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Late quaternary temperature variability described as abrupt transitions on a $1/f$ noise background

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Abstract

We show that in order to have a scaling description of the climate system that is not inherently non-stationary, the rapid shifts between stadial and interstadial conditions during the last glaciation cannot be included in the scaling law. The same is true for the shifts between the glacial and interglacial states in the quaternary climate. When these events are omitted from a scaling analysis we find that the climate noise is consistent with a $1/f$ law on time scales from months to 10^5 years.

1 Introduction

The temporal variations in Earth's surface temperature are well described as *scaling* on an extended range of time scales. In this parsimonious characterisation, a single parameter specifies how the fluctuation levels on the different time scales are related to each other. This parameter is often called the Hurst parameter H , but it is also common to use the parameter $\beta = 2H - 1$. The latter is convenient since it can be defined via the scaling of the spectral density function of the signal by the relation $S(f) \sim f^{-\beta}$. An alternative definition of β is using the variances of the set of signals $T_{\Delta t}(t)$ constructed by taking mean temperatures in time windows of length Δt :

$$\text{Var}[T_{\Delta t}(t)] \sim \Delta t^{\beta-1}. \quad (1)$$

In this description, the temperature fluctuations would decrease with scale if $\beta < 1$, implying that the climate fluctuations become less prominent as we consider longer time scales, a picture which is somewhat different than the rich long-range variability indicated by proxy reconstructions of past climate. On the other hand, a value $\beta > 0$ would imply that variability increases with scale, a property that (if it were valid on a large range of time scales) would lead to levels of temperature variability inconsistent with reality. It is therefore a natural a priori working hypothesis, that Earth's typical

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since the events are not uniformly distributed over time, there is no uniquely defined scaling exponent for the last glacial period. Moreover, the scaling law would not be useful as a climate-noise model for determining the significance of particular trends and events, such as the anthropogenic warming in the last century.

The main message of this paper is that the $1/f$ noise characterisation of the temporal fluctuations in global mean surface temperature is very robust. It is an accurate description for the Holocene climate, but it is also valid under both stadial and interstadial conditions during glaciations, and during both glacial and interglacial conditions in the quaternary climate. The $1/f$ character of the climate noise provides us with robust estimates of future natural climate variability, even in present state of global warming. Such an estimate would of course be invalidated by a future regime shift (a tipping point) to a warmer climate state provoked by anthropogenic forcing. A future observed change in the $1/f$ character of the noise could therefore be taken as an early warning signal for such a shift.

2 Data, methods and results

The analysis in this work is based on four data sets for temperature fluctuations: the HadCRUT4 monthly global mean surface temperature (Morice et al., 2012) in the period 1880–2011 CE (Common Era), the Moberg Northern Hemisphere reconstruction for annual mean temperatures in the years 1–1978 CE (Moberg et al., 2005), as well as temperature reconstructions from the North Greenland Ice Core Project (NGRIP) (Andersen et al., 2004) and the European Project for Ice Coring in Antarctica (EPICA) (Augustin et al., 2004). For the NGRIP ice core we have used 20 year means of $\delta^{18}\text{O}$ going back 60 kyr. For the EPICA ice core we have temperature reconstructions going back over 300 kyr, but the data is sampled at uneven time intervals and the time between subsequent data points becomes very large as we go back more than 200 kyr. In addition we have used annual data for radiative forcing in the time period 1880–

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2011 CE (Hansen, 2005) to remove the anthropogenic component in HadCRUT4 data. Plots of all four data records are shown in Fig. 1.

2.1 Global vs. local scaling

On the face of it, it is difficult to discern scaling laws for the climate noise on time scales longer than millennia, since we do not have high-resolution global (or hemispheric) temperature reconstructions for time periods longer than two kyr. The ice core data available only allow us to reconstruct temperatures locally in Greenland and Antarctica, and we know from the instrumental record that local and regional continental temperatures scale differently from the global mean surface temperatures on time scales shorter than millennial. The differences we find are that local temperature scaling exponents β_l are smaller than global temperature exponents β_g , and that the ocean temperatures scale with higher exponents than land temperatures. Since there are strong spatial correlations in the climate system, it is possible that all local temperatures are scaling with a lower exponent than the global. In Rypdal et al. (2015) this phenomenon is illustrated in an explicit stochastic spatio-temporal model. In this model, which is fitted to observational instrumental data, we find the relationship $\beta_g = 2\beta_l$. This relationship is derived under the highly inaccurate assumption that all local temperatures scale with the same exponent, but it is still a useful approximation in the following, where we will argue that we can use local and regional temperature records to discern the scaling of the global mean surface temperature on time scales of 10 kyr and longer. We do this by showing that the assumption that $\beta_g/\beta_l > 1$ is valid on very long times scales leads to the impossible result that the variance of global averages become larger than the mean variance of local averages. Thus we conclude that β_l converges to β_g on sufficiently long time scale, and we estimate an upper limit on that time scale.

Let us denote by σ_g and σ_l the standard deviations of the global surface temperature and a local temperature respectively, on a monthly time scale. From Eq. (1) it follows that the ratio between the variances for the global and local temperatures at time scale Δt is

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$$\rho = \left(\frac{\sigma_g}{\sigma_l} \right)^2 \left(\frac{\Delta t}{\tau} \right)^{\beta_g - \beta_l},$$

where $\tau = 1$ month. Unless we expect global temperatures to have larger variations than the local temperature at time scale Δt (the global temperature can not have a larger standard deviation than the average standard deviation of the local temperatures) we must have $\rho > 1$, or equivalently,

$$\Delta t < \tau \left(\frac{\sigma_l}{\sigma_g} \right)^{2/(\beta_g - \beta_l)}.$$

On the time scale of months, the fluctuation levels of continental temperatures is about two orders of magnitude larger than the fluctuation level for the global mean temperature. If we also use $\beta_g = 1$ and $\beta_l = 1/2$ we obtain the condition $\Delta t < 10^5$ months ~ 10 kyr, i.e. on time scales longer than 10 kyr the ratio β_g/β_l can no longer be larger than unity. A similar estimate can be obtained from the NGRIP ice core data. In the Holocene the 20 year resolution temperature reconstructions from Greenland has a standard deviation which is about five times greater than the 20 year moving average of the Moberg reconstruction for the Northern Hemisphere. Applying the same argument restricts the time scale for which Greenland scaling exponent is smaller than the global scaling exponent to approximately 10 kyr.

Based on the reasoning above, we expect scaling of the ice core data to be similar to the global scaling on sufficiently long time scales. In the remainder of this paper we demonstrate that the scaling in the ice core data on time scales up to hundreds of kyr is similar to the $1/f$ scaling we observe in global temperature up to a few millennia. This suggests that the $1/f$ scaling on very long time scales in ice core data is a reflection of the scaling in global temperatures on these scales.

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2.2 Methods for estimation of scaling

We use two methods to analyse the scaling of temperature records. The first is a simple periodogram estimation of the spectral power density $S(f)$. This estimator can also be applied to data with uneven time sampling using the Lomb-Scargle method (Lomb, 1976). The other method is to take the wavelet transform of the temperature data:

$$W(t, \Delta t) = \frac{1}{\sqrt{\Delta t}} \int T(t') \psi \left(\frac{t-t'}{\Delta t} \right) dt' \quad (2)$$

and construct the mean square of the wavelet coefficients. This is a standard technique for estimating the scaling exponent β (Malamud and Turcotte, 1999), and it is known that $\langle |W(t, \Delta t)|^2 \rangle \sim \Delta t^\beta$. We choose to use the so-called Haar wavelet

$$\psi(t) = \begin{cases} 1 & t \in [0, 1/2) \\ -1 & t \in [1/2, 1) \\ 0 & \text{otherwise} \end{cases}$$

and the integral in Eq. (2) is computed as a sum. The method can be adapted to the case of unevenly sampled data using the method described in Lovejoy (2014). In this work, we obtain very similar results using the periodogram and the wavelet transform.

2.3 Results

In Fig. 2 we show the wavelet fluctuation $\langle |W(t, \Delta t)|^2 \rangle$ estimated for two different segments of the NGRIP data. Both time series have the same number of data points and both represent time intervals of 8500 years. The differences between the two time series is that one contains DO cycles, whereas the other does not. The estimated wavelet fluctuations and the spectral density scale very differently for the two time series, and this motivates us to separate stadial and interstadial conditions when we analyse the

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scaling in NGRIP data. This separation is shown in Fig. 1a, where the red curve represents the $\delta^{18}\text{O}$ concentration in interstadial periods and the blue curve represents the $\delta^{18}\text{O}$ concentration in stadial periods. We have followed Svensson et al. (2008) in defining the dates for the onsets of the interstadials and we have defined the start dates for the stadial periods to be just after the rapid temperature decrease that typically follows the slow cooling in the interstadial periods. In Fig. 3 we show the spectral density function and the wavelet scaling function for the stadial data (red diamonds) and the interstadial periods (purple triangles), which both display an approximate $1/f$ scaling, but where the fluctuation variance in the stadial data is larger than in the interstadial data. These results are different from what is obtained when considering the NGRIP data (during the last glaciation) as a single time series (shown as blue diamonds). If we were to define a single scaling exponent for the whole time series, then we would obtain an estimate $\beta \approx 1.4$.

Figure 3 shows that the scaling of the stadial and interstadial NGRIP data are similar to the scaling of global temperatures on shorter time scales. We have included an analysis of the instrumental temperature record both with (green triangles) and without the anthropogenic component (green disks). The anthropogenic component can be removed by subtracting the response to the anthropogenic forcing in a simple linear response model of the type considered in Rypdal and Rypdal (2014). We have also included an analysis of the Moberg Northern Hemisphere reconstruction (black squares), and we observe that the composite scaling wavelet variance function and the composite spectral density function obtained by combining the instrumental data with the Moberg reconstruction, is consistent with a $1/f$ model on time scales from months to centuries. Since the NGRIP data also shows $1/f$ scaling, and since we believe that the scaling of the NGRIP data is a reflection of global scaling on time scales longer than a millennium, it is illustrative to adjust the fluctuation levels of the NGRIP data so that its Holocene part has a standard deviation close to that of the standard deviation of the 20 year means of the Moberg reconstruction in the same time period. This means that we use the adjusted NGRIP data as a proxy for global temperature

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on millennial scales. The effect of this adjustment is only a vertical shift of the wavelet scaling function and the spectral density functions in the double-logarithmic plots, so that it becomes easier to compare the scaling of the NGRIP data with the Moberg reconstruction and the instrumental data. We do not apply any adjustments of the fluctuation levels of the stadial and interstadial periods relative to each other. The same adjustment is applied to the EPICA data, and here we also consider the scaling of the glacial and interglacials separately as shown in Fig. 1b. The scaling estimated from the EPICA data for glacial periods (black crosses in Fig. 3) follow the almost the same scaling as the NGRIP data analysed as a single time series (blue diamonds). This shows that the glacial climates have similar characteristics in Greenland and in Antarctica. Careful examination of the figure shows that the fluctuations grow slightly faster with the scale Δt in the NGRIP time series than for the glacial periods of the EPICA time series. This is expected since the regime shifting events in Antarctica (that are known to be connected with the DO cycles, WAIS Divide Project Members, 2015) are much less pronounced than on Greenland. In the EPICA data we cannot estimate a scaling exponent for the dynamics in periods without regime shifts, but our results for the EPICA data are consistent with a description of the climate as a $1/f$ climate noise plus regime shifts. If we analyse the EPICA data without omitting the interglacials, then the fluctuations increase even faster with the scale Δt (orange stars in Fig. 3). This effect is completely analogous to the effect of shifting between the stadial and interstadial conditions during glaciations.

3 Discussion and concluding remarks

The characteristics of the climate noise is essential for the detection and evaluation of anthropogenic climate change. For instance, when we apply standard statistical methods for estimating the significance of a temperature trend, the result depends crucially on the so-called error model, i.e. the model for the climate noise. There is strong evidence that the temperature fluctuations are better described by scaling models than by

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so-called red-noise models (or AR(1)-type models). However, simply characterizing the climate noise as scaling does not specify an error model. The exponent in the scaling law (the β parameter) must also be determined, and it is usually determined from the same signal as we are testing for trends. Most estimators of β are sensitive to trends, providing too large β estimates when applied to signals with strong trends. So if we estimate β under the assumption that our null hypothesis (no trend) is true, then we are being led to a model with a value of β which is too large if the alternative hypothesis is true (there is a real trend). The large value of β will then lead us to believe that the climate noise can produce pseudo-trends comparable to the estimated trend in the signal, and the result is that we have a test with low statistical power (the probability of detecting a significant trend is low even if there is a real trend). It is important to realize that there is nothing formally wrong with using a trend-detection test with low power. It only means that it will be difficult to make statistical significant conclusions about the observed trends. The lack of statistical significance under a weak test does not mean that the trends are not real. It would however be incorrect to give a general characterization of the climate noise by the exponent β if it is likely that the β estimate is strongly biased by the presence of a trend.

One approach to the problem described above is to apply some type of de-trending to the signal prior to the estimation of β . This may seem to be an inconsistent step, since the β should be estimated under the assumption that the null hypothesis is true. However, since de-trending only has a small effect if the null hypothesis is true, de-trending is valid under both the null hypothesis and the alternative hypothesis. If de-trending is applied, the statistical powers of the standard trend-detection techniques for scaling processes are greatly improved.

Another approach, which is the motivation for this paper, is to characterize the scaling of the climate noise from pre-industrial temperature records. If we are to use the scaling exponent estimated from pre-industrial records to demonstrate the anomalous climate event associated anthropogenic influence, we must be confident that the temperature scaling does not change significantly over time. We must also be confident

that the scaling is robust, in the sense that it is not too sensitive to moderate changes in the climate state. The main result of this paper is that unless the climate system experiences dramatic regime shifting events, we can be confident that the natural fluctuations in global surface temperature is approximated by $1/f$ -type scaling on a large range of time scales. This result makes it easy to determine, on any time scale, if the observed increase in global mean surface temperature is inconsistent with the natural variability, and by how much.

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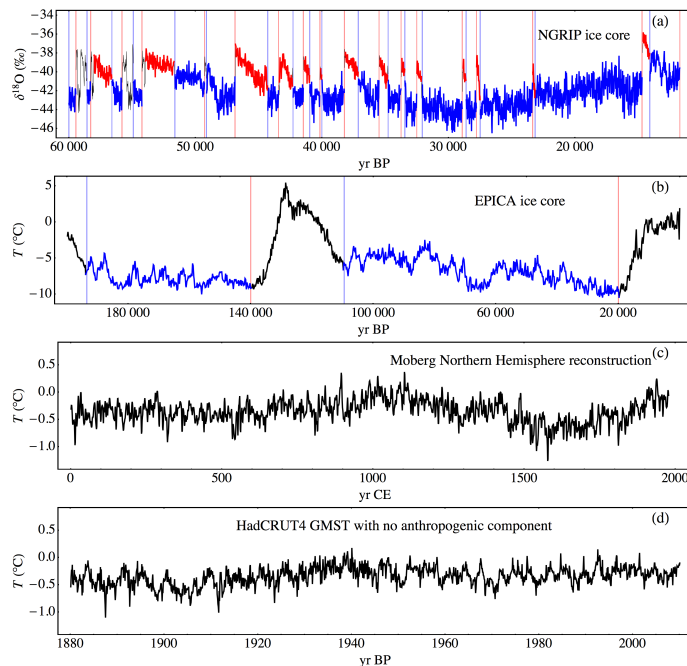


Figure 1. (a) The $\delta^{18}\text{O}$ concentration in the NGRIP ice core dating back to 60 kyr before present (BP). Here present means AD 2000 (= 2000 CE). The data is given as 20 year mean values. The time series is split into stadial (blue) and interstadial (red) periods. (b) The temperature reconstruction from the EPICA ice core. The shown time series is sampled with a time resolution of roughly 200 years. The temperature curve in the glacial periods is given in a blue color. (c) The Moberg reconstruction for the mean surface temperature in the Northern Hemisphere. The data is given with annual resolution. (d) The HadCRUT4 monthly global mean surface temperature where the anthropogenic component has been removed using a linear-response model.

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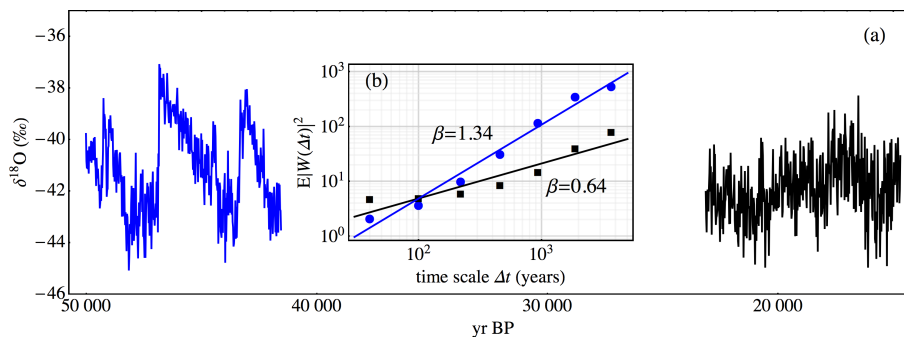


Figure 2. (a) The $\delta^{18}\text{O}$ concentration in the NGRIP ice core. The data is given as 20 year mean values. Two different parts of the the times series is shown. The blue curve represents the $\delta^{18}\text{O}$ concentration in a time period starting approximately 50 kyr before present (BP) and has a duration of approximately 8500 years. As in Fig. 1, present means AD 2000 (= 2000 CE). The black curve represents the $\delta^{18}\text{O}$ concentration in a long stadial period that started about 22 kyrs BP and has a duration of approximately 8500 years. (b) The wavelet scaling functions estimated from the two parts of the NGRIP data set. The blue points are the estimates from the part of the NGRIP ice core that is shown as a blue curve in (a), and which contains DO cycles. The black points are the estimates from the the part of the NGRIP ice core that is shown as a black curve in (a), and which does not contain any DO cycles.

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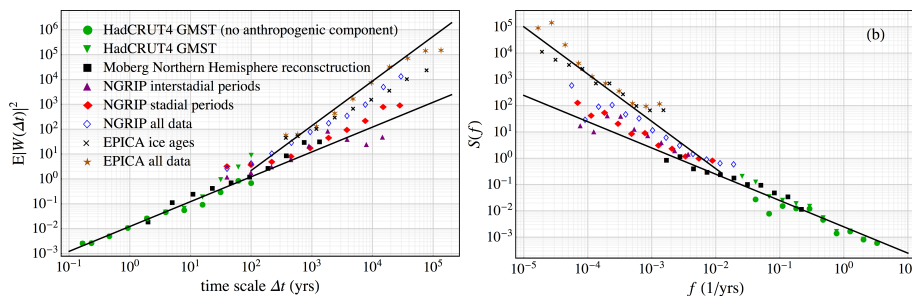


Figure 3. (a) For each time series considered in this paper we show double-logarithmic plots of the wavelet fluctuation $\langle |W(t, \Delta t)|^2 \rangle$ as a function of the time scale Δt . The green triangles and the green circles represent the the HadCRUT4 monthly global mean surface temperatures with and without the anthropogenic component respectively. The black circles is the analysis of the Moberg Northern Hemisphere reconstruction. The analysis of the 20 year mean NGRIP data is shown as the blue diamonds, the purple triangles and the red diamonds. The blue diamonds show the results of the analysis of the entire dataset dating back to 60 kyrs BP. The red diamonds are the results of the analysis performed on the stadial periods only, and the purple triangles are the results of the analysis of the interstadial periods only. The results for the EPICA ice core data are shown as the orange stars and the black crosses. The orange stars are obtained by analysis of the entire data set dating back 200 kyrs, and the black crosses are obtained by only analysing the two most recent glaciations. The two solid lines have slopes $\beta = 1$ and $\beta = 1.8$. (b) As in (a), but instead of the wavelet fluctuation function we show the spectral density function $S(f)$. The two solid lines have slopes $-\beta$ with $\beta = 1$ and $\beta = 1.8$.

