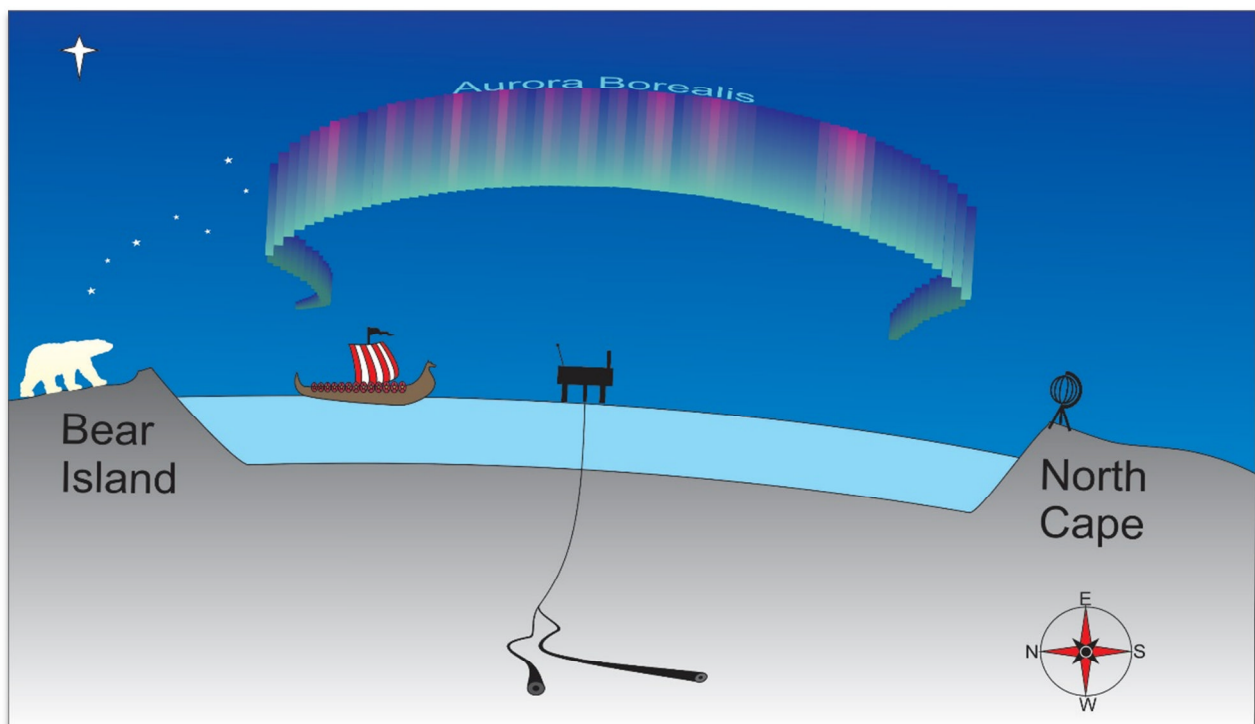


# Effects of Geomagnetic Disturbances on Offshore Magnetic Directional Wellbore Positioning in the Northern Auroral Zone

**Inge Edvardsen**

*A dissertation for the degree of Philosophiae Doctor – November 2015*





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**Inge Edvardsen**

Tromsø, November 2015

UiT – The Arctic University of Norway

Faculty of Science and Technology

Department of Physics and Technology



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# Abstract

This thesis focuses on how disturbances in the geomagnetic field, offshore northern Norway, may affect the accuracy of magnetic directional wellbore surveying. The topics discussed are, however, applicable to the entire northern auroral zone. Suggestions on how to manage the effect of increased geomagnetic activity on magnetic directional wellbore survey operations in and near the auroral zone are described.

The results from our study of the geomagnetic conditions in the Norwegian Sea clearly indicate that the direction from a monitoring station to a drilling site matters when using data from the monitoring station to quality control or correct downhole directional measurements affected by geomagnetic disturbances. While the deviations in total field and dip correspond well over large distances along the same geomagnetic latitude in east-west direction, the declination variations correlate better in the north-south direction.

When analysing observatory data in the vicinity of the Barents Sea we discovered that the deviations in declination during disturbed periods can be constantly offset from the quiet level for several hours. Magnetic directional surveys taken during such conditions will result in an unmodeled position-bias, if not corrected for.

Based on observatory data from all over the northern auroral zone, and a new application of an external magnetic field model, we show that there can be significant differences in the geomagnetic conditions along concurrent geomagnetic latitudes, in addition to the known latitudinal variations.

The use of monitoring station data to correct wellbore surveys for geomagnetic field disturbances has been carried out for many years at some onshore locations in the northern auroral zone. However, the methods and procedures for achieving valid magnetic directional surveys on land are not always applicable to distant offshore locations. Today, magnetic monitoring stations on land are used to predict or interpolate the effect of the external magnetic field at offshore drilling sites. However, the maximum distance that monitoring stations can be from a rig site, while still providing a valid estimate of the disturbance, generally decreases with increasing geomagnetic latitude. As drilling activity increases in areas within the northern auroral zone, there has been increased interest in how to effectively monitor and correct for disturbance field effects.



# Preface

This doctoral thesis is the result of research performed in the period from 2011-2015. Since this is an Industrial Ph.D., the studies have been jointly funded for a four-year period by the Norwegian Research Council and my employer Baker Hughes; three years for research and a fourth year as engineer at the Baker Hughes Survey Management department in Norway. The degree awarding institution is the University of Tromsø (UiT) - The Arctic University of Norway.

The background for starting on the doctoral studies was a 10 years long cooperation between Tromsø Geophysical Observatory and Baker Hughes and ongoing discussions on how to best handle the external magnetic field in the Norwegian waters. With the common objective of providing high quality magnetic directional surveys also at offshore locations in the auroral zone, it was agreed to start a Ph.D. project. Based on the work from our research we have established routines and procedures on how to best manage the challenges related to magnetic directional surveying in the northern auroral zone.

During the four years period this work has taken, several persons have been essential for this doctoral research to be accomplished.

First of all I would like to thank my supervisors at UiT Unni Pia Løvhaug, Truls Lynne Hansen and Magnar Gullikstad Johnsen for giving me guidance and feedback. Without you I would never have been able to complete this work.

At Baker Hughes I would like to thank my colleagues in the Survey Management department in Norway for their encouragement and support throughout the period. Mille Nielsen, Ildiko Langaker, Kieran Parrett and Runar Abelseth; you all deserve a big thank you. Harry Wilson has been my internal supervisor. Your assessment, interest and help have been crucial. Nina Heen and Irene Selland have been my project leaders and carried out the practical cooperation with the Norwegian Research Council. Thank you both.

I would like to thank all my co- authors for their contributions to this thesis. In addition to the contribution from my supervisors valuable input has been given by Morten Gjertsen, Erik Nyrnes and Jürgen Matzka.

Additional thanks go to my friends Livia Kathleen Kother and Trond Henanger for their support and valuable feedback throughout these four years.

Finally, I am grateful to my family for supporting me and letting me fulfil this project. Especially thanks to my parents in-law who make my life easier in so many ways and help me orientate so I do not lose target.





# Contents

- Abstract..... iii
- Preface ..... v
- Nomenclature ..... ix
  - List of acronyms ..... ix
- 1 Introduction ..... 1
  - 1.1 Wellbore positioning challenges at auroral latitudes ..... 1
  - 1.2 A brief history of directional surveying..... 6
  - 1.3 The connection between space physics and magnetic directional surveying..... 8
  - 1.4 Outline of the thesis ..... 9
- 2 The Earth's magnetic field ..... 11
  - 2.1 The internal geological structure of the Earth..... 11
  - 2.2 The main field (Bm) ..... 12
  - 2.3 The crustal field (Bc)..... 13
  - 2.4 The disturbance field (Bd) ..... 13
  - 2.5 Components of the Earth's magnetic field..... 15
  - 2.6 Spherical harmonic analysis..... 17
  - 2.7 Geomagnetic coordinates and time..... 19
  - 2.8 Geomagnetic field models ..... 20
- 3 Error models and quality control..... 24
- 4 Summary of papers ..... 27
- Bibliography ..... 31
- Paper 1 ..... 33
- Paper 2 ..... 45
- Paper 3 ..... 59
- Appendix..... 91
- Appendix A: Absolute orientation in magnetic wellbore directional surveying ..... 93
  - A.1 The Earth-fixed N-E-V coordinate system..... 95
  - A.2 Tool-fixed x-y-z directional sensor coordinate system ..... 95
  - A.3 Derivation of magnetic directional surveying equations ..... 97
    - A.3.2 Equations for the directional survey sensor outputs..... 101
    - A.3.3 Equations for dip ( $\theta$ ), inclination (I) and azimuth (A) ..... 102
    - A.3.4 Equations for toolface ( $\alpha$ ) ..... 105



# Nomenclature

## List of acronyms

|        |   |
|--------|---|
| BGGM   | British Geological Survey Global Geomagnetic Model      |
| BHA    | Bottom hole assembly                                    |
| CGM    | Corrected geomagnetic                                   |
| GPS    | Global positioning system                               |
| HTF    | Highside toolface                                       |
| HDGM   | High definition geomagnetic model                       |
| IFR    | In-field referencing                                    |
| IIFR   | Interpolated in-field referencing                       |
| IGRF   | The international geomagnetic reference field           |
| IMF    | Interplanetary magnetic field                           |
| ISCWSA | Industry Steering Committee on Wellbore Survey Accuracy |
| MD     | Measured depth  |
| MLT    | Magnetic local time                                     |
| MTF    | Magnetic toolface                                       |
| MWD    | Measurement while drilling                              |
| N-E-V  | The Earth-fixed north-east-vertical reference system    |
| nT     | nanoteslas = $1 \times 10^{-9}$ Tesla                   |
| QC     | Quality control   |
| RAM    | Random access memory                                    |
| SH     | Spherical harmonics                                     |
| SPE    | Society of petroleum engineers                          |
| TD     | Total depth   |
| TFDT   | Total magnetic field intensity and dip test             |
| TVD    | True vertical depth                                     |



# 1 Introduction

## 1.1 Wellbore positioning challenges at auroral latitudes

Oil wells vary widely in scale, but are commonly between 1500m and 3000m in vertical extent and can be as much as 10000m or more in total length. They must be steered to intersect distant geological targets and to avoid subsurface hazards. The wells are drilled section wise, often starting with a section 36" wide and ending with one or several 8.5" sections in the reservoir as shown in **Figure 1**. The temperature and pressure in the wells usually increase with depth. At one of the major oil and gas fields in Norway where the reservoir is at about 1575m vertical below mean sea level, the temperature and pressure is about 70° C and 130bar, respectively.

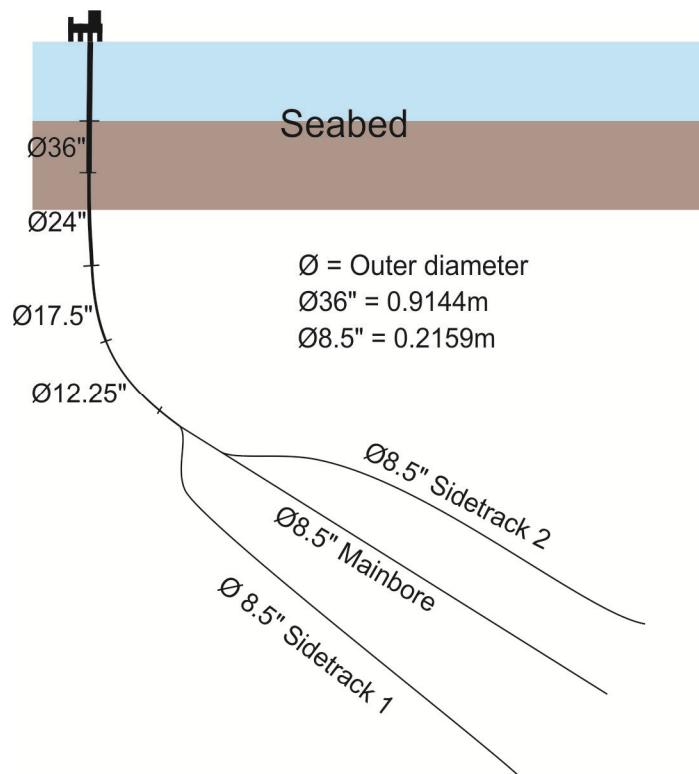


Figure 1 - A simple schematic of an oil well with different sections.

To make sure the well path is drilled according to the pre-defined plan, a method called directional surveying is applied. Global positioning systems (GPS) are not an option for use underground, as the satellite signals cannot penetrate into the ground. Instead we must rely on the fundamental geophysical quantities of the Earth's gravity field and either the Earth's spin vector or magnetic field to define the orientation of the downhole surveying device, and thereby the orientation of the wellbore. Accelerometers are used to measure the gravity field, gyros the spin vector, and magnetometers the magnetic field. All three can be used to measure tool rotation around the borehole axis. Downhole survey tools record and transmit their attitude at successive points along the wellbore and these measurements are used to calculate well path coordinates. In magnetic directional surveying a measurement is normally taken approximately every 30m. In **Appendix A**, a typical bottom hole assembly (BHA) together with sensor operation and attitude calculations (azimuth, inclination and toolface) are described in more detail.

Compared to gyros, magnetometers are robust and inexpensive, but they are susceptible to interference from steel components in the drilling assembly and nearby wells, and magnetic north is an unstable azimuth reference compared to geographic north which is referenced by gyros. Despite these shortcomings, magnetometer based survey tools are still considered to be the most reliable and cost effective option, and are the most widely used tools for surveying while drilling. Such tools are commonly referred to as measurement while drilling (MWD) tools. Gyro tools are most often run in hole after an interval has been drilled, to validate the MWD or provide a more accurate survey. However, there are cases when higher accuracy is required while drilling. This has resulted in the development of enhanced survey techniques such as in-field referencing (IFR) (Williamson et al. 1998) and new survey methods such as gyro MWD.

As for all measurements, wellbore surveys have errors. The constraints associated with making remote measurements in a harsh environment with severe space restrictions mean that the errors associated with downhole surveying can be significant with respect to the positioning accuracy required for a successful well. An estimate of the accuracy of the measurements is required to be able to calculate the positional uncertainty of a wellbore, and determine if the surveying programme meets the directional objectives. Management of positional uncertainty is crucial for many purposes such as hitting the geological targets, avoiding collision with nearby wells and maximizing recovery.

For the calculation of azimuth, when using magnetic MWD, the geomagnetic field provides the north reference. When gyro tools are applied, the spin vector is the north reference. In both cases the magnitude of the horizontal component of the reference field decreases in magnitude with increasing latitude (geomagnetic and geographic), making azimuth measurements less accurate at higher latitudes (Bang et al. 2009). In addition to the decrease in the horizontal component of the Earth's magnetic field ( $H$ ), magnetic MWD surveys also suffer from fluctuations in the geomagnetic field caused by the external magnetic field, which are larger and last longer in the auroral zones.

In downhole surveying, the geomagnetic field is normally described by the three components declination ( $D$ ), dip angle ( $I$ ) and total field strength ( $F$ ). The nominal values for these components must be known from an external source in order to correct and quality check each magnetic MWD survey. The standard practice is to obtain the values from a global geomagnetic model. More details are provided in section 2.7. Uncertainties related to the estimated parameters of the geomagnetic field given by these models are reflected in the wellbore positional uncertainty. Magnetic directional surveying is especially sensitive to uncertainty in the declination, as any error in the declination translates directly into an error in the final azimuth.

At high latitudes the declination becomes very sensitive to magnetic disturbances and may vary by several degrees during periods with increased external magnetic field variations. However, although the declination is important to navigators, little relevant research has been carried out on the physical connection between magnetic directional surveying and external magnetic field variations at higher latitudes. Researchers in ionospheric physics are typically not concerned with the geomagnetic effects the external magnetic field has on the declination as they usually operate in a coordinate system where the declination is not used.

For several years, a correction method called interpolated in-field referencing (IIFR) has been applied to correct for variations in the external magnetic field. This method works well in some locations, such onshore in Alaska, where the distances from the observatories to the drilling sites are short. However, for locations far from the reference magnetometer (>200km), especially in the auroral zone, the validity of the method is doubtful.

A statistical analysis of magnetometer data from observatories and variometers along the Norwegian coast and in Norwegian waters was carried out by Torkildsen et al. (1997). The results indicated that the use of an interpolation technique in the auroral zone will reduce the deviation in declination by about 60% over a distance of 130km and about 30% when the distance is 540km. The general trend is that the improvement decreases with increasing latitude. This work demonstrated the need for further research to determine alternative methods and procedures to correct for the external magnetic field variation. Bang et al. (2009) concluded that lateral position uncertainty increases by a factor of two when moving from North Sea to Barents Sea latitudes.

Rastogi et al. (2001) and Rastogi (2005) investigated the effect that the external magnetic field has on the horizontal magnetic field component and the declination at lower latitudes. They confirmed the long-established assumption that the currents creating the external magnetic field are guided by the Earth's dipole field and not the magnetic field measured at ground level. Even though large variations are less frequent and generally smaller at low and mid-latitudes than in the auroral zone, the techniques used to study the connection between the external magnetic field and the declination on the ground are the same. This theory was applied by Edvardsen et al. (2014) when explaining the declination variation in the Barents Sea.

The surveying of wellbore trajectories in the oil and gas industry is one of the few remaining navigation tasks where the Earth's magnetic field is used. The accuracy of the geomagnetic reference parameters used in magnetic directional surveying is reflected in the magnitude of the uncertainty associated with the wellbore trajectory. Statistical information about the accuracy of the directional survey sensors and how well the geomagnetic field is known are used in the design of error models that quantify surveyed position uncertainty. Regarding the uncertainty related to models of the geomagnetic field, the contribution from both the main and crustal field parts are today fairly well documented. These models continue to improve due to collection of more and better data, predominantly from satellites. On the other hand, the external magnetic field contribution is much more complex and more difficult to model. The accuracy of magnetic directional wellbore surveying decreases with increasing latitude. The two main reasons are the diminishment of the horizontal magnetic field component and a more turbulent external magnetic field.

Magnetic directional surveying in the Arctic, especially at distant offshore locations, introduces new challenges that need to be solved to be able to drill deviated wells in a safe and efficient manner. The increased azimuth uncertainty caused by a smaller horizontal magnetic field component and larger fluctuations in the geomagnetic parameters are significant and could limit the development of oil and gas fields. Experience from drilling operations in the sub-auroral and auroral zone along the Norwegian coast also put the focus on the difficulty of real-time directional control. Actually, how to manage the drilling operations while a magnetic storm is ongoing (Edvardsen et al. 2013) may be a more critical

issue than the increase in the wellbore positional uncertainty itself. In the northern auroral zone, single events of deflections in the geomagnetic field may have such a devastating effect on drilling operation that they cannot be neglected based on statistical analysis. In some cases, an appropriate action might be to estimate the effect on the azimuth reading and continue drilling. The survey interval with surveys taken outside the predefined specifications could be re-surveyed when pulling out of hole. On the other hand, there might be situations when drilling cannot commence without valid surveys and the only options would be to wait for the external magnetic field to calm down or make corrections for it. To illustrate the complexity in the decision-making process that the directional drilling survey team has to handle when no valid directional surveys are available due to external magnetic field variations, we will look at two case histories. Both cases describe situations where the presumption that the fluctuations in the geomagnetic field can be treated purely as acceptable noise, is impracticable from a drilling operations perspective.

### **Case history 1**

While drilling a well in a field located in the auroral zone of Norway, the directional driller was increasing the inclination to bring the well to horizontal and also initiating a right turn required by the plan before entering a geological target. At a certain point, the results from the survey quality control (QC), total magnetic field intensity and dip test (TFDT), deteriorated and were almost outside the standard margin. At the same time, an automated e-mail from the responsible Survey Management department was received, warning of increased geomagnetic activity. The last surveys had indicated that the wellbore had apparently made a left turn and not the right turn steered by the directional driller. If that was the case, the geological target might be missed. The directional driller stopped drilling and called the Survey Management department. The on-call engineer checked the magnetogram from a nearby variometer station and confirmed that a magnetic substorm was ongoing. The magnetograms from the nearby variometer stations indicated that the external magnetic field was especially affecting the declination. The basic QC-tests, applied as standard are not effective under all conditions. The TFDT may classify a bad survey station as good, even though a gross error in the declination is present (Ekseth et al. 2006). The directional surveys that almost failed the QC on TFDT were adjusted based on the variations seen at the nearby variometer. The affected surveys were corrected by about 1.5° in azimuth, which were significant in the specific drilling phase. The azimuth adjustment made the surveys fall back on the expected azimuth and drilling to section total depth (TD) could commence. This case illustrates that magnetic directional surveys taken during a period with increased geomagnetic activity can be significantly affected even though the standard QC-limits might not be exceeded.



## Case history 2

In the Norwegian Sea, the drilling of a complex well was about to reach final depth. It was in the middle of the night and there was only about 200m left to drill. Due to an unstable formation where the hole could easily collapse, a method called *steerable drilling liner* was used. This means that the steel tube liner, normally run into the hole after drilling to secure the well, was inserted while drilling. Without any warning, the magnetic directional surveys started to fail the TFDT. While the MWD engineer checked that the geomagnetic settings in the drilling software were correct, several repeat surveys were taken. All of the new directional surveys failed the QC-test. Drilling continued slowly for three hours without any valid surveys. According to the oil company's rule and "Norsk Sokkels Konkuranseposisjon" (Norsok) standard recommendations, surveys should be taken frequently enough to describe the wellpath. Irrespective of this, the maximum distance between two valid directional surveys should not exceed 100m. The directional driller stopped drilling when they had reached 80m from the last good survey and called the Survey Management department. No e-mail warnings regarding increased solar activity were sent out, but according to standard QC-procedure the Survey Management engineer tried to open the link to the magnetogram from a nearby variometer station. Surprisingly, the internet link did not work. Later it was discovered that there was a power supply shutdown in the building containing the server which stored the magnetogram data. By searching at different space weather websites, the Survey Management engineer observed a long period of negative interplanetary magnetic field (IMF), which now had turned positive. This was a very good indication of a possible large substorm. In addition, a magnetogram from an observatory in a neighbouring country, a few degrees north of the drilling location, showed that there was obviously an ongoing disturbance. There were deviations of  $\pm 300\text{nT}$  in total magnetic field,  $\pm 0.4^\circ$  in the dip and  $\pm 0.8^\circ$  in the declination. However, no corrections of the magnetic directional surveys could be made based on these data due to the long distance from the observatory to the drilling site. The directional driller and Survey Management engineer agreed that drilling could not continue. On the other hand, the oil company was clear that they did not want to stop drilling and stay too long at the current depth due to the rock formation quality. Their drilling superintendent at the rig site asked the Survey Management engineer to come up with an estimate on the azimuth error caused by the increased solar activity. This information would then be used to write a dispensation to be able to continue drilling for more than 100m from the last valid survey. Based on how much the directional surveys failed the QC-limits and the variations at the magnetogram from the observatory north of the drilling location, an estimate of  $\pm 2^\circ$  azimuth errors were given. Then, just as suddenly as the magnetic directional surveys started to fail the TFDT, a valid survey was achieved. The external magnetic field had calmed down and drilling continued to TD with good surveys and without any more interruptions from the external magnetic field. In this case, we see that drilling a well in the sub auroral zone can be challenging if appropriate procedures, regarding what to do if magnetic directional surveys start flagging are missing. One may ask; what are the chances of having a break in the power supply to the magnetogram server at the same time as drilling a complex well, in the middle of increased magnetic activity? However, there is at least one directional driller, one drilling superintendent and one Survey Management engineer who know that these things may happen.

## 1.2 A brief history of directional surveying

Wellbore surveying is quite a young discipline, where progress in technology has been driven by the operational demands of drilling longer and more complex wells in a safer and more efficient manner.

The following statements from a very early paper on the subject (Griswold 1929) are an appropriate starting point:

*"The surveying of oil wells has in recent months become a common practice in the deep fields of the Mid-Continent area. Borehole surveys have been made by mining companies for many years, but the introduction of such methods into the oil fields has been delayed until the past few years, because the value of survey data was not fully appreciated."*

*"The acid and gelatine bottle methods of obtaining the angle of deviation and the floating compass method of obtaining the direction of horizontal drift have been in use for more than 20 years. A photographic recording instrument consisting of a plumb bob, magnetic needle and electrically operated camera was devised and used in the Rand mining fields of South Africa prior to 1912."*

*"The magnetic compass is not reliable for oil-well work because of the unsymmetrical attraction of the needle to the well casing and possible magnetic formations."*

*"The gyroscopic compass has been suggested for use in direction finding, but the high cost of such instruments will prohibit their use, except in special cases."*

The acid bottle referred to was one of the first surveying tools used in the oil industry (Inglis 1987). The tool was lowered down inside the drill pipe to the depth of interest. When the tool was stationary, the solution of hydrofluoric acid and water inside the bottle slowly etched a tidemark on the glass, indicating the deviation of the wellbore from vertical. The etching time was at least 20 minutes; quite a long delay in drilling operations and without providing the direction of the deflection (azimuth). In the USA it was for a long time considered illegal to drill deviated wells as they could be used as a means of producing oil from a neighbour's property, so being able to prove that the wellbore had no significant deviation from vertical was quite important.

In the late 1920s, oil well survey instruments were developed that use a camera to photograph a mechanical inclinometer suspended over a compass. An inclinometer gives inclination and the combination of inclinometer and compass gives azimuth. The compass can be a magnetic compass or a directional gyroscope. (A directional gyroscope must be oriented to geographic north on surface and it then retains that orientation while being lowered into the well.) An additional pointer indicating the tool's internal reference is required to allow orientation of the drilling tools used to deflect the wellbore in the desired direction. In all cases, some form of timing device was required to trigger the camera when a survey record was required.

Downhole survey instrumentation remained relatively unchanged until the 1970s when drilling operations at offshore locations began. Offshore drilling is expensive and tends to require highly deviated wells originating from closely spaced surface locations. This was the incentive for developing faster, more reliable and more accurate surveying tools, an objective facilitated by increased access to aerospace technology and computing capability. Electronic devices replaced the photomechanical devices; accelerometers replaced mechanical

inclinometers, magnetometers replaced magnetic compasses, rate (north-seeking) gyros replaced directional gyros, and random access memory (RAM) or electric cable to surface replaced photographic film. Still, all survey devices tended to suffer from increased azimuth uncertainty at higher inclinations and latitudes.

As wells became more highly deviated, with horizontal drilling becoming commonplace towards the end of the 1980s, there was a strong incentive to improve azimuth performance. Azimuth correction techniques were developed for magnetometer based tools. Gyros tools were developed that continuously track changes in tool attitude after initial north-seeking, making them faster and more accurate in high inclination wells than their predecessors.

Until the late 1970's, drilling operations had to be stopped to allow surveying to take place. Due to high drilling costs, especially at offshore locations, it is desirable to minimise the time taken to survey. By then it was possible to deploy an electronic magnetic survey tool within the drilling assembly, connected to surface by electric line and providing real-time data, but this technique was only effective for short distances. This led to the development of MWD tools, using the same electronic magnetic sensor, but housed as a permanent part of the drilling assembly and transmitting data back to surface by inducing pressure pulses into the fluid column within the drill pipe. Soon after, gyro instruments were implemented in MWD tools, but the necessary trade-off between accuracy and reliability under rugged drilling conditions limited their application to relatively low inclinations. It is only in the last few years that all attitude gyro MWD tools have become available. Although gyros offer several advantages over magnetometer-based tools, their relative cost and reduced ruggedness mean magnetometer-based MWD will continue to be the most widely used survey method, especially given the very large inventory levels that exist. Therefore, directional surveying remains heavily dependent on knowledge of the Earth's magnetic field.

### 1.3 The connection between space physics and magnetic directional surveying

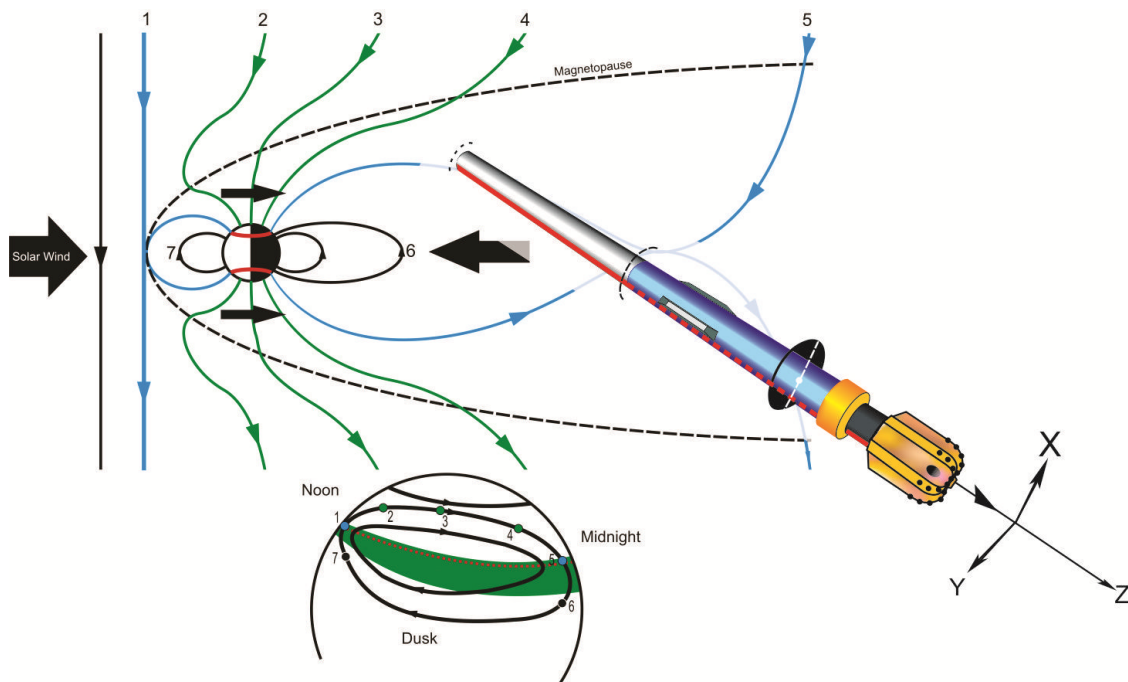


Figure 2 – The connection between space physics and magnetic directional surveying.

It is amazing that magnetic measurements used to navigate a directional drilling assembly, several thousands of meters into the ground, are affected by charged particles that originate from the inner parts of the Sun. The connection between the space physics phenomena of electrical charged particles brought to Earth as particle precipitation, and the technique of steering a drilling assembly, is the geomagnetic field. This connection is illustrated in **Figure 2**, with the Dungey cycle on the left hand side (Dungey 1961) and a directional drilling BHA on the right hand side. The electrical charged particles give rise to several current systems within the Earth's magnetosphere which may affect magnetic directional surveys. More details about the disturbance field are given in section 2.3. These currents create magnetic fluctuations which are sensed by the magnetic survey tools. Large and lasting disturbances in the geomagnetic field, especially in the auroral zones, will make the directional survey measurement corrupt or unreliable, if the effect is not adjusted for. Several geomagnetic institutions around the world offer space weather forecasts that can be useful for directional drilling companies. These forecasts are mainly based on observations of the conditions on the Sun and interplanetary space and therefore the forecast can be given for the next 2 to 3 days. The forecast definitely has some value for directional drilling operations at low and mid-high latitude. At these latitudes the drilling operations usually do not have to pay much attention to the disturbance field and would need a warning if something unusual is about to happen. However, drilling operations in the sub-auroral and auroral zone should be on the alert all the time as the surveys are likely to be affected by the disturbance field very often. The focus in these areas should be to make sure that there are observatories or variometer stations located close enough to the drilling site to be able to use them as a reference station and make correction if required. During drilling operations, the MWD engineers on the rig need to monitor the magnetogram from a nearby variometer all the time for quality control purposes.

## **1.4 Outline of the thesis**

The focus for the work in this PhD thesis has been to improve the knowledge about how magnetic directional surveying is affected by the external magnetic field variations at high latitudes. In Chapter 2 an overview of the geomagnetic field is given. The chapter is included to ensure that any reader has the basic knowledge about the Earth's magnetic field when the papers are read. Chapter 3 contains a short introduction to error models and quality control used in magnetic directional surveying. The main part of the thesis is the three papers. A summary of the papers, the first two published in the journal of SPE Drilling and Completion and the last submitted to the journal of Space Weather and Space Climate is in Chapter 4. Appendix A gives an overview of the drilling angles and the equations used in magnetic directional surveying.



## 2 The Earth's magnetic field

The geomagnetic field extends from the rotating liquid iron in the outer core of the Earth out to the magnetopause. Within a few Earth radii, the magnetic field of the Earth is similar to a dipole magnet tilted with respect to the Earth's rotation axis. Near the surface, the Earth's magnetic field can be defined as a vector quantity  $\mathbf{B}$ , expressed as the vector sum of the contributions from three main sources, see **equation (1)** and **Figure 3**.

$$\mathbf{B} = \mathbf{B}_m + \mathbf{B}_c + \mathbf{B}_d \quad (1)$$

- The main field generated in the Earth's core ( $\mathbf{B}_m$ )
- The crustal field from local rocks ( $\mathbf{B}_c$ )
- The combined disturbance field from electrical currents flowing in the upper atmosphere and magnetosphere, which also induce electrical currents in the sea and the ground ( $\mathbf{B}_d$ )

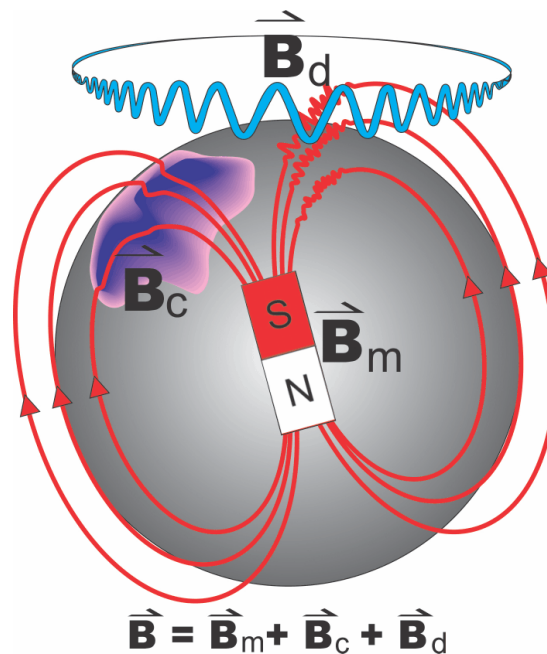


Figure 3 - Three main sources of the Earth's magnetic field.

In geomagnetism it is the SI unit Tesla that is used to describe the magnetic induction or flux density  $\mathbf{B}$ . However, gauss (G), gamma ( $\gamma$ ) and Ørsted (Oe) are also used.

### 2.1 The internal geological structure of the Earth

The internal structure of the Earth is layered in different shells, see **Figure 4**. (Inspired by: <http://geomaps.wr.usgs.gov/parks/pltec>, November 2015). From the surface to the centre of the Earth the distance is about 6371km. The first layer is the crust which is about 30km thick for the major parts of the continental regions and about 5km thick beneath the oceans (Campbell 2003). Below the crust there is an upper mantle which is divided into the lithosphere and asthenosphere. The upper mantle reaches down to about 600-700km where the lower mantle region starts. At about 2890m depth there is a core mantle boundary that defines the start of the liquid outer core. The liquid outer core extends to the depth of about 5150km where the solid inner core starts.

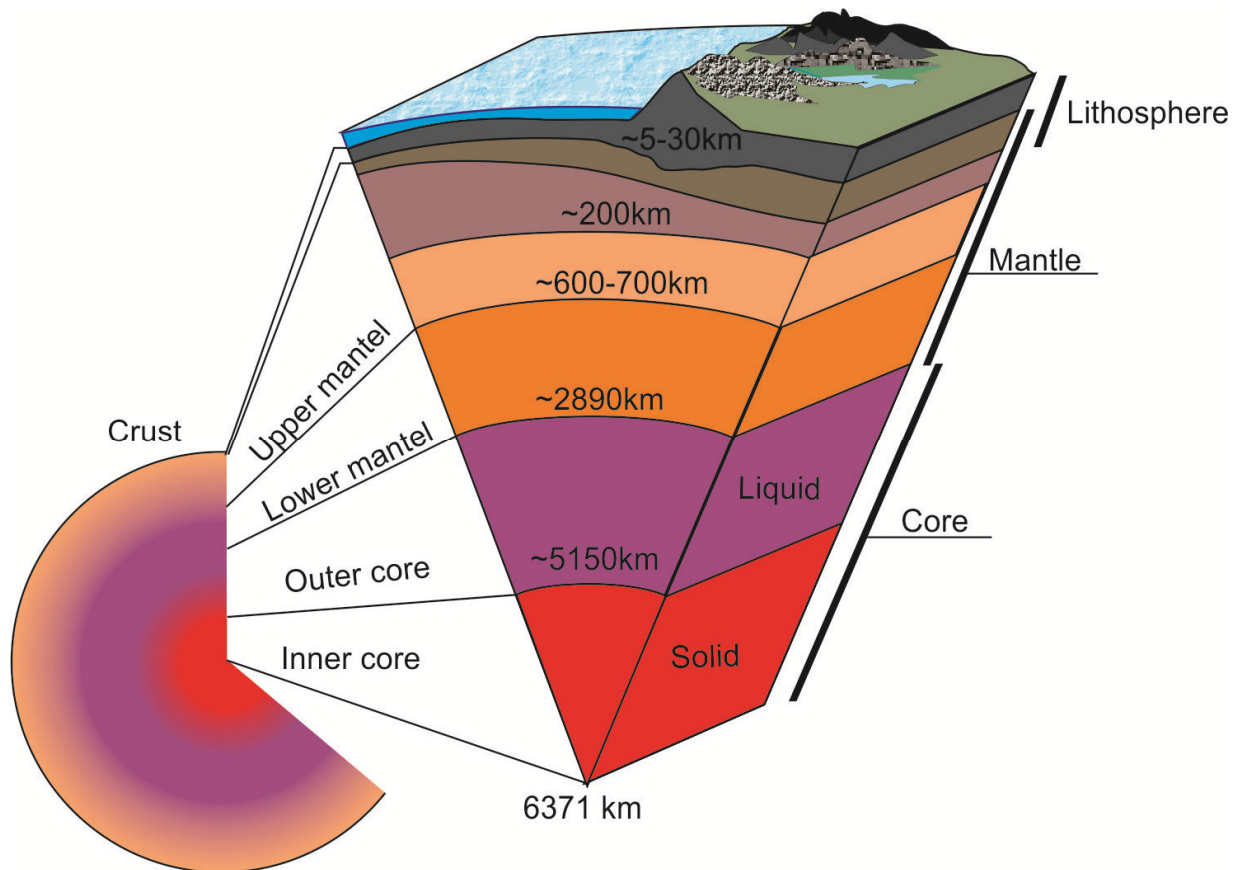


Figure 4 - The internal geological structure of the Earth. Only the left hand side of the figure is to scale. (After USGS)

## 2.2 The main field ( $B_m$ )

It is general agreed that the main part of  $B$  is produced in the interior of the Earth. The best model for explaining the existence of the geomagnetic field is that the liquid outer core of the Earth maintains an electric current, as a "self-excited dynamo" (Campbell 2003). This geodynamo creates the main field which dominates the long wavelengths of  $B$ . When making mathematical models of the main field, the main data sources are observatories and satellites. The secular or temporal variation of the mail field is slow, maximum about 1% per year (Jacobs 1987).



## 2.3 The crustal field ( $B_c$ )

The crustal field is created by magnetized rocks in the crust and upper mantle. The magnetization is caused by induction in the rocks which had sufficient magnetic susceptibility and in addition by permanent magnetization into the rocks when they cooled down (Jacobs 1987). In contrast to the main field, the crustal field may have large gradients and thereby dominates the short wavelengths. To be able to study the crustal field, closely spaced surveys taken from a ship or aircraft are required. The crustal field component cannot be separated from the other sources by measurements taken at only one location. When analysing the crustal field for a drill-site, the area actually surveyed has to be far larger to be able to determine the local magnetic field crust-vector. The calculations are performed using Fourier transformation techniques and the vertical extrapolation is called downward continuation (e.g. Waag et al., 1999). In directional drilling the method of deducing the crustal components of the geomagnetic field is known as IFR. The crustal field is often referred to as the anomaly field and is stable on geological time scales.

## 2.4 The disturbance field ( $B_d$ )

The Sun emits plasma which spreads out through the solar system as the solar wind. When the solar wind reaches the Earth's magnetosphere, it interacts with the geomagnetic field, giving rise to electric currents. Variations in solar wind result in fluctuations in these currents which eventually create the disturbance field. In **Figure 5** the main currents within the magnetosphere are illustrated.

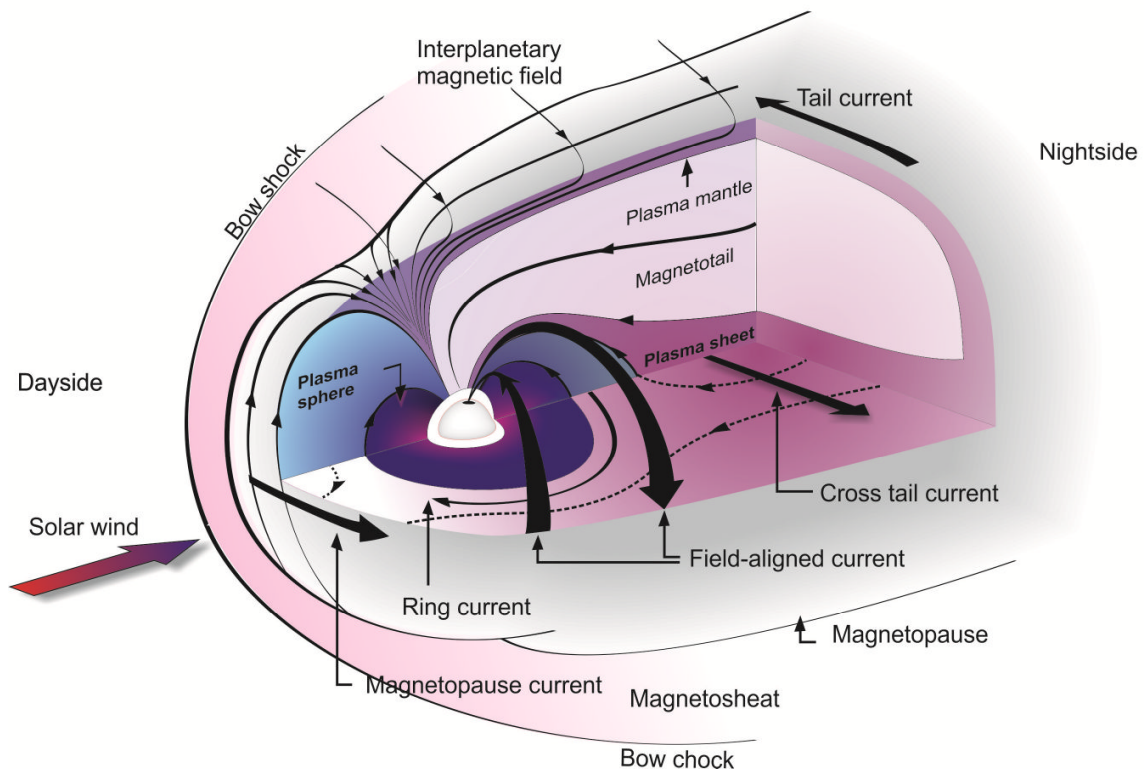


Figure 5 – Illustration of the main currents in the outer magnetosphere; the magnetopause current, the tail current, the ring current and the field-aligned currents. The illustration is after Russel (2001).

On its way from the Sun to the Earth the solar wind carries along some of the Sun's magnetic field, the IMF (e.g. Cowley 2007). The electric conductivity of the solar wind is very high and thus a magnetic field appears as being "frozen" into its plasma. The solar wind compresses the Earth's magnetic field on the dayside and drags it out on the night side, thereby shaping the magnetosphere. The magnetosphere becomes asymmetric with size about 10 Earth radii on the sunward side and beyond 200 Earth radii on the side of the magnetotail. Furthermore, the IMF couples to the Earth's magnetic field when they are antiparallel, i.e. the IMF is pointing southward. The effect of this magnetic reconnection is a transport of magnetic flux from the dayside, over the polar caps to the magnetotail. In the night side the opened field lines are closed, again through magnetic reconnection, the magnetic lines return to their normal dipole-like shape and returns to the dayside. This cyclic behaviour was first described by Dungey (1961). The Dungey cycle gives rise to electric fields in the polar regions of the ionosphere and drives a two-cell convection pattern with an anti-sunward flow in the polar cap and sunward flow further south. In the upper ionosphere this is only plasma convection, but in the lower ionosphere, the E-layer, charge separation is experienced and an electric current is generated along and oppositely directed to the plasma circulation (e.g. Prölss 2004). The sunward part of this convection gives is known as convection or auroral electrojets.

The current system within the magnetosphere includes several interconnected current, see **Figure 5**. The main currents are; the magnetopause current on the dayside, the cross magnetotail currents and the westward flowing equatorial ring current. Both the magnetopause current flowing on the boundary of the magnetopause and the magnetotail currents are relatively distant and thus create little disturbances on the Earth's surface. However, the ring current which flows in equatorial plane at a distance of about five Earth's radii may create fluctuations in the geomagnetic field. Large magnetic disturbances at lower latitudes are usually associated with global magnetic storms and are presumably caused by the ring current. The currents in the magnetosphere are connected to currents in the ionosphere through the field-aligned currents, also known as Birkeland currents.

An important feature of magnetospheric physics creating large geomagnetic disturbances at auroral latitudes is the magnetospheric substorm. Under given circumstances where the magnetosphere has accumulated sufficient energy from the solar wind, the magnetotail current is disrupted and "short-circuited" in the ionosphere via the field-aligned currents. Inside the auroral oval in the midnight sector, a current channel flowing from east towards west is created, the so-called substorm electrojet. On the Earth's surface the largest geomagnetic disturbances are caused by the substorm electrojets. During the expansion phase of a substorm, 40% of the geomagnetic disturbances are caused by induced currents in the crust (Tanskanen et al. 2001).

## 2.5 Components of the Earth's magnetic field

The flux density  $\mathbf{B}$  of the Earth's magnetic field can be described in several ways. In spherical coordinates, the vector is completely defined by the elements  $D$ ,  $I$  and  $F$ . This is illustrated in **Figure 6**, where  $M_N$  is magnetic north and  $T_N$  is true north.

$D$  is the declination, which is the angle between magnetic north and true north, measured in the horizontal plane. The declination is positive when magnetic north lies to the east of true north. As illustrated in **Figure 10**, the declination can also be defined in degrees East (E) or West (W).

Inclination ( $I$ ) is the angle between the local magnetic field and the horizontal plane. North of magnetic dip equator the inclination is positive and negative to the south. In **Figure 11** the sign of the magnitude is replaced by degrees "up" or "down".

Finally,  $F$  is the strength of the magnetic field, with units in Teslas or nanoteslas. A total intensity map is shown in **Figure 11**.

In directional surveying the DIF frame of reference is commonly used, but the terminology for  $I$  is "dip angle" and not inclination.

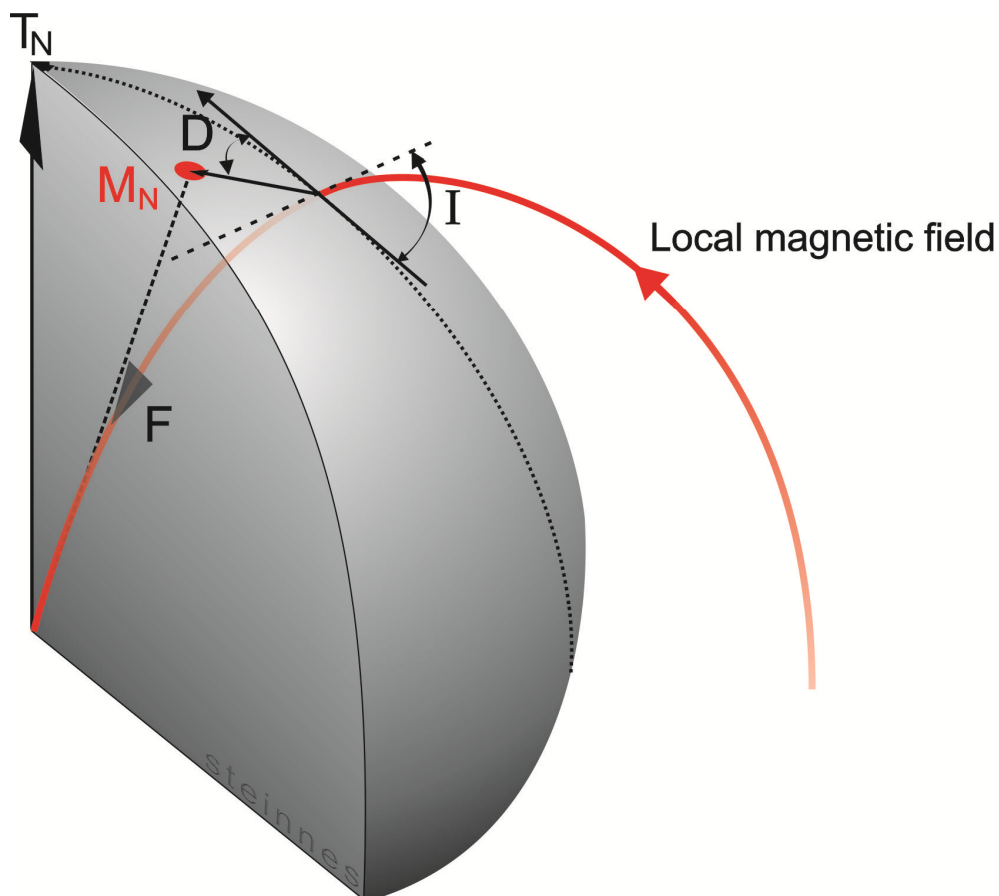


Figure 6 - The Earth's magnetic field parameters in a spherical coordinate reference frame.

In a Cartesian coordinate system the geomagnetic field parameters are defined as shown in **Figure 7**.

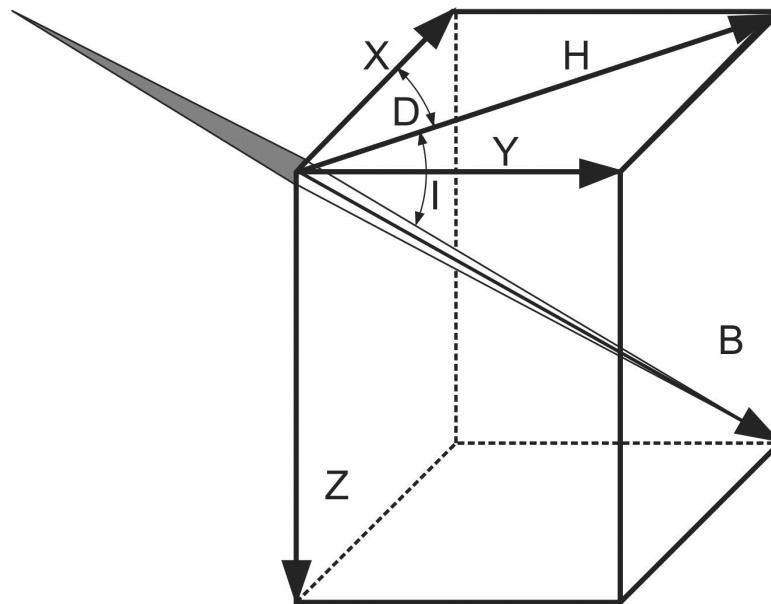


Figure 7 - The Earth's magnetic field parameters in Cartesian and cylinder coordinates.

The elements X, Y and Z make up the orthogonal set of geographic north (X), geographic east (Y) and vertical intensity (Z), all with units in nanoteslas. The cylinder coordinates D, H and Z can also be used to describe **B**. The **equations (2) to (5)** describe the relationship between the elements used in the different coordinate systems.

$$D = \arctan \frac{Y}{X} \quad (2)$$

$$I = \arctan \frac{Z}{H} \quad (3)$$

$$F = \sqrt{H^2 + Z^2} \quad (4)$$

$$H = \sqrt{X^2 + Y^2} \quad (5)$$

## 2.6 Spherical harmonic analysis

Observations have shown that the geomagnetic field changes with location and time. These measurements are used when describing the properties of the Earth's magnetic field and the most common way to model it is to make use of spherical harmonic analysis (SHA). The two Maxwell **equations (6)** and **(7)** define the starting point in such a process.

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \quad (6)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (7)$$

$\mathbf{B}$  is the magnetic induction,  $\mathbf{J}$  is the current density and  $\frac{\partial \mathbf{D}}{\partial t}$  is the time derivative of the electric flux density. In a region free of electrical currents, such as between the Earth's surface and the ionosphere,  $\mathbf{J}$  is negligible and  $\frac{\partial \mathbf{D}}{\partial t}$  is a slow process. This results in a field that is curl-free,  $\nabla \times \mathbf{B} = 0$  which allows us to represent  $\mathbf{B}$  as the gradient of a scalar,  $V$ , as the curl of a gradient is zero.

$$\mathbf{B} = -\nabla V \quad (8)$$

The divergence of  $\mathbf{B}$ , **equation (7)**, is also zero, as magnetic monopoles do not exist in space. Thus, the magnetic potential  $V_M$  must satisfy the Laplace equation:

$$\nabla \cdot \mathbf{B} = 0 \rightarrow \nabla \cdot (-\nabla V_M) = \nabla^2 V_M = 0 \quad (9)$$

The following description on how to use magnetic potential theory to calculate the Earth's magnetic field can be found in e.g. Jacobs (1987) and Blakely (1996).

In spherical coordinates  $(r, \theta, \phi)$ , **equation (9)** can be written as:

$$\frac{1}{r} \frac{\partial^2 (rV)}{dr^2} + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial V}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 V}{\partial \phi^2} = 0, \quad (10)$$

where  $r$  is the radial distance,  $\theta$  is geocentric co-latitude and  $\phi$  is the longitude, respectively. **Figure 8** shows the layout of the spherical coordinate frame.

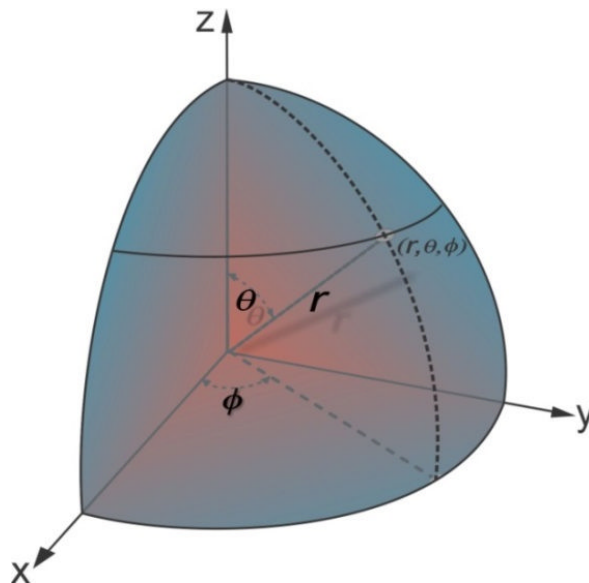


Figure 8 - Spherical Coordinates.

The solution of **equation (10)** becomes a product of three expressions, respectively, as functions of  $r$ ,  $\theta$  and  $\varphi$ . A general solution to this is given by spherical harmonic (SH) functions and can be written as:

$$V(r, \theta, \varphi) = a \sum_{n=1}^{\infty} \sum_{m=0}^n \left[ (A_{n,m} \cos m\varphi + B_{n,m} \sin m\varphi) \left(\frac{a}{r}\right)^{n+1} + (C_{n,m} \cos m\varphi + D_{n,m} \sin m\varphi) \left(\frac{r}{a}\right)^n \right] P_{n,m}(\theta), \quad (11)$$

where  $A_{n,m}$ ,  $B_{n,m}$ ,  $C_{n,m}$  and  $D_{n,m}$  are SH coefficients and  $P_{n,m}(\cos \theta)$  are the Legendre-functions of integer degree  $n$  and order  $m$ . Within geomagnetism, the partial normalised Schmidt functions  $P_n^m(\theta)$  are used. When SH functions are used to describe the geomagnetic field, the scalar magnetic potential is given as a sum of two sources, namely the internal  $V_{\text{int}}$  and external  $V_{\text{ext}}$  sources. The potential is then given by:

$$V = V_{\text{int}} + V_{\text{ext}} \quad (12)$$

As SH functions, the following scalar expressions are used:

$$V_{\text{int}} = a \sum_{n=1}^{N_i^{\text{max}}} \left(\frac{a}{r}\right)^{n+1} \sum_{m=0}^n ((g_n^m \cos m\varphi + h_n^m \sin m\varphi)) P_n^m(\theta) \quad (13)$$

$$V_{\text{ext}} = a \sum_{n=1}^{N_e^{\text{max}}} \left(\frac{r}{a}\right)^n \sum_{m=0}^n ((q_n^m \cos m\varphi + s_n^m \sin m\varphi)) P_n^m(\theta) \quad (14)$$

In **equations (13)** and **(14)**,  $a$  is the radius of reference, usually a bit smaller than the Earth's radius at equator. The Gaussian SH coefficients  $(g_n^m, h_n^m)$  and  $(q_n^m, s_n^m)$  (in nanoteslas) are estimated based on actual measurements. When the measurements are globally evenly spread, the infinity sum of **equations (13)** and **(14)** can be approximated to  $N^{\text{max}}$ . As it is practically impossible to measure  $\mathbf{B}$  all over the world, these coefficients are achieved by the use of the least square method adjustment. At the time when Carl Friedrich Gauss invented this method in 1839, observatory data from large parts of the world were missing (Jacobs 1987). Gauss used  $N_i^{\text{max}} = 4$  and  $N_e^{\text{max}} = 0$ . The internal magnetic parts of the magnetic field measured on the Earth's surface is dominated by components from the core and defined by the degree up to  $N_i = 13$ .

Most of today's geomagnetic models are based on SH. In section **2.8** some of the most common geomagnetic models are described.

## 2.7 Geomagnetic coordinates and time

### The tilted dipole and dipole coordinates

The simplest SH model of the geomagnetic field is a so-called dipole ( $V^D$ ). To describe the potential of a geocentric dipole field, only the first grade coefficients ( $g_1^0, g_1^1, h_1^1$ ) are taken into account. By calculating the potential of a dipole centred as the core, the dipole moment can be achieved. Trigonometric considerations of the dipole moment components ( $m_x, m_y$  and  $m_z$ ) can then be used to calculate the orientation of a centred dipole, as shown in **Figure 9**.

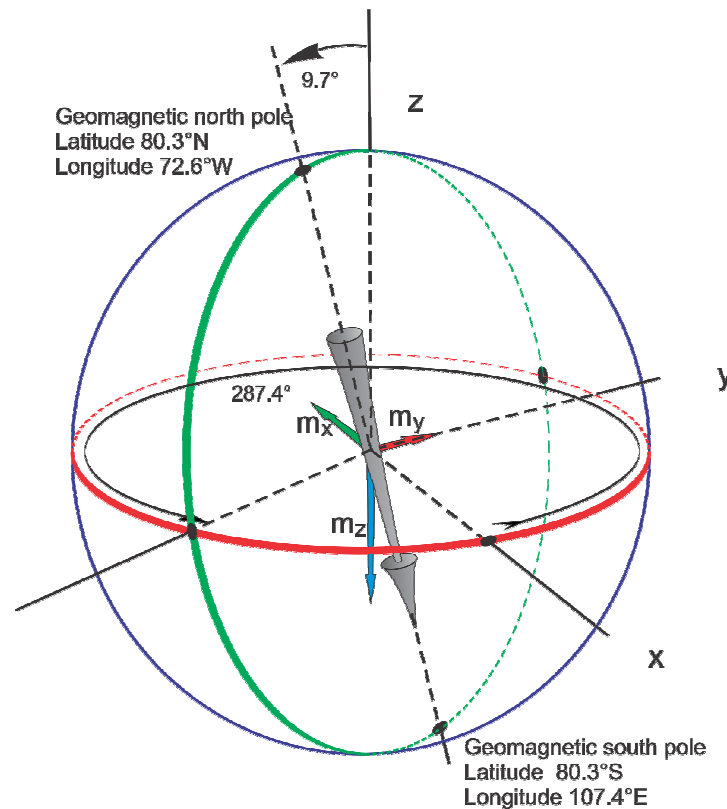


Figure 9 - The orientation of the tilted dipole field based, on IGRF-12 for 2015, with the calculated locations for the geomagnetic north and south poles. The illustration is after Blakely (1996).

### CGM coordinates

A coordinate system that is often used when analysing geophysical phenomena is the corrected geomagnetic (CGM) coordinate system.

By definition the CGM coordinates of a point P in space is calculated by tracing the geomagnetic field line through P using a geomagnetic field model such as IGRF all the way to the dipole geomagnetic equator. When reaching the crossing point between the magnetic field line and dipole geomagnetic equator, return to the same altitude along the dipole field line and assign the obtained latitude and longitude as the CGM coordinates to P, see e.g. ([http://omniweb.gsfc.nasa.gov/vitmo/cgmm\\_des.html](http://omniweb.gsfc.nasa.gov/vitmo/cgmm_des.html), October 2015). CGM coordinates take into account higher SH terms than coordinates based on the simple tilted dipole field model (e.g. MacMillan and Grindrod, 2010).

## Magnetic local time

When studying geomagnetic field variations caused by the external magnetic field, magnetic local time (MLT) is often used as the time reference. MLT is defined relative to the dipole coordinate system (e.g. Jacobs 1987). For a location "L" on the Earth's surface geomagnetic noon is when the subsolar point "S" is on the geomagnetic meridian of L. S is where the Sun is in zenith. In degrees, MLT for location L is defined as:

$$\text{MLT} = 180^\circ + \varphi_L + \varphi_S \quad (15)$$

Where  $\varphi_L$  and  $\varphi_S$  are the geomagnetic longitudes for L and S, respectively.

In simple terms midnight MLT is when L, the nearest geomagnetic pole and the Sun are on the same plane, respectively.

## 2.8 Geomagnetic field models

There exist several mathematical models of the Earth's magnetic field. Common for all models is that they can be used to calculate both the magnitude and direction of the geomagnetic field (**B**) anywhere on the Earth's surface at a given time. One of the main reasons to why models of the geomagnetic field are made is to be able to separate the contributions from the different sources. Some of the most advanced models take into account all the three sources **B<sub>m</sub>**, **B<sub>c</sub>** and **B<sub>d</sub>**, while others might only include the contribution from **B<sub>m</sub>**.

The first model of the Earth's magnetic field was a physical model with the name "Terrella" made by William Gilbert around the year 1600 (e.g. Chapman 2007). In his book "De Magnete", Gilbert illustrated the model with small magnetised pins on the surface of a globe. By this major progress of Gilbert, he was able to study and model different phenomena in a laboratory that actually agreed with actual measurements made at different places on the Earth.

In the 1830's Carl Friedrich Gauss developed a method on how to make absolute measurements of the F-component of the geomagnetic field (e.g. Linthe 2007). In addition, Gauss demonstrated how to perform the measurements and formed a system for absolute units based on the three fundamental units: length, mass and time. Gauss invented a method where he represented the geomagnetic field with the gradient of a potential, extended in series of SH functions. He designed this method to obtain the SH coefficients, the so-called Gaussian coefficients. The model was constructed by calculating values from charts of the DIF-components. The SH potential of up to degree  $n = 4$  was calculated using the least squares method on both observed and calculated values of the field components. Even though today's models are more advanced, the mathematical approach is still based on the method established by Gauss almost 200 years ago.



## IGRF

The International Geomagnetic Reference Field (IGRF) is a well-known and often used mathematical model which accounts for the inner parts of the Earth (MacMillan and Finlay 2011). The model is a product of cooperation between several experts within magnetic field modelling and different institutes that gather magnetic data from satellites and ground based stations all over the world. IGRF is made and updated to offer users easy access to a model of the large scale parts of the geomagnetic field, for locations close to the surface of the Earth. The source of this part of the magnetic field is, as described earlier, mainly caused by currents in the liquid part of the Earth's core. IGRF does not take into account the small scale fields caused by the magnetised parts of the crust or the rapid fluctuations in the electrical current system in the magnetosphere and ionosphere. The IGRF models are updated every fifth year and have spherical degree  $n = 13$  for  $\mathbf{B}_m$ . When an IGRF model is defined as definitive, the name of the model changes to Definitive Geomagnetic Reference Field (DGRF). The 12<sup>th</sup> generation of the IGRF model was published in 2014. The **Figures 10 to 13** show the global maps of D, I, F and H for the period 2015.0-2020.0, based on IGRF 12: (<http://www.geomag.bgs.ac.uk/education/earthmag.html>, July 2015).

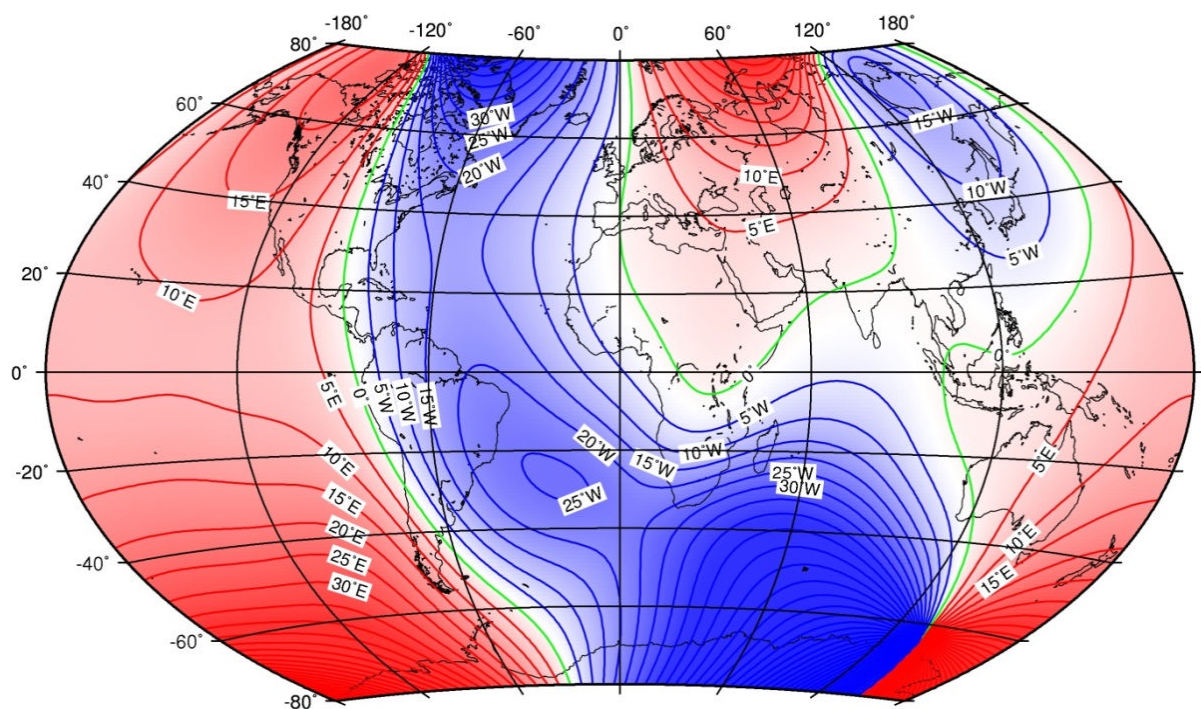


Figure 10 - Map of D (degrees East or West of true north) at 2015.0. (Illustration from BGS)

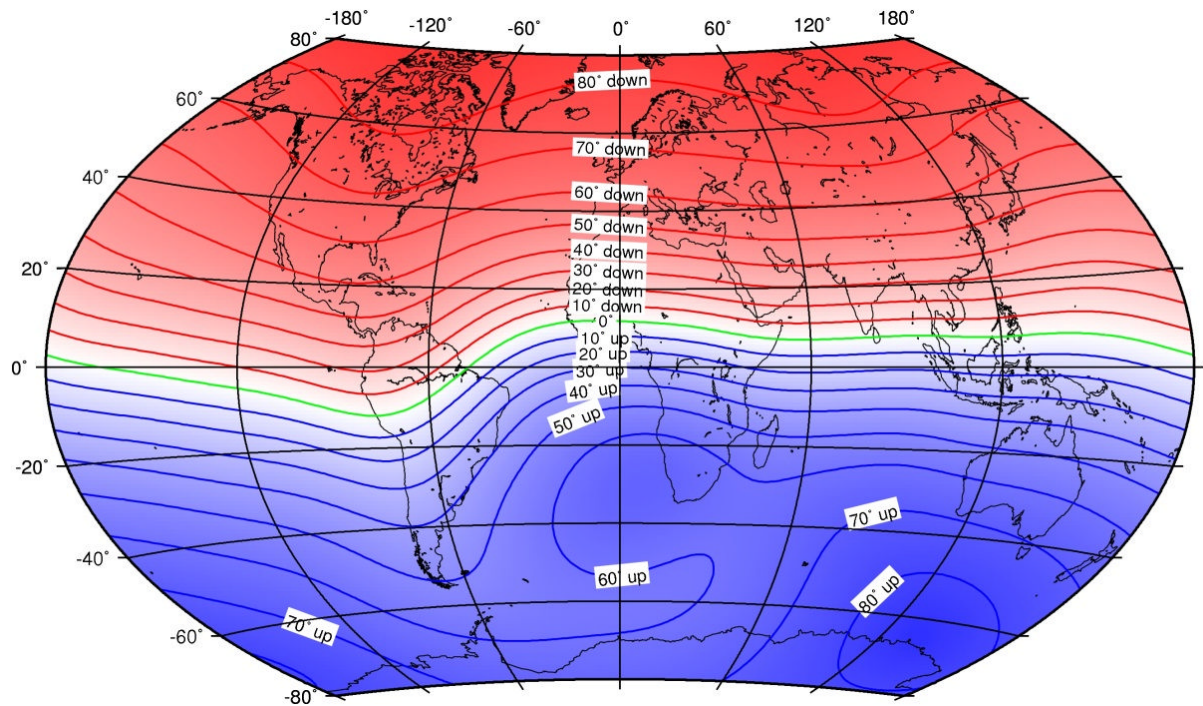


Figure 11 - Map of I (angle in degrees up or down from H) at 2015.0. (Illustration from BGS)

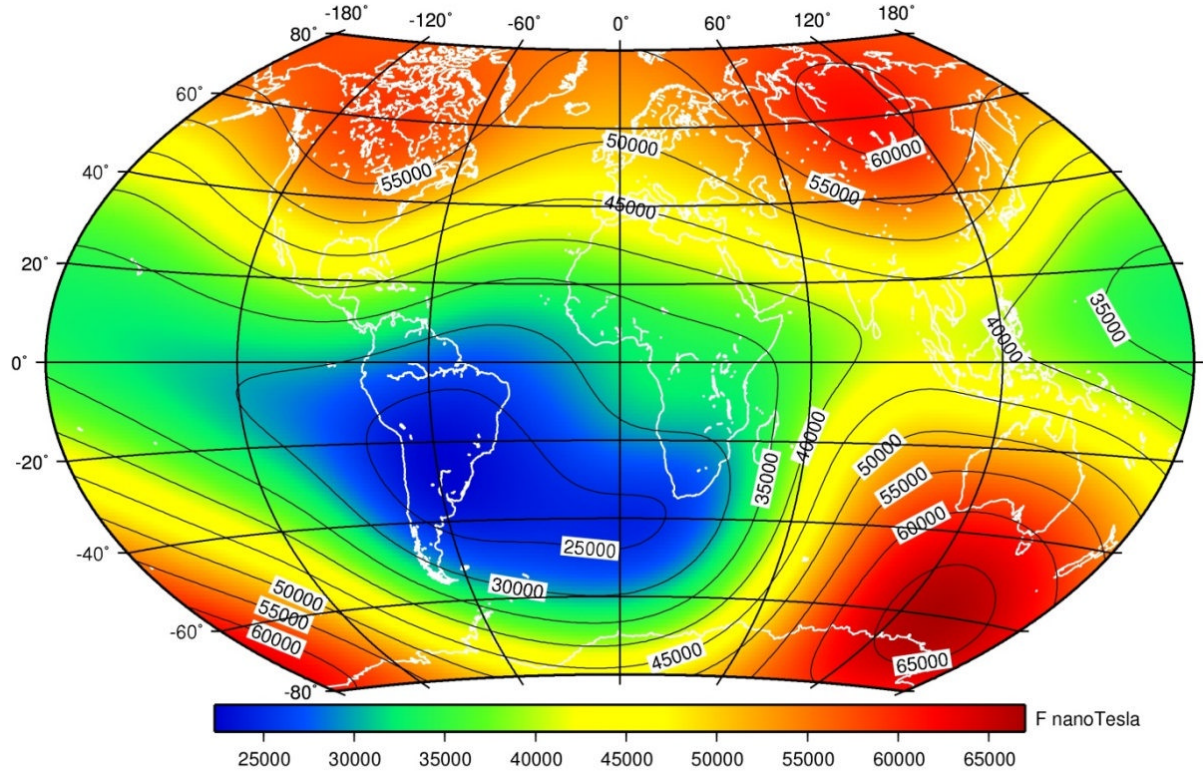


Figure 12 - Map of total intensity at 2015.0. (Illustration from BGS)

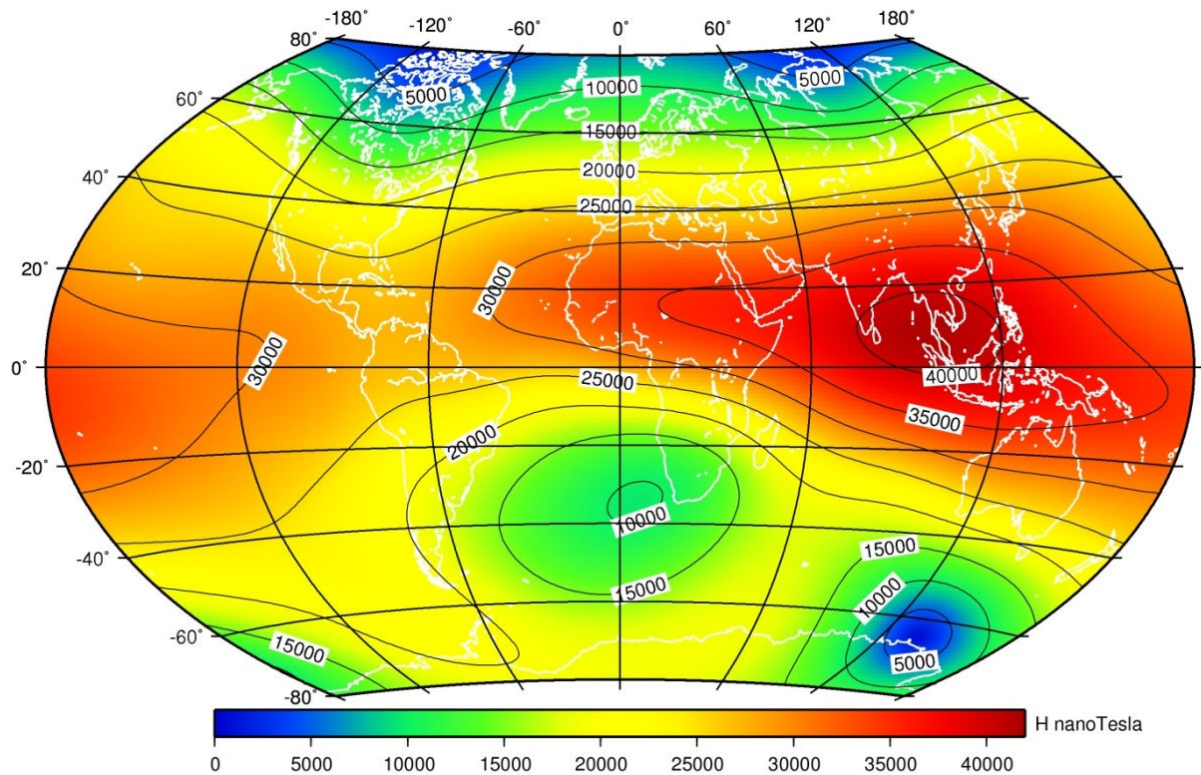


Figure 13 - Map of horizontal intensity at 2015.0. (Illustration from BGS)

### Geomagnetic models used in magnetic directional wellbore surveying

In magnetic directional wellbore surveying, global geomagnetic models are used to calculate the nominal field values for the DIF components along the planned wellbore. The declination ( $D$ ) is used to correct the azimuth from magnetic north to true north, while the dip angle and total magnetic field ( $I$  and  $F$ ) are used as reference values for quality checks of the directional surveys and as input in survey correction algorithms. At drilling locations where there are large magnetic gradients, several sets of DIF calculation along the wellbore might be used to maintain survey accuracy.

The most commonly used geomagnetic model is the BGS Global Geomagnetic Model (BGGM) provided by British Geological Survey (BGS). The model is updated annually (e.g. Lesur et al. 2011). The frequent updates help to reduce the error in the estimate of the magnetic field for any period in the near future. For the BGGM models up to the 2014 revision, the spherical harmonic degree  $n$  for the different sources of  $\mathbf{B}$  were  $n = 15$  for  $\mathbf{B}_m$  and  $n = 50$  for  $\mathbf{B}_c$ .

Another model option is the High Definition Geomagnetic Model (HDGM) made at the National Oceanic and Atmosphere Administration (NOAA). This model goes all the way up to degree  $n = 720$  for  $\mathbf{B}_c$ , and therefore has a better resolution than the BGGM model. However, for  $\mathbf{B}_m$  the degrees  $n$  are similar to the one describing the BGGM model.

### 3 Error models and quality control

In the error models used to quantify the uncertainty associated with the surveyed position, assumptions about adherence to surveying standards and procedures are made. To have confidence in the models' uncertainty estimate it is necessary to test the actual survey data for compliance with those assumptions. The strongest test is the comparison with a completely independent survey, but this is not usually possible in real time and may not be possible at all. It also adds cost. A more limited validation of the survey quality is possible by comparison of the survey tool's measurement of the reference fields (gravity **G** and magnetic **B**) with the equivalent nominal values obtained from global models. The survey tool error model predicts how well the two values should agree. If these comparisons are within specification, there is increased confidence that the calculated drilling angles are also within specification, since the same sensors are used to determine reference fields and drilling angles. The error model generates uncertainties, so the pass/fail limit for such tests must be set at a particular confidence interval, typically  $\pm 3$  standard deviations ( $3\sigma$ ). These internal tests are applied at each survey station and are generally considered to be the minimum requirement in terms of survey quality control (QC). It is common practice to assess the three parameters, **G**, **B** and **I**, but some companies use **G** and a combined **BI** value. The limit values for the geomagnetic parameters will vary with location, the attitude of the wellbore and the source of the nominal reference field. If IFR is applied to reduce the wellbore lateral uncertainty, the **B** and **I** QC-test limits are smaller than if the geomagnetic parameters were calculated by a global model, such as BGGM, only.

For example, taking the location of Tromsø, Norway, a well drilled at  $60^\circ$  inclination and  $60^\circ$  azimuth would be subject to limit values ( $3\sigma$ ) of 525nT for the **B** test and  $0.6^\circ$  for the **I** test, if the BGGM was the source of the nominal reference field. If the reference field was determined from IFR, and therefore the IFR variant of the error model was to be assigned, the  $3\sigma$  limit values decrease to 385nT and  $0.3^\circ$ . The random components of these limit values, accounting for the disturbance field, are 150nT and  $0.18^\circ$ , respectively. These values are in accordance with the disturbance limits listed by Edvardsen et al. (2013). The limit for the gravity-test varies little, and in this example would be  $0.0017\text{m/s}^2$ .

When drilling a well, all the magnetic directional surveys are put into one, or several, survey logs. Often every hole section, see **Figure 1**, is treated as a separate survey log. Each survey log gets an appropriate error model assigned that describes the quality of the surveys. With the error models applied, the wellbore positioning uncertainty can be calculated. Usually the uncertainty is represented by an ellipsoid of uncertainty. This information is essential in well collision avoidance calculations and in determining that there is an acceptably high probability of intersecting the geological target. The Industry Steering Committee on Wellbore Survey Accuracy (ISCWSA) has developed generic error models for both magnetic and gyroscopic surveys; [www.iscwsa.org](http://www.iscwsa.org) and [www.iscwsa.net](http://www.iscwsa.net). Regarding the uncertainties related to the geomagnetic field, the currently used values were calculated by BGS in the early 1990s and are still used in the ISCWSA generic MWD model. The model was first described in 2000 (Williamson 2000) and Rev.3 of the mode I is shown in **Figure 14**.

| Term      | Value (1 sigma)        | Description                              | Azimuth weighting function  |
|-----------|------------------------|--|---|
| ABXY-T1LR | 0.004 m/s <sup>2</sup> | xy accelerometer bias, random            | $(\tan q^* \cos l^* \sin Am)/Gt$  |
| ABXY-T1ZR | 0.004 m/s <sup>2</sup> | xy accelerometer bias, random            | $(- \tan q^* \cos Am + \cot l)/Gt$                                      |
| ABZ       | 0.004 m/s <sup>2</sup> | z-accelerometer bias                     | $\tan q^* \sin l^* \sin Am/Gt$  |
| AMID      | 0.6 °                  | Axial magnetic interference, systematic  | $p/180^* \sin l^* \sin Am$  |
| AMIF      | 0.25 °                 | Axial magnetic interference, fixed       | $p/180$   |
| ASXY-T1IS | 0.0005 fract           | x-accel scale factor, Systematic         | $- (\tan q^* \sin l^* \cos l^* \sin Am)/\text{Sqrt } 2$                 |
| ASXY-T1ZR | 0.0005 fract           | x-accel scale factor, Random             | $- (\tan q^* \sin l^* \cos l^* \sin Am)/2$                              |
| ASXY-T1BR | 0.0005 fract           | x-accel scale factor, Random             | $(\tan q^* \sin l^* \cos Am - \cos l)/2$                                |
| ASZ       | 0.0005 fract           | z-accelerometer scale factor             | $\tan q^* \sin l^* \cos l^* \sin Am$                                    |
| AZ        | 0.36 °                 | Magnetic declination uncertainty         | $p/180$   |
| DBH       | 5000 deg-nT            | Magnetic declination uncertainty         | $p/180/(B^* \cos q)$  |
| DREF      | 0.35 m                 | Depth reference, random                  | 0   |
| DSF       | 0.00056 fract          | Depth scale factor, systematic           | 0   |
| DST       | 2.5e-007 1/m           | Depth stretch/temperature, global        | 0   |
| MBXY-T1LR | 70 nT                  | xy mag bias, random                      | $- \cos l^* \sin Am/(B^* \cos q)$                                       |
| MBXY-T1ZR | 70 nT                  | xy mag bias, random                      | $\cos Am/(B^* \cos q)$  |
| MBZ       | 70 nT                  | z-magnetometer bias                      | $- \sin l^* \sin Am/(B^* \cos q)$                                       |
| MSXY-T1IS | 0.0016 fract           | xy-magnetometer scale factor, Systematic | $\sin l^* \sin Am^*(\sin l^* \cos Am + \tan q^* \cos l)/\text{Sqrt } 2$ |
| MSXY-T1ZR | 0.0016 fract           | xy-magnetometer scale factor, Random     | $\sin Am^*(\tan q^* \sin l^* \cos l - \cos 2l^* \cos Am - \cos Am)/2$   |
| MSXY-T1BR | 0.0016 fract           | xy-magnetometer scale factor, Random     | $(\cos l^* \cos 2Am - \cos l^* \sin 2Am - \tan q^* \sin l^* \cos Am)/2$ |
| MSZ       | 0.0016 fract           | z-magnetometer scale factor              | $- \sin l^* \sin Am^*(\sin l^* \cos Am + \tan q^* \cos l)$              |
| MXYaS     | 0.06 °                 | xy tool misalignment, systematic         | 0   |
| MXYbS     | 0.06 °                 | xy tool misalignment, systematic         | $- p/180^*[1 + Flw^*(\sin l - 1)]/\sin l$                               |
| MXYcS     | 0.06 °                 | xy tool misalignment, systematic         | $- p/180^*Flw^* \cos l^* \sin At/\sin l$                                |
| MXYdS     | 0.06 °                 | xy tool misalignment, systematic         | $p/180^*Flw^* \cos l^* \cos At/\sin l$                                  |
| SAG       | 0.2 °                  | BHA sag                                  | 0   |

Figure 14 - The ISCWSA MWD, Rev.3 (Standard) error model.

Some of the terms in the error model relate to depth and inclination (Inc). However, the error terms of interest in this context are those that are dependent on geomagnetic location. Under "Azimuth weighting functions" we see that the uncertainty related to magnetic declination (D) and the magnetometer scale factors are dependent on  $(B \cdot \cos q)$ . Here  $q = I \rightarrow (B \cdot \cos q) = H$ . Regarding the declination, it is split into two terms, namely AZ and DBH. This is to take into account the increased uncertainty in D as H decreases.

For a typical location in the auroral zone where the magnitude of the horizontal magnetic field (H) is about 10 000nT, the uncertainty in nominal D ( $1\sigma$ ) is  $\sqrt{(AZ^2 + (DBH/10000)^2)} = 0.76^\circ$ . If H = 5000nT, the estimated uncertainty in declination increases to  $1.06^\circ$ .

To account for progress in geomagnetic modelling during the last decade, BGS has suggested a new set of geomagnetic uncertainty estimates to be used in the magnetic MWD error models (MacMillan and Grindrod 2010). These so-called "look-up tables" reflect the non-Gaussian distribution of the errors and model the geographic location dependency better than the term weighting functions used in the current model.

## 4 Summary of papers

### Paper 1

Edvardsen, I., Hansen, T. L., Gjertsen, M., & Wilson, H.

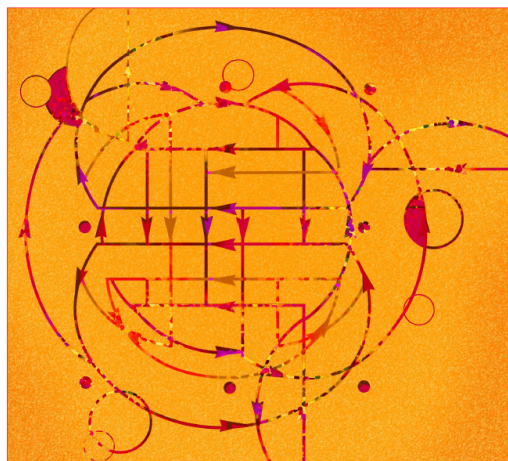
#### **Improving the Accuracy of Directional Wellbore Surveying in the Norwegian Sea.**

Society of Petroleum Engineers. doi:10.2118/159679-PA

In connection with the analyses of the external geomagnetic variations in the Norwegian Sea area, it was confirmed that data from landbased variometer stations could be used to correct magnetic directional wellbore measurements far from land. The paper describes the challenges related to magnetic directional surveying in the sub auroral zone. The external magnetic field and especially the electrojet system at high latitudes are described. A new method of calculating the quiet level for the different geomagnetic parameters measured by a variometer is outlined. Furthermore, a simple method on how to correct for external magnetic field variations is described. Based on analysis of data from several variometer stations in Norway, Sweden and Finland it is shown that the deviations in the total magnetic field and dip angle readings during moderate disturbed periods are similar over several hundreds of kilometres along the same geomagnetic latitude. This is explained by the current in the ionosphere flows along the auroral oval, which is more or less aligned with the geomagnetic latitude. Regarding the declination, the correlation over large distances is less obvious, especially during the highly disturbed periods. A practical way to use magnetogram data from a variometer in the vicinity of a drilling site to correct directional survey measurements is shown.

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Figure 15 - SPE Drilling & Completion Journal June 2013. The illustration is based on the figure 2 in the article.

## Paper 2

Edwardsen, I., Nyrnes, E., Johnsen, M. G., Hansen, T. L., Løvhaug, U. P., & Matzka, J.

### **Improving the Accuracy and Reliability of MWD/Magnetic-Wellbore-Directional Surveying in the Barents Sea.**

Society of Petroleum Engineers. doi:10.2118/166226-PA

Areas within the auroral zone are on a daily basis affected by the electrojet currents in the ionosphere. The results from analysis of the geomagnetic conditions in the Barents Sea area, show that the variations in the geomagnetic field can be completely different on distances over some hundred kilometres. This is often the case when comparing the observed disturbances at Sørøya (Norwegian main land) and Bjørnøya. The distance between the two locations is 400km in the north-south direction. Furthermore, the geomagnetic variations during a day are not necessarily randomly distributed in this area. Often the declination may have a continuous offset from quiet level for several hours. This impact of the electrojet currents may have a devastating effect on magnetic wellbore directional surveying. The network of geomagnetic monitoring stations in the Barents Sea area is very sparse, due to the land/sea configuration. Thus, this reduces the ability to apply geomagnetic data from landbased monitoring stations to estimate the variations in the geomagnetic field at offshore locations. Especially this applies to the components total magnetic field and dip angle. To account for the disturbance field effects in the Barents Sea area, during drilling operations, special caution should be made in using survey correction algorithms which are dependent on accurate input of the parameters total magnetic field and dip angle. This issue can be solved by installing seabed magnetometer stations in the vicinity of the drilling site. However, it is recommended to apply uncorrected/standard surveys to reduce the impact from the external magnetic field. This means that bottom hole assemblies need to be designed with sufficient amount of non-magnetic steel components to satisfy requirements regarding drillstring interference.



### **Paper 3**

Edvardsen, I., Johnsen, M. G., Løvhaug, U. P.

#### **Effects of substorm electrojet on declination along concurrent geomagnetic latitudes in the northern auroral zone.**

*Submitted to Journal of Space Weather and Space Climate*

The largest geomagnetic disturbances at auroral latitudes are caused by magnetic substorms. Such disruptions of the geomagnetic field pose challenges for activities such as oil well directional drilling that rely on the geomagnetic field as a directional survey reference. In magnetic directional surveying the declination is used to correct the azimuth measurement from magnetic north to true north. Therefore we are especially interested in the effect the substorm electrojet has on the magnetic declination angle. The deflection in the horizontal magnetic field component is used to determine the signatures of the geomagnetic fluctuations and decide whether the disturbance is caused by a substorm or not. In principle, the magnitude of geomagnetic disturbances from two identical substorms along concurrent geomagnetic latitudes, at different local times, will be the same. However, as a combination of the chosen coordinate system and the background geomagnetic field, one may anticipate that the signature of a substorm will vary as a function of geomagnetic longitude. The longitudinal distribution of geomagnetic disturbances due to the substorm electrojet has not yet been analysed in detail. To investigate and quantify this, a substorm current wedge model of the Earth's magnetic field is applied. Line currents are used to simulate geomagnetic substorms of different morphologies and at different local times within the northern auroral zone. The results from the modelling confirm that the major fluctuations in the geomagnetic field are produced by the ionospheric current, while the effects from the field-aligned currents and the equatorial current are much smaller. The largest deviations in the declination during a substorm are expected to occur in Siberia, while the smallest in Alaska. To quality check the results given by the current wedge substorm model, geomagnetic data from several observatories and variometers lying in the northern auroral zone are analysed with respect to variations in the horizontal magnetic field and the declination. The analysis mostly verifies the results given by the current wedge substorm model and confirms that there are large differences in how the geomagnetic field is affected by a substorm electrojet along concurrent geomagnetic latitudes. For directional drilling companies this means that the auroral zone cannot be treated as a region where the geomagnetic conditions are the same everywhere. The industry should take into account the longitudinal variations when deciding how best to manage substorm effects on survey measurements.



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# Appendix



# Appendix A: Absolute orientation in magnetic wellbore directional surveying

In this context, a thorough description of the drilling angles might be seen as unnecessary. However, the freely available literature on magnetic directional surveying contains little or no information on how the equations commonly used in magnetic directional surveying can be deduced. Various patents contain different ways of calculating for instance inclination and azimuth, but the derivations can be difficult to follow. Thus, this appendix can be seen as an introduction to the reader who is not familiar with the directional survey terminology and the concurrent drilling angle equations. Some of the layout and derivations in this appendix are the same as in the internal Baker Hughes document "Advanced Magnetic Survey School".

In magnetic wellbore directional surveying the three parameters used to describe the profile of a wellpath are: Measured depth (MD), inclination and azimuth. MD is the length of the drill pipes put in the bore hole together with transition pipe and bottom hole assembly (BHA). Inclination is the angle between vertically down and the path of the borehole in the vertical plane. Azimuth is the clockwise angle between a defined north reference and the projection of the borehole path onto the horizontal plane. The absolute orientation of a survey tool in a bore hole is finally completed by the toolface angle. Both highside toolface (HTF) and magnetic toolface (MTF) are used, but they have different applications. More details about toolface are given in section **A.3.4**. The described parameters and the lower part of a BHA are illustrated in **Figure A. 1**.

In a BHA, the most common components are: drill bit, motor and/or rotary steerable system, directional MWD and formation logging tools.

To calculate inclination, data from three orthogonal accelerometer sensors ( $A_x$ ,  $A_y$  and  $A_z$ ) measuring the Earth's gravity field vector, are used. The azimuth is calculated using measurements of the geomagnetic field vector from three orthogonal magnetometers ( $M_x$ ,  $M_y$  and  $M_z$ ) in addition to output from the three accelerometer sensors.

From an initial location with known coordinates, inclination and azimuth measurements are made at regular intervals along the wellbore, typically every 30m which is the most common length of drillpipe section drilled with. The positions where the measurements are taken are called survey stations and are defined by the parameters MD, inclination and azimuth. To determine the coordinates for a new survey station, the displacements in north (N), east (E) and true vertical depth (TVD or V) between a known station and a new one is calculated. The wellpath between two survey points is assumed to be a smooth curve and the most commonly used algorithm assumes a path of curvature. The geometrical profile of a wellbore is usually given by a set of attributes; MD, inclination, azimuth, north, east and TVD.

The main coordinate systems that are used in magnetic directional survey are the earth-fixed North-East-Vertical (N-E-V) frame and the tool-fixed x-y-z sensor coordinate frame. In this overview, the north referenced is local magnetic north. Both of the coordinate systems are right-handed and orthogonal.

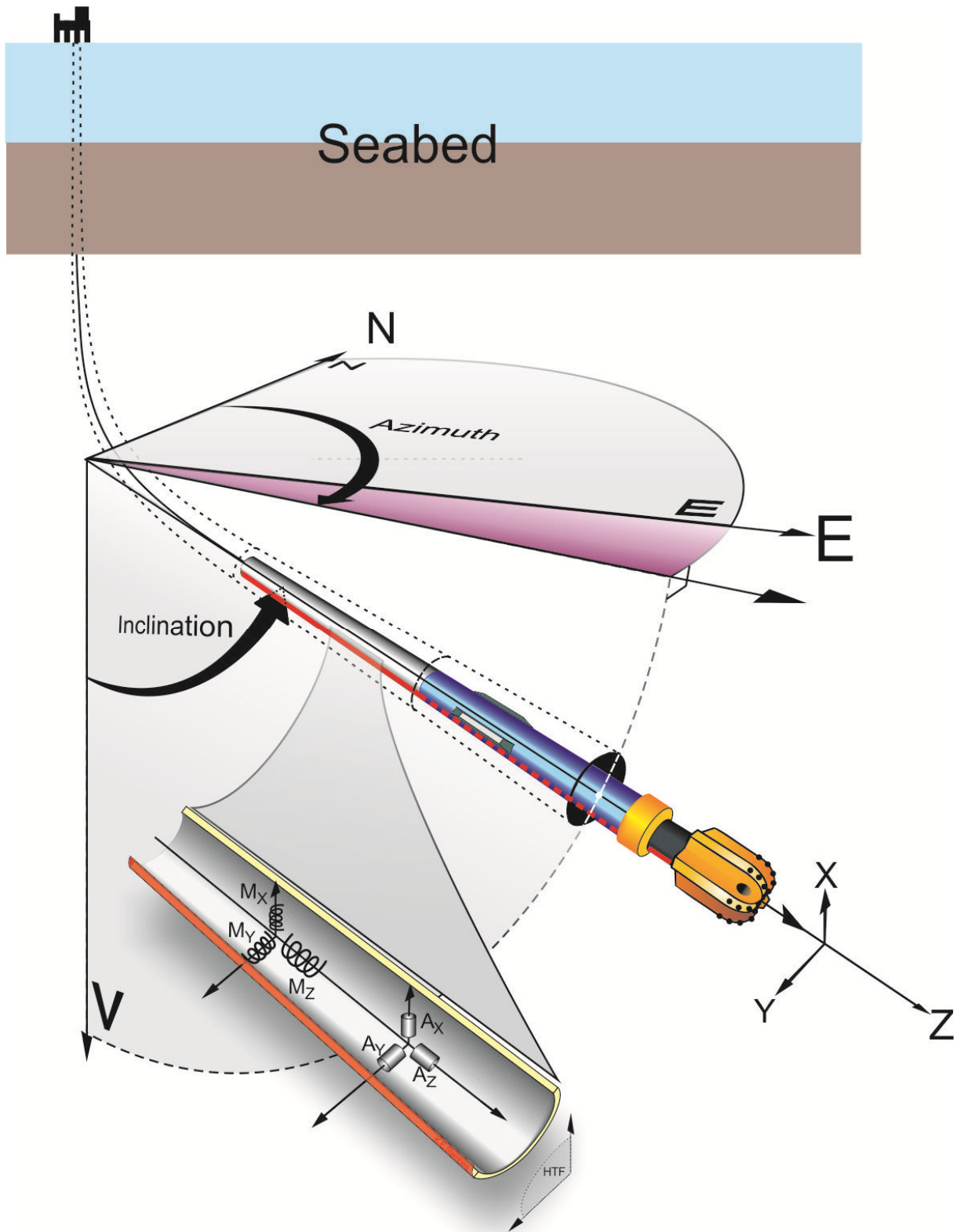


Figure A. 1 - The coordinate systems and drilling angles used in magnetic directional surveying including the lower parts of the bottom hole assembly.



## A.1 The Earth-fixed N-E-V coordinate system

The two physical quantities used to navigate the BHA in a hole in the ground are, as earlier stated, the Earth's gravity and magnetic fields. The tool sensor readings of these quantities are used to calculate the drilling angles. Below, the gravity and magnetic fields of the Earth are referred to the N-E-V frame.

### A.1.1 Earth gravity field ( $\mathbf{G}$ )

The gravity field vector  $\mathbf{G}$  acts only in the vertical plane and can therefore be expressed as the following vector in the N-E-V system:

$$\mathbf{G} = \begin{bmatrix} G_N \\ G_E \\ G_V \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \mathbf{G} \end{bmatrix} \quad (\text{A.1})$$

### A.1.2 Earth magnetic field ( $\mathbf{B}$ )

The Earth's magnetic field acts in the north-vertical plane. Therefore, the magnetic flux density vector  $\mathbf{B}$  is defined by:

$$\mathbf{B} = \begin{bmatrix} B_N \\ B_E \\ B_V \end{bmatrix} = \begin{bmatrix} \mathbf{B} \cdot \cos\theta \\ 0 \\ \mathbf{B} \cdot \sin\theta \end{bmatrix} \quad (\text{A.2})$$

According to vector (A.2), the magnitude of the Earth's magnetic field has a north component defined by  $\mathbf{B} \cdot \cos\theta$ , the east component is zero and the vertical component is  $\mathbf{B} \cdot \sin\theta$ . In this appendix, the notation for dip angle is  $\theta$  and not  $I$ . In directional drilling terminology  $I$  is used as the notation for the drilling angle inclination. The dip angle  $\theta$  ranges from  $-90^\circ$  to  $90^\circ$ , where  $0^\circ$  is along magnetic equator and  $90^\circ$  is at the magnetic north pole.

## A.2 Tool-fixed x-y-z directional sensor coordinate system

It is standard practice within the directional drilling industry to define the axis pointing along the hole as the positive z axis and the x and y axes as the cross axial axes. Any orientation of the BHA can be described by the three rotations yaw, pitch and roll. The directional drilling terminologies for these rotations are azimuth for yaw, inclination for pitch and toolface for roll. In most electronic magnetic survey tools an orthogonal array of three accelerometers and three magnetometers are typically used, as illustrated in **Figure A. 1**. In the tool-fixed x-y-z coordinate system the directional survey sensors, accelerometers and magnetometers, are mutually aligned in the BHA.

### A.2.1 Directional survey sensors

The magnetic directional survey sensors used to measure the vectors  $\mathbf{G}$  and  $\mathbf{B}$  have to be adequately accurate and robust. In a BHA the MWD sensor package is typically located from 10m to 30m behind the drill-bit. The directional sensors are placed in a non-magnetic housing and usually with at least 3m of non-magnetic spacing between the sensors and the nearest steel component. A transmitter that is included in the same downhole tool is used to send the sensor data to the surface via telemetry system. Usually the data are encoded as series of pressure pulses in the drilling fluid column inside the drillstring. On the surface, the data are detected, decoded and processed into drilling angles.

#### Accelerometers

The gravity vector  $\mathbf{G}$  is measured by accelerometer sensors. The sensors typically contain a proof mass suspended on a hinge with freedom to move in one axis only. Acceleration causes the proof mass to move and then the movement is sensed. The direction of motion is along the axis which acceleration is sensed. A preventing torque is applied electrically to return the proof mass to its initial position. Measurements of the current required to provide restraining of the torque is used to calculate the acceleration. The accelerometer sensor package in a survey tool is typically a triaxial array with one sensor along each of the three tool-fixed axes.  $\mathbf{G}$  is therefore completely defined by the tool's accelerometer outputs according to:

$$\mathbf{G} = (G_x^2 + G_y^2 + G_z^2)^{\frac{1}{2}} \quad (\text{A. 3})$$

#### Magnetometers

The magnetometer sensors are used to detect and measure the magnetic field vector  $\mathbf{B}$ , surrounding the BHA. The sensors used in directional survey tools are of the fluxgate type, in which a pair of parallel coils with ferromagnetic cores is excited in opposite directions by an alternating current. A pickoff coil is wrapped around the outside of both cores and responds to any asymmetry in the induced fields. In the absence of an ambient field, the field from one core opposes that from the other and there is no net signal in the pickoff coil. When there is an outside field, the effect in one core is not exactly equal and opposite of the other. Therefore, the pickoff coil detects a current which is proportional to the component of the surrounding magnetic field which is aligned with the axes of the cores. Thus, a triaxial array of fluxgate magnetometer can be used to measure the components of  $\mathbf{B}$  along the three tool-fixed axes.  $\mathbf{B}$  is completely defined by the tool's magnetometer outputs according to:

$$\mathbf{B} = (B_x^2 + B_y^2 + B_z^2)^{\frac{1}{2}} \quad (\text{A. 4})$$

Other names for  $\mathbf{B}$  are magnetic induction and magnetic flux density.

### A.3 Derivation of magnetic directional surveying equations

The drilling angles are calculated from the directional survey sensor outputs. The drilling angle equations can be determined by a transformation between the Earth-fixed N-E-V coordinate frame and the survey tool-fixed x-y-z sensor coordinate frame. The transformation is conducted by rotating a survey tool by the drilling angles to be determined. The rotations may be done in several ways. Rotations can be clockwise or counter clockwise and the start orientation of the survey tool can be vertical or aligned with the actual orientation of the survey tool in the borehole. In this derivation, the start orientation of the survey tool is vertical with the tools x-axis aligned with north (N) and z-axis with vertical (V), see **Figure A. 2**. When the three rotations in the plane of azimuth, inclination and toolface are completed, the orientation of the survey tool in the bore hole is fully defined by the drilling angles. In the remaining part of this appendix, the notation for azimuth is A, inclination is I and toolface is  $\alpha$ .  $\alpha$  is HTF in this case.

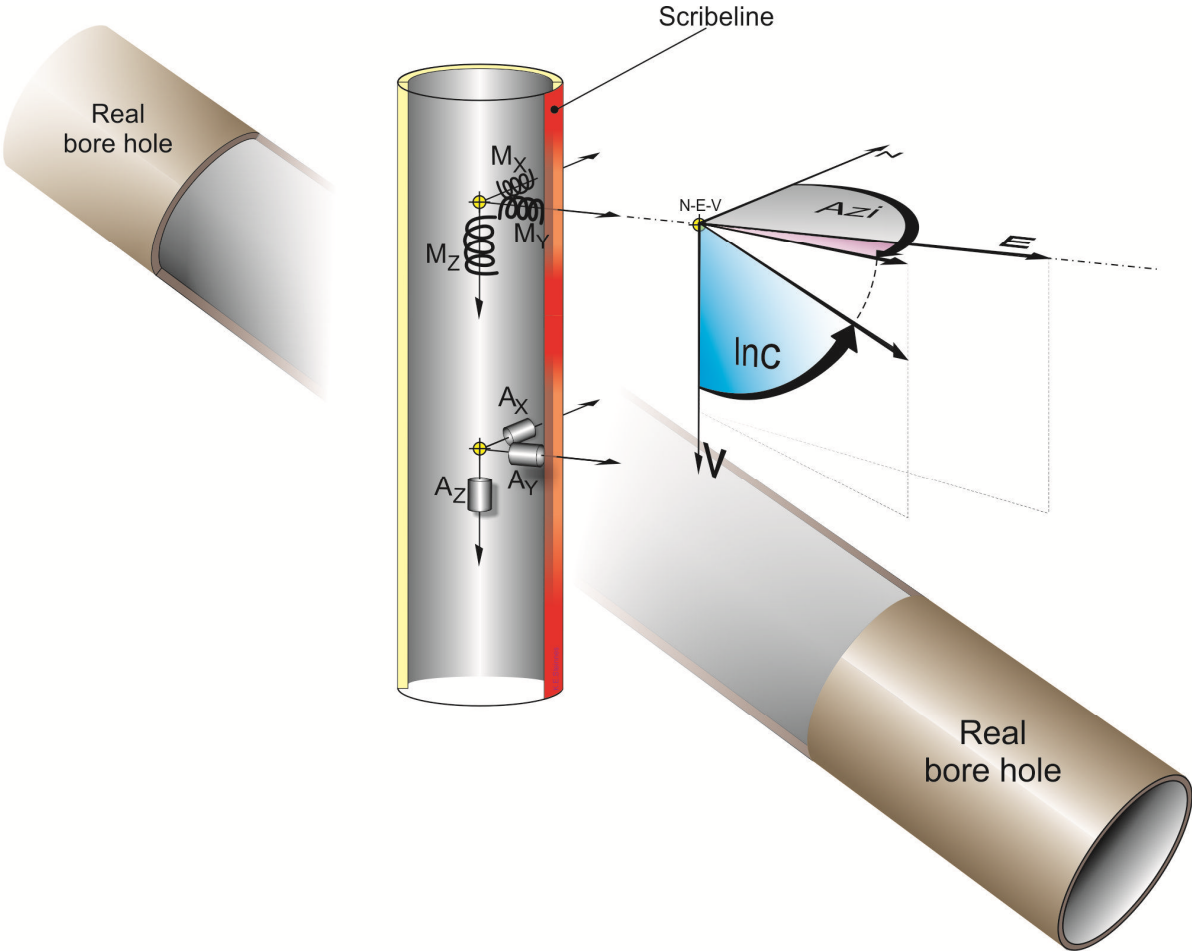


Figure A. 2 - The survey tool in its start position before any rotations are performed.

**A.3.1 Rotation between the N-E-V and x-y-z coordinate frames**

**Rotation in the plane of the azimuth (A)**

In the first rotation, the survey tool is turned clockwise by  $(A - 90^\circ)$  about the z-axis, so that its y-axis points in the azimuth direction of the borehole, see illustration in **Figure A. 3**. The rotation  $R(A)$  about the z-axis has the following form:

$$R(A) = \begin{bmatrix} \sin A & -\cos A & 0 \\ \cos A & \sin A & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{A.5}$$

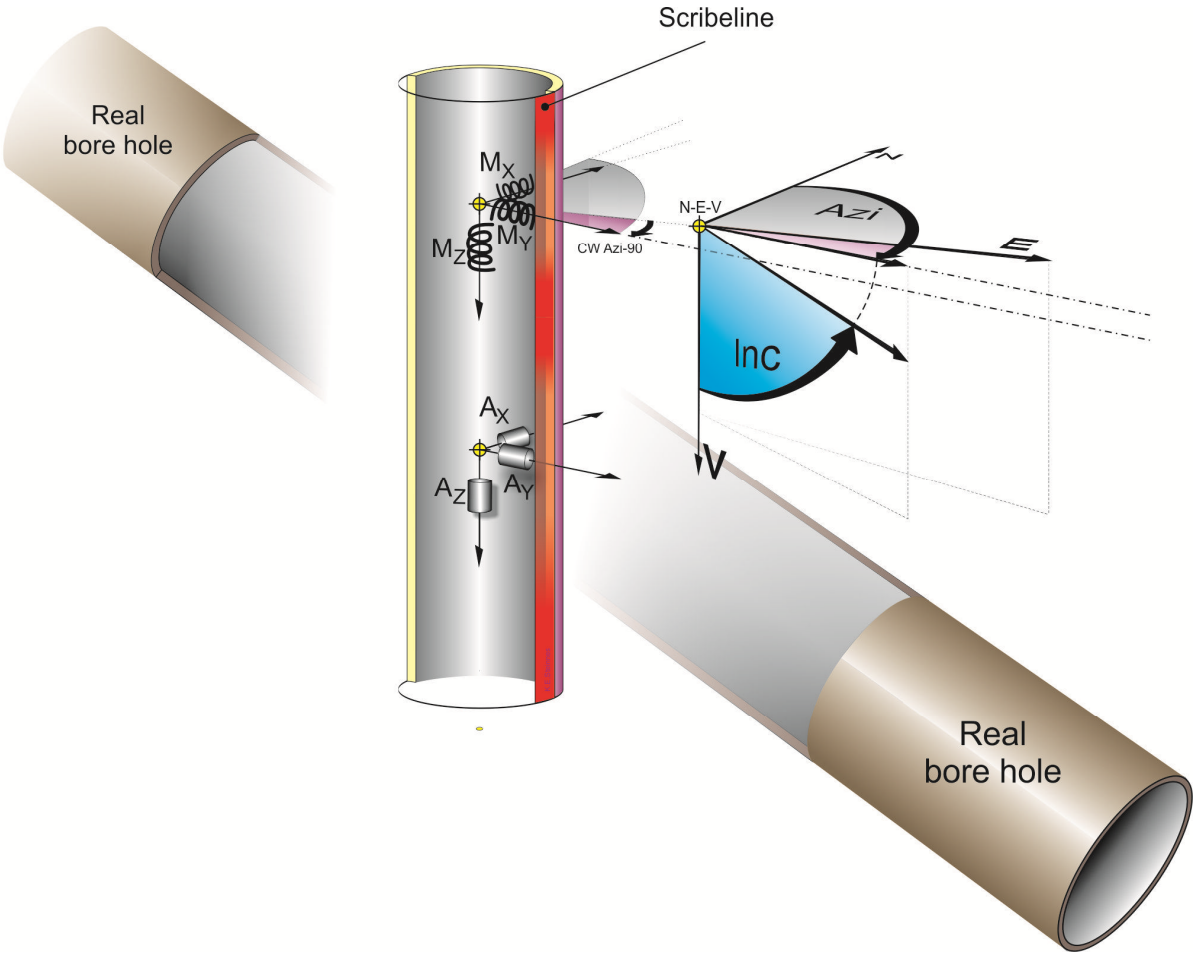


Figure A. 3 - Rotation in the plane of the azimuth.

**Rotation in the plane of the inclination (I)**

In the second rotation, the survey tool is rotated counter clockwise by (I) about its x-axis, to align the tool with the hole inclination and with its y-axis pointing up or towards highside of the tool, as illustrated in **Figure A. 4**. The rotation **R (I)** about the x-axis has the following form:

$$R(I) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos I & -\sin I \\ 0 & \sin I & \cos I \end{bmatrix} \tag{A.6}$$

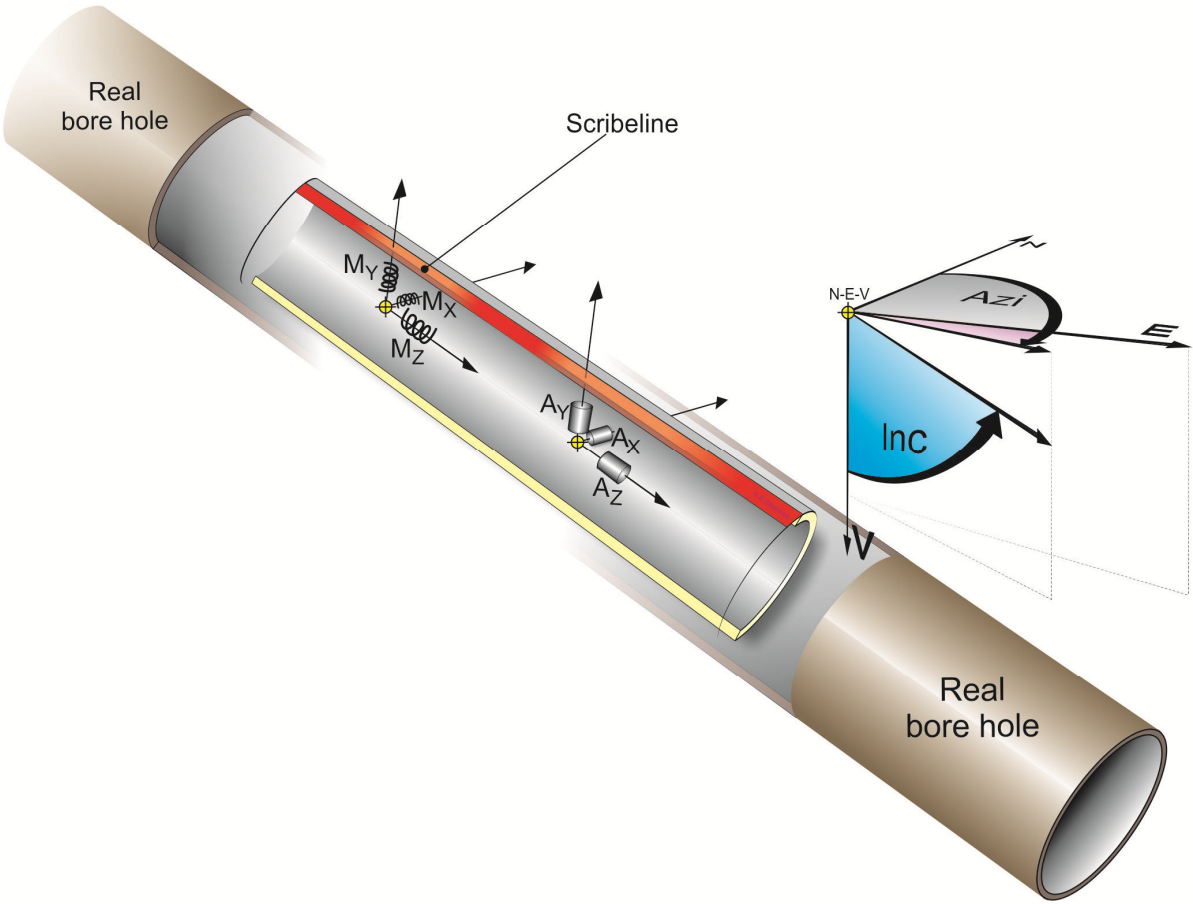


Figure A. 4 - Rotation in the plane of the inclination.

**Rotation in the plane of toolface ( $\alpha$ )**

In the third rotation, the tool is rotated clockwise about the z-axis by ( $\alpha$ ), so that it reaches HTF, as illustrated in **Figure A. 5**. The rotation  $R(\alpha)$  about the z-axis has the following form:

$$R(\alpha) = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{A.7}$$

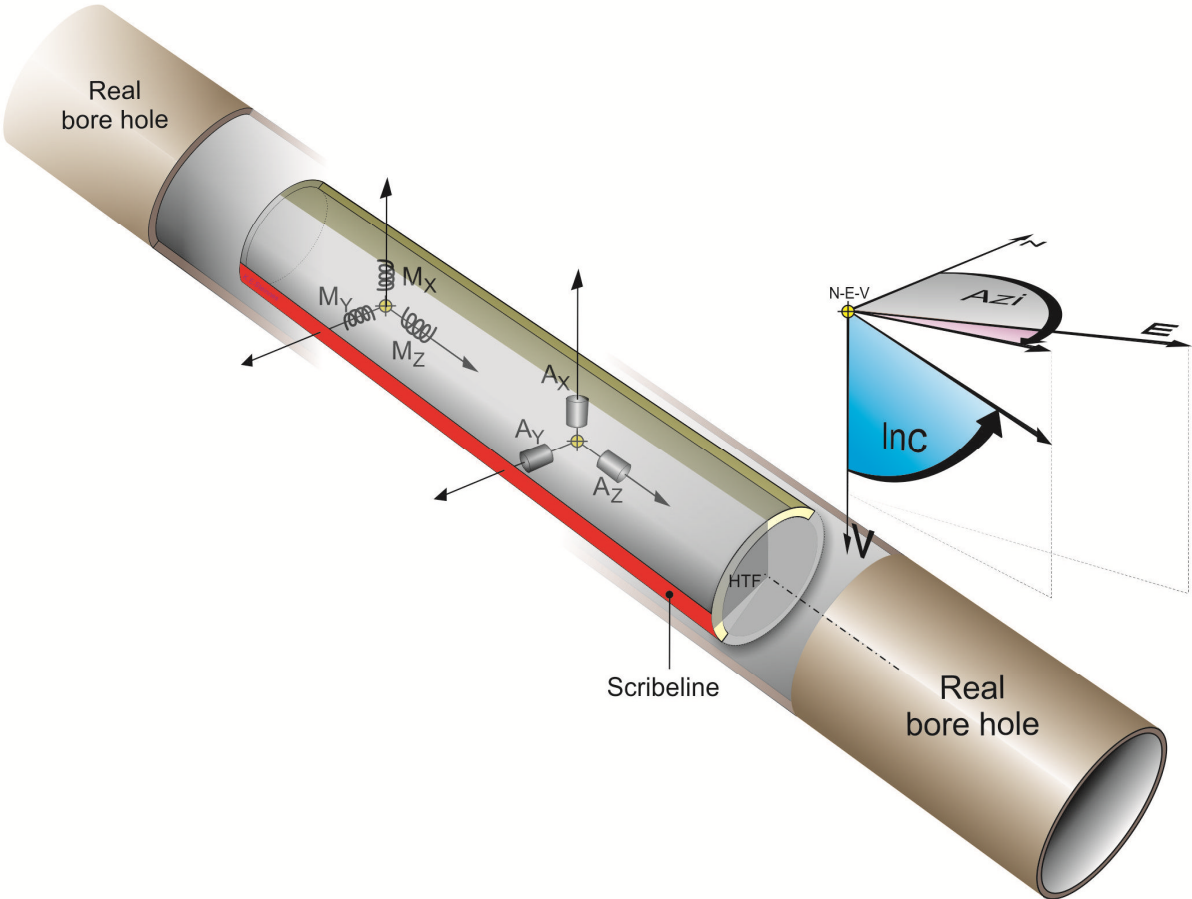


Figure A. 5 - Rotation in the plane of  $\alpha$ . In the figure  $\alpha = HTF$ .

### A.3.2 Equations for the directional survey sensor outputs

When the rotations  $\mathbf{R}(\mathbf{A})$ ,  $\mathbf{R}(\mathbf{I})$  and  $\mathbf{R}(\boldsymbol{\alpha})$  are applied as a series of matrix multiplications, the following transformation matrix  $\mathbf{T}$  between the tools-fixed x-y-z and Earth-fixed N-E-V coordinate frames can be achieved:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \mathbf{R}(\boldsymbol{\alpha}) \cdot \mathbf{R}(\mathbf{I}) \cdot \mathbf{R}(\mathbf{A}) \begin{bmatrix} N \\ E \\ V \end{bmatrix} = \mathbf{T} \begin{bmatrix} N \\ E \\ V \end{bmatrix}, \quad (\text{A.8})$$

where

$$\mathbf{T} = \begin{bmatrix} \cos I \cos A \sin \alpha + \sin A \cos \alpha & \cos I \sin A \sin \alpha - \cos A \cos \alpha & -\sin I \sin \alpha \\ \cos I \cos A \cos \alpha - \sin A \sin \alpha & \cos I \sin A \cos \alpha + \cos A \sin \alpha & -\sin I \cos \alpha \\ \sin I \cos A & \sin I \sin A & \cos I \end{bmatrix} \quad (\text{A.9})$$

The x-y-z components of  $\mathbf{G}$  seen by the accelerometer sensors  $A_x$ ,  $A_y$  and  $A_z$  are given by the following expression:

$$\begin{bmatrix} G_x \\ G_y \\ G_z \end{bmatrix} = \mathbf{T} \begin{bmatrix} 0 \\ 0 \\ \mathbf{G} \end{bmatrix} \quad (\text{A.10})$$

The x-y-z components of  $\mathbf{B}$  seen by the magnetometer sensors  $M_x$ ,  $M_y$  and  $M_z$  are given by the following expression:

$$\begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \mathbf{T} \begin{bmatrix} \mathbf{B} \cdot \cos \theta \\ 0 \\ \mathbf{B} \cdot \sin \theta \end{bmatrix} \quad (\text{A.11})$$

Along each of the tools axes, the component of  $\mathbf{G}$  and  $\mathbf{B}$  is the dot product of the appropriate row of the **matrix (A.9)** and the **vectors (A.10)** or **(A.11)**. The equations for the accelerometer and magnetometer sensor outputs can therefore be calculated as a function of  $\mathbf{G}$  and or  $\mathbf{B}$  and the drilling angles according to:

$$G_x = -\mathbf{G} \sin I \sin \alpha \quad (\text{A.12})$$

$$G_y = -\mathbf{G} \sin I \cos \alpha \quad (\text{A.13})$$

$$G_z = \mathbf{G} \cos I \quad (\text{A.14})$$

$$B_x = \mathbf{B}(\cos \theta \cos I \cos A \sin \alpha + \cos \theta \sin A \cos \alpha - \sin \theta \sin I \sin \alpha) \quad (\text{A.15})$$

$$B_y = \mathbf{B}(\cos \theta \cos I \cos A \cos \alpha - \cos \theta \sin A \sin \alpha - \sin \theta \sin I \cos \alpha) \quad (\text{A.16})$$

$$B_z = \mathbf{B}(\cos \theta \sin I \cos A + \sin \theta \cos I) \quad (\text{A.17})$$

### A.3.3 Equations for dip ( $\theta$ ), inclination (I) and azimuth (A)

The equations for dip ( $\theta$ ) and Azimuth (A) cannot be calculated from the **equations (A.12) to (A.17)** alone. Expressions of **B** in N-E-V coordinates as functions of sensor outputs and drilling angles are needed. To achieve this, the inverse transformation from the x-y-z tool coordinate system to N-E-V coordinate system is done. The transformation matrix in this operation is the inverse of **T**,  $\mathbf{T}^{-1}$ . Since **T** has the property of orthogonality, the inverse  $\mathbf{T}^{-1}$  equals its transpose  $\mathbf{T}^T$ :

$$\mathbf{T}^{-1} = \mathbf{T}^T = \begin{bmatrix} \cos I \cos A \sin \alpha + \sin A \cos \alpha & \cos I \cos A \cos \alpha - \sin A \sin \alpha & \sin I \cos A \\ \cos I \sin A \sin \alpha - \cos A \cos \alpha & \cos I \sin A \cos \alpha + \cos A \sin \alpha & \sin I \sin A \\ -\sin I \sin \alpha & -\sin I \cos \alpha & \cos I \end{bmatrix} \quad (\text{A. 18})$$

The components of the **B** defined in the N-E-V frame are therefore given by the following expression:

$$\begin{bmatrix} B_N \\ B_E \\ B_V \end{bmatrix} = \begin{bmatrix} \mathbf{B} \cdot \cos \theta \\ 0 \\ \mathbf{B} \cdot \sin \theta \end{bmatrix} = \mathbf{T}^{-1} \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} \quad (\text{A. 19})$$

This leads to the following expressions for the N-E-V components of **B**:

$$\begin{bmatrix} B_N = \mathbf{B} \cos \theta = (\cos I \cos A \sin \alpha + \sin A \cos \alpha)B_x + (\cos I \cos A \cos \alpha - \sin A \sin \alpha)B_y + \\ (\sin I \cos A)B_z \end{bmatrix} \quad (\text{A. 20})$$

$$\begin{bmatrix} B_E = 0 = (\cos I \sin A \sin \alpha - \cos A \cos \alpha)B_x + (\cos I \sin A \cos \alpha + \cos A \sin \alpha)B_y + \\ (\sin I \sin A)B_z \end{bmatrix} \quad (\text{A. 21})$$

$$\begin{bmatrix} B_V = \mathbf{B} \sin \theta = (-\sin I \sin \alpha)B_x + (-\sin I \cos \alpha)B_y + (\cos I)B_z \end{bmatrix} \quad (\text{A. 22})$$

#### Dip angle ( $\theta$ )

The equation for the dip angle  $\theta$  can be derived by solving **equation (A.22)** with input from **equations (A.12) to (A.14)**.

$$\mathbf{B} \sin \theta = (-\sin I \sin \alpha)B_x + (-\sin I \cos \alpha)B_y + (\cos I)B_z$$

$$= B_x \frac{G_x}{G} + B_y \frac{G_y}{G} + B_z \frac{G_z}{G} \rightarrow$$

$$\theta = \sin^{-1} \left[ \frac{G_x \cdot B_x + G_y \cdot B_y + G_z \cdot B_z}{\mathbf{G} \cdot \mathbf{B}} \right] \quad (\text{A. 23})$$

The range of the dip angle is from  $-90^\circ$  and  $90^\circ$ .



## Inclination (I)

The equation for inclination can be derived directly from **equation (A.14)**:

$$I = \cos^{-1}\left(\frac{G_z}{G}\right) \quad (\text{A. 24})$$

The inclination angle can have any value between  $0^\circ$  and  $180^\circ$ .

In this expression, **G** can either be the nominal field gravity value or the root sum squared of the x, y and z reading as expressed in **equation (A.3)**.

Sometimes it is convenient to express inclination as a function of tangents instead of cosine. This can be achieved by combining **equations (A.12)** and **(A.13)** and knowing that:

$$\sin^2\alpha + \cos^2\alpha = 1 \quad (\text{A. 25})$$

From **equations (A.12)** and **(A.13)** we get:

$$\sin^2\alpha = \frac{G_x^2}{G^2\sin^2I}$$

$$\cos^2\alpha = \frac{G_y^2}{G^2\sin^2I}$$

Inserting into **equation (A.25)** this gives:

$$\frac{G_x^2 + G_y^2}{G^2} = \sin^2I \quad (\text{A. 26})$$

From **equation (A.14)** we have:

$$G = \frac{G_z}{\cos I} \rightarrow$$

$$\cos^2I = \frac{G_z^2}{G^2}$$

Inserted into **equation (A.26)** this gives:

$$\frac{(G_x^2 + G_y^2)\cos^2I}{G_z^2} = \sin^2I \rightarrow$$

$$\frac{\sin^2I}{\cos^2I} = \frac{G_x^2 + G_y^2}{G_z^2} \rightarrow$$

Finally, the expression of inclination as a function of tangents is:

$$I = \tan^{-1}\left(\frac{\sqrt{G_x^2 + G_y^2}}{G_z}\right) \quad (\text{A. 27})$$

### Azimuth (A)

An expression for azimuth (A) can be found by solving **equation (A.21)**:

$$B_E = (\cos I \sin A \sin \alpha - \cos A \cos \alpha)B_x + (\cos I \sin A \cos \alpha + \cos A \sin \alpha)B_y + (\sin I \sin A)B_z = 0 \rightarrow$$

$$\sin A [\cos I (B_x \sin \alpha + B_y \cos \alpha) + B_z] + \cos A (-B_x \cos \alpha + B_y \sin \alpha) = 0 \rightarrow$$

$$\frac{\sin A}{\cos A} = \frac{B_x \cos \alpha - B_y \sin \alpha}{\cos I (B_x \sin \alpha + B_y \cos \alpha) + B_z \sin I} \rightarrow$$

$$A = \tan^{-1} \frac{B_x \cos \alpha - B_y \sin \alpha}{\cos I (B_x \sin \alpha + B_y \cos \alpha) + B_z \sin I} \quad (\text{A.28})$$

The azimuth angle ranges from 0° to 360°.

Another equation for azimuth, expressed only by the magnetometer and accelerometer sensor values, can be found by inserting **equations (A.12)** to **(A.14)** into **equation (A.28)**.

$$A = \tan^{-1} \frac{B_x \cos \alpha - B_y \sin \alpha}{\cos I (B_x \sin \alpha + B_y \cos \alpha) + B_z \sin I} \rightarrow$$

$$= \tan^{-1} \left[ \frac{B_x \frac{-G_y}{G \sin I} - B_y \frac{-G_x}{G \sin I}}{\frac{G_z}{G} \left( B_x \frac{-G_x}{G \sin I} + B_y \frac{-G_y}{G \sin I} \right) + B_z \sin I} \cdot G \sin I \right] \rightarrow$$

$$= \tan^{-1} \left[ \frac{-B_x G_y + B_y G_x}{G_z \left( B_x \frac{-G_x}{G} + B_y \frac{-G_y}{G} \right) + G B_z \sin^2 I} \right] \rightarrow$$

$$= \tan^{-1} \left[ \frac{G(G_x B_y - G_y B_x)}{-G_z(G_x B_x + G_y B_y) + G B_z \left( 1 - \frac{G_z^2}{G^2} \right)} \right] \rightarrow$$

$$= \tan^{-1} \left[ \frac{G(G_x B_y - G_y B_x)}{-G_z(G_x B_x + G_y B_y) + G^2 B_z \left( 1 - \frac{G_z^2}{G^2} \right)} \right] \rightarrow$$

$$A = \tan^{-1} \left[ \frac{G \cdot (G_x \cdot B_y - G_y \cdot B_x)}{B_z \cdot (G_x^2 + G_y^2) - G_z \cdot (G_x \cdot B_x + G_y \cdot B_y)} \right] \quad (\text{A.29})$$

**A.3.4 Equations for toolface ( $\alpha$ )**

The roll orientation of a survey tool in the borehole is given by  $\alpha$  and may have any angle between  $0^{\circ}$  and  $360^{\circ}$ . In general, HTF is used for deviated drilling, and for vertical holes, the orientation of the survey tool is given by MTF. Below is a detailed description of both HTF and MTF.

**Highside toolface, HTF**

HTF is used to define the roll orientation of the survey tool when drilling deviated wells with inclination  $> 3^{\circ}$ . When the scribeline is on the highside of the survey tool, HTF is  $0^{\circ}$ . The scribeline is an invisible line on the survey tool where the y-axes of the accelerometer and magnetometer arrays would penetrate. The HTF angles increases when looking along the survey tool from the upper end and rotating it clockwise. HTF is mathematically given by **equation (A.30)** and defined to be the angle between the tools y-axis and the projection of  $\mathbf{G}$  onto the x-y plane, see **Figure A. 6**. For vertical boreholes, the calculations of azimuth and HTF become singular as  $G_x = G_y = 0$

$$HTF = \tan^{-1} \left( \frac{-G_x}{-G_y} \right) \tag{A.30}$$

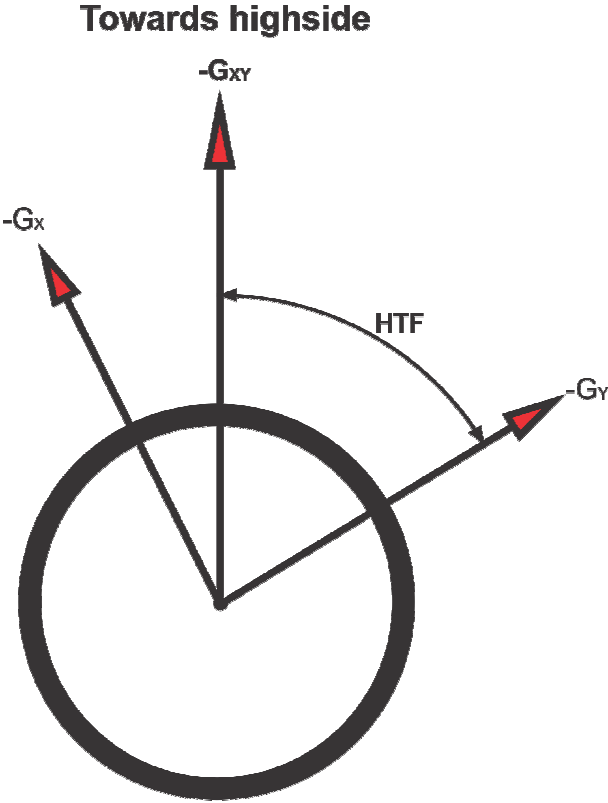


Figure A. 6 - Definition of HTF, looking down the tool from the upper end.

**Magnetic toolface, MTF**

In cases when azimuth and HTF become singular the orientation of the drillstring can be calculated using MTF. Mathematically MTF is given by **equation (A.31)** and is defined as the angle between the tools y-axis and the projection of the **B** onto the x-y plane, see **Figure A. 7**. MTF is measured clockwise from the projected magnetic field vector to the y-axis. Equation (A.31) shows that in cases when the tool is aligned in the direction of **B**,  $B_x = B_y = 0$  and thereby MTF becomes undefined.

$$MTF = \tan^{-1}\left(\frac{B_x}{B_y}\right) \tag{A.31}$$

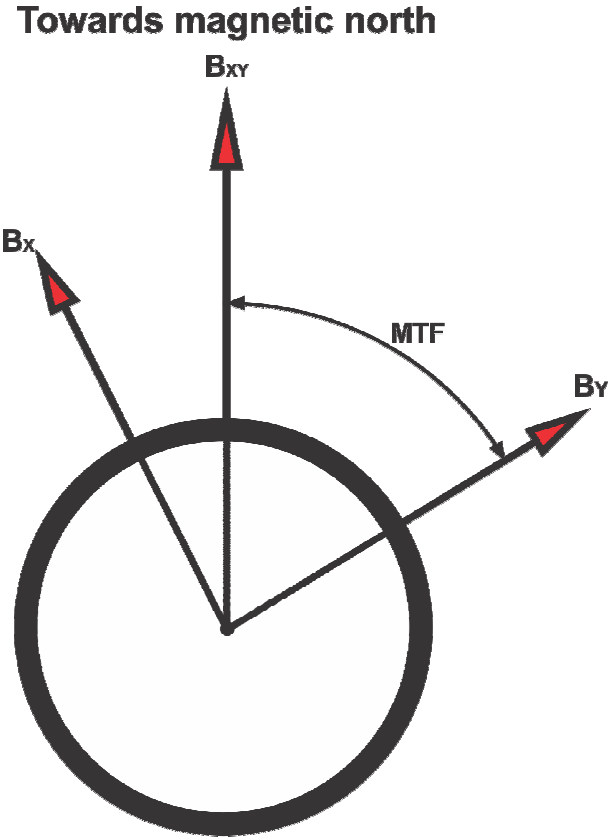


Figure A. 7 - Definition of MTF, looking down the tool from the upper end.

