 tested portfolios could with a satisfactory statistical significance outperform the risk-return relationship of the index. We find that this strategy can be useful if we were able to incorporate fundamentals like rating, duration and inflation and were able to impose restrictions with regard to the weights in each bond.

1 Introduction

1.1 Background

The modern portfolio theory presented by Harry Markowitz (1952) is one of the cornerstones of portfolio selection and management. Whereas the Markowitz model has been tested thoroughly on the stock market, it has not been used in the same extent on the bond market. A wide knowledge has been accumulated about the performance, strengths and weaknesses of this model when it is applied on equity portfolios. However, much less is known about this approach to bond portfolios.

Korn and Koziol (2006) find at least two reasons for this. First of all, at the time when the Markowitz model became more widely recognized as a useful tool for portfolio management, the interest rates were not particularly volatile. Hence, a portfolio approach was considered unnecessary. This has changed over the last decades. Possible changes in interest rates leads to substantial risks in bond investments. There are many different bonds with different maturities available and it is natural to think about the potential of risk diversification. A still present problem is the high correlation between bonds which reduces the diversification effect.

Second, difficulties to implement Markowitz's approach on bond portfolios might have discouraged further research. There are two major problems, one general problem and one bond-specific problem. The general problem concerns the large number of parameters needed in the Markowitz model if the number of assets increases. Here, recent research has shown that restrictions with respect to the parameters and portfolio weights are useful tools to improve the performance.

The bond specific issue concerns the variation of moments over time, which precludes simple historical estimation based on the assumption of stationarity. If interest rates decline during one period, the average return during this period can overstate the return during the subsequent period which might have lower interest rates. Moreover, the characteristics of bonds change over time.

1.2 Research problem

The purpose of the master thesis is to evaluate the results of a bond trading strategy, based on the Markowitz optimization algorithm. The model is developed by Harry Markowitz (1952)

and attempts to maximize portfolio expected return for a given amount of portfolio risk, or equivalently minimize the risk for a given level of expected return. Generally it has been less research on bond markets compared to stock markets, and especially when it comes to bond trading strategies compared to stock trading strategies. This may therefore be an important study and will hopefully be a contribution to research on this topic.

1.3 Problem description

The question is whether an active bond trading strategy based on the Markowitz model generates higher risk-adjusted return than an equally weighted index.

1.4 Purpose

The purpose is to evaluate the strategy and compare to an equally weighted bond index, and test if you can outperform the bond index with this trading strategy.

1.5 Benefit

The results of this research might be useful for fund managers and the buyer side of the bond markets. If the research shows that this trading strategy can generate return greater than the bond index, this approach could be a useful strategy when managing portfolios. Using a tool like this would be significantly cheaper than using an analyst or investing in a hedge fund.

1.6 Data and methodology

The data in this paper are the daily bond closing prices from Oslo Stock Exchange from the 1.1.2006 to 31.12.2009. The four year period is chosen to capture the cyclical behavior of interest rates.

We apply the Markowitz portfolio optimization algorithm to optimize a bond portfolio after the Sharpe ratio criteria (Sharpe, 1964).

The portfolios are optimized with four different constraints. The models are first tested on all bonds and then on corporate bonds only. All model are tested with monthly, quarterly, semi-annually and annually adjustments. The monthly adjusted portfolios are based on the historical data from the month prior to the optimization point; the quarterly adjusted portfolios are based on the historical data from the three past months and so on. The portfolios risk-adjusted returns are then compared against an equally weighted index to see if the portfolios renders a better reward-to-volatility relationship than the index.

1.7 Previous reaserch

Korn and Koziol (2006) applied the Markowitz approach of portfolio selection to government bond portfolios. Term structure models were used to estimate expected returns, return variances and covariances of different bonds. The study for the German market shows that a small number of risky bonds is sufficient to reach very promising predicted risk-return profiles. The study also finds that if the number of risky bonds in the portfolio is not too large and the term structure model does not contain more than two factors, these predictions are confirmed by the realized risk-return profiles.

Bansal, Dahlquist and Harvey (2004) used a mean-variance dynamic trading strategy and developed a method for constructing optimally managed portfolios, which exploit the possibility that asset returns are predictable. They find that investing in actively managed funds produces economic gains in relation to fixed weight strategies which imply that active funds should benefit the investors. Furthermore, the authors find evidence that buy and hold portfolios performs even worse than fixed weight strategies. The portfolios are based on the US bond and stock market.

Gripenvik (2006) evaluates the possibility of making relatively reliable forecasts of future variances, covariances and returns of a portfolio of risky assets. The forecasts are then used as input in a trading strategy based on the Markowitz portfolio optimization algorithm. The results show that the trading strategy, although achieving on average a 14 percent higher monthly Sharpe ratio than the benchmark index, cannot with a satisfactory statistical significance outperform the FTSE100. However, the results should be considered to be of economic significance to an investor since, on average, the strategy renders a three percent higher return while maintaining the same level of risk.

1.8 Outline and reader's guide

The remainder of this thesis is structured as follows: **Chapter 2** discusses the problems occurring when theory is applied on reality. **Chapter 3 and 4** reviews essential bond theory and the concepts of portfolio optimization. **Chapter 5** addresses the methodology and explains the construction and programming executed in Excel 2010 and Visual Basic for Applications (VBA). **Chapter 6** presents the results and the empirical investigations. **Chapter 7** sum up the paper with a conclusion of the findings and states whether the models are appropriate or not in the case of bond management.

2 From Theory to reality

2.1 The rational theory and the heuristic reality

Decisions made in the hustle and bustle of everyday life are often based on simple rules of thumb that can be either genetic spinal reflexes based on evolution (Darwin, 1968), or learned behaviour. Economic theory is often based on the assumption about the rational man which implies that we choose based on the normative answer to what we should do (Jacobsen & Torsvik, 2007).

Our decision making process is within cognitive psychology described as heuristic which means that we make decisions based experience and not based on normative and rational techniques. Research by Gilovich, Griffin and Kahneman (2002), shows that we often prefer a simple heuristic decision making process above normative and rational techniques. We have one particular heuristic rule in portfolio selection; do not put all your eggs in one basket (Løhre, 2007).

The rule is described as a naive strategy, or 1/N-rule. We note in particular that Harry Markowitz, the father of modern portfolio theory even used this rule for their own investments (Zweig, 1998). There seems to be the difference between theory and practice, even for the one who conceived the theory.

2.2 Investor uses a portfolio to maximize the utility under uncertainty

The basic assumption in financial theory is as mentioned earlier that investors are rational. Investors seek to maximize their utility. The maximum utility is related to the amount that is left in the end of the investment period.

Investment choices are made under uncertainty. If we can choose among assets in which each asset gives a certain return under 100 % certainty, we would be rational and choose the asset that gives the highest return. In such a case the investor do not need a portfolio to maximize utility. However, historical data shows that different assets gives different returns and those assets with high expected return often are connected with high risk, where risk is described as variation of the returns.

Different assets have different correlation between each other and by combining assets with low correlation the investor might achieve a better risk-return relationship.

2.3 Induction problem

The scientific method is common to formulate a theory based on empirical data. However, we cannot with 100% confidence know that a phenomenon we have observed 100% of the cases in the past would repeat itself in the future. We cannot prove a theory based on such induction (Hawking, 1998). This problem is called the induction problem.

Theory based on empirical data assumes that the inductive principle is valid for the theory to hold. In practice, inductive inference is often used in the development of financial theory, first described by Charles Peirce (1992, 1998) for about 100 years ago. Inductive inference means that we start with a data set based on facts, observations and definitions. Based on these characteristics a hypothesis is made and if the hypothesis explains the dataset and no other hypothesis can explain it better, the hypothesis is likely to be correct.

2.4 Modelling and properties of time-series

When we want to describe reality through a model, it is necessary to assume that certain relationships are valid in order to not make the model too complex. These relationships may be in the form of decomposition of one size in a number of other sizes, or in terms of the effect on size has on another size (Hill, Griffiths, & Lim, 2007).

Time series data are often used as basis when building a model. The model and the parameters of the model would be affected of properties of the data (Hill, et al., 2007).

One of the most common simplifications is the assumption of mean reversion. Mean reversion means that the series over time will fluctuate around their mean. This implies that the yield curves will have a tendency to go back to their average after they have reached their maximum or minimum. Another implication is that if the risk can be said to have mean reversion, then risk decrease over time. Thus, the risk would be lower by holding a share in 15 years, instead one year. Exley, Mehta and Smith (2004) shows that it is primarily in fixed income markets in which they have a case of mean reversion. They show that the stock markets do not have mean reversion to the same extent as fixed income markets. Others argue that mean reversion in equity markets is strong enough to form the basis for a strategy that provides better investment results than an index portfolio (Gropp, 2004).

Another feature that is often assumed is that the variance is a constant process. It is assumed that the variance is homoscedastic, which means, that it is evenly distributed over the time series. The challenge is that financial data series are often heteroskedastic, that is, that the

underlying asset may be observed to have a stable variance over a longer period, before it then comes a period of particularly high impact on the variance before the "calm" again. When we calculate risk to an asset with the variance measured over a period that shows heteroskedasticity, we can overestimate risk in a quiet period and underestimate the risk in a turbulent period. In such cases, Auto Regressive Conditional Heteroskedasticity (ARCH) models are used to characterize and model observed time series. They are used whenever there is reason to believe that, at any point in a series, the terms will have a characteristic size, or variance (Hill, et al., 2007).

2.5 Summary

In this section, we discussed the challenges of transferring theory to practice.

In many contexts it is a difference between what the investor actually chooses, and that he should choose. Experiments show that the investor is not necessarily rational in their decisions, but emphasizes experience and heuristic rules just as much as the model results which can be seen through how Markowitz (1959) even invests his funds.

The heuristic approach to decisions is challenged by theoretical science. Even if all observed data has exactly the same outcome, we cannot, however, with 100% confidence know that this is also the outcome in the future. Therefore, finance theory is developed in the form of hypotheses. The correct hypothesis is defined in that no other hypothesis can explain the observed phenomena better. It is interesting that science thus has great similarities with the heuristics, and that theory can be considered almost described practices.

To build models, it is often necessary to make simplifications in relation to the observed reality. One example is that risk is defined as a fixed throughout the time period, even though that is not always so. Another example is that we assume a normal distribution of return data, while we often can observe asymmetry and heavy tails in the historical data. Such simplification leads to the results of the modelling becomes more uncertain. If we base our decisions on these model results, we may choose positions with significantly greater risk than intended.

3 Bond theory

3.1 Definition

A bond is a debt instrument issued for a period of more than one year. The issuer of the bond is obligated to make specified payments to the holder of the bond. The bond is issued with the purpose of raising capital by borrowing. It is a formal contract to repay the principal with interest in accordance with fixed intervals (Bodie, Kane, & Marcus, 2009). The loans are securitized and traded in a secondary market, which means that the debt is distributed among several investors (Choudhry, 2005). Bonds are issued by governments, states, cities, corporations, and many other types of institutions with the purpose of borrowing money.

Bonds and stocks are both securities, but the major difference between the two is that stockholders have an equity stake in the company, whereas bondholders have a creditor stake in the company. Another difference is that bonds usually have a defined term, or maturity, after which the bond is redeemed, whereas stocks may be outstanding indefinitely (Bodie, et al., 2009).

3.2 Characteristics of bonds

The holder of a bond receives a certain cash flow from the issuer at specified dates during the life of the bond together with the principal when the bond matures (Fabozzi, 2010). The principal value of a bond or its face value is not necessarily the same as the price of the bond, as the bond can be traded at a premium or a discount in the secondary market (Fabozzi, 2010).

The bond's nominal yield or the coupon is the stated rate of interest of the bond. This yield percentage is the percentage of the face value. The coupon is usually paid semi-annually (Choudhry, 2005).

As stated above a bond may sell for less or more than par value. Hence, there will be differences between the nominal and current yield of the bond. The price of bonds moves in the opposite direction of interest rates. If the rates go up, the price of bonds goes down and vice versa. If you buy a bond at a discount, your current yield would be higher than the nominal yield and the other way around (Choudhry, 2005).

Bonds are often categorized after term to maturity, which is the number of years the issuer has promised to meet the conditions of the obligation. The bond ceases to exist at the maturity date and the issuer redeems the bond by paying the principal. The term to maturity is simply

referred to as the maturity in the bond market (Fabozzi, 2010). The maturities on bonds are important for three reasons. The first and obvious reason, is that it indicates (1) the period over which the holder of the bond can expect to receive coupon payments and (2) when to receive the principal of the bond. Second, the bonds' yield to maturity depends on it. Finally, the volatility of a bond's price depends on the maturity of the bond. With other factors held constant, the longer the maturity of a bond, the more sensitive the bond is to changes in the market yield. Hence, the volatility increases with the maturity (Fabozzi, 2010).

3.3 Bond pricing

The price of a financial instrument is equal to the present value of the expected cash flows from the financial instrument. The expected cash flows are simple to compute for some financial instruments, whereas the task is more difficult for others. The theoretical value of a bond is determined by its coupon, yield and term to maturity (Fabozzi, 2010). This can be illustrated by the equation below where P is the price, n is the number of periods, C is the semi-annual coupon payments, r is the periodic interest rate (required yield divided by 2), M is the maturity value and t is the time period when the payment is to be received.

$$P = \sum_{t=1}^n \frac{C}{(1+r)^t} + \frac{M}{(1+r)^n}$$

It follows from the formula above that there is an inverse relationship between the bond price and the interest rate. Hence, changes in interest rates are the main reason fluctuations in the bond price. In addition, expected inflation and default risk will influence the bond price.

These variables are difficult to predict (Fabozzi, 2010).

3.4 Interest rates theory

3.4.1 Current yield

The effective rate of return is the return an investor receives when investing in a bond. It is called the bonds' current yield. In contrast to the coupon, the yield is not locked in over the maturity of the bond, but will rather change reflecting changes in the bond price due to changes in supply and demand for the bond as well as changes in the interest rates. The current yield calculation takes no consideration to capital gain that the investor achieves from buying the bond at a discount. It only takes into account the coupon interest on the bond. The time value of money is also ignored when calculating the current yield. For example, the

current yield for a 10-year 5 % coupon bond with par value of \$1000 sold for \$800 is 6.25 % because the bond is selling at a discount (Fabozzi, 2010).

3.4.2 Yield to maturity

The yield to maturity is the yield that will make the present value of the cash flows equal to the investment or price. Hence, the yield to maturity is the internal rate of return. If the bond sells on a discount the yield to maturity will be higher than the yield since the bond price considers the capital gain as well as the coupon payments. If the bond sells at a premium it will be the other way around. The yield to maturity is calculated from the equation below where P is the price of the bond, C is the coupon payments, M is the maturity value, n is the number of periods and y is the internal rate of return or yield to maturity (Fabozzi, 2010).

$$P = \sum_{t=1}^n \frac{C}{(1+y)^t} + \frac{M}{(1+y)^n}$$

3.5 Market structure

The global bond market consists of domestic and international bonds. A bond denominated in the currency where it is issued is called a *domestic bond*. Bonds issued in a country by a non-domestic entity are called an *international bond*. Domestic bonds accounts for around 70 % of the global bond market and international bonds for the rest (Fabozzi, 2010).

The world's largest bond markets are the markets for government bonds. Government bonds comprise over half the bonds outstanding. A government bond is issued by a national government denominated in the country's own currency. These bonds are usually considered as risk free. The remainder of the bond market consists of corporate bonds, municipal bonds, mortgages and mortgage-backed securities (Fabozzi, 2010).

3.5.1 The U.S Bonds market

The US bond market is the largest and most important bond market in the world. The U.S Treasury has over \$14 trillion dollars of marketable bonds, notes and bills outstanding, which makes it the largest issuer of debt in the world. In addition, as indicated above, the size, liquidity and volume of the U.S corporate bond market is considered to be the biggest in the world. Overall the U.S market contributes to around 44 % of the global bond market. The U.S bond market is approximately the same size as the U.S stock market (Securities Industry and Financial Markets Association, 2011).

3.5.2 The European market

Globalization and the rise of different types of European bond markets such as the corporate bond market have changed the opportunities for European bond investors. Bonds contribute to 2/3 of the total amount outstanding of securities in bonds and shares in Europe. About 60 % of the European bond market is government bonds, 30 % is corporate and last 10% is asset-backed bonds. The corporate sector in Europe is relatively smaller compared to the US corporate sector (Association for Financial Markets in Europe, 2011).

3.5.3 The Norwegian market

The Norwegian bond market is dominated by national issuers. In the private sector, financial institutions are the main contributor. The dominance of the financial sector is partly due to the fact that borrowing has grown more than lending the latest years. Furthermore, the global financial crises led to an increase in bonds as funding for financial institutions (Government.no, 2011).

Compared with Denmark and Sweden, the turnover in the Norwegian bond market is relatively low. The Norwegian market is also small compared to the total economic activity. This applies particularly to the ratio of bonds issued by the state. This of course reflects that the government borrowing needs are far smaller in Norway than in most other countries (Government.no, 2011).

Low liquidity in the secondary market for bonds affects pricing. The lower the price is in the secondary market, the higher the effective interest rate on the loan. That is, the more liquid the debt paper is, the lower, *ceteris paribus*, the company's marginal borrowing cost will be. In the Norwegian market, it is therefore only companies that borrow larger amounts that will be able to borrow cheaper in the bond market than by borrowing from financial institutions (Government.no, 2011).

3.6 Bond rating

Credit ratings or bond ratings agencies are tools that investors can use when making decisions about purchasing bonds. These agencies are not investment advisors, but provide assessments of relative credit risk. The ratings express the rating agencies opinions about the ability and willingness of an issuer to meet its financial obligations.

Rating agencies like Standard & Poor, Moody's and Fitch apply their own methodologies in measuring the creditworthiness and use a specific rating scale to publish their ratings

opinions. Ratings are typically expressed as letter grades ranging from “AAA” to “D” - where “AAA” is the best possible rating (Appel, 2010).

Credit ratings are useful for both the buyers and sellers in the bond market. The ratings may facilitate the process for the issuer as they can use the widely recognised and long standing measure of credit risk to communicate to the investors. On the other side, investors may use the ratings as a screening device to match the credit risk of an issuer with their own risk tolerance (Standard & Poor's, 2011).

3.7 Bond portfolio management strategies

3.7.1 Passive strategies

There are two specific passive management strategies. The first is the buy-and-hold strategy, in which the bond manager selects a portfolio based on the objectives and constraints given by the client with the purpose of holding the bonds to maturity. This strategy is considered as the simplest strategy. It involves finding bonds with desired quality, coupon levels, and term to maturity, and indenture provisions, such as call features. Buy-and-hold strategies do not involve active trading, but rather, looks for prospects whose maturities approximate their stipulated investment horizon (Reilly & Brown, 2002).

The second passive strategy is called indexing. The object of this strategy is to construct a portfolio that equal the performance of a bond index, such as the government bond index. Numerous empirical studies have demonstrated that most money managers are unable to outperform or match the risk-return performance of common stock or bond indexes. Hence, some managers construct portfolios that track an index closely (Reilly & Brown, 2002).

3.7.2 Active bond management

There are five well known active bond management strategies, which are discussed below in sections.

Interest rate anticipation

Interest rate anticipation is probably the most risky bond management strategy, as it relies on uncertain forecasts of future interest rates. The basic idea is to preserve capital when an increase in interest rates is expected and to achieve capital gains when the interest rates are expected to decline. This can be achieved by changing the duration of the portfolio in accordance to interest rate expectations. For instance, if a rise in the interest rates is expected,

duration of the portfolio may be lowered so as to preserve capital, - and the other way around if a decline in the interest rates is expected. If the expectations are wrong, the outcome will on one side, be loss of the opportunity for capital gains and, -on the other hand increase in capital loss (Reilly & Brown, 2002).

Valuation analysis

With the valuation analysis strategy, the portfolio manager attempts to pick bonds based on their intrinsic value. The intrinsic value is determined by the characteristics of the bonds and the average market value of these characteristics. Based on the valuation analysis, undervalued issues would be bought whereas overvalued issues would be sold. Success with this strategy requires accurate estimates of the bond characteristics (Reilly & Brown, 2002).

Credit analysis

The credit analysis strategy involves analysis of the bond issuer so as to determine expected changes its default risk, inter alia by attempting to predict changes in the rating of the bond given by Moody's, Finch and Standard & Poor. The idea is to find bonds that are likely to be upgraded and avoid bonds that are likely to be downgraded (Reilly & Brown, 2002).

Yield spread analysis

The yield spread analysis approach looks at the spread relationship between alternative bonds by comparing the maturity, liquidity and the creditworthiness of the two bonds or by comparing them to an index. Bond managers using this strategy monitors bond closely and try to detect abnormal relationships. Under this approach, it is crucial for the bond manager to have the required background information and liquidity to buy and sell quickly enough to profit from the temporary yield abnormality (Reilly & Brown, 2002).

3.8 The strategy in this paper

In this paper we shall apply an active bond selection strategy using modern portfolio theory. Our strategy is based on the Markowitz model and we shall optimize the portfolios after the maximum Sharpe ratio criteria from the Markowitz model. The optimization will be based on historical data from the period prior to the optimization point. The portfolios are optimized with monthly, quarterly, semi-annual and annual adjustments. The monthly adjusted portfolio are based on the historical data from the month prior to the optimization point, the quarterly adjusted portfolio from the quarter prior to the optimization point and so on.

3.9 Bond Index/Reference Index

In lack of an appropriate bond index to evaluate our portfolio against we decided to construct an equally weighted bond index based on the same data set as we are constructing the portfolio from. An equally weighted index is constructed by giving the same weight to each security in the index. This allows the smaller securities to be considered side by side with larger securities (Bodie, et al., 2009).

$$EWI = \frac{\sum_{i=1}^N E(R_i)}{N}$$

Where,

EWI is equally weighted index

$E(R_i)$ is expected return on each security

N is the number of securities

4 Modern portfolio theory (MPT)

Modern portfolio investment theory attempts to minimize portfolio risk for a given level of portfolio expected return, or equivalently maximize return for a given level of risk. The core of the theory was presented as early as 1952 by Harry Markowitz in an article and a subsequent book in 1959. The concept behind MPT is that the assets in a portfolio have lower risk collectively than just one asset. A diversification strategy is to have more than one security in your portfolio. By adding more securities to the portfolio, the exposure to firm specific factors is spread out, and the volatility of the portfolio decreases until the unsystematic risk eliminated. However, the risk altogether cannot be avoided, because all securities are affected by the common market risk, which can be macroeconomic factors. The risk that remains after diversifying the portfolio is called market risk, systematic risk or non-diversifiable risk. The risk that can be diversified away is called unique risk, firm-specific risk or nonsystematic risk (Elton, Gruber, Brown, & Goetzmann, 2007).

In the early 1960s, the investment community used the expression risk, but there was no specific measure for the term. To build a portfolio model, however, investors had to quantify their risk variable. The basic portfolio model was developed by Harry Markowitz, who derived the expected rate of return for a portfolio of assets and an expected risk measure.

Markowitz showed that the variance of the rate of return was a meaningful measure of portfolio risk under a reasonable set of assumptions, and he derived the formula for computing the variance of a portfolio. This portfolio variance formula indicated the importance of diversifying your investments to reduce the total risk of a portfolio. The Markowitz model is based on several assumptions regarding investor behavior:

1. Investors consider each investment alternative as being represented by a probability distribution of expected returns over some holding period.
2. Investors maximize one-period expected utility, and their utility curves demonstrate diminishing marginal utility of wealth.
3. Investors assess the risk of the portfolio on the basis of the variability of expected returns.
4. Investors base decisions exclusively on expected return and risk, so their utility curves are a function of expected return and the expected variance (or standard deviation) of returns only.
5. For a given risk level, investors prefer higher returns to lower returns. Likewise, for a given level of expected return, investors prefer less risk to more risk.

Under these assumptions, a single asset or portfolio of assets is considered to be efficient if no other asset or portfolio of assets offers higher expected return with the same (or lower) risk, or lower risk with the same (or higher) expected return (Elton, et al., 2007).

Investors should consider the covariance between the different asset, and not only the expected return and variance. The covariance is an absolute measure of the degree of association between returns for pairs of securities, so if a covariance is positive the two securities move in the same direction at the same time, where a negative covariance indicates that the two securities move inversely. Covariances close to zero, show that the securities are independent of each other, and have no tendency to move in the same or opposite directions. A further analysis of statistic relationship between two securities is the correlation coefficients (Elton, et al., 2007). By dividing the covariance between two assets by the product of the standard deviation of each asset, produces a variable with the same properties as the covariance but with an absolute value that cannot exceed 1. This value is denoted ρ_{ij} and is the correlation between securities i and j . A correlation of 1 means the securities is perfect positive correlated (move in the same direction) and a correlation of -1 means perfect negative correlated (move in the opposite direction). As it approaches to zero between the two

securities (uncorrelated) the securities are independent from each other and as it approaches -1 or 1 it is stronger relationship between the two securities (correlated) (Elton, et al., 2007).

4.1 Mean and variance calculation

The preliminary step in composing a portfolio, is to determine the key characteristics of each asset; being the returns and standard deviations of each asset.

The average of a probability distribution of possible returns, calculated by using the following formula:

$$E(R) = \sum_s p(s) r(s)$$

Where $E(R)$ is the expected or mean return for an asset, $p(s)$ is the probability of each scenario and $r(s)$ is the holding period return of each scenario. Each scenario is labelled by s .

We can write the expected return on any portfolio as:

$$\bar{R}_p = \sum_{i=1}^N X_i \bar{R}_i$$

Where X_i is weight in asset i and \bar{R}_i is the expected return of asset i (Elton, et al., 2007).

It is possible to calculate the mean of an investment in several different ways, but mainly arithmetic and geometric method is used. We have decided to use the geometric method when calculating the historic mean and the growth rate of the portfolio (Elton, et al., 2007).

Markowitz argues that the geometric method will guarantee a more realistic result compared to Arithmetic and compounding average methods. We assume that the original investment plus interest payments are reinvested the following periods. This criterion is not fulfilled by the arithmetic method (Elton, et al., 2007).

In investment analysis the mean of a population of interest is often unknown, usually because it is impossible practically to identify or take measurements from each member of the population. The population mean is estimated from a sample drawn from the population, and the sample variance and sample standard deviation is then calculated (Defusco, McLeavey, & Pinto, 2001).

The variance is a measure of the average distance between each set of observations and their mean value; the sum of the square differences between observations and their mean divided n-1, where n is denoted the sample size and the sample variance is denoted:

$$s^2 = \frac{1}{n-1} \sum_{i=1}^n (a_i - \bar{a})^2$$

Where,

s= sample variance

n=number of observations

a_i =The corresponding member of the data set where $i=1,2,..,n$

\bar{a} =The sample mean

The true variance is denoted by σ^2 . The standard deviation of a sample is the square root of the sample variance (Elton, et al., 2007).

$$s = \sqrt{s^2}$$

The variance and standard deviation are both assumed to be unbiased estimators for σ leading the random variables to be normally distributed (Elton, et al., 2007).

While the standard deviation of return on any portfolio can be written as:

$$\sigma_p = \left[\sum_{i=1}^N X_i^2 \sigma_i^2 + \sum_{i=1}^N \sum_{\substack{j=1 \\ j \neq i}}^N X_i X_j \sigma_i \sigma_j \rho_{ij} \right]^{1/2}$$

Further analysis of the risk of the portfolio requires the covariance which will be discussed in subsequent chapters.

4.2 Diversification

Diversification is a risk management technique in finance. It reduces or eliminates the unsystematic risk of a portfolio by holding a portfolio of securities that are uncorrelated with each other. A diversified portfolio is not that sensitive to impacts to single securities.

One of the great features of diversification is that it reduces the risk of a portfolio, but not necessarily the return of the portfolio (Luenberger, 1998).

In order to understand how to diversify a portfolio we need to understand the difference between systematic and unsystematic risk. Risk related to market forces that affect all assets is called un-diversifiable risk. For instance, the impact of different economic cycles for cannot be diversified away. The other risk component is unsystematic risk and this risk can be diversified away or reduced by holding a portfolio of securities that are not perfectly correlated with the other securities in the portfolio (Elton, et al., 2007).

The total risk of a portfolio is the sum of systematic and unsystematic risk. The risk of a portfolio diminishes as the portfolio gets more diversified to a point where all unsystematic risk is diversified away as figure 1 shows. When we get to this point it is impossible to reduce risk without reducing the expected return of the portfolio (Elton, et al., 2007).

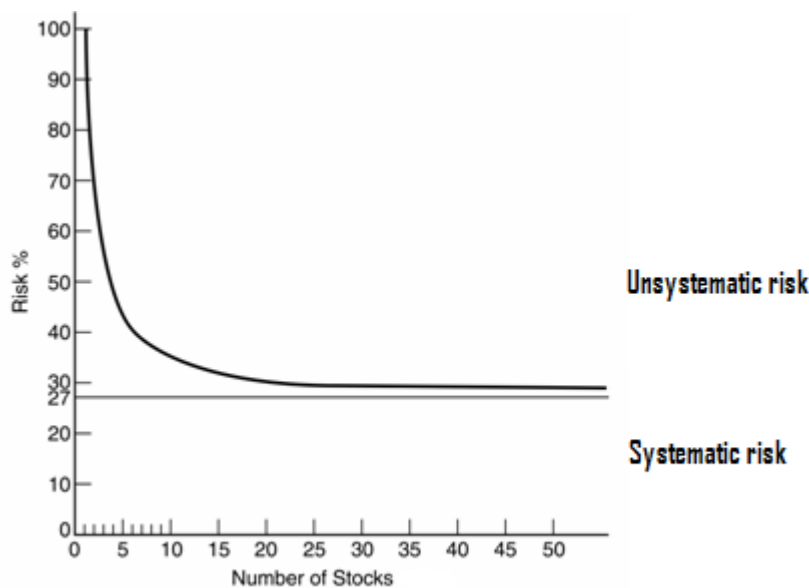


Figure 1 How diversification affect risk(Elton, et al., 2007, p. 60).

4.2.1 Diversification in the Markowitz model

Diversification in the Markowitz model is based on five basic interrelated concepts. The first concept requires that the weights of the assets in the portfolio sums to 100%. The main task for the portfolio manager is to calculate these weights to optimize the portfolio.

$$\sum_{i=1}^N X_i = 1$$

X represents the weight of asset i in a portfolio that contains N assets. The weights should sum to 1 even if we allow shortsales and unlimited borrowing and lending.

Further, the expected return of a portfolio is the weighted average of the expected returns of the assets in the portfolio.

$$E(R_p) = \sum_{i=1}^N X_i E(R_i)$$

$E(R_i)$ is the expected rate of return on asset i.

The objective for portfolio managers is to find weights that ensures the highest possible expected return at a given risk level or conversely the minimum risk at a given level of expected return. The set of portfolios that hold these features are lying on the efficient frontier. A portfolio on the efficient frontier dominates all other portfolios.

The risk of a portfolio is calculated based on historical data. It can be broken into two parts; the variance which represents the individual risk and the covariance which represents the interaction between the assets in the portfolio. The equation below with double summation represents the variance-covariance. Written in matrix form

$$VAR(R_p) = \sum_{i=1}^N \sum_{j=1}^N x_i x_j \sigma_{ij}$$

$\sigma_{ij} = \sigma_i \sigma_j \rho_{ij}$ and ρ_{ij} is the correlation between asset i and j. According to Markowitz, the assets included in the portfolio should have low correlations between each other in order to diversify the portfolio and hence, reduce the portfolio risk.

The last concept we discuss in connection to diversification in the Markowitz model is the capital allocation line. This concept discusses the possibility of borrowing and lending at a risk free rate. The Markowitz model allows us to combine risky assets and riskless assets through borrowing and lending. This concept will be clarified later when we explain the concepts of modern portfolio (Elton, et al., 2007).

4.3 The efficient frontier and market portfolio

The efficient frontier gives us a geometric interpretation of asset combinations. It shows us which asset combinations that should be preferred by different investors, given the Markowitz assumption that all investors are risk averse.

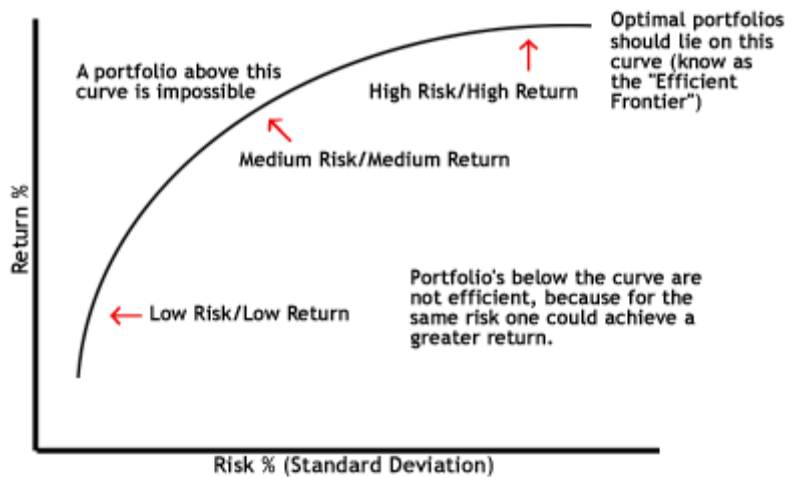


Figure 2 The efficient frontier. The optimal portfolios plotted along the curve have the highest expected return possible for the given amount of risk (Investopedia, 2003).

We can see from figure 2 that the risk-return relationship is not linear. Hence, the risk-return relationship will vary on different points on the frontier. All portfolios on the efficient frontier are superior to other portfolios with the same level of risk or the same level of expected return.

The next step is to find the optimum portfolio when we mix risky and riskless investments. We add the riskless investment to the graph and then maximize the Sharpe ratio. We will draw a straight line from the return axis to the point where it is tangent to the efficient frontier. The line represents the riskless borrowing and lending and is called the capital market line. The capital market line graphs the risk premiums of efficient portfolios as a function of portfolio standard deviation. The CML is considered to be superior to the efficient frontier since it takes into account the inclusion of a risk-free asset in the portfolio. The CML also represents the highest possible sharp ratio (Elton, et al., 2007).

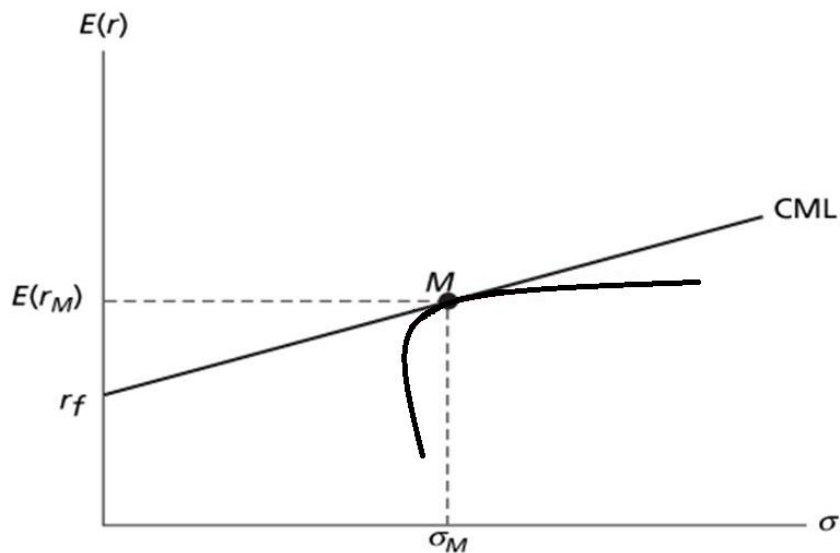


Figure 3 Efficient frontier and capital market line

$E(R)$ is the expected return

σ is the standard deviation

R_F is the risk-free rate

M is the market portfolio

CML is the capital market line

4.3.1 The Sharpe ratio

The Sharpe ratio is reward-to-volatility ratio. It measures a portfolio's excess return to standard deviation. It was developed by William F. Sharpe (1964) and is defined as,

$$S_P = \frac{E(R_P) - R_F}{\sigma(R_P)}$$

Where,

$E(R_P)$ is the expected return of the portfolio

R_F is the return on the risk free rate

$\sigma(R_P)$ is the standard deviation of the portfolio return

The Sharpe ratio simplifies portfolio evaluation and is widely used to measure portfolio performance. However, in isolation the Sharpe ratio does not give much information. It just

states how much a portfolio get “paid” in expected return for the risk incorporated in the portfolio (Bodie, et al., 2009).

4.3.2 The risk free asset

This is an asset that is said to be without any default risk. Hence, with variance and standard deviation equal to zero. It is said to be a hypothetical asset as no asset is without any risk. However, assets such as government bonds are often recognized as risk free as the chance of default or bankruptcy for a country is close to zero. This applies for government bonds issued by most of the industrialized countries. The common notation for the risk free asset is R_F (Bodie, et al., 2009).

We have chosen the Norwegian three months Treasury bill as our risk free asset. The risk free rate is calculated by calculating the average from the month prior to the calculation point.

5 Methodology

5.1 The Markowitz model and its mathematics

Markowitz (1959) developed a model to calculate the efficient frontier of a portfolio. The model is based on the assumption that the investors make investment decisions based on risk and return. The investors seek to maximize their risk reward. The model is based on expected returns, variances and covariance between the different securities and assumes that we can calculate decent values for the expected rates of return, the variances and the covariance. The objective is as stated above to maximize return and minimize risk simultaneously. The Markowitz model makes it possible to construct a portfolio with various combinations of securities where shortsales, borrowing and lending are allowed. We can also set restrictions to the model like shortsales not allowed (Benninga, 2008).

With this model we should be able to reach a single portfolio of risky assets that is preferred to all other portfolios with the same level of return. We can show this graphically by drawing it in an expected return and standard deviation space (see figure 4). We can plot all possible portfolios containing both riskless and risky investments. The optimal portfolio will be somewhere on the ray between risk free investments R_F and the optimal risky portfolio located in the tangency point on the efficient frontier. Different points on the ray shows the maximum return for a given level of risk, or conversely, minimum risk for a given level of return.

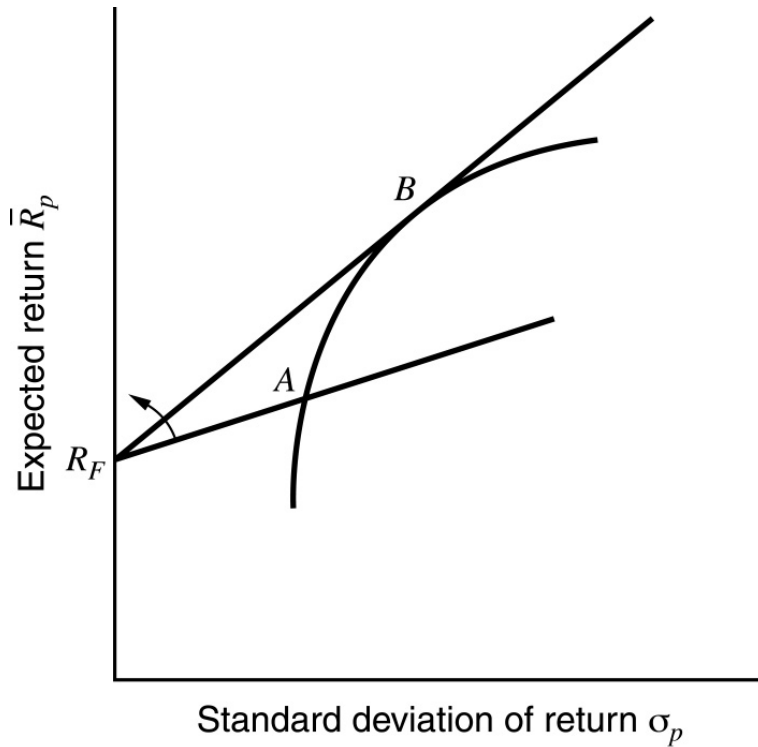


Figure 4 Combinations of the riskless asset in a risky portfolio (Elton, et., 2007, p. 100).

As mentioned above investors seek the ray with the highest slope to maximize their risk reward. The slope of the ray is called the sharp ratio θ . The sharp ratio is excess return divided by standard deviation of the portfolio σ_p . It states how much the return of a portfolio increase for every “unit” of risk we add to the portfolio.

5.2 Optimum portfolio

In equation form we have: maximize the objective function

$$\theta = \frac{\bar{R}_P - R_F}{\sigma_P}$$

Subject to the constraint,

$$\sum_{i=1}^N X_i = 1$$

$$X_i \geq 0 \text{ all } i$$

The constrained maximization problem above can be solved by Langrangian multipliers. Another alternative is to substitute the constraint into the objective function and maximize the objective function as an unconstrained problem. R_F can be written as 1 times R_F and we have,

$$R_F = 1R_F = \left(\sum_{i=1}^N X_i \right) R_F = \sum_{i=1}^N (X_i R_F)$$

Making this substitution in the objective function and stating the standard deviation and expected return in general form we get,

$$\theta = \frac{\sum_{i=1}^N X_i (\bar{R}_i - R_F)}{\left[\sum_{i=1}^N X_i^2 \sigma_i^2 + \sum_{i=1}^N \sum_{\substack{j=1 \\ j \neq i}}^N X_i X_j \sigma_{ij} \right]^{1/2}}$$

The problem stated above is a simple maximization problem and can be solved by finding the derivatives of the function with respect to the different variables and equating them to zero.

This gives us the solution to the following system of simultaneous equations.

$$1. \frac{d\theta}{dX_1} = 0$$

$$2. \frac{d\theta}{dX_2} = 0$$

⋮

$$N. \frac{d\theta}{dX_N} = 0$$

Before we can solve the maximization problem we need to take the derivative of the ratio θ .

We rewrite θ ,

$$\theta = \left[\sum_{i=1}^N X_i (\bar{R}_i - R_F) \right] \left[\sum_{i=1}^N X_i^2 \sigma_i^2 + \sum_{i=1}^N \sum_{\substack{j=1 \\ j \neq i}}^N X_i X_j \sigma_{ij} \right]^{-1/2}$$

The ratio consists of a multiplication of two functions. To derive this we need to apply the product rule and the chain rule and we get,

$$\begin{aligned}
\frac{d\theta}{dX_k} &= \left[\sum_{i=1}^N X_i (\bar{R}_i - R_F) \right] \left[\left(-\frac{1}{2} \right) \left(\sum_{i=1}^N X_i^2 \sigma_i^2 + \sum_{i=1}^N \sum_{\substack{j=1 \\ j \neq i}}^N X_i X_j \sigma_{ij} \right) \right]^{-3/2} \\
&\quad \times \left(2X_k \sigma_k^2 + \sum_{\substack{i=1 \\ j \neq k}}^N X_j \sigma_{kj} \right) + \left[\sum_{i=1}^N X_i^2 \sigma_i^2 + \sum_{i=1}^N \sum_{\substack{j=1 \\ j \neq i}}^N X_i X_j \sigma_{ij} \right]^{-1/2} \times [(\bar{R}_k - R_F)] \\
&= 0
\end{aligned}$$

If we multiply the derivative by

$$\left(\sum_{i=1}^N X_i^2 \sigma_i^2 + \sum_{i=1}^N \sum_{\substack{j=1 \\ j \neq i}}^N X_i X_j \sigma_{ij} \right)^{1/2}$$

And rearranging yields

$$-\left[\frac{\sum_{i=1}^N X_i (\bar{R}_i - R_F)}{\sum_{i=1}^N X_i^2 \sigma_i^2 + \sum_{i=1}^N \sum_{\substack{j=1 \\ j \neq i}}^N X_i X_j \sigma_{ij}} \right] \left(X_k \sigma_k^2 + \sum_{\substack{i=1 \\ j \neq k}}^N X_j \sigma_{kj} \right) + [(\bar{R}_k - R_F)] = 0$$

Where we define λ as Lagrange multiplier,

$$\frac{\sum_{i=1}^N X_i (\bar{R}_i - R_F)}{\sum_{i=1}^N X_i^2 \sigma_i^2 + \sum_{i=1}^N \sum_{\substack{j=1 \\ j \neq i}}^N X_i X_j \sigma_{ij}}$$

Yields,

$$-\lambda \left(X_k \sigma_k^2 + \sum_{\substack{i=1 \\ j \neq k}}^N X_j \sigma_{kj} \right) + [(\bar{R}_k - R_F)] = 0$$

By multiplication,

$$-\left(\lambda X_k \sigma_k^2 + \sum_{\substack{i=1 \\ j \neq k}}^N X_j \sigma_{kj} \right) + [(\bar{R}_k - R_F)] = 0$$

Now by extension

$$\frac{d\theta}{dX_i} = -(\lambda X_1 \sigma_{1i} + \lambda X_2 \sigma_{2i} + \dots + \lambda X_i \sigma_i^2 + \dots + \lambda X_{N-1} \sigma_{N-1i} + \lambda X_N \sigma_{Ni}) + \bar{R}_k - R_F = 0$$

Multiplying each X_k by a constant λ creates a new variable $Z_k = \lambda X_k$. The X_k gives us the fraction to invest in each security and the Z_k are proportional to this fraction. In order to simplify we substitute Z_k for λX_k and move the variance covariance terms to the right-hand side which gives us,

$$\begin{aligned} \bar{R}_1 - R_F &= Z_1 \sigma_1^2 + Z_2 \sigma_{12} + Z_3 \sigma_{13} + \dots + Z_N \sigma_{1N} \\ \bar{R}_2 - R_F &= Z_1 \sigma_{12} + Z_2 \sigma_2^2 + Z_3 \sigma_{23} + \dots + Z_N \sigma_{2N} \\ \bar{R}_3 - R_F &= Z_1 \sigma_{13} + Z_2 \sigma_{23} + Z_3 \sigma_3^2 + \dots + Z_N \sigma_{3N} \\ &\vdots \\ &\vdots \\ &\vdots \\ \bar{R}_N - R_F &= Z_1 \sigma_{1N} + Z_2 \sigma_{2N} + Z_3 \sigma_{3N} + \dots + Z_N \sigma_N^2 \end{aligned}$$

We have N equations with N unknowns. By solving for Zs we get X_k s which are the optimum proportions to invest in security k,

$$X_k = Z_k / \sum_{i=1}^N Z_i$$

The equations above show us how to calculate the optimal weights in general form with shortsalses allowed and riskless lending and borrowing. With other constrains like no shortsale allowed we follow the same pattern but there will be other constraints(Elton, et al., 2007).

5.2.1 Optimization with Riskless lending and borrowing with shortsale not allowed

This problem in analogous to the case with shortsalses and riskless borrowing and lending allowed. We have one optimal portfolio, which is the one that maximizes the slope of the line

connecting the riskless asset and a risky portfolio. However, in contrast to the previous case, investors cannot hold securities in negative amounts. The problem can be stated as,

$$\theta = \frac{\bar{R}_P - R_F}{\sigma_P}$$

Subject to the constraint,

- (1) $\sum_{i=1}^N X_i = 1$
- (2) $X_i \geq 0$ all i

Because of the inequality restriction on X_i this is a mathematical programming problem. This might look like a linear programming question and both constraints are in fact linear. The problem on the other hand is that the objective function is not linear. σ_P contains terms involving X_i^2 and $X_i X_j$. Equations involving both squared terms and cross-product terms are called quadratic equations. Hence, we are dealing with a quadratic programming problem (Elton, et al., 2007).

5.2.2 Quadratic programming and Kuhn-Tucker conditions

The quadratic programming algorithms are based on the Kuhn-Tucker conditions from advanced calculus. In the case of shortsales allowed and riskless borrowing and lending allowed we simply took the derivative of θ with respect to X_i and found the maximum value of θ by setting the equation equal to zero. In the case of shortsales not allowed and riskless lending and borrowing, X_i is constrained to be larger or equal to zero and we can write,

$$\frac{d\theta}{dX_i} \leq 0$$

We can make it an equality by writing,

$$\frac{d\theta}{dX_i} + U_i = 0$$

The equality above is the first Kuhn-Tucker condition for a maximum. Further, we have to notice two things about U_i . First, if the optimum occurs when X_i is positive, then the $d\theta/dX_i = 0$ and U_i are zero. If the optimum occurs at $X_i=0$, then $d\theta/dX_i < 0$ and U_i are positive. At the optimum we then have,

$$X_i > 0, U_i = 0$$

$$X_i=0, U_i >0$$

This is the second Kuhn-Tucker condition. It can be written as,

$$X_i U_i = 0$$

$$X_i \geq 0$$

$$U_i \geq 0$$

The four Kuhn-Tucker conditions are

$$(1) \frac{d\theta}{dx_i} + U_i = 0$$

$$(2) X_i U_i = 0$$

$$(3) X_i \geq 0$$

$$(4) U_i \geq 0$$

If we have a solution that satisfies the Kuhn-Tucker conditions, then we could be sure that the solution is the optimum portfolio (Elton, et al., 2007).

5.3 Model weakness and state of the art

5.3.1 Model weakness

In 1969 Review of Economic Studies published two articles with criticism to the Markowitz model.

Borch(1969) and Feldstein (1969) shows that the Markowitz model only provides optimal results if the returns of the securities are symmetrically distributed or if the investors have a quadratic utility function.

This can be interpreted as the utility of money. A quadratic utility function is not plausible as such a function implies increasing instead of decreasing absolute risk aversion, and negative marginal utility of a certain threshold value. This function would imply that investors buy less risky assets as their income increase (Løhre, 2007).

A symmetrical distribution excludes by definition an asymmetrical distribution. It is widely accepted that the returns of risky assets such as stocks and bonds tends to follow an asymmetrical distribution, especially in the long run. The distribution is close to a log-normal distribution as the value cannot go below zero and the fact that stocks and bonds on average

generates a positive return in the long run. Hence, the variance is an inferior measure of risk. The downside is limited, while the upside potential is unlimited, and variance as a measure of symmetric risk, penalizes the unlimited and desired upside equally to the undesirable downside (Løhre, 2007).

Furthermore, Green and Hollifield (1992) finds that the Markowitz model tends to choose large positions in a relatively small number of securities which leads to undiversified instead of diversified portfolio as the portfolio follows the movements in these few large positions. Thus, the portfolio would be exposed for events such as bankruptcy, natural disasters etc.

A less extensive model such as an equally weighted portfolio where every available asset has an equal stake in the portfolio, generates returns surprisingly close to the optimal mean-variance portfolio (Fisher & Statman, 1997).

5.3.2 State of the art

In line with a number of empirical studies, it is widely accepted that historical stock returns data are not normally distributed. Furthermore, empirical research states that variance and standard deviation are an inferior measure of risk as investors are most keen on avoiding losses.

Researchers have tried to deal with these challenges through two developments. The first is to find a better description of the historical return data, and the other is to find better measures of risk. Empirical research shows that mGH is superior to normal distribution dealing with historical return data.

The returns are often asymmetric and sometimes have large tails and several returns that falls on the outside of the normal distribution. Thus, it is developed a method to describe such a distribution. The method is called Multivariate generalized hyperbolic (MGH). The hyperbola is described such that it accounts kurtosis and skewness.

A widely used method to quantify risk exposure is through value at risk (VaR). A VaR statistic has three components: a time period, a confidence level and a loss amount. With these factors we can determine the maximum possible loss in a given time period at a given confidence level. In contrast to volatility, VaR separates between upside and downside risk. However, VaR does not define the upside, it only measures the downside. Technically, we are forced to, among other things, assuming a Gaussian return distribution for the VaR to be sub additive (Artzner, Delbaen, Ebner, & Heath, 1999).

As stated earlier, return distributions are often unsystematic and in some cases VaR will underestimate or overestimate the risk. Another risk measure is Expected Shortfall (ES) which is defined as expected loss in 1-x% worst cases. The ES is sub additive as VaR and describes the total risk.

VaR and ES are in many cases used along with the standard deviation. As mentioned earlier, such cases assume normal distributions. In a world with normal distribution, VaR and ES are only the standard deviation multiplied with a constant. Hence, it will not give an extra dimension beyond the standard deviation. It is when we combine the description of the return data as a mGH and uses VaR or ES to describe the risk we get a closed model where we can optimize the risk-reward dimension. This is considered the state of the art in the end of 2007(Hellmich & Kassberger, 2009).

5.4 Data

The Markowitz model is tested on real market data. The data is collected from Oslo Stock Exchange and consists of the daily closing prices for all traded bonds (Oslo Stock Exchange, 2011). We have data from 1.1.2006 to 31.12.2009. We have excluded all equity-linked bonds and bonds with put/call features. The data set consists of government bonds, corporate bonds and bonds issued by financial institutions.

The data set is split into different data sets in order to optimize different bond portfolios. The first data set consists of all bonds and the second data set consists of 28 corporate bonds. This is done to discover which risk-return relationship a portfolio consisting of only corporate bonds would give. The portfolios are optimized and adjusted monthly, quarterly, semi-annually and annually.

Due to problems with singularity in the variance-covariance matrix we divided the bonds in the first data set into nine different portfolios. The nine portfolios consist of closely correlated bonds. The bonds in each portfolio are equally weighted.

5.5 Programming and calculations

5.5.1 Sorting the raw data

Microsoft Excel 2010 and Microsoft Visual Basic for Applications 7.0 were applied to process the data and calculate the optimal portfolios.

We received daily bond closing prices from 1.1.2006 to 31.12.2009 from Oslo Stock Exchange(2011). The raw data was sorted and arranged by using PivotTable in Excel.

#	A	B	C	D	E	F	G	H	I	J	K
1	ISIN	Ticker	Name	Name_Coupon	Date	Closing price	Maturity	Settlement	Coupon	Observations	Call/Put/AIO
2	N00010521610	NSG17	Norske Skog ASA 09/14 15.50%	41820&Norske Skog ASA 09/14 15.50%_15.5	23.07.2009	99.9	30.06.2014	30.06.2009	15.5	13	FALSE
3	N00010521610	NSG17	Norske Skog ASA 09/14 15.50%	41820&Norske Skog ASA 09/14 15.50%_15.5	25.08.2009	96.5	30.06.2014	30.06.2009	15.5	13	FALSE
4	N00010521610	NSG17	Norske Skog ASA 09/14 15.50%	41820&Norske Skog ASA 09/14 15.50%_15.5	26.08.2009	97.75	30.06.2014	30.06.2009	15.5	13	FALSE
5	N00010521610	NSG17	Norske Skog ASA 09/14 15.50%	41820&Norske Skog ASA 09/14 15.50%_15.5	01.09.2009	97	30.06.2014	30.06.2009	15.5	13	FALSE
6	N00010521610	NSG17	Norske Skog ASA 09/14 15.50%	41820&Norske Skog ASA 09/14 15.50%_15.5	18.09.2009	97.45	30.06.2014	30.06.2009	15.5	13	FALSE
7	N00010521610	NSG17	Norske Skog ASA 09/14 15.50%	41820&Norske Skog ASA 09/14 15.50%_15.5	25.09.2009	100.5	30.06.2014	30.06.2009	15.5	13	FALSE
8	N00010521610	NSG17	Norske Skog ASA 09/14 15.50%	41820&Norske Skog ASA 09/14 15.50%_15.5	28.09.2009	99	30.06.2014	30.06.2009	15.5	13	FALSE
9	N00010521610	NSG17	Norske Skog ASA 09/14 15.50%	41820&Norske Skog ASA 09/14 15.50%_15.5	09.10.2009	99.5	30.06.2014	30.06.2009	15.5	13	FALSE
10	N00010521610	NSG17	Norske Skog ASA 09/14 15.50%	41820&Norske Skog ASA 09/14 15.50%_15.5	29.10.2009	98.75	30.06.2014	30.06.2009	15.5	13	FALSE
11	N00010521610	NSG17	Norske Skog ASA 09/14 15.50%	41820&Norske Skog ASA 09/14 15.50%_15.5	06.11.2009	98.5	30.06.2014	30.06.2009	15.5	13	FALSE
12	N00010521610	NSG17	Norske Skog ASA 09/14 15.50%	41820&Norske Skog ASA 09/14 15.50%_15.5	17.11.2009	99.5	30.06.2014	30.06.2009	15.5	13	FALSE
13	N00010521610	NSG17	Norske Skog ASA 09/14 15.50%	41820&Norske Skog ASA 09/14 15.50%_15.5	07.12.2009	99.25	30.06.2014	30.06.2009	15.5	13	FALSE
14	N00010521610	NSG17	Norske Skog ASA 09/14 15.50%	41820&Norske Skog ASA 09/14 15.50%_15.5	16.12.2009	99.25	30.06.2014	30.06.2009	15.5	13	FALSE
15	N00001259113	KORK11	ADJ Kraftverkene i Orkla 83/08	39528&ADJ Kraftverkene i Orkla 83/08_13.5	26.11.2007	100.1075	21.03.2008	21.03.1983	13.5	1	FALSE
16	N00001106173	AKFY17	ADJ Akershus Fylke 85/08	39787&ADJ Akershus Fylke 85/08_13.05	01.06.2007	98.65	05.12.2008	05.12.1985	13.05	1	FALSE
17	N00001137079	BLY07	ADJ Bergen Lysverker 84/09	39892&ADJ Bergen Lysverker 84/09_13	05.04.2006	99.3	20.03.2009	20.03.1984	13	1	FALSE
18	N00001389092	SFFY09	ADJ Sogn og Fjord Fylke 84/09	39968&ADJ Sogn og Fjord Fylke 84/09_12.875	26.01.2006	99.96	22.06.2009	22.06.1984	12.875	1	FALSE
19	N00001187025	FVAR02	ADJ Fredrikstad og Omegn Avlep. 84/09	40108&ADJ Fredrikstad og Omegn Avlep. 84/09_12.75	16.04.2007	97.17	22.10.2009	22.10.1984	12.75	1	FALSE
20	N00001261523	KSUEN52	ADJ Kristiansund E-verk 84/09	40168&ADJ Kristiansund E-verk 84/09_12.625	19.01.2006	100.05	21.12.2009	21.12.1984	12.625	1	FALSE
21	N00001106165	AKFY16	ADJ Akershus Fylke 84/09	40168&ADJ Akershus Fylke 84/09_12.5	04.06.2007	100.68	21.12.2009	21.12.1984	12.5	1	FALSE
22	N00001478150	OVOK15	ADJ Øvre Otra 84/09	40084&ADJ Øvre Otra 84/09_12.5	19.01.2006	101.71	28.09.2009	28.09.1984	12.5	1	FALSE
23	N00001004444	NST444	12.00% Den norske stat 85/07	39141&12.00% Den norske stat 85/07_12	19.01.2007	99.445	28.02.2007	28.02.1985	12	1	FALSE
24	N00010532765	MORG29	Spb Møre 09/PERP ADJ C HYBRID	767010&Spb Møre 09/PERP ADJ C HYBRID_11.7	11.09.2009	106.25	31.12.3999	10.09.2009	11.7	7	FALSE
25	N00010532765	MORG29	Spb Møre 09/PERP ADJ C HYBRID	767010&Spb Møre 09/PERP ADJ C HYBRID_11.7	17.09.2009	107	31.12.3999	10.09.2009	11.7	7	FALSE
26	N00010532765	MORG29	Spb Møre 09/PERP ADJ C HYBRID	767010&Spb Møre 09/PERP ADJ C HYBRID_11.7	18.09.2009	107.5	31.12.3999	10.09.2009	11.7	7	FALSE
27	N00010532765	MORG29	Spb Møre 09/PERP ADJ C HYBRID	767010&Spb Møre 09/PERP ADJ C HYBRID_11.7	21.09.2009	107.5	31.12.3999	10.09.2009	11.7	7	FALSE
28	N00010532765	MORG29	Spb Møre 09/PERP ADJ C HYBRID	767010&Spb Møre 09/PERP ADJ C HYBRID_11.7	24.09.2009	108.5	31.12.3999	10.09.2009	11.7	7	FALSE
29	N00010532765	MORG29	Spb Møre 09/PERP ADJ C HYBRID	767010&Spb Møre 09/PERP ADJ C HYBRID_11.7	25.09.2009	109.25	31.12.3999	10.09.2009	11.7	7	FALSE
30	N00010532765	MORG29	Spb Møre 09/PERP ADJ C HYBRID	767010&Spb Møre 09/PERP ADJ C HYBRID_11.7	22.10.2009	110.98	31.12.3999	10.09.2009	11.7	7	FALSE
31	N00010391642	SEVAN05	FRN Sevan Marine 07/12 CALL	41206&FRN Sevan Marine 07/12 CALL_11.15	11.12.2007	103.75	24.10.2012	24.10.2007	11.15	65	TRUE
32	N00010391642	SEVAN05	FRN Sevan Marine 07/12 CALL	41206&FRN Sevan Marine 07/12 CALL_11.15	13.12.2007	104	24.10.2012	24.10.2007	11.15	65	TRUE
33	N00010391642	SEVAN05	FRN Sevan Marine 07/12 CALL	41206&FRN Sevan Marine 07/12 CALL_11.15	18.12.2007	104.5	24.10.2012	24.10.2007	11.15	65	TRUE
34	N00010391642	SEVAN05	FRN Sevan Marine 07/12 CALL	41206&FRN Sevan Marine 07/12 CALL_11.15	07.01.2008	103.12	24.10.2012	24.10.2007	11.15	65	TRUE
35	N00010391642	SEVAN05	FRN Sevan Marine 07/12 CALL	41206&FRN Sevan Marine 07/12 CALL_11.15	11.01.2008	103.85	24.10.2012	24.10.2007	11.15	65	TRUE
36	N00010391642	SEVAN05	FRN Sevan Marine 07/12 CALL	41206&FRN Sevan Marine 07/12 CALL_11.15	18.01.2008	103.125	24.10.2012	24.10.2007	11.15	65	TRUE
37	N00010391642	SEVAN05	FRN Sevan Marine 07/12 CALL	41206&FRN Sevan Marine 07/12 CALL_11.15	21.01.2008	101.125	24.10.2012	24.10.2007	11.15	65	TRUE
38	N00010391642	SEVAN05	FRN Sevan Marine 07/12 CALL	41206&FRN Sevan Marine 07/12 CALL_11.15	22.01.2008	100.375	24.10.2012	24.10.2007	11.15	65	TRUE
39	N00010391642	SEVAN05	FRN Sevan Marine 07/12 CALL	41206&FRN Sevan Marine 07/12 CALL_11.15	31.01.2008	100.75	24.10.2012	24.10.2007	11.15	65	TRUE
40	N00010391642	SEVAN05	FRN Sevan Marine 07/12 CALL	41206&FRN Sevan Marine 07/12 CALL_11.15	01.02.2008	100.75	24.10.2012	24.10.2007	11.15	65	TRUE
41	N00010391642	SEVAN05	FRN Sevan Marine 07/12 CALL	41206&FRN Sevan Marine 07/12 CALL_11.15	06.02.2008	100.5	24.10.2012	24.10.2007	11.15	65	TRUE
42	N00010391642	SEVAN05	FRN Sevan Marine 07/12 CALL	41206&FRN Sevan Marine 07/12 CALL_11.15	05.03.2008	102.2	24.10.2012	24.10.2007	11.15	65	TRUE
43	N00010391642	SEVAN05	FRN Sevan Marine 07/12 CALL	41206&FRN Sevan Marine 07/12 CALL_11.15	10.03.2008	102	24.10.2012	24.10.2007	11.15	65	TRUE

Figure 5 Closing prices raw data from Excel (Oslo Stock Exchange, 2011).

#	A	B	C	D	E	F
1						
2						
3						
4	Observations	(Multiple Items)				
5	Call/Put/AIO	FALSE				
6						
7	Sum of Closing price	Column Labels				
8	Row Labels	387918,0% Statskasevsel 15.03.06_0	388998,0% Statskasevsel 21.06.06_0	389980,0% Statskasevsel 20.09.06_0	390418,01% Telenor cert. 22.11.06_1.01	390668,6,05% Spareb. NOR 98/06_6.05_390711
9	02.01.2006	99.575		98.204	100.0253	102.84
10	03.01.2006	99.5775				
11	04.01.2006	99.5825	98.925			
12	05.01.2006	99.6125	98.95	98.25		
13	06.01.2006	99.61		98.24		
14	09.01.2006	99.614	98.945			
15	10.01.2006	99.6275		98.28		
16	11.01.2006					
17	12.01.2006		99.001			
18	13.01.2006	99.6525	99			102.859
19	16.01.2006					
20	17.01.2006	99.6675	99.015			102.86
21	18.01.2006	99.67	99.025	98.3629	100.1691	
22	19.01.2006	99.688	99.0225			
23	20.01.2006					
24	23.01.2006	99.692				
25	24.01.2006	99.706	99.06	98.4254		102.8
26	25.01.2006					102.82
27	26.01.2006	99.72	99.0725	98.41		
28	27.01.2006	99.736	99.09			
29	30.01.2006	99.7312	99.085	98.43		
30	31.01.2006	99.741				
31	01.02.2006	99.754				
32	02.02.2006	99.77	99.115	98.475		
33	03.02.2006	99.7785	99.14			
34	06.02.2006	99.779				102.6
35	07.02.2006					
36	08.02.2006	99.7887	99.155	98.49	100.123	
37	09.02.2006					
38	10.02.2006			98.5524	99.9819	100.1884
39	13.02.2006	99.823	99.1815			
40	14.02.2006	99.828	99.193			
41	15.02.2006	99.839	99.19			
42	16.02.2006	99.8535	99.21			
43	17.02.2006					102.61
44						102.56

Figure 6 PivotTable in Excel

On the left is the shell of the PivotTable; this is where it will be displayed after it is defined. On the right is the *PivotTable Field List*, and it is here you define what data will be in the PivotTable and how it will be arranged (Aitken, 2007).

Date is displayed in the Row Labels section of the *PivotTable Field List*, and the PivotTable itself changes to display the dates — in column A.

Name_Coupon category is displayed in the Column Label section of the PivotTable Field List, and the PivotTable itself changes to display Name_Coupon- in row 8 and column B and onwards.

In the *Report Filter* section of the *PivotTable Field List*, we applied the filtering capability in the PivotTable. Observations filter is set to 25 – only bonds with more than 25 observations is chosen. Call/Put/AIO filter is set to *False* – bonds with call or put features and equity-linked bonds are excluded.

Finally, Sum of closing price is displayed in the Values box of the PivotTable Field List, and the PivotTable itself changes to display the sum of closing prices.

5.5.2 Processing the data

To calculate the daily return of each bond, the algorithm has to know if the corresponding date is within the bonds maturity. This is done by creating a sheet "Traded" (see figure 7) that contains trading Dates in the rows and Name_Coupon in columns and a value *True* if it is within the maturity of the bond and a value *False* if it is before the bond issue or after the bond is redeemed.

		40192&FRN Storebrand Bank 05/10__2.18	40212&FRN Spb. Møre 05/10__2.02	40212&FRN Spb. Vest 05/10__2.01	40224&FRN Landkreditt 05/10__2.04	40226&FRN Hafslund 05/10__0	40239&FRN Akker 05/10__5.689999999999999	40246&FRN Prosafe 05/10__3.11	40286&3.80% Spb. Hedmark 05/10__3.8	40294&3.75% Sparebanken Pluss 05/10__3.75
38719	02.01.2006	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
38720	03.01.2006	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
38721	04.01.2006	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
38722	05.01.2006	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
38723	06.01.2006	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
38726	09.01.2006	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
38727	10.01.2006	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
38728	11.01.2006	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
38729	12.01.2006	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
38730	13.01.2006	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
38733	16.01.2006	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
38734	17.01.2006	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
38735	18.01.2006	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
38736	19.01.2006	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
38737	20.01.2006	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
38740	23.01.2006	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
38741	24.01.2006	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
38742	25.01.2006	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
38743	26.01.2006	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
38744	27.01.2006	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
38747	30.01.2006	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
38748	31.01.2006	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
38749	01.02.2006	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
38750	02.02.2006	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE

Figure 7 The sheet "Traded".

Next step is to create a sheet “Last Traded”(see figure 8) which contains trading dates in the rows and Name_Coupon in columns and last closing price to the corresponding date and Name_Coupon. Since the PivotTable sheet contains closing price only when it has been a real trade, we have to adjust so if the bond has been traded for NOK 100.12 one day and it is three days to next trade, NOK 100.12 will be the last traded price for day two and three.

	Sum of Sluttkurs					
		40192&FRN Storebrand Bank 05/10__2.18	40212&FRN Spb. Møre 05/10__2.02	40212&FRN Spb. Vest 05/10__2.01	40224&FRN Landkredit 05/10__2.04	40226&FRN Hafslund 05/10__0
	first traded					
38719	02.01.2006	-	-	-	100.1200	-
38720	03.01.2006	-	-	-	100.1200	104.2600
38721	04.01.2006	-	-	-	100.1200	104.2600
38722	05.01.2006	-	-	-	100.1660	100.7920
38723	06.01.2006	-	-	-	100.1660	100.8330
38726	09.01.2006	-	-	-	100.1660	100.8330
38727	10.01.2006	-	-	-	100.1700	100.8330
38728	11.01.2006	-	-	-	100.1700	100.8330
38729	12.01.2006	-	-	-	100.1700	100.8330
38730	13.01.2006	-	-	100.2000	100.1623	100.8330
38733	16.01.2006	-	-	100.2000	100.1623	100.8330
38734	17.01.2006	-	-	100.2000	100.1623	100.8330
38735	18.01.2006	100.2700	-	100.2000	100.1623	100.8330
38736	19.01.2006	100.2700	100.1000	100.2000	100.1623	100.8330
38737	20.01.2006	100.2700	100.1000	100.2000	100.1623	100.8330
38740	23.01.2006	100.2700	100.1000	100.2000	100.1623	100.8330
38741	24.01.2006	100.2700	100.1000	100.2000	100.1623	100.8330
38742	25.01.2006	100.2700	100.1000	100.2000	100.1623	100.8330
38743	26.01.2006	100.2700	100.1000	100.2000	100.1623	100.8330
38744	27.01.2006	100.2700	100.1000	100.2000	100.1623	100.8330
38747	30.01.2006	100.2700	100.1000	100.2000	100.2540	100.8330
38748	31.01.2006	100.2700	100.1000	100.2000	100.2540	100.9400
38749	01.02.2006	100.2700	100.1000	100.2000	100.2540	100.9400
38750	02.02.2006	100.2700	100.2500	100.2000	100.2540	100.9400
38751	03.02.2006	100.2700	100.3000	100.2000	100.2540	100.9400
38754	06.02.2006	100.2700	100.3000	100.2000	100.2540	100.9400

Figure 8 The sheet “Last Traded”

Next step is to create a sheet “Return without coupons” (see figure 9) which contains trading dates in the rows and Name_Coupon in columns and percentage return from day to day to the corresponding date and Name_Coupon if the corresponding cell in the sheet “Traded” contains a *True* value. If the sheet ”Traded” contains a *False* value, then the return is 0.

		Sum of Sluttkurs	40192&FRN Storebrand Bank 05/10__2.18	40212&FRN Spb. Møre 05/10__2.02	40212&FRN Spb. Vest 05/10__2.01	40224&FRN Landkredit 05/10__2.04	40226&FRN Hafslund 05/10__0	40239&FRN Aker 05/10__5.689999999999999
38719	02.01.2006							
38720	03.01.2006	0.0000 %	0.0000 %	0.0000 %	0.0000 %	0.0000 %	0.0000 %	0.0000 %
38721	04.01.2006	0.0000 %	0.0000 %	0.0000 %	0.0000 %	0.0000 %	0.0000 %	0.0000 %
38722	05.01.2006	0.0000 %	0.0000 %	0.0000 %	0.0459 %	0.0000 %	0.0000 %	0.0000 %
38723	06.01.2006	0.0000 %	0.0000 %	0.0000 %	0.0000 %	0.0407 %	0.0000 %	0.0000 %
38726	09.01.2006	0.0000 %	0.0000 %	0.0000 %	0.0000 %	0.0000 %	0.0000 %	0.0000 %
38727	10.01.2006	0.0000 %	0.0000 %	0.0000 %	0.0040 %	0.0000 %	0.0000 %	0.0000 %
38728	11.01.2006	0.0000 %	0.0000 %	0.0000 %	0.0000 %	0.0000 %	0.0000 %	0.3261 %
38729	12.01.2006	0.0000 %	0.0000 %	0.0000 %	0.0000 %	0.0000 %	0.0000 %	-0.0956 %
38730	13.01.2006	0.0000 %	0.0000 %	0.0000 %	-0.0077 %	0.0000 %	0.0000 %	0.0000 %
38733	16.01.2006	0.0000 %	0.0000 %	0.0000 %	0.0000 %	0.0000 %	0.0000 %	-0.0287 %
38734	17.01.2006	0.0000 %	0.0000 %	0.0000 %	0.0000 %	0.0000 %	0.0000 %	0.0000 %
38735	18.01.2006	0.0000 %	0.0000 %	0.0000 %	0.0000 %	0.0000 %	0.0000 %	-0.3350 %
38736	19.01.2006	0.0000 %	0.0000 %	0.0000 %	0.0000 %	0.0000 %	0.0000 %	0.0000 %
38737	20.01.2006	0.0000 %	0.0000 %	0.0000 %	0.0000 %	0.0000 %	0.0000 %	0.8452 %
38740	23.01.2006	0.0000 %	0.0000 %	0.0000 %	0.0000 %	0.0000 %	0.0000 %	0.0000 %
38741	24.01.2006	0.0000 %	0.0000 %	0.0000 %	0.0000 %	0.0000 %	0.0000 %	-0.4286 %
38742	25.01.2006	0.0000 %	0.0000 %	0.0000 %	0.0000 %	0.0000 %	0.0000 %	0.0000 %
38743	26.01.2006	0.0000 %	0.0000 %	0.0000 %	0.0000 %	0.0000 %	0.0000 %	0.0000 %
38744	27.01.2006	0.0000 %	0.0000 %	0.0000 %	0.0000 %	0.0000 %	0.0000 %	-0.2391 %

Figure 9 The sheet “Return without coupons”

To calculate the daily return of each bond, we have to add the daily coupon to the percentage change in price from day to day. This means that we have to make a sheet “Traded1” (see figure 10) that is almost the same as the sheet “Traded” except that this sheet contains not just the traded dates, but the weekend dates as well, as the coupon is calculated on all days.

		40192&FRN Storebrand Bank 05/10 __2.18	40212&FRN Spb. Møre 05/10 __2.02	40212&FRN Spb. Vest 05/10 __2.01	40224&FRN Landkredit 05/10 __2.04	40226&FRN Hafslund 05/10 __0	40239&FRN Aker 05/10 __5.6899999999999999	40246&FRN Prosafe 05/10 __3.11	40288&3.80% Spb. Hedmark 05/10 __3.8	40294&3.75% Sparebanken Pluss 05/10 __3.75
38719	02.01.2006	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
38720	03.01.2006	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
38721	04.01.2006	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
38722	05.01.2006	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
38723	06.01.2006	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
38724	07.01.2006	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
38725	08.01.2006	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
38726	09.01.2006	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
38727	10.01.2006	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
38728	11.01.2006	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
38729	12.01.2006	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
38730	13.01.2006	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
38731	14.01.2006	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
38732	15.01.2006	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
38733	16.01.2006	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
38734	17.01.2006	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
38735	18.01.2006	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
38736	19.01.2006	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
38737	20.01.2006	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
38738	21.01.2006	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
38739	22.01.2006	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
38740	23.01.2006	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE

Figure 10 The sheet ”Traded1”

parentheses after a word in the *dim* statement define a dynamic array and its size is set when the procedure runs. An *UBound* functions is used to determine the highest index values for the array. To process all the data values they are assigned to a VBA array by applying a *Datav=Data.Value* function. Next a *ReDim* statement uses variables to define the required size of the array at run time. Next the *Filter* function is used (Described in an own sections below).When running a sub procedure from another procedure, you can use the *Call* statement(Bullen, et al., 2007). In this case the *Call* statement calculates the covariance, mean and also which data that will be passed to subsequent calculations. Then if there is a risk free rate in the *Function Arguments*, the algorithm chooses between two different function depending if the *ShortSaleOK Function Arguments* contains a *True* value or not. If *False* it runs the function *OptPortShortRF* and if *True* it runs *OptPortNoShortRF*. If there is not a risk free rate in the *Function Arguments*, the algorithm chooses between two different function depending if the *ShortSaleOK Function Arguments* contains a *True* value or not. If *False* it runs the function *OptPortShort* and if *True* it runs *OptPortNoShort*. We will however not use the functions without a risk free rate.

```
Function OptPort(Data, Traded, Optional RiskParam As Double = 1#, Optional RiskFree As Boolean = True, Optional ShortSaleOK As Boolean = False)
    Dim Cov, Mean, Datav(), Tradedv(), a(), tmp(), k As Long, i As Long |

    Datav = Data.Value
    Tradedv = Traded.Value

    Datav = Filter(Datav, Tradedv)

    n = UBound(Datav, 1) 'UBound(Datav,1)The 1 means the first dimension (rows or records).
    k = UBound(Datav, 2) 'Similarly, UBound(Datav,2) for the second dimension(fields or columns) .
    ReDim a(1 To k, 1 To 1)
    Call CalcMoments(Datav, Cov, Mean)
    If RiskFree Then
        If ShortSaleOK Then
            tmp = OptPortShortRF(Cov, Mean, RiskFree)
        Else
            tmp = OptPortNoShortRF(Cov, Mean, RiskFree)
        End If
    Else
        If ShortSaleOK Then
            tmp = OptPortShort(Cov, Mean, RiskParam)
        Else
            tmp = OptPortNoShort(Cov, Mean, RiskParam)
        End If
    End If
    For i = 1 To k
        a(i, 1) = tmp(i, 1)
    Next i
    OptPort = a
End Function
```

Figure 12 VBA snapshot of the main function OptPort(Sirnes, 2011)

Function Arguments

The *Data* field in the Function Argument window is where the return data matrix is put, which is used to calculate the variance-covariance matrix and the mean.

The *Traded* field in the Function Argument window is where the Traded1 matrix is put, which passes the data to the *Filter* function.

The *RiskParam* field in the Function Argument window is where the investor can choose his risk preference. The risk parameter is set to 1, which means that the risk parameter does not affect our calculation since multiplying a number with one is the same.

The *RiskFree* field in the Function Argument window is where the risk free rate is put, which is used to calculate our portfolios.

The *ShortSaleOK* field in the Function Argument window is where you choose if shortsale is allowed, this is set to *False* as default and shortsale is not allowed. If you put the value *True* here, shortsale is allowed.

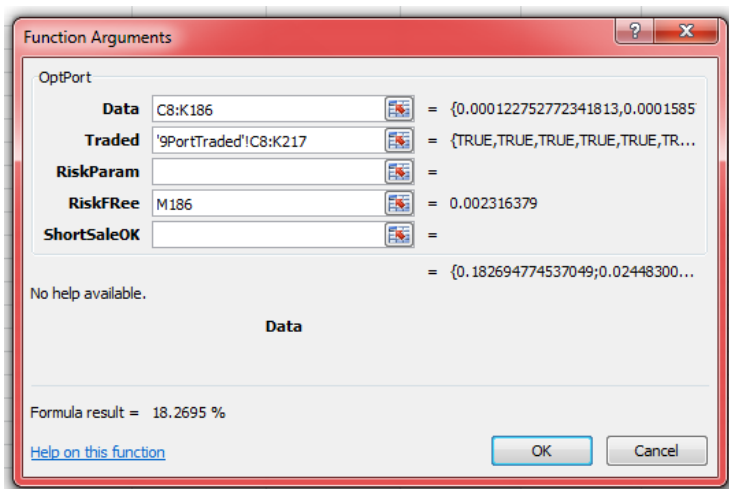


Figure 13 Function Argument window in Excel

Filter function

We had to tell the algorithm what data to use by creating a *For...Next* loop that is used to process all the objects in our collection and the loop execute until it exceeds a predefined value, or an upper bound in our case(Bullen, et al., 2007).Again we refer to the sheet “Trading1” and if the column or row in the collection contains a value *True*, we use the data, and if it contains *False* the bond is outside the trading period of the bond and hence not used. Then a new matrix is created that just contains the return to traded bonds.

```

Function Filter(Data(), Traded())
    Dim i As Long, j As Long, UseColumn() As Boolean, n As Long, NewData()

    ReDim UseColumn(1 To UBound(Traded, 2))

    For j = 1 To UBound(Traded, 2)
        UseColumn(j) = True
        For i = 1 To UBound(Traded, 1)
            If Traded(i, j) = False Then
                UseColumn(j) = False
                Exit For
            End If
        Next i
        If UseColumn(j) Then n = n + 1
    Next j

    ReDim NewData(1 To UBound(Data), 1 To n)
    n = 0
    For j = 1 To UBound(Data, 2)
        If UseColumn(j) Then
            n = n + 1
            For i = 1 To UBound(Data, 1)
                NewData(i, n) = Data(i, j)
            Next i
        End If
    Next j

    Filter = NewData
End Function

```

Figure 14 VBA snapshot of the Filter function (Sirnes, 2011)

CalcMoments sub function

The new matrix from the *Filter* function is used to calculate the mean return for each bond in the prevailing period by using a *For...Next* loop. Then we used *mmult* worksheet functions in Excel to calculate the covariance and mean from the matrix (Jackson & Staunton, 2001), by using a *For...Next* loop.

```

Sub CalcMoments(Data, ByRef Covar, ByRef Mean)
    Dim n As Long, k As Long, retMatr(), i As Long, j As Long, QMean
    n = UBound(Data)
    k = UBound(Data, 2)
    ReDim Covar(1 To k, 1 To k)
    ReDim Mean(1 To k, 1 To 1)
    For i = 1 To k
        For j = 1 To n
            Mean(i, 1) = Mean(i, 1) + Data(j, i)
        Next j
        Mean(i, 1) = Mean(i, 1) / n
    Next i

    With WorksheetFunction
        Covar = .MMult(.Transpose(Data), Data)
        QMean = .MMult(Mean, .Transpose(Mean))
    End With
    For i = 1 To k
        For j = 1 To k
            Covar(i, j) = (Covar(i, j) - n * QMean(i, j)) / (n - 1)
        Next j
    Next i
End Sub

```

Figure 15 VBA snapshot of the CalcMoments function (Sirnes, 2011)

OptPortShortRF function

We want to optimize the portfolio by using a risk free rate, hence we use a *For...Next* loop to calculate the mean minus the risk free rate. Then we used the *mmult* worksheet functions in

Excel to multiply the inverse of the covariance matrix with the mean minus the risk free rate to optimize the portfolio. Further we use a *For...Next* loop to calculate the bond weights.

```

Function OptPortShortRF(Cov, Mean, RiskFree)
'portfolio optimization with risk free asset
'mean is excess return over risk free rate
Dim Covar(), Mn(), i As Integer, Port(), n, sum, returnt As Double, Md()
If IsObject(Cov) Then
    Covar = Cov.Value
Else
    Covar = Cov
End If
If IsObject(Mean) Then
    Mn = Mean.Value
Else
    Mn = Mean
End If
n = UBound(Mn)

For i = 1 To n 'subtract risk free rate before calculating optimum portfolio
    Mn(i, 1) = Mn(i, 1) - RiskFree
Next i
With WorksheetFunction
    If n < UBound(Mn, 2) Then
        Mn = .Transpose(Mn) 'Returns a vertical range of cells as a horizontal range, or vice versa
        n = UBound(Mn)
    End If
    Port = .MMult(.MInverse(Covar), Mn) 'markowitz optimum portfolio formula as a matrix
    sum = Abs(.sum(Port))
End With
For i = 1 To n
    Port(i, 1) = Port(i, 1) / sum
Next i

OptPortShortRF = Port
End Function

```

Figure 16 VBA snapshot of the OptPortShortRF function (Sirnes, 2011)

OptPortNoShortRF function

We want to optimize the portfolio by using a risk free rate, hence we use a *For...Next* loop to add the Lagrange multipliers for the shortsale constraints for the first order condition. It also calculates the mean minus the risk free rate. Then we used the *mmult* worksheet functions in Excel to multiply the inverse of the covariance matrix with the mean minus the risk free rate to optimize the portfolio. Further we use a *For...Next* loop to calculate the bond weights and also check if the shortsale constraint is obliged or if the portfolio weight is greater than zero.

```

Function OptPortNoShortRF(Cov, Mean, RiskFree)
'portfolio optimization with risk free asset and short sale constraints
Dim ParamMatr() As Double, i As Integer, j As Integer, r() As Double, n As Integer, MeanVar(), CovVar()
Dim sum As Double
On Error Resume Next
CovVar = Cov
MeanVar = Mean
n = UBound(CovVar, 1)
ReDim r(1 To 2 * n, 1 To 1)
ReDim ParamMatr(1 To 2 * n, 1 To 2 * n)
For i = 1 To n
    For j = 1 To n
        ParamMatr(i, j) = CovVar(i, j)
    Next j
    ParamMatr(i, n + i) = 1 'Adds the Lagrange multipliers for the Shortsale constraints (gamma) for the first order conditions.
    ParamMatr(n + i, n + i) = 1 'Adds that the shortsale constraints is obliging (gamma=0)
    r(i, 1) = MeanVar(i, 1) - RiskFree
Next i
With WorksheetFunction
    For i = 1 To n * 25
        Changed = False
        a = .MMult(.MInverse(ParamMatr), r)
        For j = 1 To n 'passes through all "a" values to check if the shortsale constraint is fulfilled
            If a(j, 1) < 0 Then 'if the shortsale constraints are fulfilled, the constraints is set as obliging
                ParamMatr(n + j, n + j) = 0 'gamma set free
                ParamMatr(n + j, j) = 1 'a=0
                Changed = True
            ElseIf a(n + j, 1) > 0 Then 'if gamma>0 then gamma=0 and the derivative is positive and indicate that a>0
                ParamMatr(n + j, n + j) = 1 'gamma=0
                ParamMatr(n + j, j) = 0 'a set free
                Changed = True
            End If
        Next j
        If Not Changed Then Exit For
    Next i
End With
For i = 1 To n
    sum = sum + a(i, 1)
Next i
For i = 1 To n
    a(i, 1) = a(i, 1) / Abs(sum)
Next i
OptPortNoShortRF = a
End Function

```

Figure 17 VBA snapshot of the OptPortNoShortRF function (Sirnes, 2011)

CalcCovar function

We want to calculate the covariance variance matrix, hence we use a *For...Next* loop to calculate the mean. Then we used *mmult* worksheet functions in Excel to calculate the covariance and mean from the matrix, by using a *For...Next* loop.

```

Function CalcCovar(Data As Range)
Dim n As Long, k As Long, retMatr(), i As Long, j As Long, QMean, Mean()
n = Data.Rows.Count
k = Data.Columns.Count
Datav = Data.Value
ReDim Covar(1 To k, 1 To k)
ReDim Mean(1 To k, 1 To 1)
For i = 1 To k
    For j = 1 To n
        Mean(i, 1) = Mean(i, 1) + Datav(j, i)
    Next j
    Mean(i, 1) = Mean(i, 1) / n
Next i
With WorksheetFunction
    Covar = .MMult(.Transpose(Datav), Datav)
    QMean = .MMult(Mean, .Transpose(Mean))
End With
For i = 1 To k
    For j = 1 To k
        Covar(i, j) = (Covar(i, j) - n * QMean(i, j)) / (n - 1)
    Next j
Next i
CalcCovar = Covar
End Function

```

Figure 18 VBA snapshot of the CalcCovar function (Sirnes, 2011)

5.5.4 Calculation of expected return, return, variance and standard deviation and the Sharpe ratio

When we have the portfolio weights we want to calculate the expected return, return, variance and standard deviation, the Sharpe ratio and, expected return and return when you uses the risk free rate instead of holding bonds when the risk free rate is greater than the expected return. Since we need to calculate all these data for each portfolio we wrote a macro in VBA to automate the calculations. This is what the macro was set up to do:

Step 1: Use the portfolio weights in percentages and label the percentage as "w". This represents the proportion of the portfolio that is in a particular bond.

Step 2a: Calculate the sample mean (average return for the individual bond) in the previous holding period.

Step 2b: Calculate the return (average return for the individual bond) in the holding period.

Step 3a: Label the sample mean as "mu".

Step 3b: Label the return as "re".

Step 5: Use the CalcCovar macro to calculate the variance-covariance matrix for the previous holding period and name the covariance matrix results "sigma". After you have entered that formula, you have to hit "shift, ctrl, and enter" at the same time because it is a special function in Excel.

Step 6a: Use this formula to calculate the "expected return": $=\text{mmult}(\text{transpose}(w),\text{mu})$. After you have entered that formula, you have to hit "shift, ctrl, and enter" at the same time because it is a special function in Excel (Benninga,2000).

Step 6b: Use this formula to calculate the "realized return": $=\text{mmult}(\text{transpose}(w),\text{re})$. After you have entered that formula, you have to hit "shift, ctrl, and enter" at the same time because it's a special function in Excel(Benninga, 2008).

Step 7: Use this formula to calculate the "Variance": $=\text{mmult}(\text{mmult}(\text{transpose}(w),\text{sigma}),w)$. Again, after you have entered that formula, you have to hit "shift, ctrl, and enter" at the same time because it is a special function in Excel(Benninga, 2008).

Step 8: Use the *sqr*t function in Excel on the Variance to get the Standard deviation for the portfolio(Benninga, 2008).

Step 9: Calculate the expected Sharpe ratio and realized Sharpe ratio.

Step 10a: Use the IF function in Excel to change the risk free rate with the expected return:
=if(rf>Expected return;rf;Expected return)

Step 10b: Use the IF function in Excel to change the risk free rate with the return:
=if(rf>return;rf;return)

5.6 Testing the level of significance

We are testing whether the realized Sharpe ratios are statistically different from the Sharpe ratio of the index. Only Sharpe ratios that is higher than the index is tested.

The equation below can be used to develop the confidence interval estimate for the difference between two population means.

$$(\bar{x}_1 - \bar{x}_2) \mp t s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

Where,

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}} = \text{pooled standard deviation}$$

t = critical t-value from the t-distribution, with degrees of freedom equal to $n_1 + n_2 - 2$

To use the first equation, we must compute the pooled standard deviation, s_p . We, therefore, use a weighted average of s_1^2 and s_2^2 , denoted as s_p^2 to estimate σ^2 , where the weights comes from the degrees of freedom associated with each sample.

We are using a t-distribution and find the critical t-value at 5% significance level. If the confidence interval crosses zero, it indicates that there is no difference. Hence, we keep the null hypothesis (Groebner, 2008).

We applied PHstat2 in Excel 2010 to test the level of significance (see Table 1).

Table 1 Testing the level of significance.

Testing the level of significance	
Data	
Hypothesized Difference	0
Level of Significance	0.05
Population 1 Sample	
Sample Size	42
Sample Mean	0.2157
Sample Standard Deviation	6.859
Population 2 Sample	
Sample Size	42
Sample Mean	0.1646
Sample Standard Deviation	0.3515
Intermediate Calculations	
Population 1 Sample Degrees of Freedom	41
Population 2 Sample Degrees of Freedom	41
Total Degrees of Freedom	82
Pooled Variance	23.58472
Difference in Sample Means	0.0511
t Test Statistic	0.048219
Two-Tail Test	
Lower Critical Value	-1.98932
Upper Critical Value	1.989319
p-Value	0.961659
Do not reject the null hypothesis	
Confidence Interval Estimate for the Difference Between Two Means	
Data	
Confidence Level	95%
Intermediate Calculations	
Degrees of Freedom	82
t Value	1.989319
Interval Half Width	2.108191

6 Summary of results and empirical investigation

In this section we present the results of four different models. The four models are all based on the Markowitz model and the maximum Sharpe ratio criteria, but have different constraints. The essential question concerning the success of different portfolios is whether the predicted risk-return profiles are attainable out of sample. Therefore, we compare predicted and realized results. The results are presented in tabular form by comparing the expected return with the actual return, the standard deviation with absolute deviation and the expected Sharpe ratio with the actual Sharpe ratio. We have presented the returns as monthly return with its belonging deviations. The Sharpe ratios are calculated as an average over all holding periods.

The first model is optimized with the constraint, shortsale not allowed. The model is first applied on all available bonds and then on corporate bonds only. In both cases we have constructed four portfolios which are adjusted monthly, quarterly, semi-annually and annually.

The second model is optimized without the shortsale constraint. The model is optimized on all bonds and then on corporate bonds and adjusted as the first model.

There were only two of the 16 portfolios in model one and two that produced a higher risk-adjusted return than the index. This was not very surprising as the model for some periods picked portfolios with expected returns for some period that was less than the risk free rate. On the basis of this weakness we constructed two replicas of model one and two where we hold the risk free asset in periods where the expected return is less than the risk free rate. Models three and four are presented in same terms as models one and two.

6.1 Model 1

All the tables below include the expected monthly returns with their belonging standard deviation and Sharpe ratios for the efficient portfolios. The tables also include the realized monthly return with its belonging deviations and Sharpe ratios. The realized values come from holding the efficient portfolio in the period subsequent to the optimization point. An asterisk indicates that the realized Sharpe ratio is higher than the index on a 5% significance level.

6.1.1 Shorts sale not allowed all bonds

Table 2 Shorts sale Not allowed all bonds

Shorts sale Not allowed all bonds			
	Expected return	Standard deviation	Expected Sharpe ratio
Montly adjusted	0.2994 %	0.0849 %	-0.0939
Quarterly adjusted	0.2993 %	0.0819 %	-0.0988
Semi-annually adjusted	0.3000 %	0.0846 %	-0.0864
Annually adjusted	0.2681 %	0.0847 %	-0.4629
Index	0.3238 %	0.2056 %	0.0800
Risk free	0.3073 %		
	Actual return	Actual deviation	Actual Sharpe Ratio
Montly adjusted	0.3040 %	0.1639 %	-0.0205
Quarterly adjusted	0.3146 %	0.1732 %	0.0420
Semi-annually adjusted	0.3275 %	0.1975 %	0.1022
Annually adjusted	0.2979 %	0.2450 %	-0.0386
Index	0.3652 %	0.3515 %	0.1646

We can see that the deviations between the expected and actual returns increase as the holding period increases. The realized risk-return relationship is significantly better than the expected

risk-return relationship. The portfolios are well diversified. However, only the quarterly and semi-annually adjusted portfolios generate returns greater than the risk free rate. The expected returns for all of the portfolios are less than the rate of return on the risk free asset. Thus, the risk-return profiles for these portfolios are not very attractive. Based on the expected returns, it is not very surprising that none of the portfolios produces a better risk-adjusted return than the index.

6.1.2 Shortsale not allowed Corporate Bonds

Table 3 Shortsale not allowed Corporate Bonds

Shortsale Not allowed Corporate Bonds			
	Expected return	Standard deviation	Sharpe ratio
Monthly adjusted	0.3803 %	0.2342 %	0.3115
Quarterly adjusted	0.4136 %	0.2761 %	0.3847
Semi-annually adjusted	0.3119 %	0.3291 %	0.0140
Annually adjusted	0.3523 %	0.5187 %	0.0867
Index	0.3238 %	0.2056 %	0.0800
Risk free	0.3073 %		
	Actual return	Actual deviation	Sharpe ratio
Monthly adjusted	0.2729 %	0.9225 %	-0.0373
Quarterly adjusted	0.3237 %	0.9999 %	0.0164
Semi-annually adjusted	0.2871 %	1.4400 %	-0.0141
Annually adjusted	0.3073 %	1.5498 %	-0.0000
Index	0.3652 %	0.3515 %	0.1646

In contrast to the portfolios chosen among all the bonds, the corporate bond portfolios have negative deviations from the expected returns. The actual deviations on the other hand are significantly higher. As in model one, the deviations increase with the holding period. The realized returns for all portfolios, except the quarterly adjusted are less than risk free rate. Thus, we end up with negative Sharpe ratios for some of the portfolios.

The portfolios are not well diversified for all holding periods due to the near singularity in the variance-covariance matrix. The portfolios consist of single bonds for some of the holding periods which give the portfolios excess risk. None of the portfolios produces a higher risk-adjusted return than the index.

6.2 Model 2

6.2.1 Shortsale allowed all bonds

Table 4 Shortsale allowed all bonds

Shortsale allowed all bonds			
	Expected return	Standard deviation	Expected Sharpe ratio
Montly adjusted	0.2977 %	0.0849 %	-0.1133
Quarterly adjusted	0.2972 %	0.0814 %	-0.1246
Semi-annually adjusted	0.2984 %	0.0829 %	-0.1074
Annually adjusted	0.2650 %	0.0793 %	-0.5333
Index	0.3238 %	0.2056 %	0.0800
Risk free	0.3073 %		
	Actual return	Actual deviation	Actual Sharpe Ratio
Montly adjusted	0.3040 %	0.1559 %	-0.0214
Quarterly adjusted	0.3147 %	0.1647 %	0.0446
Semi-annually adjusted	0.3308 %	0.1888 %	0.1242
Annually adjusted	0.2943 %	0.2431 %	-0.0537
Index	0.3652 %	0.3515 %	0.1646

The table shows that the actual returns are higher than the expected returns for all portfolios. The realized deviations are also higher than the standard deviation and the deviations increase with the holding period. The actual Sharpe ratios for the portfolios are significantly higher than the expected Sharpe ratios and all portfolios are well diversified. However, they are no way near the Sharpe ratio of the index. The portfolios are well diversified, but none of them generates a better risk adjusted return than the index.

6.2.2 Shortsale allowed corporate bonds

Table 5 Shortsale allowed corporate bonds

Shortsale allowed Corporate Bonds			
	Expected return	Standard deviation	Sharpe ratio
Montly adjusted	0.6533 %	0.4570 %	0.7570
Quarterly adjusted	0.6155 %	0.8054 %	0.3826
Semi-annually adjusted	1.0058 %	1.6603 %	0.4207
Annually adjusted	1.0594 %	1.6912 %	0.4447
Index	0.3238 %	0.2056 %	0.0800
Risk free	0.3073 %		
	Actual return	Actual deviation	Sharpe ratio
Montly adjusted	1.7052 %	6.8773 %	0.2033
Quarterly adjusted	1.1854 %	6.7902 %	0.1293
Semi-annually adjusted	1.4570 %	7.1884 %	0.1599
Annually adjusted	1.2773 %	4.9374 %	0.1965
Index	0.3652 %	0.3515 %	0.1646

A striking point to note is that the realized returns of the portfolio strongly deviate from the predictions with respect to both the expected return and the volatility. The near singularity of the variance-covariance matrix leads to rather extreme portfolios with a high volume of shortsales. The monthly and annually adjusted portfolios produce a higher risk-adjusted return than the index, even though the actual Sharpe ratios are significantly lower than the expected Sharpe ratios. However, the results are not statistically significant at a 5% significance level.

6.3 Model 3

6.3.1 Shortsale not allowed on all bonds with risk-free asset when the expected return is less than the risk free rate.

Table 6 Shortsale not allowed on all bonds with risk-free asset when the expected return is less than the risk free rate.

Shortsale not allowed all bonds - RF			
	Expected return	Standard deviation	Expected Sharpe ratio
Monthly adjusted	0.3864 %	0.0680 %	1.1625
Quarterly adjusted	0.3851 %	0.0670 %	1.1604
Semi-annually adjusted	0.3858 %	0.0700 %	1.1206
Annually adjusted	0.3562 %	0.0978 %	0.5000
Index	0.3238 %	0.2056 %	0.0800
Risk free	0.3073 %		
	Actual return	Actual deviation	Actual Sharpe Ratio
Monthly adjusted	0.3832 %	0.1163 %	0.6526 *
Quarterly adjusted	0.3885 %	0.1327 %	0.6114 *
Semi-annually adjusted	0.4030 %	0.1581 %	0.6051 *
Annually adjusted	0.3691 %	0.1746 %	0.3534 *
Index	0.3652 %	0.3515 %	0.1646

The actual return for the portfolios falls close to the expectations. The realized deviations are roughly twice the expected standard deviations. Thus, the Sharpe ratios are lower than the expectations. However, all of the portfolios produce Sharpe ratios significantly higher than the index. All portfolios are well diversified in all periods. By comparing the results with the results from model one, we can see major improvements in performance. The realized returns are higher and the realized deviations are lower. This is no surprise as periods with expected returns less than the risk free rate are replaced with the risk free asset. This generates both higher returns and lower risk since the risk free asset has a standard deviation of zero. The asterisks indicate that results are statistically significant at a 5 % significance level.

6.3.2 Shortsale not allowed corporate bonds with risk-free asset when the expected return is less than the risk free rate.

Table 7 Shortsale not allowed corporate bonds with risk-free asset when the expected return is less than the risk free rate.

Shortsale Not allowed - Corporate Bonds - RF			
	Expected return	Standard deviation	Expected Sharpe ratio
Montly adjusted	0.4597 %	0.1823 %	0.8353
Quarterly adjusted	0.5014 %	0.1982 %	0.9790
Semi-annually adjusted	0.4673 %	0.1406 %	1.1379
Annually adjusted	0.5528 %	0.3577 %	0.6863
Index	0.3238 %	0.2056 %	0.0800
Risk free	0.3073 %		
	Actual return	Actual deviation	Actual Sharpe Ratio
Montly adjusted	0.3671 %	0.9136 %	0.0654
Quarterly adjusted	0.3816 %	0.9869 %	0.0752
Semi-annually adjusted	0.2257 %	1.1749 %	-0.0695
Annually adjusted	0.2646 %	1.3129 %	-0.0326
Index	0.3652 %	0.3515 %	0.1646

The monthly and quarterly adjusted portfolios produces both higher expected and actual returns than the corresponding portfolios from model one. The volatilities are approximately the same as in model one. Thus, the risk-return relationships are better than in model one. However, they are far below the index. The problem with the near singularity in the variance-covariance matrix is still present and we end up with undiversified portfolios for some of the holding periods.

The semi-annually and annually adjusted portfolios perform worse than the corresponding portfolios in model one. Both Sharpe ratios are below the corresponding Sharpe ratios from model one. This is surprising given the fact that we choose the risk free asset for periods with expected returns less than the risk free rate.

Again, the near singularity in the variance-covariance matrix leads to holding periods with single bonds. Thus, the portfolios are undiversified for some of the holding periods.

6.4 Model 4

6.4.1 Shortsale allowed all bonds with risk-free asset when the expected return is less than the risk free rate.

Table 8 Shortsale allowed all bonds with risk-free asset when the expected return is less than the risk free rate.

Shortsale allowed all bonds - RF			
	Expected return	Standard deviation	Expected Sharpe ratio
Montly adjusted	0.3859 %	0.0680 %	1.1554
Quarterly adjusted	0.3842 %	0.0669 %	1.1493
Semi-annually adjusted	0.3852 %	0.0683 %	1.1389
Annually adjusted	0.3540 %	0.0954 %	0.4884
Index	0.3238 %	0.2056 %	0.0800
Risk free	0.3073 %		
	Actual return	Actual deviation	Actual Sharpe Ratio
Montly adjusted	0.3845 %	0.1034 %	0.7463 *
Quarterly adjusted	0.3884 %	0.1226 %	0.6609 *
Semi-annually adjusted	0.4065 %	0.1459 %	0.6794 *
Annually adjusted	0.3657 %	0.1735 %	0.3362 *
Index	0.3652 %	0.3515 %	0.1646

The portfolios produce an actual return close to the expected return. The actual deviations are slightly higher than the standard deviations. The realized Sharpe ratios are below the expected Sharpe ratios. However, the realized Sharpe ratios give a very attractive reward-to-volatility relationship. Moreover, the reward-to-volatility on the portfolios is far above what the index achieve. The asterisks indicate that the results are statistically significant at a 5 % significance level.

6.4.2 Shortsale allowed corporate bonds with risk-free asset when the expected return is less than the risk free rate.

Table 9 Shortsale allowed corporate bonds with risk-free asset when the expected return is less than the risk free rate.

Shortsale allowed - Corporate Bonds - RF			
	Expected return	Standard deviation	Expected Sharpe ratio
Montly adjusted	0.7148 %	0.3972 %	1.0259
Quarterly adjusted	0.7343 %	0.7128 %	0.5989
Semi-annually adjusted	1.1441 %	1.5290 %	0.5473
Annually adjusted	1.2208 %	1.5675 %	0.5827
Index	0.3238 %	0.2056 %	0.0800
Risk free	0.3073 %		
	Actual return	Actual deviation	Actual Sharpe Ratio
Montly adjusted	1.7867 %	6.8590 %	0.2157
Quarterly adjusted	1.1843 %	6.7613 %	0.1297
Semi-annually adjusted	0.9612 %	6.1908 %	0.1056
Annually adjusted	1.2231 %	4.8939 %	0.1871
Index	0.3652 %	0.3515 %	0.1646

As in the corresponding portfolios from model two, the realized returns strongly deviates from the predictions with respect to both expected returns and the volatility. The near singularity of the variance-covariance matrix leads to rather extreme portfolios with a high volume of shortsales. Thus, the portfolios are not well diversified for all holding periods. The monthly and annually adjusted portfolios produce a higher risk-return profile than the index even though actual Sharpe ratios are significantly lower than the expected Sharpe ratios. However, the results are not statistically different from the risk-return relationship of the index.

6.5 Summery of results

The results from models one and two show generally poor portfolio performance although some of the portfolios provide an ex ante performance above the index. Only two of the 18 portfolios generate a better risk-return relationship than the index. However, the results are not statistically different from the index at a 5 % significance level. The performances of the portfolios are not very robust out of sample, as we see large deviations between the expected and actual performance.

Model 3 and 4 generates significantly better results than model 1 and 2 as we replaced every holding period with expected return below the risk free rate with the risk free asset. The portfolios chosen among all bonds generate a better risk-return relationship than the index. The results are statistically significant at a 5 % significance level. We can see that we have small deviations between predicted and realized values. The high accuracy of the predictions not only refers to the Sharpe ratios. Also values for expected return and volatility, that determine the Sharpe ratio, are accurately predicted.

The portfolios chosen among the corporate bonds show a very promising ex ante risk-return relationships. However, it is not very surprising that their performance is not robust out of sample. As the near singularity leads to rather extreme portfolios with single bonds in model 3 and large short positions in model 4. We find a clear pattern in the deviations between the expected and the realized values. The deviations increase with the holding period of the portfolios. This is not surprising as the parameters change over time. The further we move away from the optimization point, the less reliable the predictions are.

A well-known problem with the Markowitz model is that it tends to choose relatively large positions in a small number of bonds, which leads to rather undiversified portfolios. This problem is also present in our study and is very clear in the case of corporate bond portfolios. The model chooses a single bond for some holding periods and extreme short positions for some holding periods. The problems with singularity in the variance-covariance matrix were also present in the case of all bonds. We coped with the problems by dividing the bonds into parts based on their correlation characteristics. This enabled us to find well diversified efficient portfolios.

7 Conclusion

The research problem of this paper is whether an active bond trading strategy based on the Markowitz model generates higher risk-adjusted return than an equally weighted index.

The results regarding the usefulness of this trading strategy are a bit inconsistent. This implies that no reliable conclusions regarding the potential abilities of the trading strategy can be made. The results from model one and two are generally poor. None of the portfolios from these models produce a better risk-return relationship than the index, although some of the portfolios have very promising ex ante returns. The results from model three and four are significantly better. The portfolios chosen among all bonds produce risk-adjusted return above

the index. The results are statistically significant at a 5 % significance level. The corporate bond portfolios have very attractive ex ante returns, but their performance out of sample is not very good. The main reason for this problem is the near singularity in the variance-covariance matrix, which leads to rather extreme portfolios of single assets and large short positions.

As mentioned earlier, the Markowitz model has some shortcomings when it is applied on bond portfolios. Without weight restrictions, the near singularity in the variance-covariance matrix cause the model to choose large amounts in few bonds, leading the portfolios to be undiversified. Furthermore, factors like duration, interest rate changes, inflation and bond rating are not incorporated in the model. The only factor measuring risk in this model is price fluctuations. Strategies like interest rate anticipation, credit analysis, valuation analysis and yield spread analysis considers the fundamentals of bonds. However, these factors are difficult to predict and such strategies can be a bit cumbersome and costly.

We find that this strategy could be useful if we were able to incorporate fundamentals like rating, duration and inflation and were able to impose restrictions with to regard to the weights in each bond. It is plausible that this strategy is more cost efficient than other bond strategies as you do not need a troop of analysts to make the investment choices.

Finally, some concerns about the results are also justified. In the process of excluding trading costs, such as bid-ask spreads and commissions, the results will of course be biased upwards.

8 LIST OF REFERENCES

- Aitken, P. G. (2007). *Excel® 2007 PivotTables and PivotCharts*. Indianapolis, USA: Wiley Publishing.
- Appel, M. (Ed.). (2010). *Higher Returns from Safe Investments: Using Bonds, Stocks and Options to Generate Lifetime Income*. New Jersey, USA: FT Press.
- Artzner, P., Delbaen, F., Ebner, J. M., & Heath, D. (1999). Coherent measures of risk. *Mathematical Finance*, 9, 203-228.
- Association for Financial Markets in Europe, A. (2011). AFME Investing in bonds Europe. Retrieved 17 January 2011, from <http://www.investinginbondseurope.org/>
- Bansal, R., Dahlquist, M., & Harvey, C. R. (2004). Dynamic Trading Strategies and Portfolio Choice. *SIFR Research Report Series 31*.
- Benninga, S. (2008). *Financial modeling* (3rd ed.). Cambridge, MA: MIT Press.
- Bodie, Z., Kane, A., & Marcus, A. J. (2009). *Investments* (8th ed.). New York, USA: McGraw-Hill.
- Borch, K. H. (1969). A note on Uncertainty and Indifference Curves. *The Review of Economic Studies*, 36(1), 1-4.
- Bullen, S., Green, J., Bovey, R., & Alexander, M. (2007). *Excel® 2007 VBA Programmer's Reference*. Indianapolis, USA: Wiley Publishing.
- Choudhry, M. (2005). *Fixed Income Securities and Derivatives Handbook: Analysis and Valuation*. Princeton, USA: Bloomberg Press.
- Darwin, C. (1968). *The origin of the species by means of natural selection; or, The preservation of favoured races in the struggle for life*: Penguin.
- Defusco, R., McLeavey, D. W., & Pinto, J. E. (2001). *Quantitative Methods for Investment Analysis*. Baltimore, USA: United Book Press.
- Elton, E. J., Gruber, M. J., Brown, S. J., & Goetzmann, W. N. (2007). *Modern Portfolio Theory and Investment Analysis* (7th ed.). New Jersey, USA: John Wiley & Sons.
- Exley, J., Mehta, S., & Smith, A. (2004). Mean Reversion. Retrieved from <http://www.actuaries.org.uk/research-and-resources/documents/mean-reversion>
- Fabozzi, F. J. (2010). *Bond markets, Analysis and Strategies* (7th ed.). New Jersey, USA: Prentice Hall.
- Feldstein, M. (1969). Mean-Variance Analysis in the Theory of Liquidity Preference and Portfolio Selection. *The Review of Economic Studies*, 36(1), 5-12.
- Fisher, K. L., & Statman, M. (1997). Investment Advice from Mutual Fund Companies. Closer to the Talmud than to Markowitz. *The Journal of Portfolio Management*, 24(1), 9-25.
- Gilovich, T., Griffin, D., & Kahneman, D. (2002). *Heuristics and Biases: The Psychology of Intuitive Judgment*. London, UK: Cambridge University Press.
- Government.no. (2011). The Norwegian Bondmarket (Norwegian site). Retrieved 17 January, 2011, from <http://www.regjeringen.no/nb/dep/nhd/dok/nou-er/2001/nou-2001-29/10/3/2.html?id=365632>
- Green, R. C., & Hollifield, B. (1992). When will mean-variance efficient portfolios be well diversified? *Journal of Finance*, 47(5), 1785-1809.
- Gripenvik, H. (2006). Portfolio optimization, A trading strategy based on time varying volatility. Retrieved from <http://www.essays.se/essay/ffbda6ef10/>
- Groebner, D. F. (2008). *Business statistics : a decision-making approach* (7th ed.). Upper Saddle River, N.J.: Pearson/Prentice Hall.
- Gropp, J. (2004). Mean reversion of industry stock returns in the U.S. *Journal of Empirical Finance*, 11(4), 537-551.

- Hawking, S. (1998). *A Brief History of Time*. New York City, USA: Bantam Dell Publishing Group.
- Hellmich, M., & Kassberger, S. (2009). Efficient and robust portfolio optimization in the multivariate Generalized Hyperbolic framework. Retrieved from http://papers.ssrn.com/sol3/Delivery.cfm/SSRN_ID1018241_code739640.pdf?abstractid=1018241&mirid=1
- Hill, R. C., Griffiths, W. E., & Lim, G. C. (2007). *Principles of Econometrics* (3rd ed.). Sydney, Australia: Wiley.
- Investopedia. (2003). Efficient frontier. Retrieved 20 January, 2011, from <http://www.investopedia.com/terms/e/efficientfrontier.asp>
- Jackson, M., & Staunton, M. (2001). *Advanced modeling in finance using Excel and VBA* (1st ed.). London, UK: JOHN WILEY & SONS.
- Jacobsen, D. I., & Torsvik, J. (2007). Hvordan organisasjoner fungerer.
- Korn, O., & Koziol, C. (2006). Bond Portfolio Optimization: A Risk-Return Approach. *The Journal of Fixed Income*, 15 (4), 48-56.
- Luenberger, D. G. (1998). *Investment Science*. New York, USA: Oxford University Press.
- Løhre, M. (2007). Fra Markowitz til Statens Pensjonsfond – Utland; Porteføljevalg i teori og praksis. Retrieved from <http://bora.nhh.no/bitstream/2330/1778/1/Morten%20Lohre%202007.pdf>
- Markowitz, H. (1952). Portfolio Selection. *The Journal of Finance*, 7(1), 77-91.
- Markowitz, H. (1959). *Portfolio Selection: Efficient Diversification of Investments*. New Jersey, USA: John Wiley & Sons.
- Oslo Stock Exchange. (2011). Bond closing prices. In C. Pettersen (Ed.). Oslo, Norway.
- Peirce, C. S. (1992). *The Essential Peirce, Selected Philosophical Writings, Volume 1 (1867–1893)*. Bloomington and Indianapolis, USA: Indiana University Press.
- Peirce, C. S. (1998). *The Essential Peirce, Selected Philosophical Writings, Volume 2 (1893-1913)*. Bloomington and Indianapolis, USA: Indiana University Press.
- Reilly, F., & Brown, K. (2002). *Investment Analysis and Portfolio Management* (7th ed.). Cincinnati Ohio, USA: South-Western Pub.
- Securities Industry and Financial Markets Association, S. (2011). SIFMA Investing in bonds. Retrieved 17 January, 2011, from <http://www.investinginbonds.com/>
- Sharpe, W. F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. *Journal of Finance*, 19(3), 425-442.
- Sirnes, E. (2011). Optimal Portfolio. Tromsø, Norway.
- Standard & Poor's. (2011). Credit Ratings Definitions & FAQs. Retrieved 25 January, 2011, from <http://www.standardandpoors.com/ratings/definitions-and-faqs/en/us>
- Zweig, J. (1998). How the Big Brains Invest at TIAA-CREF. *Money Magazine*, 27(1), 114.