Determination of Thermal Properties of Fresh Water and Sea Water Ice using Multiphysics Analysis

T. Rashid*, H. Khawaja, K. Edvardsen
Department of Engineering and Safety, UiT The Arctic University of Norway, Tromsø, Norway

ABSTRACT
This paper presents a methodology to determine the thermal conductivity of ice using Multiphysics analysis. This methodology used a combination of both experimentation and numerical simulation. In the experimental work, an ice block is observed using an infrared camera. The results reveal the variation in temperature over the surface. These results are dependent on two primary heat transfer parameters, namely, conductivity of ice within the ice cuboid and overall heat transfer coefficient. In addition to these two parameters, the surrounding temperature also affects the observed temperature profile. In the numerical simulation, the same behaviour is simulated using Multiphysics tools. In this work, the finite difference method is used to discretize the heat equation and is solved using an FTCS (Forward-Time Central-Space) method in MATLAB® software. The inputs to the simulation are the thermal properties of ice. These parameters are varied to match with the experimental results, hence revealing the real-time thermal properties of ice and surroundings.

1. INTRODUCTION

The thermal properties of ice have been under investigation since the early 1900s and many studies have reported various physical properties of ice. For example, [1, 2] studied heat capacity, [3, 4] studied variation in ice density, [4, 5] studied the latent heat of fusion, [6] reported variation in absorption coefficient, [7, 8] described variation in the coefficient of thermal expansion.

The focus of this paper is to determine the thermal conductivity of fresh water and sea water ice. Various researchers have studied the thermal conductivity of fresh water ice.

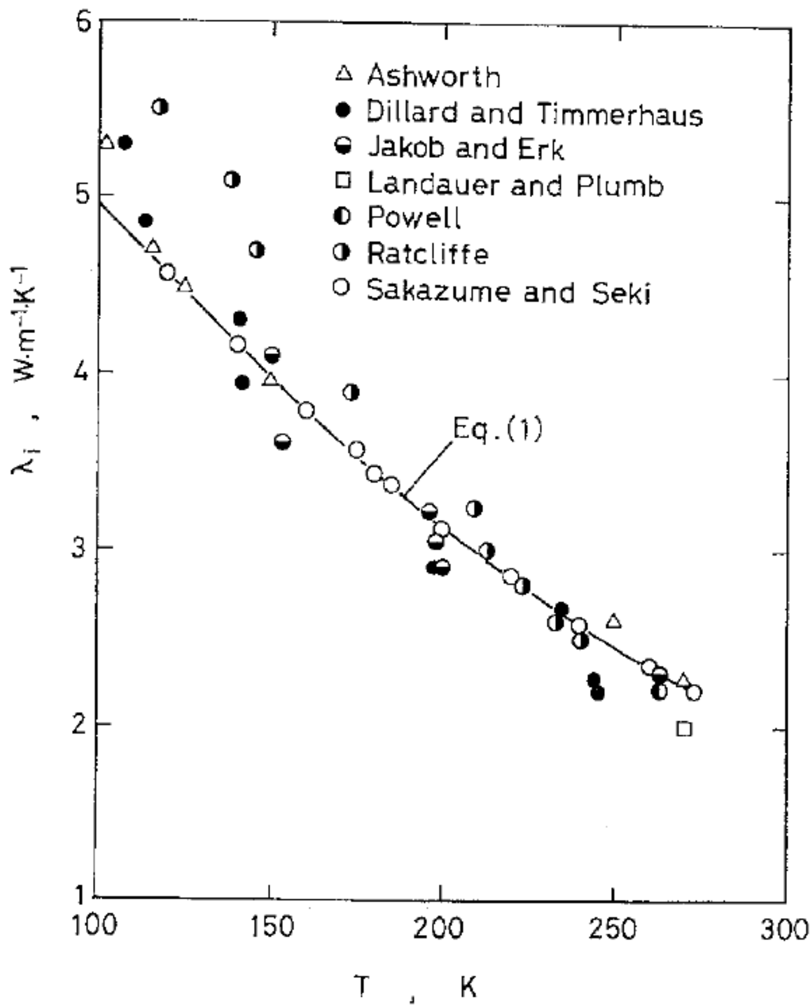


Figure 1 shows the variation in thermal conductivity of fresh water ice as reported by [7, 9-13] and summarized by [5]. In addition, researchers have given fresh water ice thermal conductivity functions that vary with temperature [13-16], as summarized by [17]. These functions are given in Equations (1) to (4).

[13] $$\lambda_{i_{(Sakazume-Seki)}} = 1.16(1.91 - 0.00866\, T_i + 0.0000297\, {T_i}^2) \quad (1)$$

[14] $$\lambda_{i_{(Choi-Okos)}} = 2.2196 - 0.0062489\, T_i + 0.00010154\, T_i^2 \quad (2)$$

*Corresponding Author: taimur.rashid@uit.no

[15] $$\lambda_{i(\text{Yen})} = 6.727\, exp(-0.0041(T_i + 273.15)) \quad (3)$$

[16] $$\lambda_{i(\text{US Army})} = 2.21 - 0.011T_i \quad (4)$$

where λ_i is the coefficient of thermal conductivity of fresh water ice in W/ (m.K) and T_i is temperature in ^{o}C.

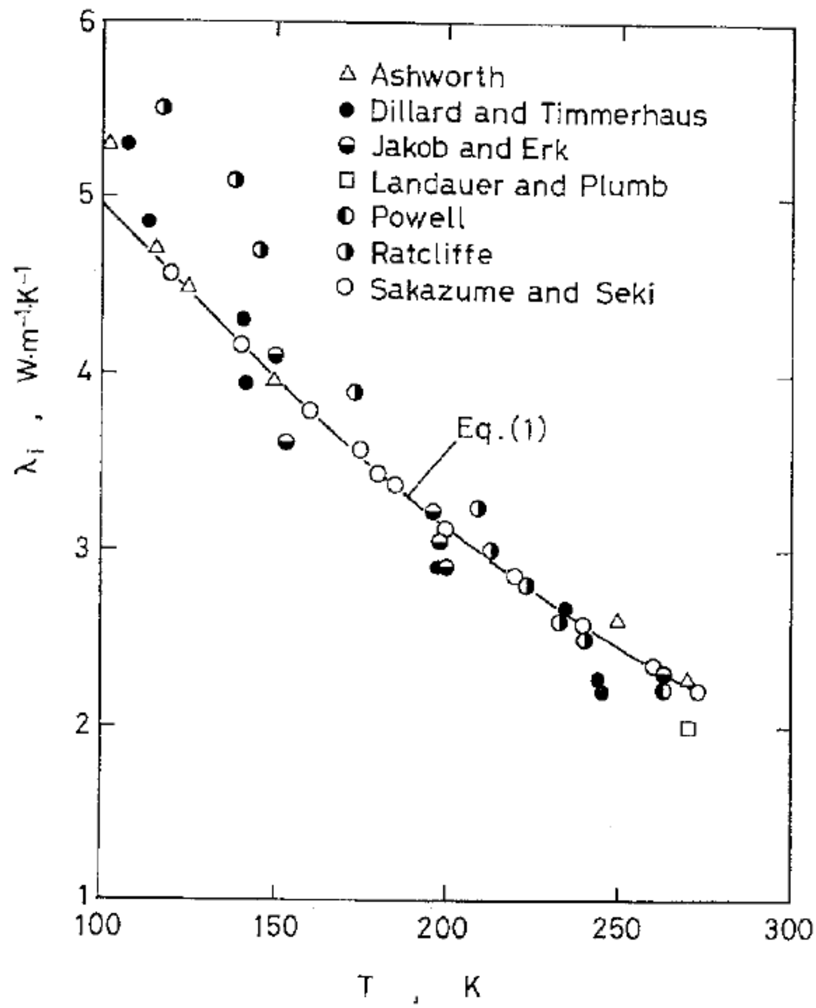


Figure 1: Variation in λ_i , the Thermal Conductivity (Wm^{-1}K^{-1}) of Fresh Water Ice with T the Temperature (K), Reproduced from [5].

According to [5], the variation between different sets of thermal conductivity results is due to the fact that each researcher used different techniques in the preparation of the samples and collection of the experimental data.

The thermal conductivity of sea water ice is different from that of fresh water ice due to the salinity factor. Figure 2 shows the effect of salinity on sea water ice [5]. Sea water ice as reported by [18-20] is also shown.

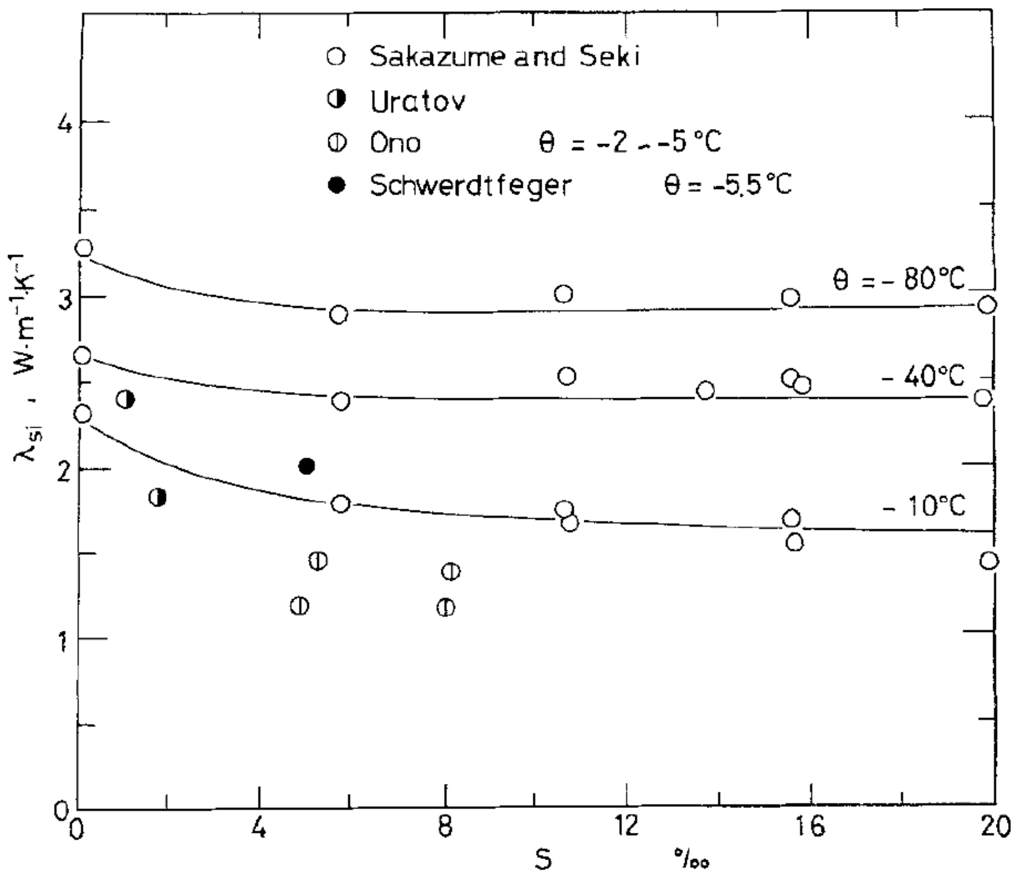


Figure 2: Variation in λ_i , the Thermal Conductivity ($Wm^{-1}K^{-1}$) of Sea Water Ice with S the Salinity (%), Reproduced from [5].

In addition, the conductivity of sea water ice is affected by air bubbles and density, as shown in Figure 3 [15].

Researchers have also used advanced experimentation techniques to evaluate the conductivity of sea water ice [21].

This work focuses on the determination of the coefficient of thermal conductivity and the overall heat transfer coefficient, using infrared experimental technique [22, 23] and FTCS simulation method using MATLAB® [24]. The results obtained are compared with those in the literature.

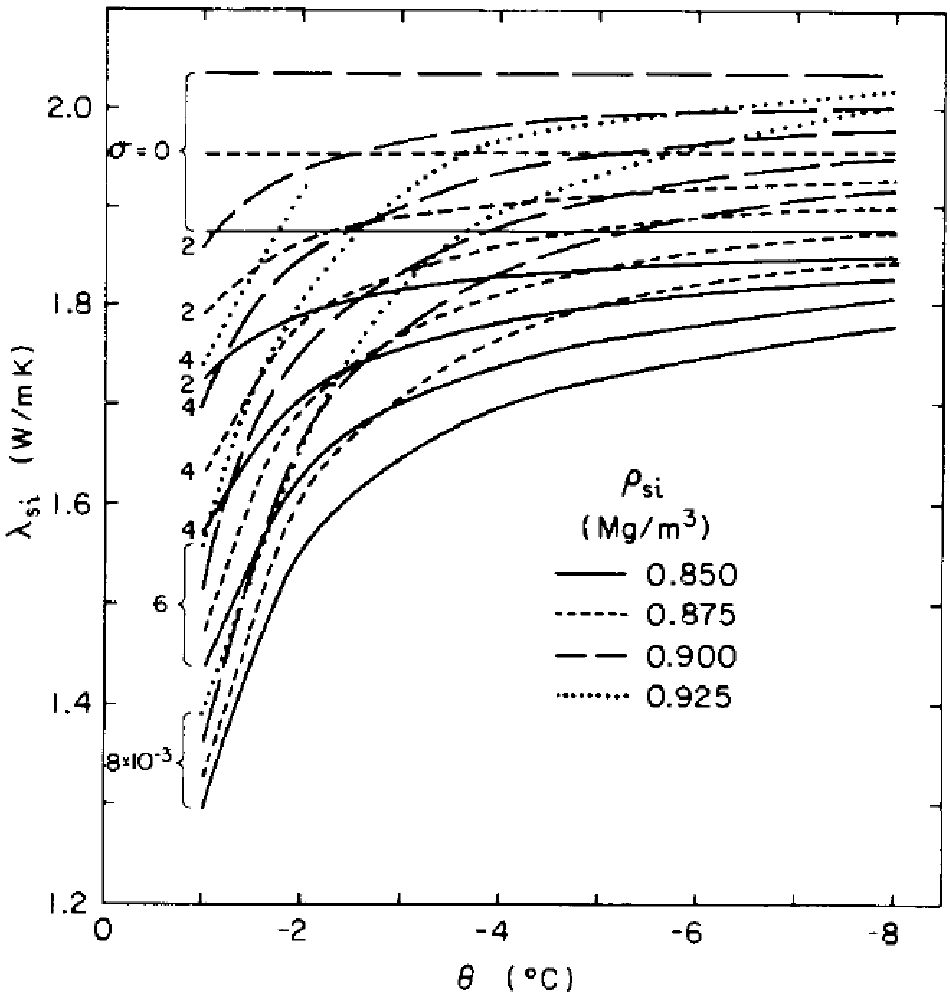


Figure 3: Variation in λ_{si}, the Thermal Conductivity of Sea Water Ice ($Wm^{-1}K^{-1}$) with Salt Contents σ (mg of salt/mg of water), ρ_{si} the Density of Sea Water Ice (Mg/m^3) and θ the Temperature (°C), Reproduced from [15].

2. METHODOLOGY

The underlying physics of heat transfer through conduction in a solid medium can be solved mathematically using the heat equation [5] as given in Equation (5).

$$\rho c \frac{\partial T}{\partial t} = \dot{q} + \frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) \tag{5}$$

where ρ is density of the medium (kg/m^3), c is specific heat capacity (J/(kg K)), $\dot{q}$ is the volumetric energy generation term (W/m^3), k is coefficient of thermal conductivity (W/ (m.K)), T is temperature (K), x refers to spatial position (m) and t is the time (s).

The extended form of the above equation in three spatial dimensions with no energy generation term [6] is given in Equation (6).

$$\frac{\partial T}{\partial t} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) \tag{6}$$

where x, y and z refer to spatial positions (m) in three dimensions and α is the thermal diffusivity term (m^2/s) as given in (7).

$$\alpha = \frac{k}{\rho c} \tag{7}$$

To solve Equation (6), the boundary, and the initial conditions are required. The convective boundary conditions [27] are applied on each external surface of the cubical geometry as given in Equation (8).

$$-k\frac{\partial T_s}{\partial x} = h(T_\infty - T_s) \tag{8}$$

where T_s is the surface temperature (K), T_∞ is the surrounding temperature (K) and h is convective heat transfer coefficient ($W/(m^2.K)$).

2.1 Finite Difference Method (MATLAB®)

The Finite Difference Method (FDM) is a numerical method for solving differential equations such as the heat equation, as given in Equation (5). This method approximates the differentials by discretizing the dependent variables (temperature, in this case) in the independent variable domains (space and time, in this case) [28]. Each discretized value of the dependent variable is referred to as a nodal value. In this case, the heat equation, as given in Equation (6), is discretized using a Forward-Time Central-Space (FTCS) FDM. The discretized equation is given in Equation (9).

$$\begin{aligned} T_{i,j,k}^{t+1} = T_{i,j,k}^{t} &+ \alpha\frac{\left(T_{i+1,j,k}^{t} - 2T_{i,j,k}^{t} + T_{i-1,j,k}^{t}\right)}{(\Delta x)^2}\Delta t \\ &+\alpha\frac{\left(T_{i,j+1,k}^{t} - 2T_{i,j,k}^{t} + T_{i,j-1,k}^{t}\right)}{(\Delta y)^2}\Delta t \\ &+\alpha\frac{\left(T_{i,j,k+1}^{t} - 2T_{i,j,k}^{t} + T_{i,j,k-1}^{t}\right)}{(\Delta z)^2}\Delta t \end{aligned} \tag{9}$$

where superscript t and subscript i, j, k refer to time and position, respectively for a value of nodal temperature. Δt is a timestep size (s) and $\Delta x, \Delta y, \Delta z$ are the differences in the spatial positions of the temperature nodes.

The boundary condition is also discretized using the FDM but only applied to the outer surfaces as given in Equation (10).

$$-\lambda\frac{\left(T_{i+1,j,k}^{t} - T_{i,j,k}^{t}\right)}{\Delta x} = h(T_\infty - T_{i,j,k}^{t}) \tag{10}$$

It is vital for the stability and accuracy of the FDM to choose the correct time step value. In this work, the Courant–Friedrichs–Lewy (CFL) condition [28, 29] is used to decide the time step size. The CFL condition for the heat equation is given in Equation (11).

$$2\alpha\Delta t \leq (\Delta x)^2 \tag{11}$$

Equations (9) and (10) are solved and post-processed in MATLAB® [30]. The results are discussed in Section 3.

2.2 Infrared imaging experiment

Infrared imaging experiments are performed using an A310 FLIR® infrared camera. The data is analyzed by FLIR research software [31]. The schematic of the experimental setup is shown in Figure 4. The actual experimental setup is shown in Figure 5.

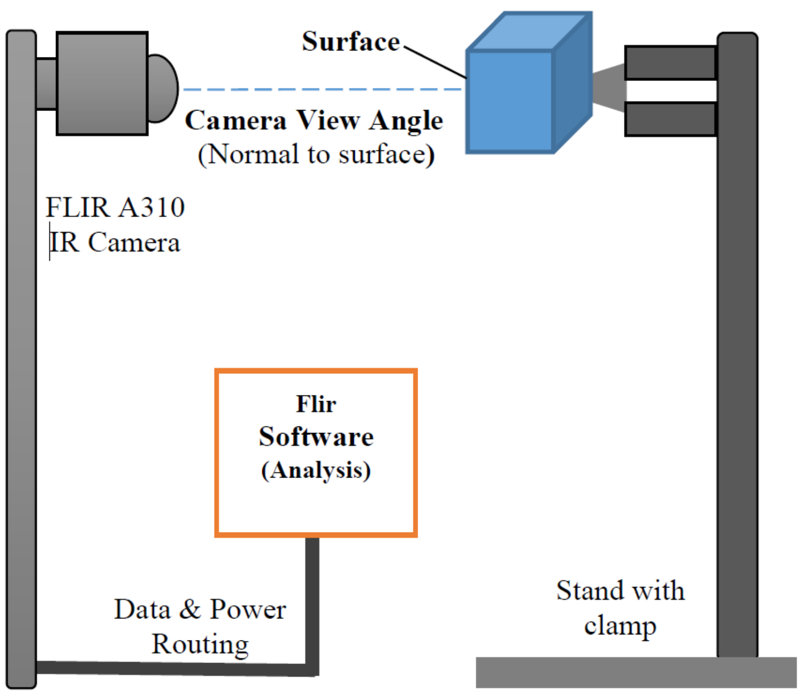


Figure 4: Infrared Imaging Experiment Schematics. Infrared camera is facing the surface of ice block [22, 23].

As shown in Figure 4, the camera is placed facing to the surface of the ice block. The sample of ice is clamped in a stand by means of a piece of wood, which had been frozen into the ice. Because the thermal conductivity of wood [32] is quite low in comparison to ice [33], it is safe to assume that the wood will not interfere significantly with the thermal behavior of the ice.

The sample of ice was taken out of the freezer at -31 ^{o}C and was then allowed to warm under room conditions. The dimensions of the ice cube were 13.5 x 13.5 x 10 cm^3. As it warmed, temperature profiles formed on the surface of the ice, which were recorded using an infrared imaging camera. The results reveal the variations in temperature over time.

The experiments were repeated with fresh water and saline ice cubes. The saline water was collected from Norskhavet (GPS 69°41'07.2"N 19°00'23.3"E) with salinity around 25%. The salinity in Norwegian fjords is less than in the coastal waters and varies over the year [34].

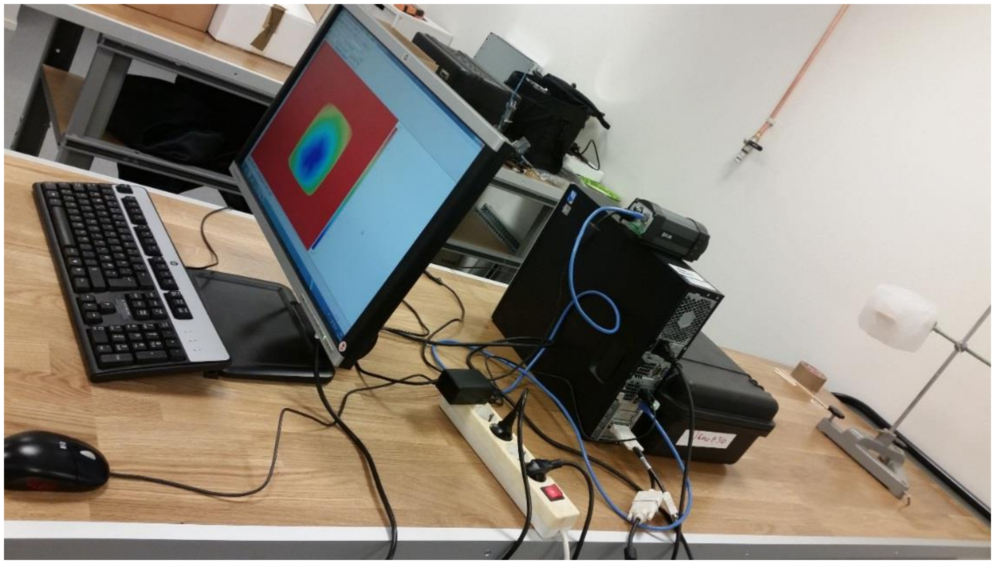


Figure 5: Actual Infrared Imaging Experiment Setup

3. RESULTS AND DISCUSSION

In order to determine the thermal properties of ice, a piece of ice was taken from a cold environment (a freezer set at -31 ºC) and allowed to warm under room temperature conditions. These conditions established thermal gradients within the ice cube. Two important features can be observed: 1) variation in temperature on the ice surface and 2) increase in the temperature of the ice cube. Keeping in view the mechanism of heat transfer, thermal conductivity is mainly responsible for the first feature, since it determines the amount of heat that is transferred within the ice cube. The second feature is associated with the overall heat transfer coefficient. This feature determines how rapidly heat is being released (or absorbed as in this case) by the ice surfaces from the surroundings.

3.1 Coefficient of thermal conductivity

The coefficient of thermal conductivity (also known as thermal conductivity) determines the amount of heat transfer based on the temperature difference between the two points. Figure 6 and Figure 7 show the variation in the temperature of the fresh water and saline water ice cubes, respectively. Table 1 gives the corresponding values of thermal conductivity.

Table 1: Coefficient of Thermal Conductivity of Fresh Water and Saline Water Ice

Coefficient of Thermal Conductivity of Ice (λ)	**Value (W/(m.K))**
Fresh Water Ice	2.35
Saline Water Ice	0.8

The coefficient of thermal conductivity of fresh water ice is found to be in agreement with the literature [33]. The coefficient of thermal conductivity of saline water ice is found to be less than the values stated in the literature. The reason for such behavior can be explained by the fact that the saline ice started to melt far earlier in the experiments. This created a layer of water around the ice cube, hence reducing the effective thermal conductivity of the ice cube. The thermal conductivity of saline water is in the range of 0.5-0.7 (W/(m.K)) [35]. The thermal conductivity varies with temperature; however, this study shows the average values over a temperature range (-30 ^{o}C to 0 ^{o}C).

The temperature contours are not symmetric in experiments as can be seen in Figure 6 and Figure 7. This can be associated with the influence of buoyancy.

3.2 Coefficient of overall heat transfer

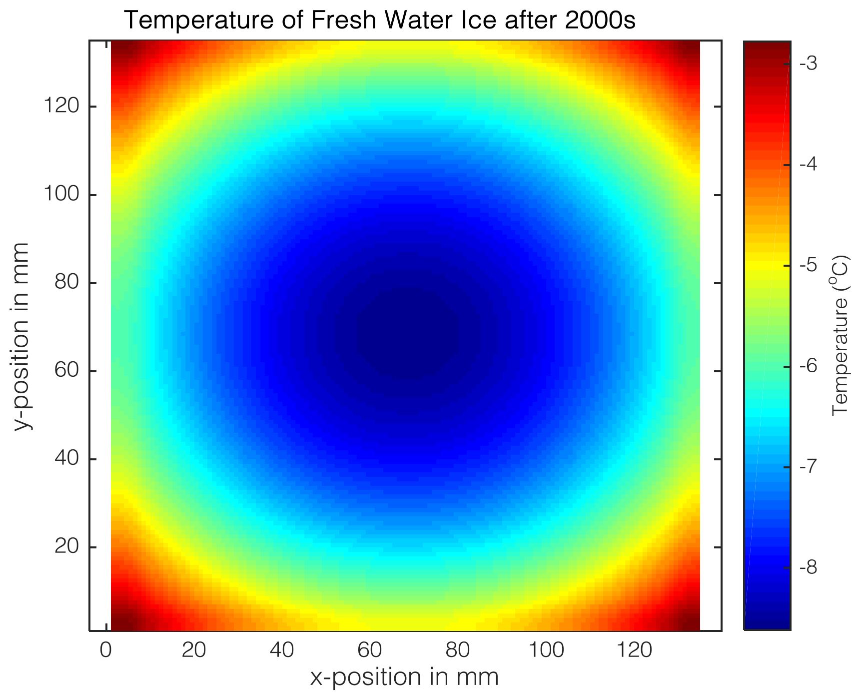
The coefficient of overall heat transfer (also known as the overall heat transfer coefficient) determines heat flux from one body to another. In the given case, the overall heat transfer coefficient determines the amount of heat energy being transferred from the surroundings to the ice cubes. In order to calculate the heat transfer coefficient, the variation in temperature is monitored on the ice cube surface. Figure 8 and Figure 9 show the variation in temperature for the fresh water and saline water ice cubes in time. Table 2 gives the corresponding values of thermal conductivity.

Table 2: Coefficient of Overall Heat Transfer of Fresh Water and Saline Water Ice

Coefficient of Overall Heat Transfer of Ice (h)	Value (W/(m^2.K))
Fresh Water Ice	9.2
Saline Water Ice	4.2

The results indicate that the heat transfer coefficient of fresh water ice is almost twice that of saline water ice under the same room conditions. This can further be linked to the observation that saline water ice started to melt in the initial stages and hence built a coat of water on the ice, consequently reducing its heat transfer. The coefficient of overall heat transfer varies with temperature; however, this study shows the average values.

(a)



(b)

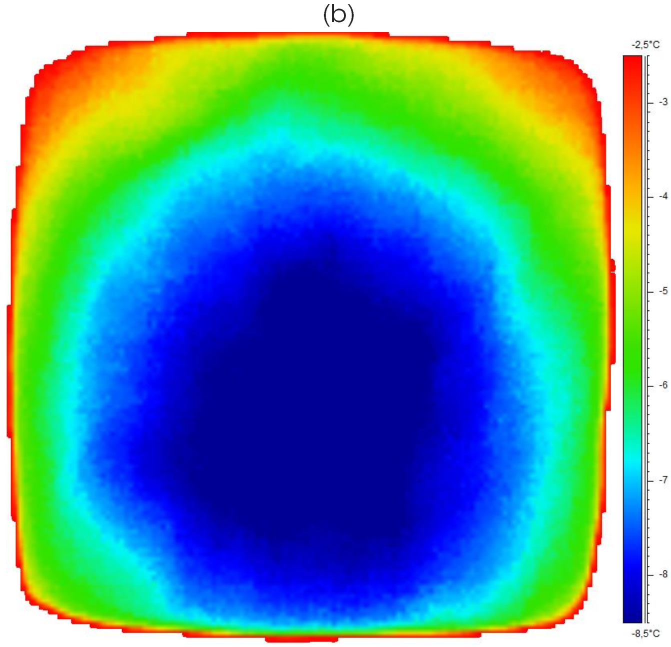
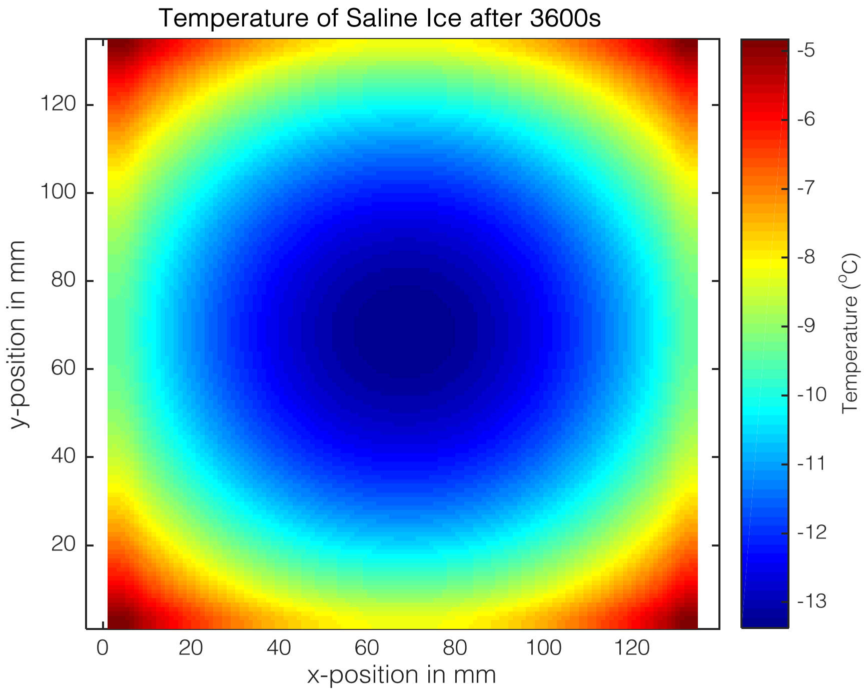


Figure 6: Variation in Temperature of the Fresh Water Ice Cube after 2000 s; (a) Finite Difference Method (MATLAB®); (b) False Infrared Image (A310 FLIR®)

(a)



(b)

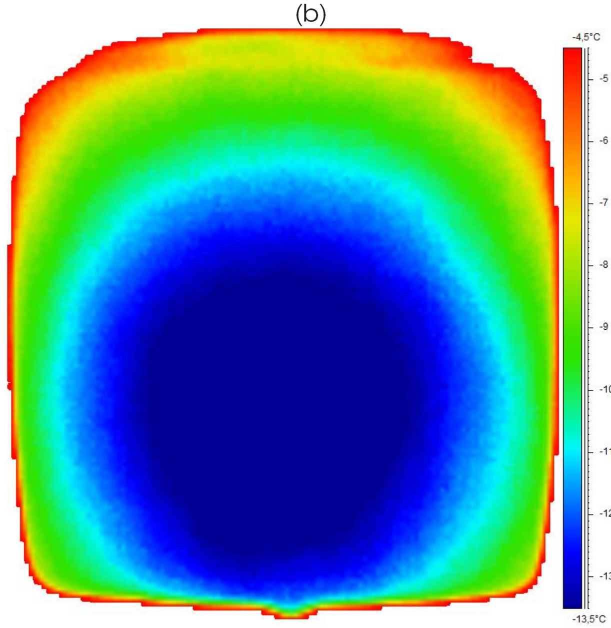


Figure 7: Variation in Temperature of the Saline Water Ice Cube after 3600s; (a) Finite Difference Method (MATLAB®); (b) False Infrared Image (A310 FLIR®)

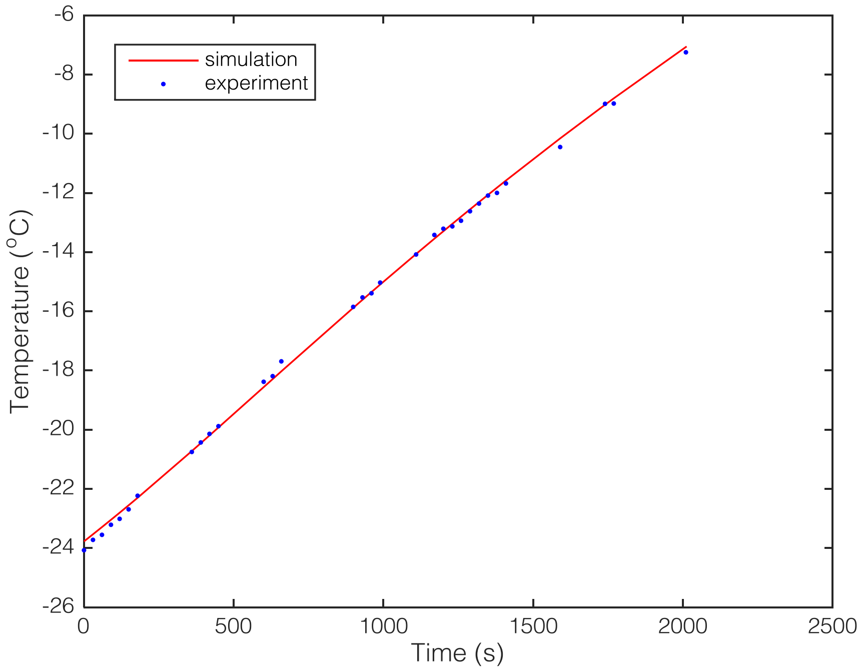


Figure 8: Variation of Temperature with Time for Fresh Water Ice Cube

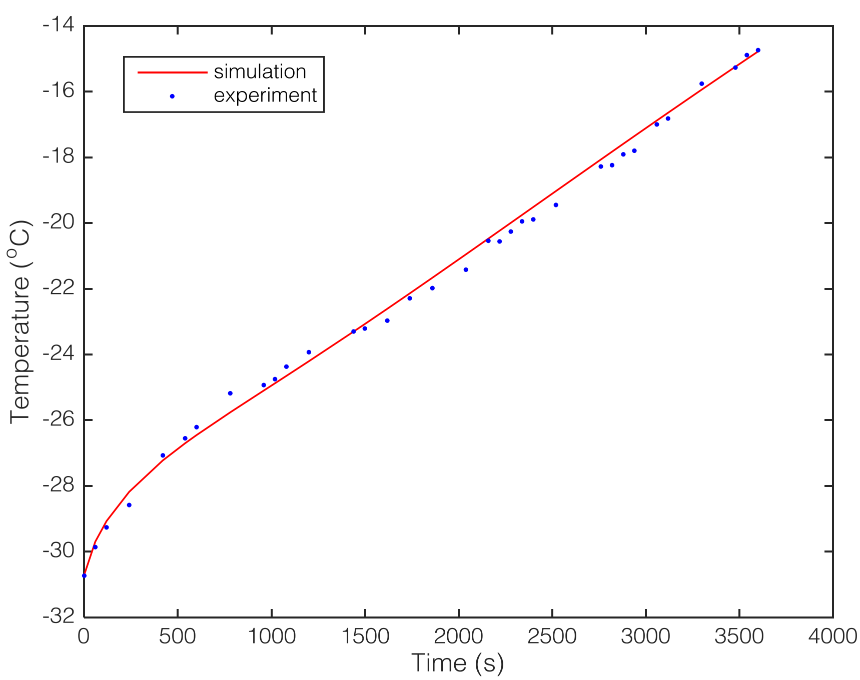


Figure 9: Variation of Temperature with Time for Saline Water Ice Cube

7. CONCLUSION & FUTURE WORK

A Multiphysics finite difference methodology has been employed to determine the thermal properties of freshwater and saline ice. The real-time results were obtained through an infrared imaging technique. These results were matched with a finite difference model. Using this method, two thermal properties of fresh water ice and saline water ice were identified, namely, thermal conductivity and overall heat transfer coefficient. Thermal conductivity was obtained by observing the temperature profiles on the surface of the ice cubes. The overall heat transfer coefficient was determined by observing the variation of temperature over the surface in time.

The results reveal that the fresh water ice has an average conductivity of about 2.35 (W/(m.K)) in a temperature range of about -30 ^{o}C to 0 ^{o}C, which is in agreement with the literature. This value dropped significantly for saline ice to 0.8 (W(m.K)) in the same temperature range. In addition, it is observed that the melting of saline ice contributed to a reduction in the thermal conductivity; hence, it can be said to be effective thermal conductivity rather than an absolute value.

The average value of the overall heat transfer coefficient of fresh water ice was found to be about 9.2 (W/(m^2.K)). The average value of the overall heat transfer coefficient of saline ice was found to be 4.2 (W/(m^2.K)), approximately half that of fresh water ice. This may also be associated with the layer of liquid water around the ice cube.

ACKNOWLEDGMENT

The work reported in this paper is funded by the Norges forskningsråd, project no. 195153/160 in collaboration with Faroe Petroleum. We would also like to acknowledge the support given by Prof. James Mercer at the UiT The Arctic University of Norway.

REFERENCES

[1] Dorsey, N.E., Properties of ordinary water-substance. 1940.

[2] Giauque, W. and J. Stout, The Entropy of Water and the Third Law of Thermodynamics. The Heat Capacity of Ice from 15 to 273° K. Journal of the American Chemical Society, 1936. 58(7): p. 1144-1150.

[3] Lonsdale, D.K. The structure of ice. in Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences. 1958. The Royal Society.

[4] Ginnings, D.C., An Improved Ice Calorimeter-the Determination of its Calibration Factor and the Density of Ice at ooe. 1947.

[5] Fukusako, S., Thermophysical properties of ice, snow, and sea ice. International Journal of Thermophysics, 1990. 11(2): p. 353-372.

[6] Irvine, W.M. and J.B. Pollack, Infrared optical properties of water and ice spheres. Icarus, 1968. 8(1): p. 324-360.

[7] Powell, R.W., Thermal conductivities and expansion coefficients of water and ice. Advances in Physics, 1958. 7(26): p. 276-297.

[8] Jakob, M. and S. Erk, Warmedehnung des Eises zwischen 0 und-253. Z Ges. Kalte-Ind., 1928. 35: p. 125-130.

[9] Landauer, J. and H. Plumb, Measurements on anisotropy of thermal conductivity of ice US Army SIPRE Research Report No. 16. 1956.

[10] Jakob, M. and S. Erk, Die Wärmeleitfähigkeit von Eis zwischen 0 und-125. Zeitschr. f. techn Physik, 1929. 10(12): p. 623-624.

[11] Ratcliffe, E., The thermal conductivity of ice new data on the temperature coefficient. Philosophical Magazine, 1962. 7(79): p. 1197-1203.

[12] Dillard, D.S. and K. Timmerhaus, Low temperature thermal conductivity of solidified H2O and D2O. Pure and Applied Cryogenics, 1966. 4: p. 66.

[13] Sakazume, S. and N. Seki, Thermal properties of ice and snow at low temperature region. Bulletin of the Japanese Society of Mechanical Engineering, 1978. 44: p. 2059-2069.

[14] Choi, Y.H., Effects of Temperature and Composition on the Thermal Conductivity and Thermal Diffusivity of Some Food Components. Korean Journal of Food Science Technology, 1986. 18(5).

[15] Yen, Y.-C., Review of thermal properties of snow, ice and sea ice. 1981, DTIC Document.

[16] Engineering & Design –Ice Engineering: Manual No. 1110-2-1162. 1996, U.S. Army Corps of Engineers: Washington D.C.

[17] Tarnawski, V.R., W.H. Leong, and T. Momose, Modeling the Thermal Conductivity of Frozen Foods, in Encyclopedia of Agrophysics. 2014, Springer. p. 483-489.

[18] Sakazume, S. and N. Seki, Bulletin of J. S. M. E, 1980. 46 p. 1119-1126.

[19] Schwerdtfeger, P., The thermal properties of sea ice. Journal of Glaciology, 1963. 4: p. 789-807.

[20] Ono, N., Thermal properties of sea ice. i- measurements of the thermal conductivity of young winter ice. Low Temperature Science, 1965. 23: p. 167-176.

[21] Pringle, D., H. Trodahl, and T. Haskell, Direct measurement of sea ice thermal conductivity: No surface reduction. Journal of Geophysical Research: Oceans (1978–2012), 2006. 111(C5).

[22] Rashid, T.K., Hassan Abbas; Edvardsen, Kåre; Mughal, Umair Najeeb, Infrared Thermal Signature Evaluation of a Pure Ice Block, in Sensorcomm 2015, International Academy, Research and Industry Association (IARIA): Venice ,Itlay.

[23] Rashid, T., et al., Infrared Thermal Signature Evaluation of a Pure and Saline Ice for Marine Operations in Cold Climate. Sensors & Transducers, 2015. 194(11): p. 62-68.

[24] H. Khawaja, T.R., O. Eiksund, E. Broadal, K. Edvardsen, Multiphysics Simulation of Infrared Signature of an Ice Cube. The International Journal of Multiphysics, 2016. In Review.

[25] Cannon, J.R., The One-Dimensional Heat Equation. 1984: Cambridge University Press.

[26] Widder, D.V., The Heat Equation. 1976: Elsevier Science.

[27] Moran, M.J., Introduction to thermal systems engineering: thermodynamics, fluid mechanics, and heat transfer. 2003: Wiley.

[28] Patankar, S., Numerical Heat Transfer and Fluid Flow. 1980: Taylor & Francis.

[29] Courant, R., K. Friedrichs, and H. Lewy, Über die partiellen Differenzengleichungen der mathematischen Physik. Mathematische Annalen, 1928. 100(1): p. 32-74.

[30] MATLAB®, version 8.5.0.197613. 2015, The MathWorks Inc.: Natick, Massachusetts.

[31] ThermaCAM, Researcher Pro 2.9. 2007, FLIR Systems AB: Sweden.

[32] Laboratory, F.P., Centennial Edition: Wood Handbook: Wood As an Engineering Material. 2013: CreateSpace Independent Publishing Platform.

[33] Petrenko, V.F. and R.W. Whitworth, Physics of Ice. 2002: OUP Oxford.

[34] Helland-Hansen, B. and F. Nansen, The Norwegian Sea: Its Physical Oceanography Based Upon the Norwegian Researches 1900-1904. 1909: Det Mallingske bogtrykkeri.

[35] Ramires, M.L.V., et al., Standard Reference Data for the Thermal Conductivity of Water. Journal of Physical and Chemical Reference Data, 1995. 24(3): p. 1377-1381.