The Translog Neoclassical Growth Model

by

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Abstract
The macroeconomic growth equation based on the translog aggregate production function is derived and compared to the growth equation based on Cobb-Douglas both with and without human capital. The model is estimated directly in structural form, using international panel data. Results are compared to the Cobb-Douglas case and a conventional fixed effect model.

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1. Introduction

Empirical convergence studies in the neoclassical Solow-tradition following Mankiw, Romer and Weil (1992), have so far exclusively been interpreted in terms of the Cobb-Douglas aggregate production function. This is unfortunate since recent work on stochastic aggregation suggests that the translog functional form could be a better choice. Garderen, Lee and Pesaran (2000) ask what functional form should be estimated for an aggregate production function when industry production functions are given and the primary objective is to obtain optimal forecasts. They show that when the industry production functions can be represented by different Cobb-Douglas functions, an analytical solution to the model selection problem does exist, provided a generalized version of the Hicks' aggregation condition is fulfilled. Moreover, the optimal functional form turns out to be the translog.

An important argument for Cobb-Douglas is that it can be used to derive a growth equation in terms of investment rates and makes it possible to forge a rigorous link between production structure parameters and the rate of convergence. There appears to be no similar results available in the literature for flexible forms - the least restrictive functional form that has been analysed seems to be the CES (see Barro and Sala-i-Martin, 2004, p.68). The main purpose of the present paper is to remove this objection against using the translog as an alternative to Cobb-Douglas by deriving the growth equation in the translog case. The growth equations consistent with Cobb-Douglas and translog are estimated in structural form based on panel data and compared to a conventional regression model with country and time specific effects. The estimates for the average rate of convergence turn out to be rather similar, but the models give different predictions for country specific convergence rates.

The paper is organized in 5 sections. Following this introduction, the traditional model with labor and physical capital is presented in Section 2. The augmented model of Mankiw et al. (1992) with human capital is presented in Section 3. Using panel data from the World Penn Tables, the Cobb-Douglas and the translog are estimated and the results are compared in Section 4. Section 5 concludes.

2. The basic translog growth model

Employing lower cases for variables per effective labor unit, we may write the aggregate Cobb-Douglas production function in intensive form, using logarithms and standard notation,

$$\ln y = \alpha \ln k, \quad 0 < \alpha < 1 \tag{1}$$

By Young's theorem, imposing linear homogeneity, the analogue to (1) in translog form is,

$$\ln y = \alpha \ln k + 2^{-1}\gamma \ln^2 k \tag{2}$$

Differentiating (2) logarithmically, we obtain the marginal product, $dy/dk = y\left(\alpha + \gamma \ln k\right)k^{-1}$. Subtracting the logarithm of capital per effective labor unit from both sides, and again differentiating logarithmically, we obtain $d\left(y/k\right)/dk = y\left(\alpha + \gamma \ln k - 1\right)k^{-2}$. Hence, the production function exhibits positive and diminishing marginal product when

$$0 < \alpha + \gamma \ln k < 1 \tag{3}$$

We can only expect a flexible form like translog to satisfy the Inada (1963) conditions locally, since (3) will only hold if k is constrained. For γ positive, $-\alpha\gamma^{-1} < \ln k < \left(1-\alpha\right)\gamma^{-1}$, and reversing the inequalities for γ negative. Hence, we should only regard the translog as an approximation to a neoclassical production structure.

The dynamics of the neoclassical growth model are given by

$$\dot{k} = sy - (n + g + \delta)k \tag{4}$$

where the rates of saving, labour force growth, technological progress and depreciation are s, n, g and δ, respectively. Solving for the steady state level of capital per unit output, we have

$$k^* y^{*-1} = s\left(n + g + \delta\right)^{-1} \tag{5}$$

Taking logs on both sides of (5) and rearranging, we obtain $\ln^2 k^* + 2\left(\alpha - 1\right)\gamma^{-1}\ln k^* = 2\gamma^{-1}\ln\left(\left(n + g + \delta\right)s^{-1}\right)$. Completing the square and solving, we have

$$\ln k^* = \left(1-\alpha\right)\gamma^{-1} \pm \gamma^{-1}\sqrt{\left(\alpha - 1\right)^2 + 2\gamma \ln\left(\left(n + g + \delta\right)s^{-1}\right)} \tag{6}$$

However, for (3) to hold in steady state, only the negative root is feasible,

$$\ln k^* = \left(1-\alpha\right)\gamma^{-1} - \gamma^{-1}\sqrt{\left(\alpha - 1\right)^2 + 2\gamma \ln\left(\left(n + g + \delta\right)s^{-1}\right)} \tag{7}$$

Substituting $\ln k^*$ for $\ln k$ in (2), we obtain the steady state level of output per effective worker,

$$\ln y^* = \ln k^* (\alpha + 2^{-1} \gamma \ln k^*) \tag{8}$$

The steady state solution is illustrated in Figure 1.

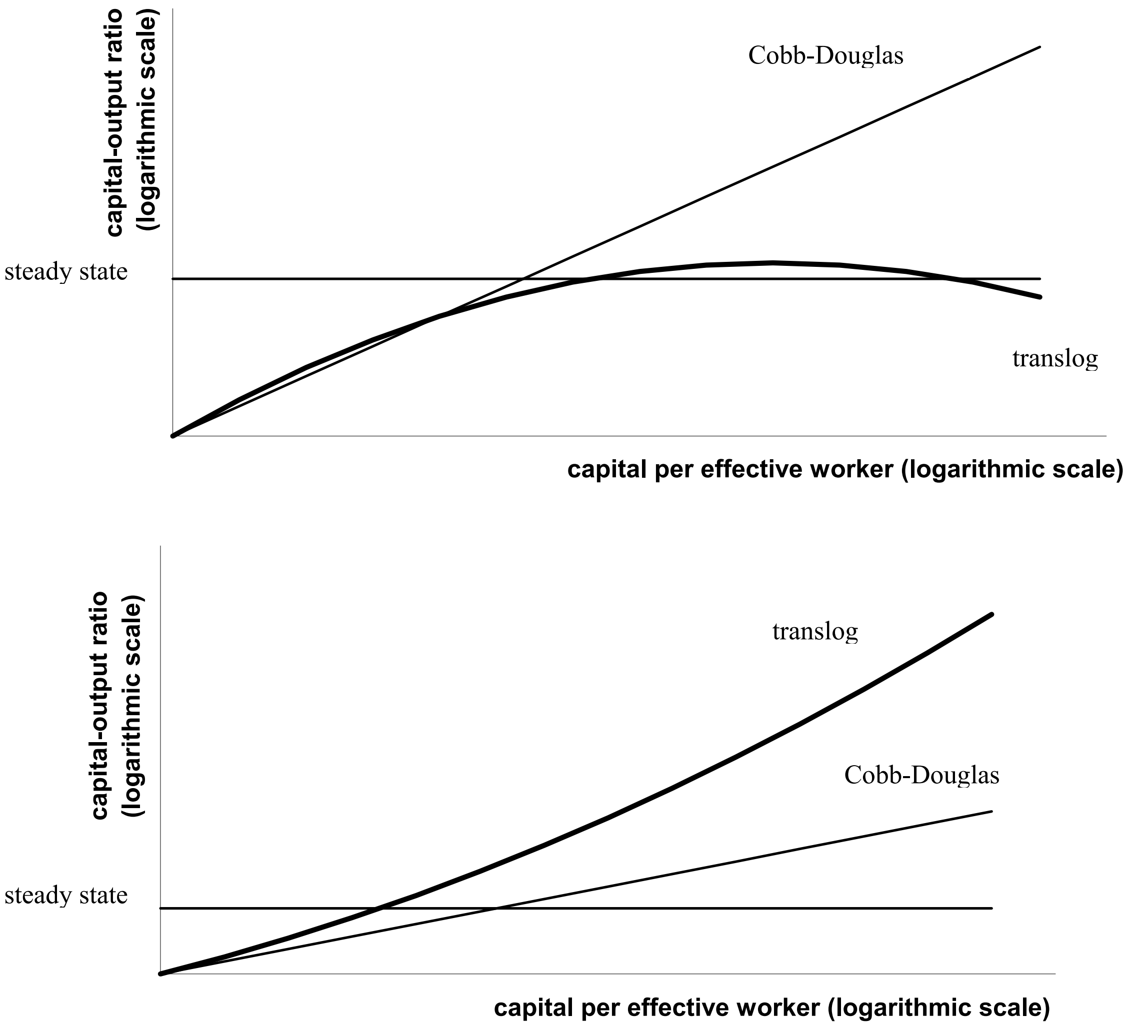

Figure 1. Steady-state

The horizontal line represents the capital-output ratio in steady-state (equation (5)). The intersection with the ray, representing the Cobb-Douglas capital-output ratio, gives the steady-state level of capital per effective worker in the Cobb-Douglas case. The intersection with the curve, representing the translog capital-output ratio, gives the equivalent in the translog case. For positive γ, the curve is concave (the upper panel) and for negative γ it is convex (the lower panel). The intersection with the concave curve to the far right in the upper panel represents the solution to equation (6) that violates the restriction imposed by (3). If we had extended the convex curve to the left in the lower panel, we would have seen the infeasible solution to the far left.

Let us now look at the dynamics outside steady state and return to equation (4).[1] Instead of working with a specified form, it is now convenient to write $y = f(k)$. Around steady state,

$$\dot{y} = f'(k^*)\dot{k} \tag{9}$$

Approximating the true functional form around steady state by a first order Taylor expansion, we have approximately,

$$f(k) = f(k^*) + f'(k^*)(k - k^*) \tag{10}$$

or

$$y^* - y = f'(k^*)(k^* - k) \tag{11}$$

The steady state level of capital is given by

$$sf(k^*) = (n + g + \delta)k^* \tag{12}$$

and the dynamics may be written $\dot{k} = sf(k) - (n + g + \delta)k$. Substituting for $f(k)$ from (10) and for s from (12),

$$\dot{k} = \left[\frac{f'(k^*)k^*}{f(k^*)} - 1\right](n + g + \delta)(k^* - k) \tag{13}$$

Substituting for $k - k^*$ from (11) in (13), and then substituting for $\dot{k}$ from (13) in (9),

[1]The approach here is in principle the same as used by Mankiw et al. (1992), among others.

$\dot{y} = \left[\frac{f'(k^*)k^*}{f(k^*)} - 1 \right] (n+g+\delta)(y - y^*)$. The capital share, $f'(k^*)k^* / f(k^*)$ is equal to α under Cobb-Douglas, and $\alpha + \gamma \ln k^*$ under translog.[2] Switching back to translog, we may therefore approximately write $\dot{y} = \left[1 - \alpha - \gamma \ln k^*\right](n+g+\delta)(y^* - y)$.This is an ordinary first-order linear differential equation that is easily solved. The solution may be written,

$$\frac{y_t - y_{t-T}}{y_{t-T}} = \left[1 - e^{-\beta T}\right] \left(\frac{y^* - y_{t-T}}{y_{t-T}} \right), \quad \beta \equiv (1 - \alpha - \gamma \ln k^*)(n+g+\delta) \tag{14}$$

It is convenient to approximate the growth rates, using logarithms, so we rewrite (14) as

$$\ln\left(y_t / y_{t-T}\right) = \left[1 - e^{-\beta T}\right] \ln\left(y^* / y_{t-T}\right) \tag{15}$$

For empirical applications we would like to have the variables expressed in terms of labor units, not effective labor units. Define output per labor unit by, $\overline{y}_t \equiv Y_t / L_t = A_t y_t$. Since efficiency by assumption grows at the constant rate, g, $\ln A_t = \ln A_0 + gt$. Hence, $\ln y_t = \ln \overline{y}_t - \ln A_0 - gt$ and $\ln y_{t-T} = \ln \overline{y}_{t-T} - \ln A_0 - g(t-T)$. Substituting in (15), and dividing by the length of the time period, we get the average growth rate of output per labour unit,

$$\frac{1}{T} \ln\left(\overline{y}_t / \overline{y}_{t-T}\right) = g \frac{t - e^{-\beta T}(t-T)}{T} + \frac{1 - e^{-\beta T}}{T} \ln\left(y^* A_0 / \overline{y}_{t-T}\right) \tag{16}$$

Equation (16) could be used as basis for panel data estimation or simple cross section regressions. In the latter case, t is equal to T, and (16) more compactly written as

$$\frac{1}{T} \ln\left(\overline{y}_T / \overline{y}_0\right) = g + \frac{1 - e^{-\beta T}}{T} \ln\left(y^* A_0 / \overline{y}_0\right) \tag{17}$$

The model can be extended to allow for human capital effects, similar to the extension of the Cobb-Douglas version of the neoclassical growth model by Mankiw et al. (1992).

3. The augmented translog growth model

Introducing human capital in addition to physical capital and labor input, the equivalent to equation (2) is

[2] We now see that the restriction given by equation (3) simply means that we demand the capital share to be well-defined.

$$\ln y = \alpha_k \ln k + \alpha_h \ln h + \frac{1}{2}\left(\gamma_k \ln^2 k + \gamma_h \ln^2 h + \gamma_{kh} \ln^2\left(k/h\right)\right) \tag{18}$$

with $\alpha_k + \alpha_h < 1$. There are decreasing returns to all capital, since constant returns have been imposed on the underlying production function. Human capital per effective worker is denoted by h.

The dynamics of the model is now governed by two equations of motion, one for each type of capital. For ease of comparison, we adopt the same system as used by Mankiw et al. (1992),

$$\begin{aligned} \dot{k} &= s_k y - (n+g+\delta)k \\ \dot{h} &= s_h y - (n+g+\delta)h \end{aligned} \tag{19}$$

where s_k and s_h are the fractions of income invested in physical and human capital. This means that both types of capital depreciate at the same rate.

In steady state, capital per effective worker is constant. The steady state level of physical capital, given by (10) when we had one type of capital, is now given by

$$\begin{aligned} \ln k^* = {} & \frac{1-\alpha_k-\alpha_h+\gamma_h \ln(s_h/s_k)}{\gamma_k+\gamma_h} \pm \frac{1}{\gamma_k+\gamma_h} Sqrt[(\alpha_k+\alpha_h-1-\gamma_h \ln(s_h/s_k))^2 \\ & +2(\gamma_k+\gamma_h)(\ln(n+g+\delta)-\ln s_k+\alpha_h \ln(s_h/s_k)-(\gamma_h+\gamma_{kh})\ln^2(s_h/s_k)/2)] \end{aligned} \tag{20}$$

Imposing a zero-restriction on the human capital variable, (20) is reduced to (6). The positive root can therefore be ruled out for the same reason as before. Once we have obtained the steady state level of physical capital, the steady state level of human capital is simply given by

$$\ln h^* = \ln k^* + \ln(s_h/s_k) \tag{21}$$

Substituting for steady state levels from (20) and (21) in the production function, (18), we obtain the steady state level of output per effective worker as well. The steady state solution is illustrated in the "three-dimensional" Figure 2.

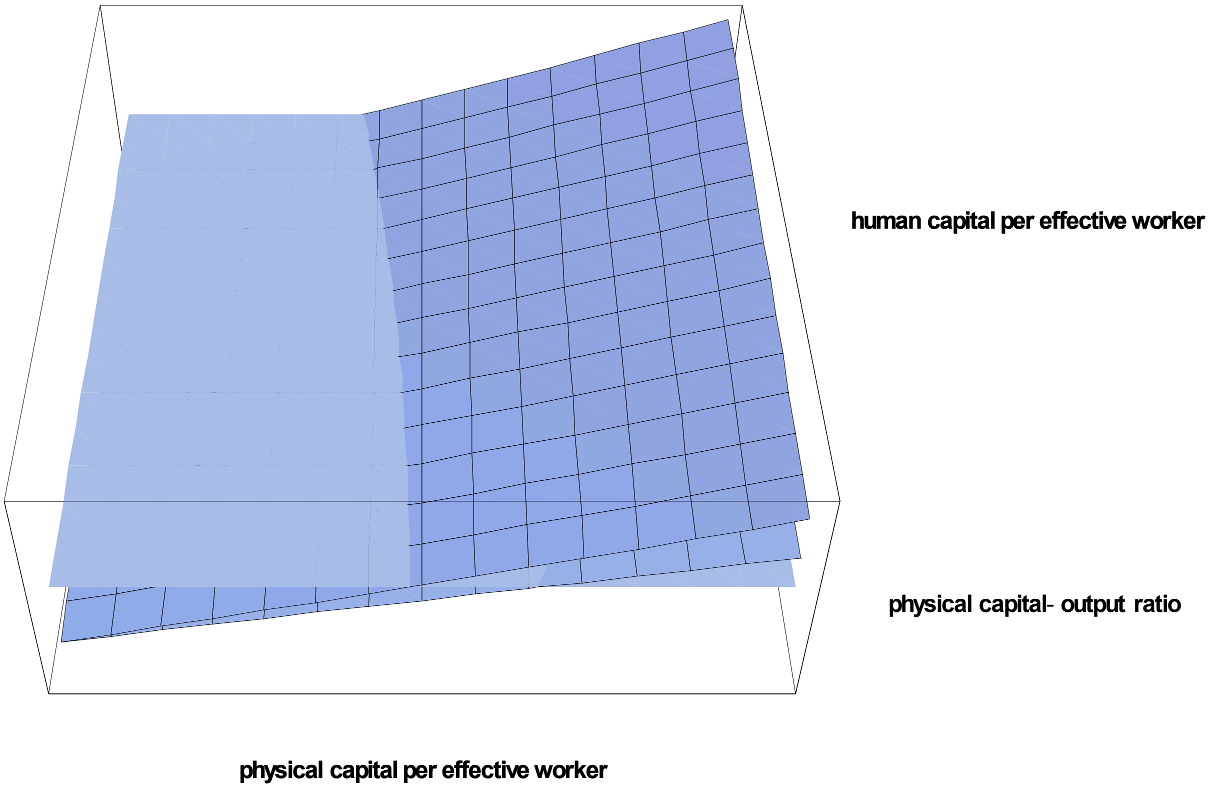

Figure 2. Steady-state with two types of capital (logarithmic scale)

The horizontal plane represents the physical capital-output ratio in steady-state. The intersection with the lower plane, rising from the left corner, gives the steady-state level of capital per effective worker in the Cobb-Douglas case. The intersection with the convex surface above the Cobb-Douglas plane, gives the equivalent in the translog case. You should recognize the image in the front plane from the lower panel of Figure 1.

Let us look at the dynamics outside steady state and return to equation (18). Applying the same approach as we used in case of one type of capital, instead of working with a specified form we choose to write $y = f\left(k,h\right)$. Around steady state,

$$\dot{y} = f_k(k^*)\dot{k} + f_h(h^*)\dot{h} \tag{22}$$

Approximating the true functional form around steady state by a first order Taylor expansion, we have approximately,

$$f\left(k,h\right) = f\left(k^*,h^*\right) + f_k\left(k^*,h^*\right)\left(k-k^*\right) + f_h\left(k^*,h^*\right)\left(h-h^*\right) \tag{23}$$

or

$$y^* - y = f_k\left(k^*, h^*\right)\left(k^* - k\right) + f_h\left(k^*, h^*\right)\left(h^* - h\right) \tag{24}$$

The fractions spent on either type of capital are constant and therefore always the same as in steady state,

$$\begin{aligned} s_k &= \frac{\left(n+g+\delta\right)k^*}{y^*} \\ s_h &= \frac{\left(n+g+\delta\right)h^*}{y^*} \end{aligned} \tag{25}$$

Substituting for $f(k,h)$ from (23) and for s_k and s_h from (25) in (19),

$$\begin{aligned} \dot{k} &= \left[\left(\frac{f_k\left(k^*,h^*\right)k^*}{y^*} - 1\right)\left(k - k^*\right) + \frac{f_h\left(k^*,h^*\right)k^*}{y^*}\left(h - h^*\right)\right]\left(n+g+\delta\right) \\ \dot{h} &= \left[\left(\frac{f_h\left(k^*,h^*\right)h^*}{y^*} - 1\right)\left(h - h^*\right) + \frac{f_k\left(k^*,h^*\right)h^*}{y^*}\left(k - k^*\right)\right]\left(n+g+\delta\right) \end{aligned} \tag{26}$$

Substituting from (26) in (22) and making use of (24), we obtain, after some manipulations,

$$\dot{y} = \left[\frac{f_k\left(k^*,h^*\right)k^*}{y^*} + \frac{f_h\left(k^*,h^*\right)h^*}{y^*} - 1\right]\left(n+g+\delta\right)\left(y - y^*\right) \tag{27}$$

Under Cobb-Douglas, $f_k\left(k^*,h^*\right)k^*/y^* + f_h\left(k^*,h^*\right)h^*/y^*$ is equal to $\alpha_k + \alpha_h$, under translog, $\alpha_k + \alpha_h + \gamma_k \ln k^* + \gamma_h \ln h^*$. In the translog case, we may therefore approximately write

$$\dot{y} = \left[1 - \alpha_k - \alpha_h - \gamma_k \ln k^* - \gamma_h \ln h^*\right]\left(n+g+\delta\right)\left(y^* - y\right) \tag{28}$$

This is a differential equation of the same kind as with one type of capital. Indeed, the solution is the same, given by (14), provided that we redefine β,

$$\begin{aligned} \frac{y_t - y_{t-T}}{y_{t-T}} &= \left[1 - e^{-\beta T}\right]\left(\frac{y^* - y_{t-T}}{y_{t-T}}\right), \\ \beta &\equiv (1 - \alpha_k - \alpha_h - \gamma_k \ln k^* - \gamma_h \ln h^*)(n+g+\delta) \end{aligned} \tag{29}$$

With this redefinition, (15), (16) and (17) remain valid, as well.

It is useful to note that the model is considerably simplified if we make the assumption that the two capital stocks, physical and human, are equal in steady state. Given the present very imperfect state of knowledge on how to measure human capital it is probably fair to say that this is as good a working hypothesis as any, and at least acceptable as a first approximation.[3] On this assumption we do not need data on human capital and (the deterministic part of) the model is almost as if there were only physical capital, the only difference being the interpretation of the parameters used to define β for the one-type capital case (equation (14)). Provided that $\alpha \equiv \alpha_k + \alpha_h$ and that $\gamma \equiv \gamma_k + \gamma_h$, equation (29) is reduced to (14). With this reinterpretation in mind, we may use (17) as the setup for simple cross-section regressions or (16) for panel data. Then, why bother about human capital at all? There are two answers. First, the data may be consistent with a low capital share as implied by the basic model or a high share as implied by the augmented model. We should let the data decide what the relevant interpretation or the relevant model should be. Second, the data sometimes suggest that higher labor force growth leads to higher growth and not lower, as predicted by the basic model. In the augmented model the prediction is not clear, and again we may let the data decide what is the appropriate model.[4] In the next Section we look at the growth equation from an empirical point of view using panel data and assuming that the simplifying condition (equal capital stocks in steady state) holds.

4. Empirical performance

In order to discriminate empirically between the two alternative growth equations based on respectively the Cobb-Douglas and the translog production structures, we are going to use the dataset from the World Penn Tables 6.1 (Heston, Summers, and Aten, 2002) We will be using data for 96 countries spanning the time period 1960 to 2000 by 10-year intervals. The data include all countries where there are available data for 1960, 1970, 1980, 1990 and 2000.We are using the purchasing-power adjusted real GDP per worker for $\bar{y}$, the investment to real GDP ratio averaged over the 10 year interval for s_{kt}, and the growth rate of workers from start year to end year in each interval assuming a constant rate, for n_t.[5] Following Mankiw et al.

[3] There are few serious attempts to actually estimate the stock of human capital. Estimates based on U.S. data, suggesting that the human capital share is somewhere between 0.4 and 0.5 (see Barro and Sala-i-Martin, 2004, p. 60) can hardly be expected to be representative for the broad group of countries making up the dataset we are using.

[4] In the terms of Shioji (2001), the composition effect due to embodied human capital may dominate the quantity effect, leading to higher growth when the labor force grows faster because of improved quality.

[5] There are two real GDP figures available in WPT 6.1 based on a Laspeyre and a chained index, resp. We are using the latter. The investment to real GDP ratio given is based on the Laspeyre version, but we have converted it to the chained one. Although the number of workers are not given explicitly, we use GDP/worker, GDP/cap and population in order to arrive at the number of workers. The workers figures appear to be the working age

(1992) the rate of technological progress, g, and the rate of depreciation, δ, are assumed common for all countries and equal to 2 and 3 per cent, respectively.

We will be considering three different specifications of the growth equation. The first may be written,

$$\frac{1}{T}\ln\left(\bar{y}_{i,t}/\bar{y}_{i,t-T}\right)=\varphi_i+\varphi_t+\varphi\ \ln\left(\frac{s_{ki,t}}{n_{i,t}+g+\delta}\right)-\frac{1-\exp\left(-\beta T\right)}{T}\ln\bar{y}_{i,t-T}+\varepsilon_{i,t} \qquad (4.1)$$

Here φ_i, φ_t, φ and β are coefficients to be estimated. This is a fixed effect model across countries and time. The reason for this choice is that equations similar to (4.1) have been widely used for panel data estimation, and the results have therefore some interest for comparisons. However, as a specification of the growth equation it has at least three short-comings. First, the rate of convergence is treated as if it were a constant. Second, the parameter restrictions implied by the structural form is not imposed. Third, it is not possible to reveal the underlying production structure. The structural form specifications to be presented next, have none of these drawbacks and we may ask why equations like (4.1) are used at all. I can think of two reasons. The first reason is that the model may be estimated as a log-linear model if we refrain from estimating the convergence rate directly, and linear models continue to be popular despite the increasing power of computers that makes non-linear estimation increasingly attractive. Another reason is that sometimes there are computational difficulties with highly non-linear models, like the structural form specifications in equation (4.2) and (4.3).[6]

The second specification of the growth equation is the structural form based on Cobb-Douglas technology,

$$\begin{aligned}\frac{1}{T}\ln\left(\bar{y}_{i,t}/\bar{y}_{i,t-T}\right)&=g\frac{t-\exp\left(-\beta_i\right)(t-T)}{T}+\frac{\alpha}{1-\alpha}\frac{1-\exp\left(-\beta_i\right)}{T}\ln\left(\frac{s_{ki,t}}{n_{i,t}+g+\delta}\right)\\&\quad+\frac{1-\exp\left(-\beta_i\right)}{T}\ln\left(A_i/\bar{y}_{i,t-T}\right)+\varepsilon_{i,t}\end{aligned} \qquad (4.2)$$

where the country specific rates of convergence, β_i, are replaced by $(1-\alpha)(n_{i,t}+g+\delta)$ so that

population for many countries in the database. For further documentation the reader is referred to WPT 6.1 and the references therein.

[6] Fingleton and McCombie (1998) is a good example. They tried to estimate a hybrid model between (4.1) and (4.2) based on cross section data for European regions, allowing for the fact that the convergence rate is not a constant, but report on p. 101: "Computational difficulties precluded the inclusion of national dummies in this regression."

α (the capital share) and A_i (initial efficiency of labor) are the only coefficients to be estimated.

The third and final specification is the structural form based on translog technology,

$$\begin{aligned}\frac{1}{T}\ln\left(\bar{y}_{i,t}/\bar{y}_{i,t-T}\right) = g\frac{t-\exp\left(-\beta_i\right)(t-T)}{T} \\ -\frac{1-\exp\left(-\beta_i\right)}{T}\left(\frac{1}{\gamma}\left(1-\alpha-\frac{\beta_i}{n_{i,t}+g+\delta}\right)+\ln\left(\frac{s_{ki,t}}{n_{i,t}+g+\delta}\right)\right) \\ +\frac{1-\exp\left(-\beta_i\right)}{T}\ln\left(A_i/\bar{y}_{i,t-T}\right)+\varepsilon_{i,t}\end{aligned} \qquad (4.3)$$

where β_i now are replaced by $sqrt\left(\left(\alpha-1\right)^2-2\gamma\ln\left(\frac{s_{ki,t}}{n_{i,t}+g+\delta}\right)\right)\left(n_{i,t}+g+\delta\right)$ so that now the parameter γ are estimated along with α and A_i. The country specific capital shares can then be computed as $1-sqrt\left(\left(\alpha-1\right)^2-2\gamma\ln\left(\frac{s_{ki,t}}{n_{i,t}+g+\delta}\right)\right)$. In actual estimation we have allowed α and γ to vary between high saving and low saving countries, defined by whether $s_{ki,t}$ exceeds $n_{i,t}+g+\delta$ or not, when estimating both (4.2) and (4.3). This is not an arbitrary choice. It is clear from (7) that a positive (negative) γ may be necessary for $\ln k^*$ to be well defined if the saving rate is low (high). Hence, low and high saving countries cannot share the same technology as in the Cobb-Douglas model. We may think about it as a world with two technologies or modes of production available: the modern economy technology and the subsistence economy technology.

The three specifications have all been estimated by means of Nonlinear Least Squares (NLS). Results are reported in Table 4.1. Country specific estimates are relegated to the Appendix.

The conventional reduced form model, with common technology imposed, suggests that countries converge to their steady state at an annual rate of 4 per cent and that the capital share equals 40 per cent. The comparable Cobb-Douglas model predicts that the rate of convergence is 4.4 per cent and the capital share is 37 per cent. When we allow technology to be different between what we called modern economies and subsistence economies, the

Table 4.1 Estimation results

Technology	Reduced Form	Cobb-Douglas	Translog
Shared	φ: .019 (.003) β: .040 (.007)	α: .366 (.046)	Not feasible
	Implied α: .404 (.094) Rate of convergence: see β	Rate of convergence: .044 (.007)	
	Log of likelihood: 1103.86	Log of likelihood: 1037.56	
Modern	φ: .022 (.004) β: .042 (.007	α: .396 (.044)	α: .427 (.084) γ: -.028 (.070)
Subsistence	φ: .005 (.008) β: .043 (.007	α: .104 (.158)	α: .046 (.187) γ: -.221 (.135)
	Implied α: .374 (.098) Rate of convergence: see β	Rate of convergence: .045 (.012)	Rate of convergence: .045 (.010)
	Log of likelihood: 1106.33	Log of likelihood: 1040.14	Log of likelihood: 1042.36

Note: NLS estimates. Standard deviation in the parenthesis after point estimate. Country specific fixed effects and country specific parameter estimates are omitted. Number of observations: 384.

results become even more similar and we observe that the two structural form models both predict a rate of convergence (evaluated at the mean) equal to 4.5 per cent, slightly higher than the reduced form. Hence, the estimate for the rate of convergence appears to be robust to the choice of functional form when evaluated by the mean. This does not imply that the choice of functional form has no substantial significance for the average rate of convergence in general, but the choice appears to be of little consequence when using this particular dataset.

However, when we move from the average to country specific estimates, there are interesting differences between the structural form models (the reduced form does not give country specific estimates except for the fixed effects, reported in Table A.1 in the Appendix, that are often crudely interpreted as differences in steady states). The correlation is far from perfect, being equal to 0.88. For a number of African countries, equation (4.2) gives a much more optimistic scenario than equation (4.3). The most extreme example is Uganda, where the Cobb-Douglas model suggests a conditional convergence rate at 6.9 per cent whereas the translog model gives only 3.8 per cent. Other examples of notable difference are Rwanda, Mozambique, Madagascar, Gambia and Ethiopia. These countries are represented to the lower right in the scatterplot presented in Figure 3. The plot is based on the information given in the Appendix, Table A.2. As an artifact of the model, the steady state capital stock does not enter the convergence rate expression in the Cobb-Douglas case. The inclusion of the steady state capital stock in the convergence rate expression in the translog case leads to the different results visible in Figure 3.

The differences in predicted conditional rates of convergence are to some extent reflected in differences in predicted deviations from steady state, illustrated in Figure 4, and in predicted initial efficiency, illustrated in Figure 5 (see also Appendix, Table A.1). The correlation between predicted deviations and the correlation between initial efficiencies are much higher than for the rates of convergence and close to perfect (0.998 and 0.999), but Uganda, the outlier in Figure 3, is clearly off the diagonal in Figure 4 and 5.

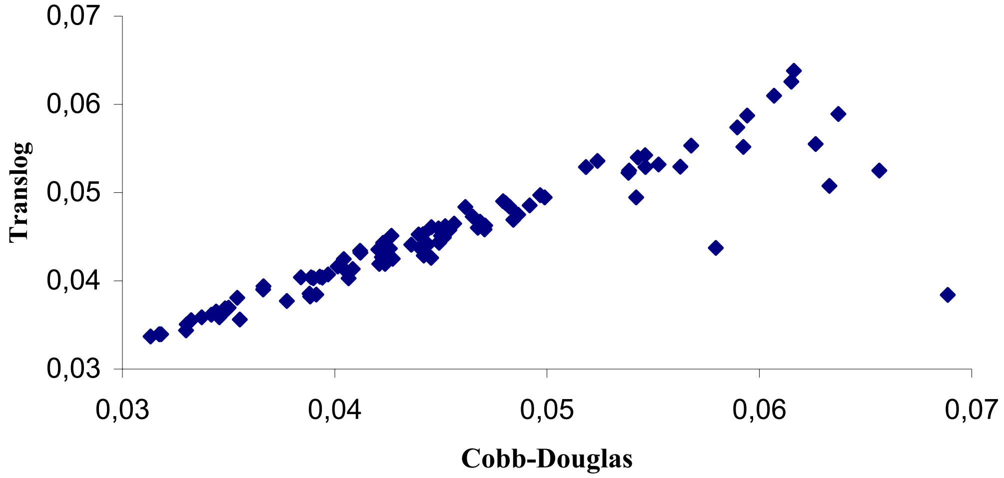

Figure 3. Country specific rates of convergence (average over the time periods)

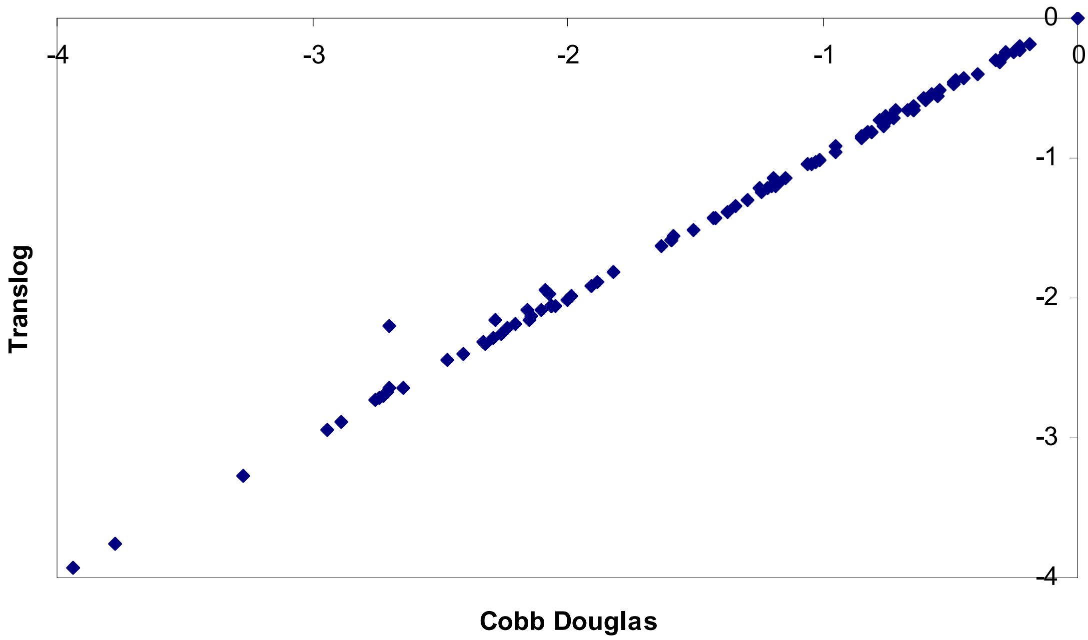

Figure 4. Country specific deviations from steady state (per cent, average over the time periods, USA assumed in steady state)

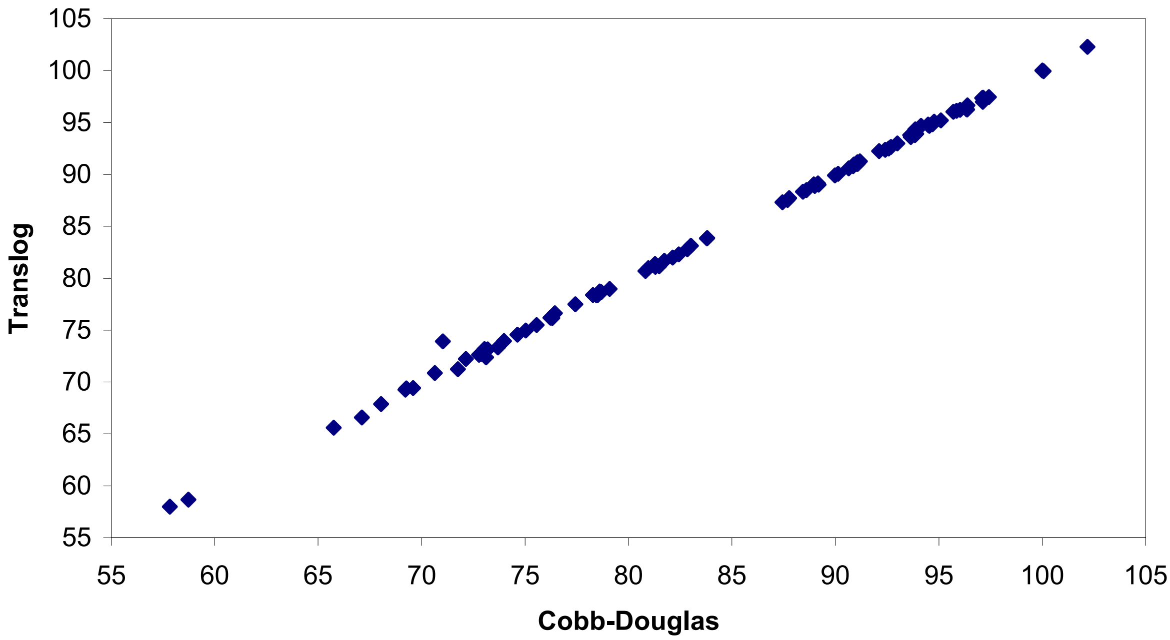

Figure 5. Country specific initial labor efficiency (average over the time periods, USA normalized to 100)

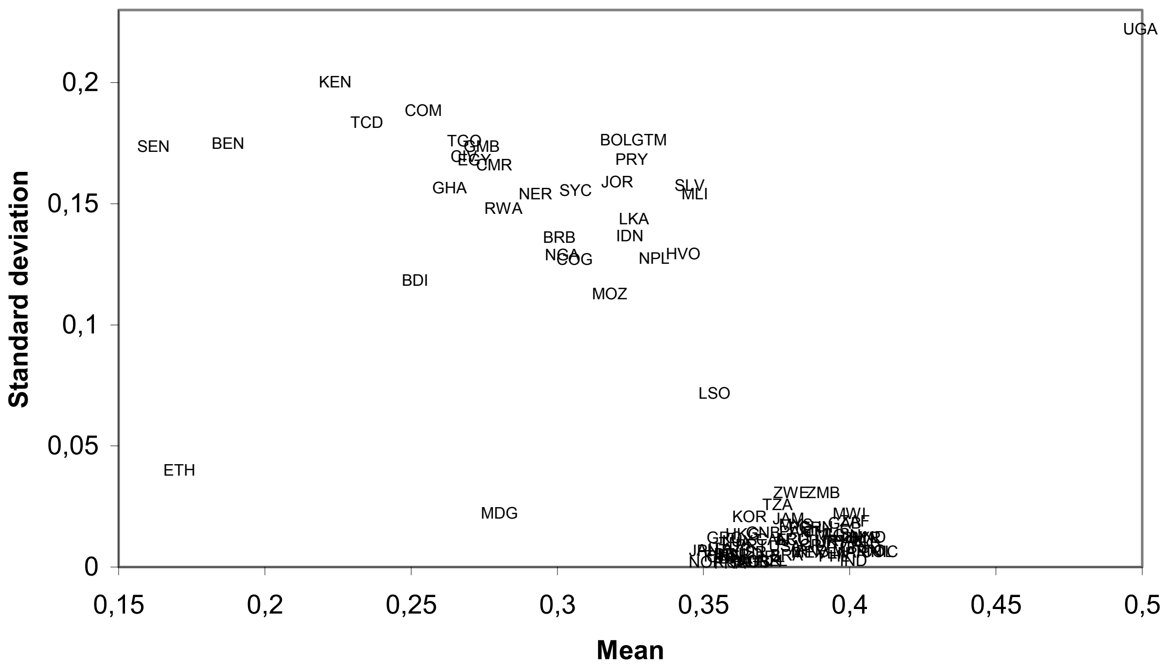

Figure 6. Translog country specific capital shares (means and standard deviations for each country over the time periods)

An interesting possibility when moving from Cobb-Douglas to translog, is country specific estimates for the capital share. Figure 6 plots standard deviations against means for individual capital shares over the 4 time periods, 1960-1970, 1970-1980, 1980-1990, and 1990-2000. For most countries and notably all OECD members, the model predicts a stable capital share somewhere around 35 to 40 per cent. However, for a considerable number of countries that we identify as less developed countries from the three-letter acronym used as labels in the plot, the predicted capital share is lower or much lower and even more strikingly, very volatile over time (observe the large standard deviations).[7]

5. Concluding discussion

We have in this paper derived the macroeconomic growth equation for the translog production function. Besides theoretical arguments for growth equations based on more flexible functional forms than the traditional Cobb-Douglas, there may also be empirical arguments. We have used the highly used dataset from the World Penn Tables in order to compare the translog setup to Cobb-Douglas.

One conclusion is that estimates for the conditional rate of convergence, evaluated at the mean, are similar regardless of the setup. The principle of Occam's razor therefore suggests that the simpler Cobb-Douglas setup should be preferred. Moreover, although the results may be interpreted in terms of capital in a broad sense, including both physical and human capital, the predicted capital share is too low to be consistent with this interpretation[8]. Hence, the basic Solow model seems to be the best choice.

If the interest is on country specific estimates, there are important differences between the models that may motivate the use of the translog, in particular when the data comprise information on less developed countries. For many of these countries, the translog model predicts lower rates of convergence than does the Cobb-Douglas. With panel data estimation, we may obtain country specific estimates for initial efficiency, but the translog has the additional advantage over Cobb-Douglas that it also predicts country specific capital shares. The predicted capital shares suggest a cleavage between many less developed countries and the rest of the world that is not visible in the simpler model.

[7] Uganda is again far out, but we are more concerned with regularities than singular cases here. However, we may ask whether the data on Uganda reflects reality or just poor data quality.

Appendix

Table A.1 Country specific effects (USA=100)

The reduced form Fixed effects		Cobb-Douglas Initial labor efficiency		Translog Initial labor efficiency	
Luxembourg	104	Luxembourg	102	Luxembourg	102
Italy	100	Ireland	100	USA	100
USA	100	USA	100	Ireland	100
Belgium	100	Canada	97	Canada	97
Ireland	100	Belgium	97	Belgium	97
Hong Kong	100	Hong Kong	97	Italy	97
Austria	99	Italy	97	Hong Kong	97
Spain	99	Netherlands	96	Netherlands	97
France	99	Trinidad &Tobago	96	Spain	96
Netherlands	99	Barbados	96	Barbados	96
Norway	98	Spain	96	Trinidad &Tobago	96
Japan	98	Australia	96	Australia	96
Finland	98	France	96	France	96
Canada	98	Austria	96	Austria	96
Switzerland	98	Israel	95	Israel	95
Australia	98	Denmark	95	Denmark	95
Denmark	98	United Kingdom	95	United Kingdom	95
Israel	97	Japan	95	Finland	95
Greece	97	Sweden	94	Sweden	95
Sweden	97	Iceland	94	Iceland	95
Iceland	97	Finland	94	Switzerland	95
United Kingdom	97	Switzerland	94	Japan	95
Portugal	96	Portugal	94	Norway	94
Korea, Republic of	96	Norway	94	Greece	94
Barbados	96	Seychelles	94	Portugal	94
New Zealand	95	Greece	94	Seychelles	94
Trinidad &Tobago	93	South Africa	94	New Zealand	94
Seychelles	92	New Zealand	94	South Africa	94
Mexico	92	Gabon	93	Gabon	93
Malaysia	92	Korea, Republic of	93	Korea, Republic of	93
Argentina	92	Mexico	93	Mexico	92
Mauritius	92	Mauritius	92	Mauritius	92
Gabon	92	El Salvador	92	El Salvador	92
South Africa	91	Guatemala	91	Guatemala	91
Chile	91	Chile	91	Egypt	91
Brazil	91	Egypt	91	Jordan	91
Iran	90	Jordan	91	Chile	91
Uruguay	90	Argentina	91	Argentina	91
Panama	89	Malaysia	91	Malaysia	91
Venezuela	89	Venezuela	91	Venezuela	91
Syria	88	Syria	90	Syria	90
Jordan	88	Uruguay	90	Uruguay	90
Turkey	88	Costa Rica	89	Brazil	89
Dominican Republic	87	Brazil	89	Paraguay	89
Costa Rica	87	Iran	89	Costa Rica	89

[8] Strictly speaking, we are only allowed to interpret the results in terms of capital in the broad sense if the maintained hypothesis that the stocks of human and physical capital are equal in steady state, holds.

Guatemala	87	Paraguay	89	Iran	89
Egypt	87	Dominican Republic	89	Dominican Republic	88
Morocco	87	Colombia	88	Colombia	88
Thailand	86	Panama	88	Panama	88
Colombia	86	Turkey	88	Turkey	88
El Salvador	86	Morocco	87	Morocco	87
Ecuador	86	Peru	84	Peru	84
Paraguay	86	Ecuador	84	Ecuador	84
Peru	85	Bolivia	83	Bolivia	83
Romania	84	Indonesia	83	Indonesia	83
Cape Verde	84	Cape Verde	82	Cape Verde	82
Indonesia	83	Philippines	82	Philippines	82
Philippines	83	Sri Lanka	82	Sri Lanka	82
Jamaica	81	Nicaragua	81	Cote d'Ivoire	81
Pakistan	81	Honduras	81	Nicaragua	81
Bolivia	81	Cote d'Ivoire	81	Honduras	81
Sri Lanka	80	Thailand	81	Thailand	81
Honduras	80	Pakistan	81	Pakistan	81
Zimbabwe	80	Cameroon	79	Cameroon	79
Nicaragua	79	Comoros	79	Romania	79
India	79	Romania	79	Comoros	79
Cote d'Ivoire	79	India	79	Jamaica	78
Bangladesh	78	Bangladesh	78	India	78
Congo, Republic of	78	Jamaica	78	Bangladesh	78
China	77	Zimbabwe	77	Zimbabwe	78
Guinea	77	Senegal	76	Senegal	77
Cameroon	76	Guinea	76	Gambia, The	76
Comoros	75	Gambia, The	76	Guinea	76
Lesotho	75	Togo	76	Togo	75
Nepal	73	Congo, Republic of	75	Congo, Republic of	75
Ghana	73	China	75	China	75
Senegal	72	Ghana	74	Ghana	74
Gambia, The	72	Lesotho	74	Uganda	74
Togo	72	Benin	73	Lesotho	73
Zambia	72	Mozambique	73	Kenya	73
Chad	72	Madagascar	73	Benin	73
Kenya	72	Kenya	73	Madagascar	73
Benin	70	Nepal	73	Nepal	73
Madagascar	68	Rwanda	73	Rwanda	73
Rwanda	68	Chad	72	Mozambique	72
Mozambique	68	Nigeria	72	Chad	72
Malawi	68	Uganda	71	Nigeria	71
Burkina Faso	68	Niger	71	Niger	71
Mali	68	Mali	70	Mali	69
Nigeria	68	Zambia	69	Zambia	69
Niger	67	Ethiopia	69	Ethiopia	69
Uganda	67	Burkina Faso	68	Burkina Faso	68
Ethiopia	66	Burundi	67	Burundi	67
Burundi	65	Malawi	66	Malawi	66
Guinea-Bissau	65	Guinea-Bissau	59	Guinea-Bissau	59
Tanzania	64	Tanzania	58	Tanzania	58

Note: Countries are sorted in descending order. The reported figures are the time means for the 4 time periods, 1960-1970, 1970-1980, 1980-1990, and 1990-2000.

Table A.2 Country specific conditional rates of convergence

Cobb-Douglas estimates: Country	Rate of convergence	Translog estimates: Country	Rate of convergence
Uganda	0.069	Kenya	0.064
Rwanda	0.066	Senegal	0.063
Ethiopia	0.064	Jordan	0.061
Madagascar	0.063	Ethiopia	0.059
Gambia. The	0.063	Benin	0.059
Kenya	0.062	Ghana	0.057
Senegal	0.062	Gambia. The	0.055
Jordan	0.061	Cote d'Ivoire	0.055
Benin	0.059	Nigeria	0.055
Nigeria	0.059	Paraguay	0.054
Ghana	0.059	Comoros	0.054
Mozambique	0.058	Congo. Republic of	0.054
Cote d'Ivoire	0.057	Cameroon	0.053
Niger	0.056	Niger	0.053
Cameroon	0.055	Chad	0.053
Mali	0.055	Mali	0.053
Paraguay	0.055	Togo	0.053
Comoros	0.054	Rwanda	0.053
Burundi	0.054	Egypt	0.052
Togo	0.054	Madagascar	0.051
Egypt	0.054	Bolivia	0.050
Congo. Republic of	0.052	Burundi	0.049
Chad	0.052	Costa Rica	0.049
Costa Rica	0.050	Zimbabwe	0.049
Bolivia	0.050	Peru	0.049
Guatemala	0.049	Guatemala	0.049
Colombia	0.049	Venezuela	0.048
El Salvador	0.048	Israel	0.048
Venezuela	0.048	Colombia	0.048
Peru	0.048	Mexico	0.047
Zimbabwe	0.048	El Salvador	0.047
Honduras	0.047	Indonesia	0.047
Nicaragua	0.047	Iran	0.047
Indonesia	0.047	Honduras	0.046
Syria	0.047	Malaysia	0.046
Mexico	0.046	Tanzania	0.046
Israel	0.046	Syria	0.046
Iran	0.046	Panama	0.046
Zambia	0.045	Nicaragua	0.046
Malaysia	0.045	Zambia	0.046
Sri Lanka	0.045	Ecuador	0.045
Philippines	0.045	Brazil	0.045
Nepal	0.045	Thailand	0.045
Dominican Republic	0.045	Philippines	0.045

Panama	0.045	Sri Lanka	0.045
Tanzania	0.045	Dominican Republic	0.044
Lesotho	0.045	Nepal	0.044
Morocco	0.044	Korea. Republic of	0.044
Burkina Faso	0.044	Morocco	0.044
Ecuador	0.044	Cape Verde	0.044
Pakistan	0.044	Mozambique	0.044
Brazil	0.044	Pakistan	0.044
Cape Verde	0.044	Seychelles	0.044
Malawi	0.043	Canada	0.044
Thailand	0.043	Iceland	0.043
Seychelles	0.043	Australia	0.043
South Africa	0.042	Burkina Faso	0.043
Korea. Republic of	0.042	Chile	0.043
Chile	0.042	Lesotho	0.043
Gabon	0.042	Malawi	0.043
Canada	0.042	Hong Kong	0.042
Australia	0.041	South Africa	0.042
Iceland	0.041	Gabon	0.042
China	0.041	New Zealand	0.042
India	0.041	China	0.041
Turkey	0.041	Turkey	0.041
Hong Kong	0.040	Jamaica	0.041
New Zealand	0.040	USA	0.040
Jamaica	0.040	Guinea-Bissau	0.040
Argentina	0.039	Netherlands	0.040
USA	0.039	Argentina	0.040
Trinidad &Tobago	0.039	India	0.040
Barbados	0.039	Barbados	0.040
Guinea-Bissau	0.039	Norway	0.039
Bangladesh	0.039	Switzerland	0.039
Guinea	0.039	Guinea	0.039
Netherlands	0.038	Trinidad &Tobago	0.038
Mauritius	0.038	Uganda	0.038
Norway	0.037	Bangladesh	0.038
Switzerland	0.037	Japan	0.038
Uruguay	0.036	Mauritius	0.038
Japan	0.035	Luxembourg	0.037
Luxembourg	0.035	Spain	0.037
Sweden	0.035	Sweden	0.037
Spain	0.035	France	0.037
Ireland	0.035	Portugal	0.036
Portugal	0.035	Denmark	0.036
France	0.034	Greece	0.036
Denmark	0.034	Ireland	0.036
Greece	0.034	Uruguay	0.036
Finland	0.033	Finland	0.036
Belgium	0.033	Belgium	0.035
United Kingdom	0.033	United Kingdom	0.034
Italy	0.032	Austria	0.034
Austria	0.032	Italy	0.034
Romania	0.031	Romania	0.034

Note: Countries are sorted in descending order according to rate of convergence. The reported rate of convergence is the time mean for the 4 time periods, 1960-1970, 1970-1980, 1980-1990-2000.

REFERENCES

Barro, R.J. and X. Sala-i-Martin 2004, Economic Growth, 2nd ed., Cambridge Ma: The MIT Press.

Fingleton, B. and J.S.L. McCombie 1998, Increasing returns and economic growth: some evidence for manufacturing from the European Union regions. Oxford Economic Papers 50: 89-105.

Garderen, K.J. van, K. Lee and M.H. Pesaran 2000, Cross-sectional aggregation of non-linear models. Journal of Econometrics 95: 285-331

Heston, A., R. Summers and B. Aten 2002, Penn World Tables 6.1. Center for International Comparisons at the University of Pennsylvania (CICUP).

Inada, K.-I. 1963, On a two-sector model of economic growth: Comments and a generalization. Review of Economic Studies 30: 119:27.

Mankiw, N.G., D. Romer and D.N. Weil 1992, A contribution to the empirics of economic growth. The Quarterly Journal of Economics 107: 407-38

Shioji, E. 2001, Composition effect of migration and regional growth in Japan. Journal of the Japanese and International Economies 15: 29-49.