UiT THE ARCTIC UNIVERSITY OF NORWAY

Faculty of Science and Technology

Optimal Ordering Strategy

of Perishable Products

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Simon Sagelv Bjørvik

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Abstract

It is challenging to plan for the future, especially if there are uncertainties. In the retail store business making the right orders at the right time might have a severe impact on the profit of the store. Maintaining the right inventory is a part of this. With fluctuations in price and competition, having the best deals, and being at the whims of the customers making sure that the store has the correct amount of items in store is a challenge.

By analyzing the historical sales data, we are able to make predictions for the expected sales. By doing this we can improve our inventory management to the point we are, at any time, able to meet the customer's demands. The thesis has used statistical methods and tools to analyze the data, resulting in both a visual representation, and a simple mathematical representation of the analysis.

In this thesis we used sales data for an item sold at Coop Obs Hypermarked Tromsø, and analyzed this to make predictions for the sales in the future. At the end of this thesis there have been made a suggestion for a simple inventory management of an item.

Keywords: Inventory, Management, Optimization, Planning, Sales

Acknowledgements

This thesis is written as a part of the 2-year master's program M.Sc. Technology and Safety in the High North at UiT the Arctic University of Norway.

Due to my interest in finding the best solutions to problems, I decided to write my thesis around a problem like this. Writing this thesis is most likely the most challenging work I have ever done. The process has taught me a lot about performing my own research and finding the needed theory.

I would like to thank my supervisor Yuan Fuqing that helped me find what I needed to write this thesis. I also want to thank Hugo Andersen and Hilde Rasmussen at Coop Obs Hypermarked Tromsø that supplied me with the data I used for this thesis.

I wish to thank all my friends and classmates throughout my education at UiT and especially my friends at the office, some whom I have studied with since the beginning of my bachelor's degree. Thank you for all the support and all the games of cards. Without you the years at UiT would not be the same.

Simon Sagelv Bjørvik

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1. Introduction

This chapter will present the theme for the thesis, and the thesis problem. Furthermore, the aim and objective of the thesis will be presented, as well as the limitations of the project, and the structure of the thesis.

The reader of this project is assumed to have basic knowledge within statistics and probability. In addition to having an understanding of basic retail management.

1.1 Background

Inventory management is the act of maintaining a proper, well stocked inventory. It is used everywhere from the food in your own fridge to the spare parts at the car mechanic. During your everyday life you may not even realize the work that some people do to ensure that your local store has the items you need, or that the printer at work always has enough paper available.

1.2 Problem

During this thesis, we will perform an analysis of the sales numbers of a retail store in the Coop enterprise. We will look at a specific item and try to find the optimal ordering amount and frequency based on the amount of sales the store has of that item. Using the sales numbers, we will find the expected sales trends of a certain product at any time.

1.3 Goal

The goal of this thesis is to present a strategy for how often a restock order should be delivered, and how much each delivery should contain.

- The final result should be to present the information needed to create a strategy for optimal inventory management of an item, and give a suggestion for a reordering strategy using the knowledge earned through the thesis.
- The safety stock should be determined; this should give the store clerks a goal to work towards.
- Each restock order should not be too large because the item has an expiration date, and the item should ideally be sold in good time before this.

- The local storage area reserved for this item is limited; this has an impact on the ordering strategy.

1.4 Scope of work and Limitations

- The data given for this thesis was limited to the necessary data for the analysis.
- The data contains number of sales per day over a lengthy timespan.
- The data may vary due to special sales where the price is lowered; this might cause a change in the number of sales for a period.
- There will be no economic analysis, as the costs in some parts of the supply chain are kept secret.
- The result of the analysis is only valid for the normal conditions, that is; a normal week without any holidays and/or events that could change the sales numbers. Use of this analysis outside of the normal conditions is done on your own regards.

2. Research Approach and Methodology

In this chapter we will define what to do in this thesis and how we should do it. The steps of the analysis will be presented, as well as what we expect to find out from the analysis.

2.1 Goal of the analysis

The main goal of the analysis is to find out how the inventory management can be improved. We will look at a single item sold in at Coop Obs Hypermarked in Tromsø, and analyze the sales numbers to find the expected sales at any time. The result of this analysis can be used to make a plan for the number of items that should be in inventory at any time. Finding the expected sales can also let us find the necessary safety stock that is needed to prevent running out of stock.

2.2 Data collection

This thesis consists of an analysis performed using sales data from Coop Obs in Tromsø. The data consists of the sales of a single dairy product. The data contains the number of sales that were made in the period 27th of April 2015 to 26th of April 2016, having a full year of data points ensures that we have a decent database for our calculations. I had the permission of store manager Hugo Andersen of Coop Obs Hypermarked to use this data for my thesis, and the help of Hilde Rasmussen for the collection of the data and for answers to my questions about the local storage.

This data used in this thesis is what we call primary data as it is gathered directly from the source, as opposed to secondary data that is gathered and/or manipulated by someone else before you. (Joop & Hennie, u.d.)

After collecting the data, the processing begins. The data is sorted by each day of the week, giving us seven datasets, one for each day of the week. This separation of the data allows us to figure out the expected demand for each day of the week.

2.3 Data evaluation and interpretation

In using the datasets for our analysis we must make some assumptions about the data, and define how it is interpreted. Seeing as each data point has six days between them we assume that the data is independently distributed of each other, and that each data point does not affect the next one.

We also make the assumption that sales that happen in the days before the day in question does not affect the sales. As well as assuming that any possible fluctuations in the price has no impact on the sales either.

Sometimes, due to certain holidays, the store has been closed for a day it normally would have been open. This is logged as zero sales that day. Since it was not possible to sell any items due to the store being closed, these numbers are ignored in the analysis.

During December the store is allowed to be open on Sundays, causing there to be sales happening on these days. Since there are very few data points from sales on Sundays, these sales are added to their respective Saturdays to include these sales for the inventory management.

There are also a lot of zeroes in the data, these represent days that the store was not open and will be removed before any calculations as they do not represent the normal conditions.

2.4 Data analysis

The analysis will be performed using MatLab's distribution fitting tool (dfittool), this allows us to use a set of data and find the fit for distributions, and show these visually along with the data. Using MatLab we will present the Probability Density Function (PDF) and Cumulative Density Function (CDF) for the data sets and use these to find the expected number of sales.

When the data has been analyzed we will have gained the necessary knowledge that we need to make a plan for the inventory management.

3. Inventory Management

This chapter will explain the basics of managing an inventory system, showing the importance of planning for the expected demand and ensuring a minimum stock level. The chapter will also present the basic theory behind the analysis done to find the sales trend and expected sales.

3.1 The inventory

Inventories are everywhere; you may not even realize where you will find most of them. Your fridge is stocked with milk, cheese, ham, and many other things, all of these are inventories. Do you have some spare lightbulbs lying around? These stocked spare parts are also an inventory. Even your shoes and clothes are inventories of items that you may choose to use. The goal of inventory management is to always have the necessary spare parts available to replace the part that is broken. In the case of a retail store, this is to restock the shelf once the item has sold.

Inventories are usually categorized into several different types: anticipation stock, cycle stock, safety stock, pipeline stock, and decoupling stock. (Muckstadt & Sapra, 2010) These will be explained below.

3.1.1 Anticipation stock

Anticipation stock is exactly what it sounds like, it is a stock used to meet the expected number of sales in the future. This could be ordering a large quantity of items that the store knows will be on sale the next week. The store could anticipate there to be a “price war” for a certain item the next holiday, and expect to sell much more of that item than they usually do. Some items might only be sold in a specific season, and so the production is maintained all year in anticipation to sell a lot of this item when the season comes.

3.1.2 Cycle stock

Cyclical stock is a type of stock that shows the same demand for each cycle. This is very important for companies that expect to sell approximately the same number of items for each time period. These cycles are mostly considered as either a day, week or month. A producer of fresh pastries and bread need to produce enough to meet their daily demands, while a retail store selling dry goods, might only need to resupply each type of item weekly or even monthly.

3.1.3 Safety stock

The safety stock is the minimum stock the store expects to have at any time. This is necessary to meet unexpectedly high demands for an item. In the case of a delay in the restock delivery, the store will still have a few days' worth of stock in inventory. The size of the safety stock should be large enough to ensure the store can still supply their customers with the item they want, even if there is an issue in the restock delivery. Figure 1. shows how the safety stock is used in combination with the cyclical stock. The reorder should be done in good time before the inventory level is expected to reach the safety stock level. The right side of the figure shows some variable demands between the reorder point and the delivery.

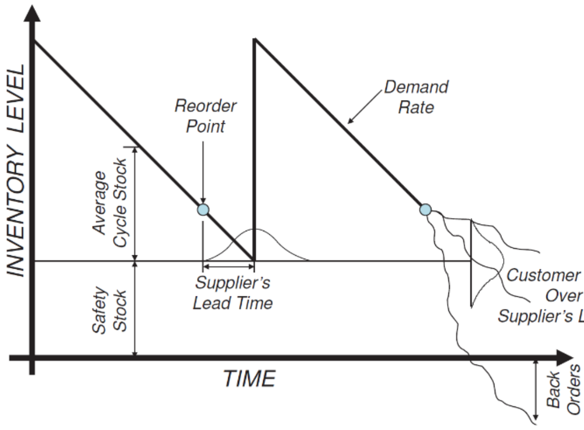

Figure 1 - Cycle stock and safety stock (Muckstadt & Sapra, 2010)

3.1.4 Pipeline stock

The pipeline stock is equal to the expected demand over the supplier's lead time. This is essentially a variant of the safety stock specifically for the waiting time before restocking. The safety stock is a broader definition considering more than just this issue.

3.1.5 Decoupling stock

The decoupling stock is almost only used in manufacturing. The decoupling stock contains the items that you find between work stations. These exist to protect the process from being influenced by variations in production at the different stations. Having an inventory between each work station lets each step of the process be mostly independent of the others. This ensures that production is not held up, by not being blocked from sending the products onwards, nor from not having the materials to work.

In this thesis, we will only use the stocks that are relevant for a retail store. The cyclical stock and the safety stock are the most important ones, and to a certain degree the anticipation stock. The pipeline stock and decoupling stock will not be used for this analysis. By performing a trend analysis of the sales, we can make a plan for the best use of the inventory.

3.2 The ordering strategy

The goal of the strategy should be to ensure that the items in question is always available when and where it is needed. To make a good ordering strategy you should be able to answer these four important questions: (Muckstadt & Sapra, 2010)

- What items should be stocked?
- Where should these items be stored?
- How much should be ordered when the order is placed?
- When should the order be placed?

The first question should be easily answered by what type of business you are running. In our case of a retail store, there should be many different wares available for the customers. The store itself will have to decide exactly which items they want to stock based on the customers' demands and the available suppliers.

The second question will depend on the item in question, as many wares have demands on the storage conditions. Especially fresh and frozen foodstuffs have demands such as being stored at a certain temperature. The storehouse needs to have available storage that is satisfactory for the best perseverance of the item.

The third and fourth questions are more difficult to answer. Most stores usually just make orders when the inventory is reduced to a certain amount. Then just order what they think they will need until it is time for the next order. While this is an acceptable practice, it is far from optimal. If the store is able to adjust their order amount and frequency according to the expected demand, they might save on costs due to less frequent transports.

Nevertheless, to answer the last two questions as best we can, we need to perform a trend analysis on the sales of an item.

3.3 Availability

The main concern the customer has when shopping is that the item they are after is available. Thus the goal for the store is to maintain an availability of 100%. A customer that does not find the item they are after might lose their trust in the store. Making sure that the item is available at all times is therefore paramount to the store. Thus creating an ordering strategy that ensures a high availability at all times should be our goal. In general, availability is computed as the ratio of the accumulated uptime and the sum of the accumulated uptime and downtime (Sandborn, 2013)

$$Availability = \frac{Uptime}{Uptime + Downtime} \quad (1)$$

This is the most basic definition of availability and is very useful when we are considering only a single item. However, this is not very useful when we consider multi-item systems. When there is a large amount of items stored at the same location, and only one item need to be present for the item to be available, finding the availability becomes more of a task of ensuring sufficient “spare parts”.

Let us say there is room for 50 items on the shelf and only one item needs to be there for the item to be available. By the time that all 50 items are sold, the store clerks will have discovered that the stock is running low and have refilled the missing parts. Refilling the shelves does not take much time, thus the MTTR can be considered minimal as long as there are available items to restock in the inventory.

3.4 Finding optimal delivery frequency and amount

As determined earlier, finding the best frequency and ordering amount is the key factor in optimizing the inventory system. The best result you can get is when your orders match the expected sales. This result might not be easy to find though, when the store has many items from the same source, the difficulty in maintaining the optimal stock increases. The goal will then be to keep the number of deliveries at a minimum, while still having enough wares in stock.

To do this, we have to be able to predict how much purchased by the customers at different times of the week. Say, if the sales on Mondays are larger than they are mid-week, then a large delivery might be delivered Monday morning to replenish stock from last week and to ensure that the stock is large enough for the expected sales.

3.5 Perishable Products

Some types of wares have a limited window for their usage. These items can be anything from foodstuffs to newspapers. We call these types of products “perishable”, due to their limited lifetime. Having these types of items in stock requires more planning to than your average dry goods, as you need to sell them before they spoil or are no longer needed. Most food products have set conditions that needs to be fulfilled for their expected lifetimes to be valid, the most common requirements are storage requirements, such as, requirements to be kept within a given temperature range, or kept away from moisture. These requirements should not be too difficult to fulfill, as long as there is room in the storage. However, some storages might be quite limited, such as the local cold storage. This is where the challenge lies; to find the best frequency of restocking, while not exceeding the storage space.

When dealing with short lifetime products, it is important that the store clerks pay attention to the expiration date of the products. Unless they make sure that the oldest wares are displayed in the front, they might come to close the expiration date before they are sold.

3.6 Substitutions

In some groups of items there is the possibility of substitutions, to replace one item with another item of similar qualities. If the store has several items for sale that will fill the customers’ needs, then it might not be as catastrophic if the store runs out of one such item.

3.7 Sales trends

By analyzing the sales data, we find the trend of the demand for an item. Using this, we can attempt to predict the sales throughout the week, optimizing our control of the inventory. By separating the data set by weekdays we are able to find the demand for each day of the week. The analysis should give us a good idea of what part of the week the demand is highest, and show us if there are parts of the year, like holidays or seasons, that has substantially large sales numbers.

3.7.1 The historical sales data

By analyzing the historical data, we are able to find trends in the demands. Knowing when the most sales happen is important to ensure that the inventory is stocked accordingly.

3.7.1.1 Finding the Safety Stock

By looking at the expected sales data we can find out how large the safety stock should be. The safety stock should at minimum contain one whole day of sales, assuming that restock is always available the next day. If the resupply is not that readily available, then we need to plan for this so that the inventory does not run out. As a general rule, the longer the resupply takes from the order being made, the larger the safety stock should be.

3.7.1.2 Finding the reorder point

By analyzing the sales trend, we are able to find the reorder point. This is the point in time when we should ideally order our next restock of the inventory. The reorder point should be set before the inventory level reaches the safety level. Ideally this should be at the point where, after the suppliers lead time, the inventory level is expected to exactly reach the safety level. However, planning for this while having variable sales is impossible. Therefore, we should find a point where reorder is done, before the inventory level is reduced to the safety level.

3.7.2 Prediction

When we know the expected number of sales for each day of the week, we can plan the inventory management. Making sure that the stock is always sufficient for the customers' demands is our priority now.

3.8 Statistics

In most of our calculations we need to know how to do some basic statistical mathematics. Mainly understanding how distributions work and how we can use them to plan ahead. We will use software to perform all our calculations since this is much better when working with larger datasets. Using MatLab we will find the best fit.

When performing statistical analyses there is a need for having a sufficient number of data points. In statistics the size of sample data is the main determining factor in the confidence of the result. Having too few data points will simply not give an answer that we can trust. In the next few subchapters we will give a very basic look at the distributions used in this thesis.

3.8.1 Normal Distribution

The most widely used continuous probability distribution in statistics is the normal probability distribution. (Britannica Academic, 2016) This is the simples form of distributions as it only uses the two parameters; Mean (μ) and standard error (σ). The mean shows the center of the distribution curve, and the standard error telling of how much the distribution spreads around the Mean. The figure below shows a basic normal curve with the mean in the center.

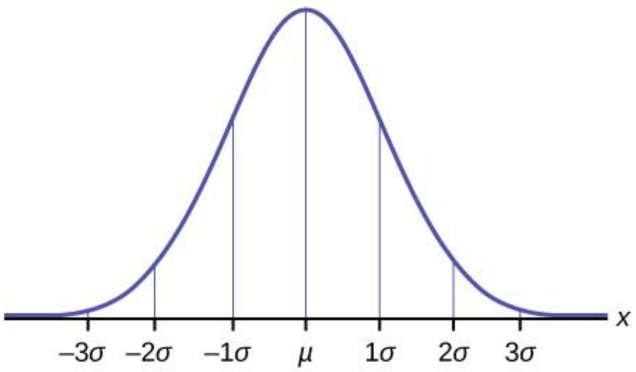

Figure 2 - The curve of a Normal Distribution (OpenStax CNX, 2016)

By using software, we can easily find the parameters used for creating a normal distribution to our data. MatLab has a very simple function that finds the mean and standard error of a normal distribution. We will use this later in the calculations.

3.8.1.1 Finding Mean and Median for the normal distribution

Finding these numbers can help us with understanding the data and its variances. The Mean is simply the average of all the numbers and has a rather simple mathematical formula to find it. (Næs, et al., 2010)

$$Mean = \frac{\sum_{i=1}^{n} x_i}{n} \quad (2)$$

Where *x* is the set of numbers, and *n* is the number of data points. Knowing the mean of the data can help a lot in understanding the data, as you now know at which value the data is centered around.

The Median tells you the middle number in a set of numbers, it can be used to tell you how the pdf is shifted around the normal distribution. The example below shows that the Median is the number in the middle of a set of sorted numbers.

$$[1, 1, 2, 6, \mathbf{7}, 9, 12, 15, 18] \quad (3)$$

The only value that matters, is that of the middle number, as it tells you where the center of the data points is located. We can compare the Median and the Mean to get an estimation of how close a fit the normal distribution is to the actual data. In the case above, we have a median of 7 and a Mean of 7.89, this means that the distribution is slightly deviating from the data. We determine how the data is skewed by comparing the Mean and Median, if the Mean is larger than the Median the distribution is right skewed, and if the Mean is less than the Median the distribution is left skewed. The figure below shows how the relationship between the Median and Mode changes the skewedness of the distribution.

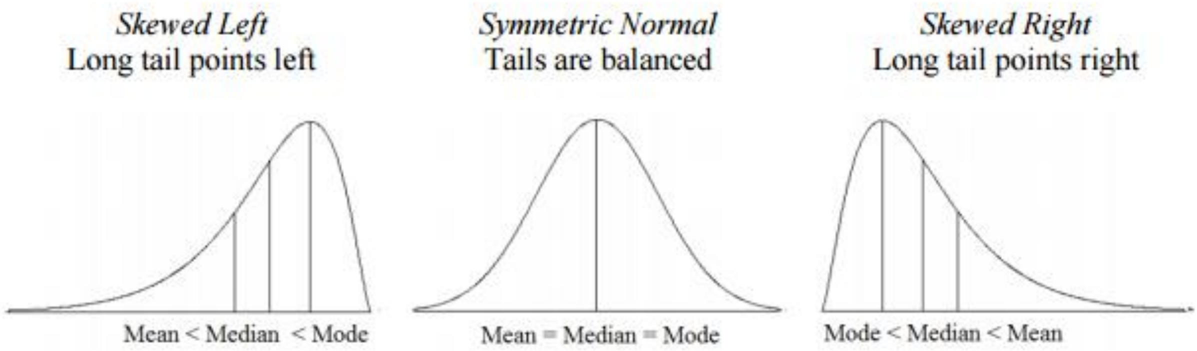

Figure 3 - Figure showing how skewedness varies according to mean and median (Doane & Seward, 2011)

3.8.2 Lognormal distribution

The lognormal distribution is a variant of the normal distribution that uses a logarithmic scale, useful when the data is skewed to the right. The figure below shows an example of how a lognormal distribution could look like.

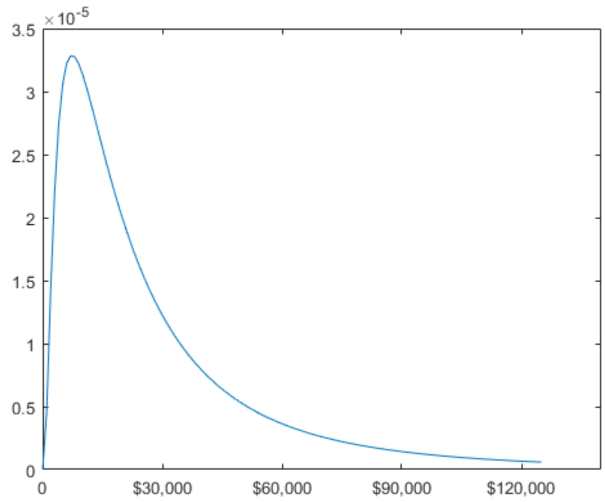

Figure 4 - Example of a lognormal distribution (MatLab, 2016)

3.8.3 Weibull distribution

The Weibull distribution is a variant of the exponential distribution. It is a very flexible distribution can be used both as a 2-parameter and 3-parameter distribution. It uses a shape and a scale parameter to define the curve. The figure below shows the Weibull distribution with a varying scale parameter.

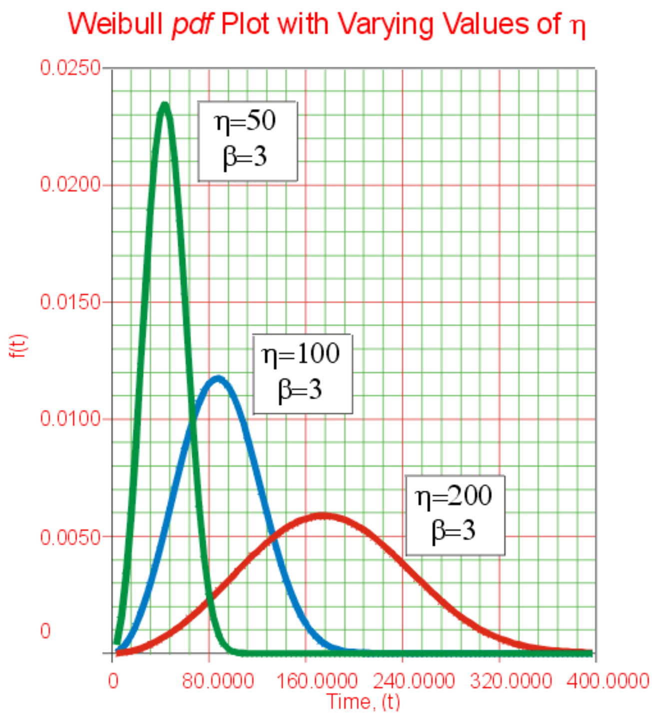

Figure 5 - Weibull distribution with a varying scale parameter (Weibull.com, 2016)

4. Procedure/Calculations

In this chapter we will perform the analysis using MatLab, the analysis will find the distribution for the datasets, and give us the information needed for creating a plan for when restocking should be done. Using software significantly reduces the workload needed to perform these analyses, but we still need an understanding of the concepts for the result to have meaning.

4.1 Case 1 – unmodified data

In the first case we will simplify the data by ignoring that they belong to separate days, and combine it all into a single dataset to find the average expected sales. We will perform this analysis twice. First, by not doing anything special to the data, except for removing zeroes. Second, by removing all extreme data, this includes the sales from days close to holidays, as they would be abnormal cases where the sales number are much larger than they would be otherwise.

4.1.1 Data Gathering and preparation

The first set of data that we use is the original data from Coop Obs, ordered into a single column, with only the zeroes removed. This will give us all the sales over one year in a single dataset to be used for our calculations. The zeroes are removed because they represent days that the store was not open, and thus do not represent that which we are looking for. The table below is an excerpt of the original data, before any manipulation is done to it.

Table 1 - Excerpt from the data from Coop Obs

Week	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
18	125	145	140	391	0	330	0
19	222	152	193	223	311	379	0
20	140	174	408	0	417	266	0
21	113	133	141	250	387	385	0
22	0	184	113	210	255	282	0
23	160	152	131	232	290	234	0
24	136	140	147	185	303	375	0
25	183	169	132	171	241	218	0
26	168	149	95	221	243	208	0
27	177	139	129	137	149	226	0

The complete set of the original data is included as an appendix due to the size of the table.

With the data combined into a single set of data we are able to use this as an input in MatLab to do our analysis. In the figure below the data is presented as columns in a probability density diagram, this is to visually show how the data is distributed. Each of the bins is defined to have a width of 20.

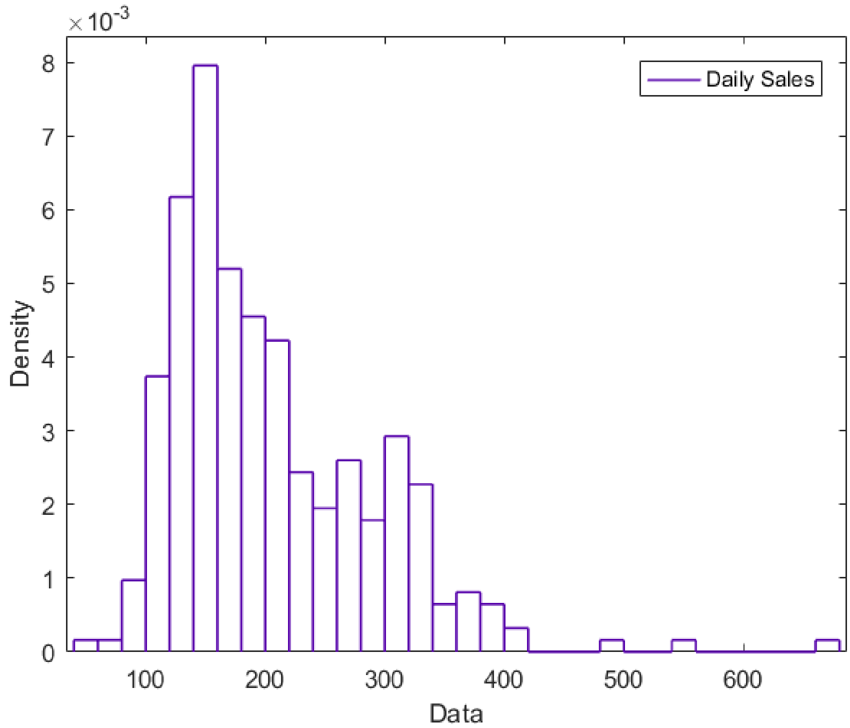

Figure 6 - The unmodified data shown in a PDF plot

We can clearly see from figure 6 where the data is centered and what numbers are the most common. From the data we can also find the mean and median that tells us of how the data is distributed. Using MatLab we find that the mean is 204.4 and that the median is 183. The mean gives us the daily average number of sales we can expect over an entire year. While the median gives us the 50% “limit” of the data. This means 50% of the number of sales were below 183. Seeing as the median is lower than the mean we can expect there to be more occurrences of sales lower than the mean, than there are higher of it. However, the times that the sales are larger than the mean we can expect them to be more extreme.

4.1.2 Selecting distribution

Using the distribution fitting tool in MatLab we are able to check many different distributions to look for the best fit. The table below will show the result for fits to the data by the different distributions.

Table 2 - list of some distributions and their fit to the data

Distribution	Normal	Weibull	Lognormal
Log likelihood	-1803.38	-1791.01	-1760.55

Seeing as the lognormal distribution had the lowest "Log likelihood" and therefore was the best fit to the data, we will be using this distribution onwards.

The parameters for this distribution are:

Table 3 - Parameters taken from MatLab

Parameter	Estimate	Std. Err.
Mu (μ) Sigma (σ)	5.24336 0.388831	0.0221557 0.0157047

These parameters are used by putting them into the formula for the Lognormal distribution.

4.1.3 Presenting the data

By plotting the lognormal distribution curve over the data we can show visually how good the fit is.

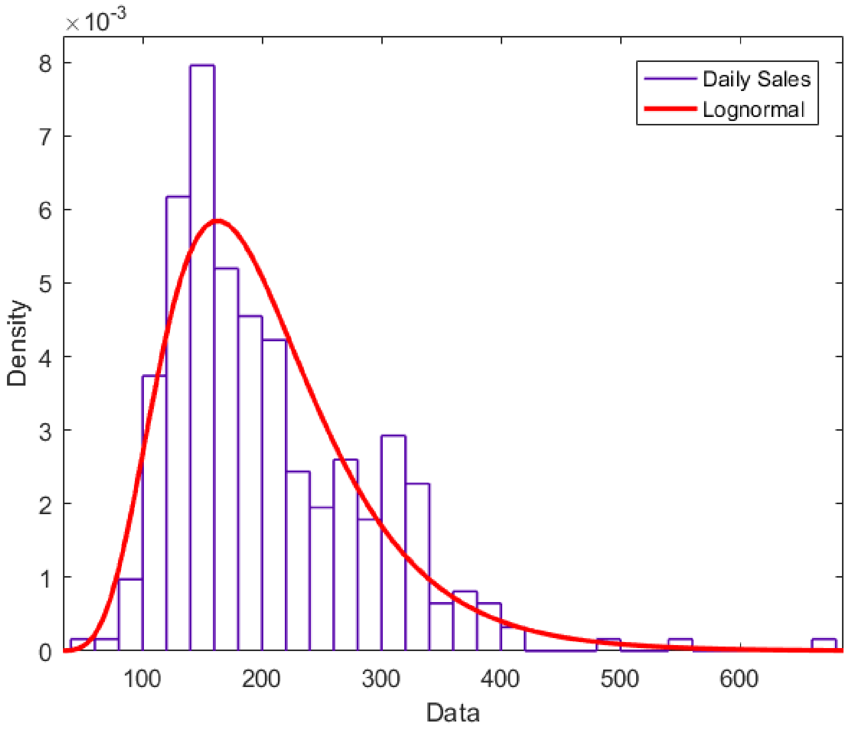

Figure 7 - The daily sales data with lognormal distribution

One can also look at the data presented as a cumulative density function (CDF) to better show when the data passes certain probability thresholds.

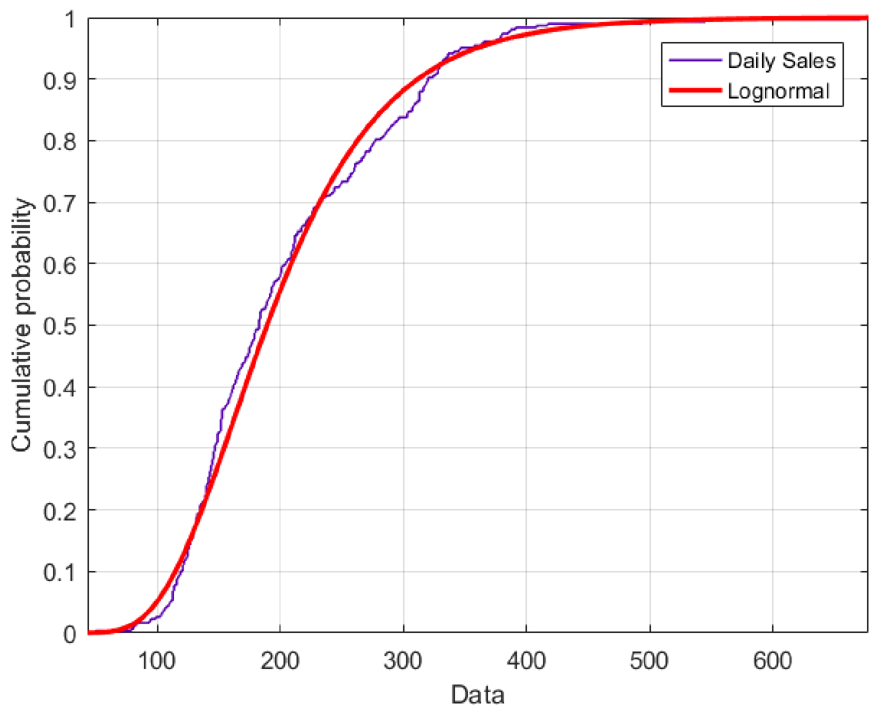

Figure 8 - The daily sales and lognormal curve shown in a CDF plot

The table below shows the probability that the value of the data is lower than the value in the X column of the CDF. Looking at the table of data from the CDF is very helpful in finding the point that the probability reaches a certain value.

Table 4 - Shows the cumulative probability compared to differing values of the data

X	F(X)
40	3,20E-05
60	0,00156306
80	0,01337346
100	0,05036667
120	0,12051579
140	0,21888454
160	0,33267138
180	0,44842842
200	0,55619708
220	0,65041978
240	0,72914553
260	0,79277437
280	0,84295501
300	0,88181596
320	0,91150716
340	0,93396764
360	0,9508356
380	0,96343838
400	0,97282129
420	0,97979125
440	0,98496234
460	0,98879714
480	0,99164151
500	0,99375273
520	0,99532151
540	0,9964889
560	0,99735906
580	0,9980089
600	0,99849521
620	0,99885992
640	0,99913408
660	0,99934064
680	0,99949666

4.2 Case 1 – Data with removed extremes

By removing some data from the dataset before we do anything with it, we can get a much better result for most cases. With the extremes removed we get fewer number of data that has values that are not common for most days. In this case we will remove the sales that are done during the Christmas holidays and those that are in close proximity to long weekends, as the sales numbers during these days do not represent the normal conditions.

4.2.1 Data Gathering and preparation

Using the dataset from Coop Obs, we remove the aforementioned days, the zeroes, and then order the data into a single column. This gives us all the sales over one year in a single dataset to be used for our calculations.

With the data combined into a single set of data, we are able to use this as an input in MatLab to do our analysis. In the figure below, the data is presented as columns in a probability density diagram. This is to visually show how the data is distributed. Each of the bins is defined to have a width of 20.

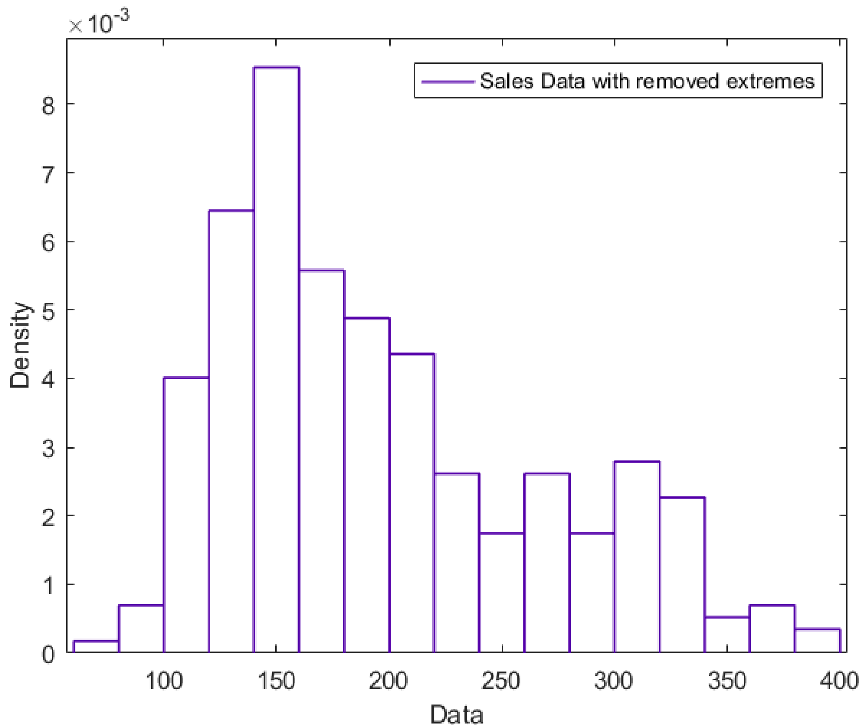

Figure 9 - Dataset with removed extremes

From figure 9, we see that the data is less scattered compared to the unmodified data. We also find the Mean to be 196.5 and Median to be 179. These are naturally lower than the unmodified data's values, as some of the high numbers are removed.

4.2.2 Selecting distribution

Using the distribution fitting tool in MatLab, we are able to check many different distributions to look for the best fit. The table below will show the result for fits to the data by the different distributions.

Table 5 - List of some distributions and their fit to the data

Distribution	Normal	Weibull	Lognormal
Log likelihood	-1630.48	-1624.74	-1602.7

We see that the lognormal has the lowest "Log likelihood", and we will be using this onwards.

We find the parameters for this distribution to be:

Table 6 - Parameters taken from MatLab

Parameter	Estimate	Std. Err.
Mu (μ) Sigma (σ)	5.21889 0.349316	0.0206195 0.0146184

These parameters are used by putting them into the formula for the Lognormal distribution.

4.2.3 Presenting the data

In figure 10 we plot the lognormal over the data to visually show the fit to the distribution.

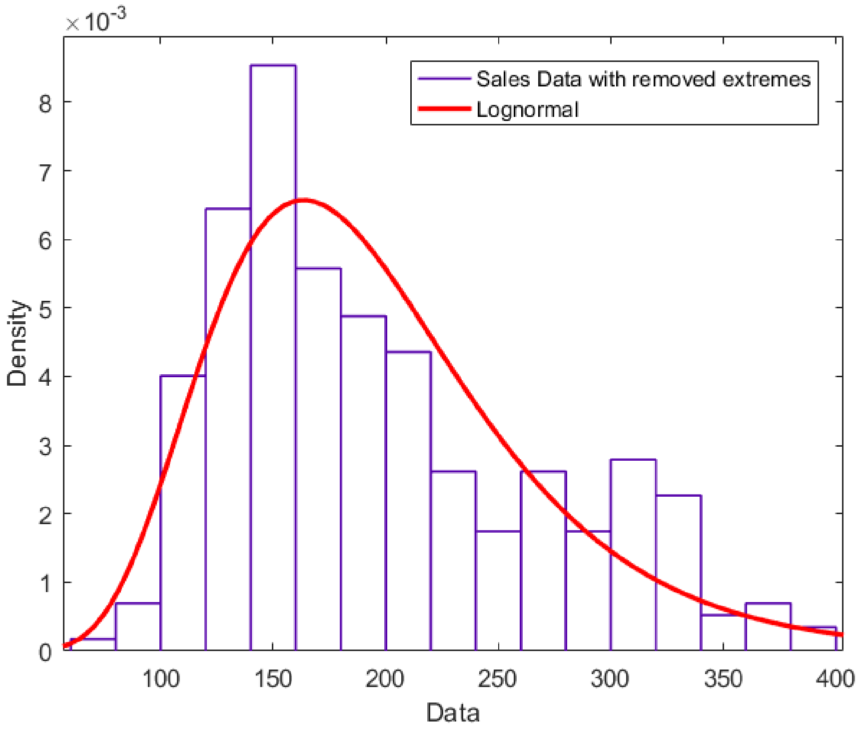

Figure 10 - Shows the dataset with the lognormal curve plotted over it

By looking at the CDF instead of the PDF, we can see at what point the data passes the different points of probabilities.

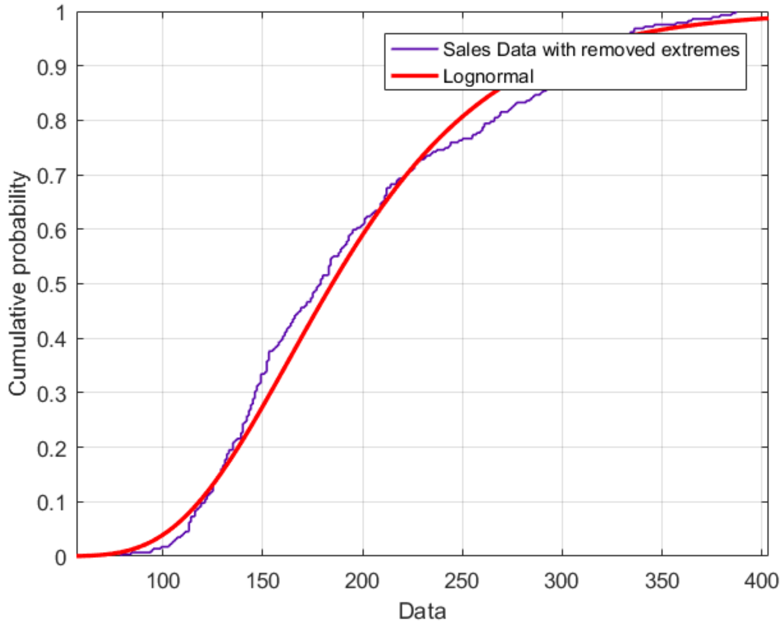

Figure 11 - CDF of the modified data along with the lognormal curve

The table below shows the probability that the value of the data is lower than the value in the X column. Looking at the table of data from the CDF is very helpful in finding the point that the probability reaches a certain value.

Table 7 - Cumulative probabilities of the modified data

X	F(X)
60	0,00064254
80	0,00829355
100	0,03946461
120	0,10841633
140	0,2136847
160	0,3403765
180	0,47040419
200	0,58993029
220	0,6915377
240	0,77316315
260	0,83607305
280	0,88309418
300	0,91744761
320	0,94212471
340	0,95963059
360	0,97193628
380	0,98053022
400	0,98650507

4.3 Case 2 – Separated days

In the second case, we will take a closer look at each day separately. With having in-depth knowledge for the expected number of sales, we are able to plan our inventory strategy better.

Now we will find the best distributions for each of the days. We will use the modified data for the weekdays as they represent the normal conditions better. Knowing the most likely number of sales, lets us ensure that we have the necessary number of items in stock. By using MatLab, we find the CDF for each day.

4.3.1 Data gathered

In this case, we will use the same dataset as before but this time we will not ignore the fact that the data represents different days. We will find the best fit for each of the days and find the PDF and CDF to show the expected sales numbers.

4.3.2 Best fits

Let us find the distributions for each of the days and show the PDF, CDF and a table for the cumulative distributed data.

4.3.2.1 Monday

The PDF of Monday's sales:

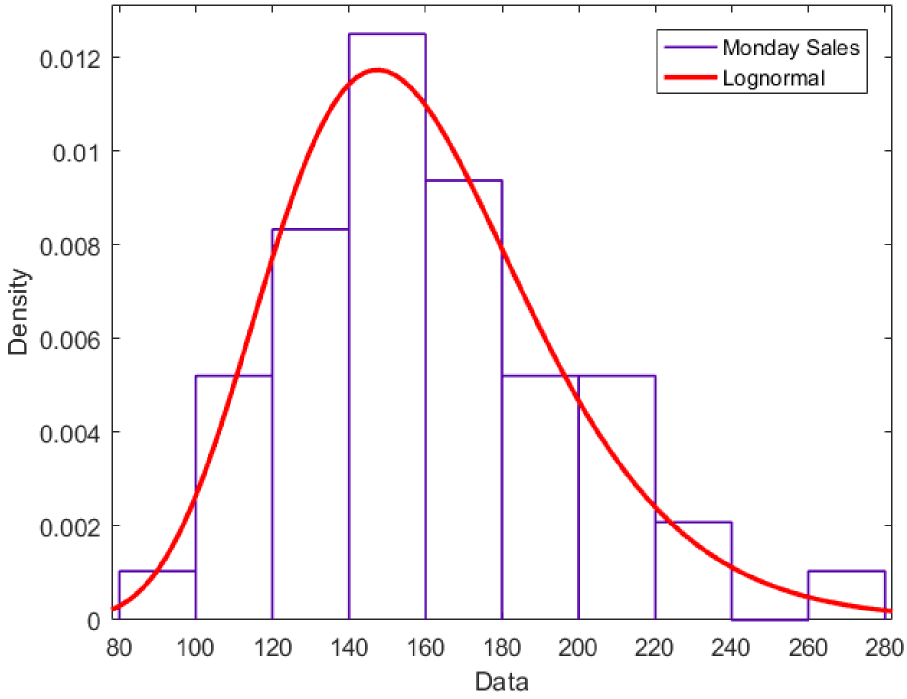

Figure 12 - PDF of Monday's sales with distribution

The data has a lognormal distribution with the parameters:

Table 8 - Parameters for the distribution

Parameter	Estimate	Std. Err.
Mu (μ) Sigma (σ)	5.04378 0.22494	0.0324673 0.0233252

The CDF of Monday's sales:

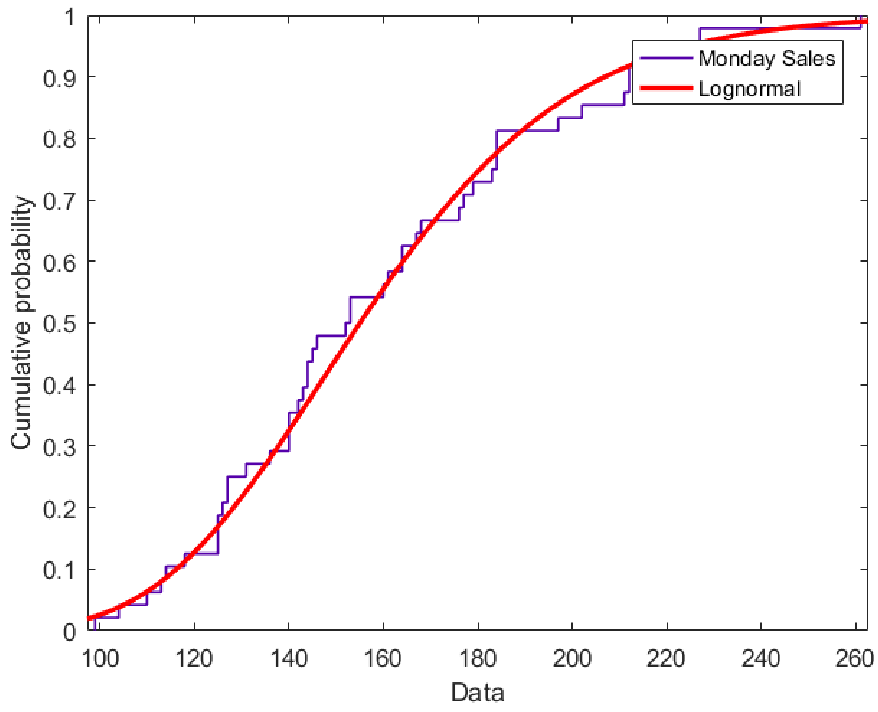

Figure 13 - CDF of Monday's sales with distribution

And the table below shows how the cumulative probability is distributed.

Table 9 - CDF values from the data

X	F(X)
100	0,02559517
120	0,12727951
140	0,32489714
160	0,55550559
180	0,74640011
200	0,87109838
220	0,94006421
240	0,97393954
260	0,98921837

4.3.2.2 Tuesday

The PDF of Tuesday's sales:

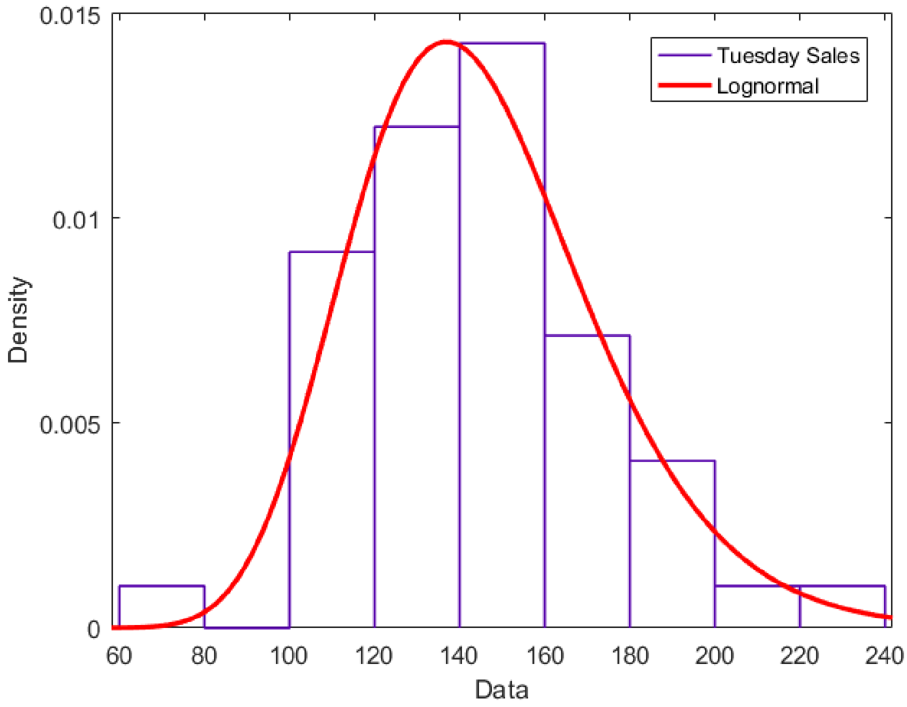

Figure 14 - PDF of Tuesday's sales with distribution

The data has a lognormal distribution with the parameters:

Table 10 - Parameters for the distribution

Parameter	Estimate	Std. Err.
Mu (μ) Sigma (σ)	4.95886 0.199508	0.0285011 0.020469

The CDF of Tuesday's sales:

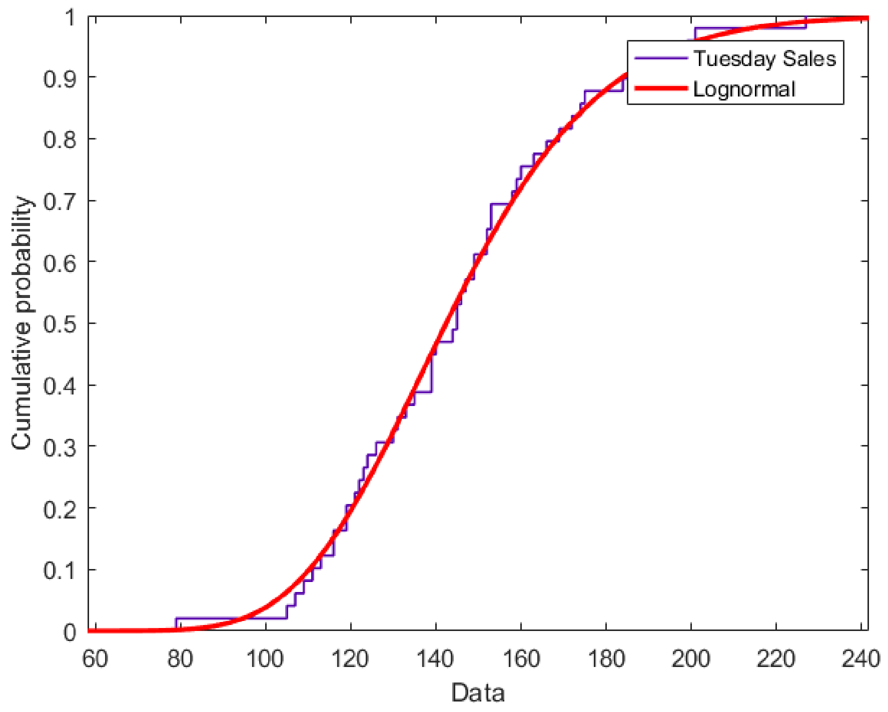

Figure 15 - CDF of Tuesday's sales with distribution

And the table below shows how the cumulative probability is distributed.

Table 11 - CDF values from the data

X	F(X)
60	7,35E-06
80	0,00191837
100	0,03813015
120	0,19518371
140	0,46561588
160	0,72005669
180	0,87967804
200	0,95557373
220	0,98534193

4.3.2.3 Wednesday

The PDF of Wednesday's sales:

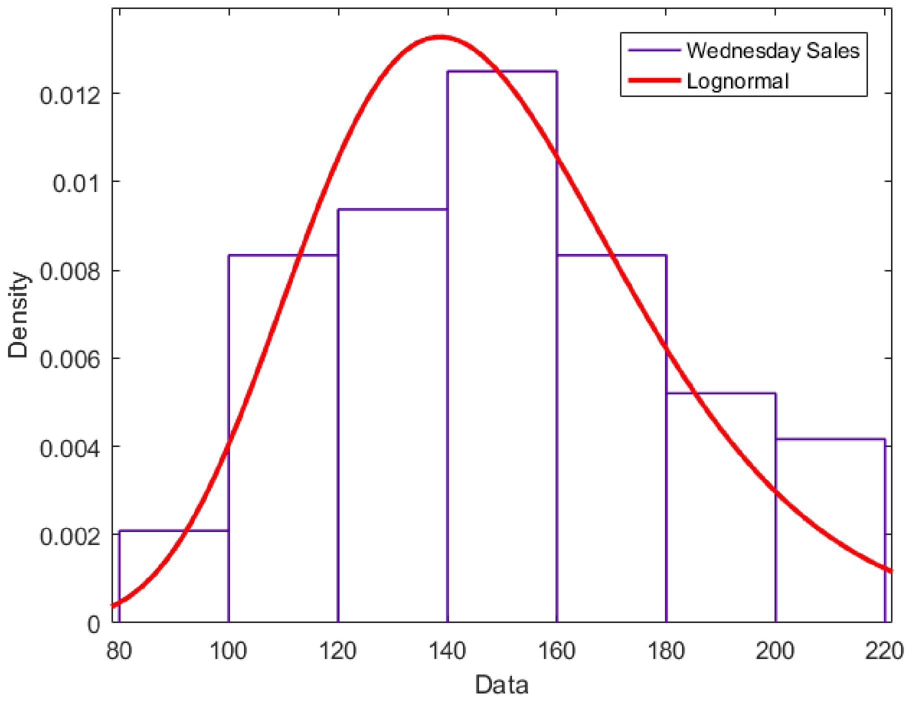

Figure 16 - PDF of Wednesday's sales with distribution

The data has a lognormal distribution with the parameters:

Table 12 - Parameters for the distribution

Parameter	Estimate	Std. Err.
Mu (μ) Sigma (σ)	4.97677 0.211809	0.030572 0.0219636

The CDF of Wednesday's sales:

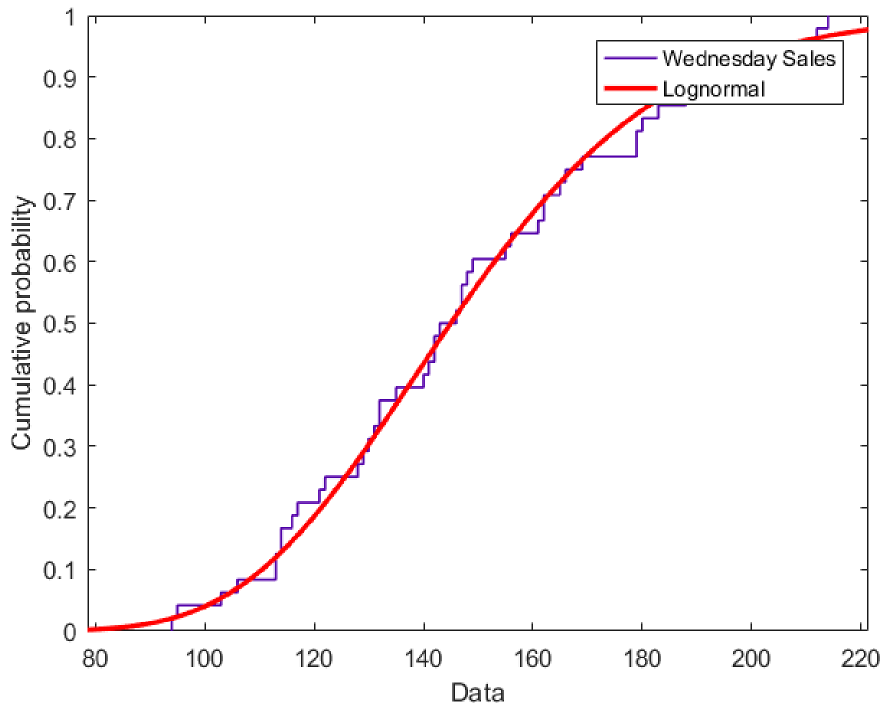

Figure 17 - CDF of Wednesday's sales with distribution

And the table below shows how the cumulative probability is distributed.

Table 13 - CDF values from the data

X	F(X)
80	2,49E-03
100	0,03967903
120	0,18575693
140	0,4341333
160	0,67888084
180	0,84629037
200	0,93550329
220	0,97546968

4.3.2.4 Thursday

The PDF of Thursday's sales:

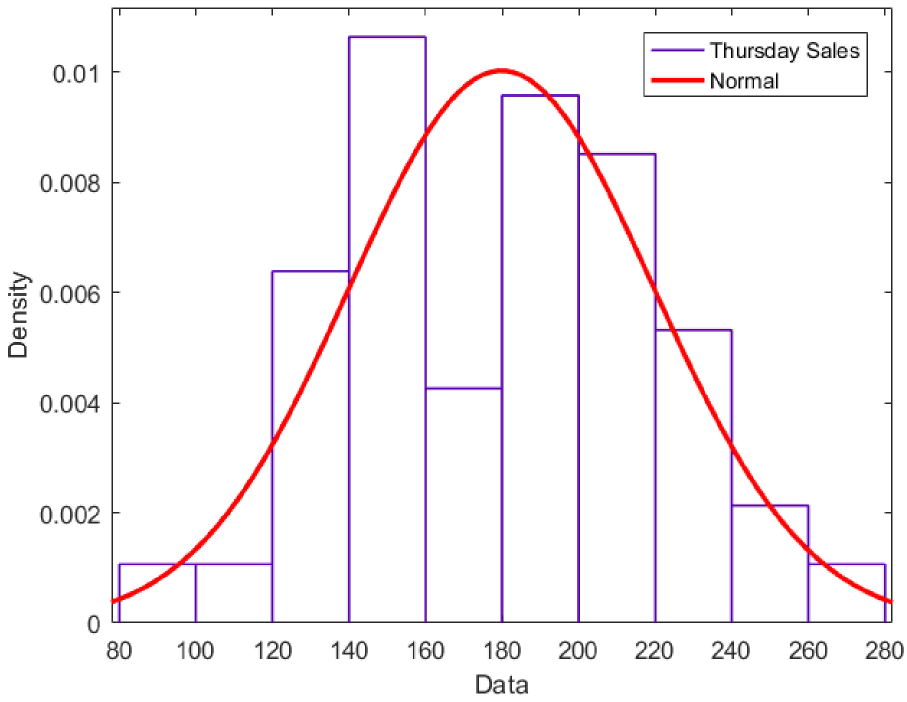

Figure 18 - PDF of Thursday's sales with distribution

In this case the normal distribution was the best fit, and has the parameters:

Table 14 - Parameters for the distribution

Parameter	Estimate	Std. Err.
Mu (μ) Sigma (σ)	179.851 39.7924	5.80432 4.17138

The CDF of Thursday's sales:

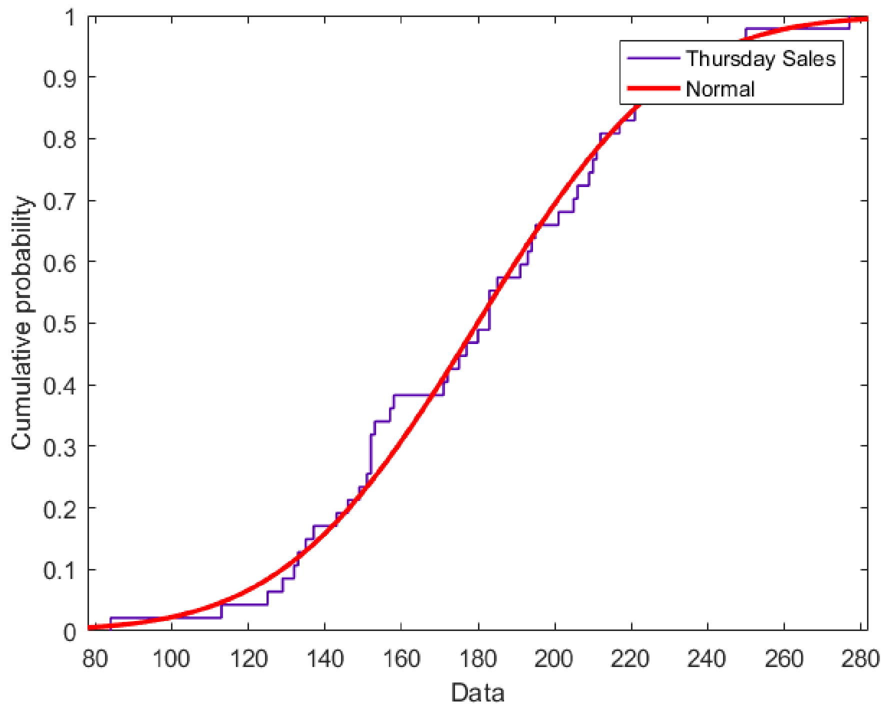

Figure 19 - CDF of Thursday's sales with distribution

And the table below shows how the cumulative probability is distributed.

Table 15 - CDF values from the data

X	F(X)
80	6,05E-03
100	0,02239123
120	0,06627995
140	0,1582987
160	0,30893693
180	0,50149317
200	0,69369507
220	0,84350316
240	0,93467823
260	0,97800456
280	0,99407847

4.3.2.5 Friday

The PDF of Friday's sales:

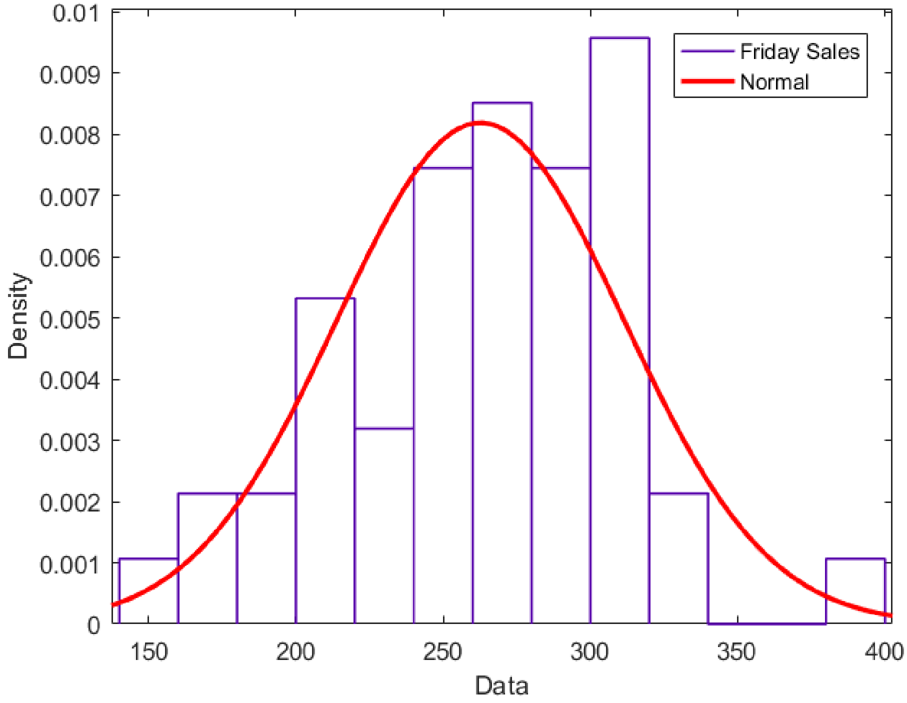

Figure 20 - PDF of Friday's sales with distribution

The data has a normal distribution with the parameters:

Table 16 - Parameters for the distribution

Parameter	Estimate	Std. Err.
Mu (μ) Sigma (σ)	262.638 48.7297	7.10796 5.10826

The CDF of Friday's sales:

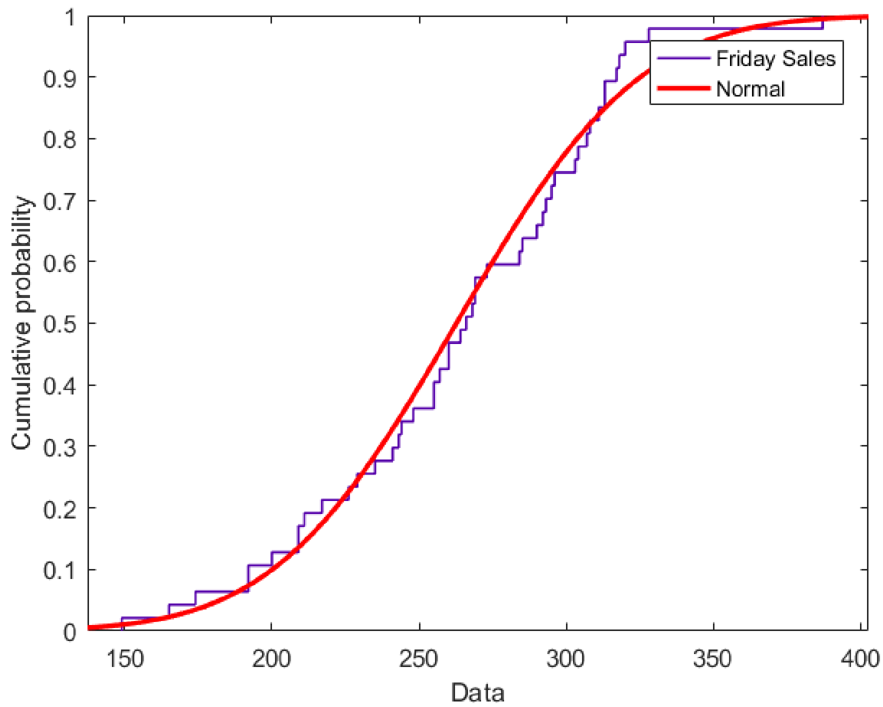

Figure 21 - CDF of Friday's sales with distribution

And the table below shows how the cumulative probability is distributed.

Table 17 - CDF values from the data

X	F(X)
100	4,23E-04
120	0,00171051
140	0,00592289
160	0,0175901
180	0,04495708
200	0,09932218
220	0,19078801
240	0,32112014
260	0,47841122
280	0,63918674
300	0,77837396
320	0,88043028
340	0,94380794
360	0,9771414
380	0,99198918
400	0,9975902

4.3.2.6 Saturday

The PDF of Saturday's sales:

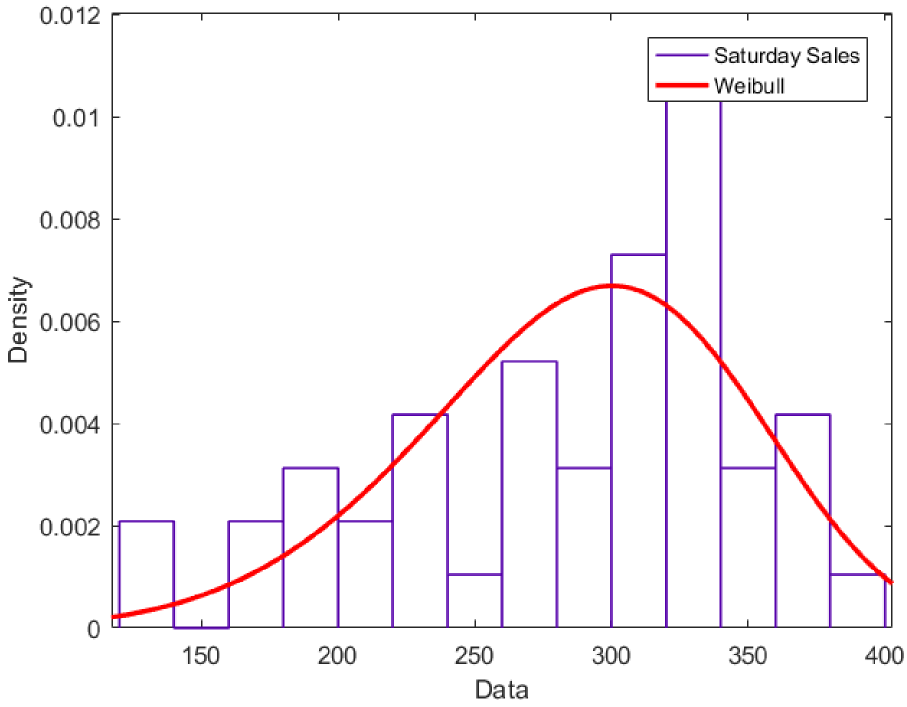

Figure 22 - PDF of Saturday's sales with distribution

The best fit for the Saturday data was a Weibull distribution with the parameters:

Table 18 - Parameters for the distribution

Parameter	Estimate	Std. Err.
Scale (η)	311.036	8.44843
Shape (β)	5.55919	0.670967

The CDF of Saturday's sales:

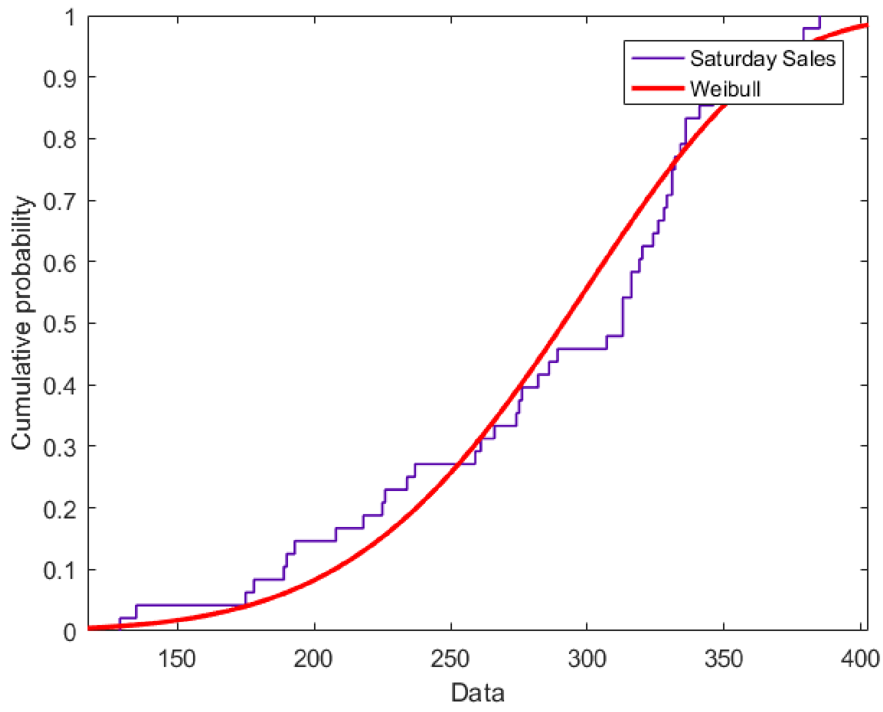

Figure 23 - CDF of Saturday's sales with distribution

And the table below shows how the cumulative probability is distributed.

Table 19 - CDF values from the data

X	F(X)
100	1,82E-03
120	0,00500569
140	0,01175326
160	0,02453178
180	0,04668082
200	0,08228846
220	0,13572883
240	0,21070214
260	0,30872653
280	0,42733231
300	0,55870619
320	0,68997414
340	0,80610602
360	0,89502502
380	0,95237326
400	0,98255809

5. Result and Conclusion

Throughout chapter 3, the basics of inventory management and statistics has been explained. In chapter 4, an analysis of the sales data has been performed. By performing this analysis, we have gathered the necessary information to create a plan for the inventory management.

5.1 Result

From chapter 4, we have analyzed the data and found the expected number of sales for the average day. Once, with an uncritical analysis by ignoring any unusual condition in the data, and once more by removing some data points that did not represent the normal conditions. In both cases, we also removed all zeroes from the data as they represent days that the store was not open.

Since the goal of the thesis is to give a suggestion for an inventory management plan, let us choose a few percentage points from the CDF to have a close look at. Since most designing should be done for “bad” or “worst” cases, let us take a closer look at 50%, 70%, 80%, and 90%.

5.1.1 Case 1

For the first case, the unmodified daily data, we find that the sales numbers for these percentages are:

Table 20 - Selected CDF percentages and their corresponding values

50%	70%	80%	90%
190	232	263	312

And for the modified data where numbers that did not represent the normal conditions were removed:

Table 21 - Selected CDF percentages and their corresponding values

50%	70%	80%	90%
185	222	248	289

The number of sales in the 50% position is the expected number of sales and is a good number to start looking at. If we were to choose any day of the year at random, excluding days the store is closed, the 50% number is the most likely number of sales to happen on that day. If we say that the store is open 6 days in a row and sells 185 items each day, the total sales that week becomes 1110 items. The item is delivered from the producer in “packages” of 160 items per “unit”, and thus the orders should be done in increments of 160 items. The 1110 items sold in our average week results in needing 6,93 units, rounded up to 7 units, delivered per week.

Performing the same check for 70%, 80%, 90% cases gives us these sales in one week. For 70% we sell 1332 items, needing 8,33 units delivered. For 80% we sell 1488 items, needing 9,3 units delivered. And for 90% we sell 1734 items, needing 10,84 units delivered. By looking at the data we see that the 50% case happens often, and the 70% and 80% case sometimes. And as expected the 90% case is very rare, and only happens during the holiday seasons.

Let us use the 70% case to plan the inventory during normal conditions. Within one week, we plan to sell about 1330 items. To have enough items in stock we need, on average, 9 units delivered in one week.

5.1.2 Case 2

In the second case, we took a look at each day separately, finding the expected number of sales for each day of the week. Let us do the same check for 50%, 70%, 80%, and 90% as we did for the daily data.

Monday:

Table 22 - Selected CDF percentages and their corresponding values

50%	70%	80%	90%
155	174	187	207

Tuesday:

Table 23 - Selected CDF percentages and their corresponding values

50%	70%	80%	90%
142	158	168	184

Wednesday:

Table 24 - Selected CDF percentages and their corresponding values

50%	70%	80%	90%
145	162	173	190

Thursday:

Table 25 - Selected CDF percentages and their corresponding values

50%	70%	80%	90%
180	201	213	231

Friday:

Table 26 - Selected CDF percentages and their corresponding values

50%	70%	80%	90%
263	288	304	325

Saturday:

Table 27 - Selected CDF percentages and their corresponding values

50%	70%	80%	90%
291	322	339	361

We see that the sales numbers are quite large on Fridays and Saturdays compared to the rest of the week. Now we need to study the expected sales numbers and find a combination of numbers distributed over the week, that are likely to be a real case, and use this to present a suggestion for the inventory management.

5.2 Conclusion

Let’s sum up the results and the demands for our inventory management. We know from earlier that the order should be made in increments of 160 items, and that our suggestion has to take this into consideration. There is also a limit to how many items can be stored locally at any time; from personal experience, restock usually happens early in the week, with a slew of different items being restocked. This fills up the storage at the beginning of the week, and reduces the possibility of “overfilling” the storage with the item in question.

5.2.1 Safety stock

Let us first find the necessary safety stock, since it is difficult to make exact orders of items due to the way orders are made we need to be flexible when this stock is determined. We see from the result that Monday, Tuesday, Wednesday and Thursday are quite similar when it comes to the expected sales, then let us use this similarity to give a common safety stock for these days. Thursday’s 90 % value is the highest at 231, so this should be set as the safety stock for the first four days of the week. Finding a safety stock for Friday and Saturday is a bit harder. Here rather than having a safety stock we should rather make an anticipation stock for what we expect to sell during the weekend. To plan for the items needed during the weekend we should decide on the amount to have at the local storage. Friday has 325 sales at the 90 % limit, while Saturday has 361 sales. This equals selling approximately 5 units over the weekend. The recommended stock should then at least cover these sales, and still make sure that there is enough left over to cover sales on Monday morning, before deliveries are done.

5.2.2 Inventory management

From the discussion above this thesis makes these recommendations:

- The store should at minimum have about one and a half unit of items at the local storage at the start of each Monday, Tuesday, Wednesday and Thursday.
- If the local storage can support it, a good ordering frequency is to deliver 2 units on Mondays. 3 units on Wednesdays. Depending on the sales at the beginning of the week; 2 or 3 units on Fridays, and 3 units on Saturdays.
- These suggestions are only valid for the normal conditions, that is; normal weeks without holidays or other events that can influence the sales.

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Appendices

Appendix 1

This is the original data I was given by Hilde Rasmussen; the data contains all the sales for a single item from Coop Obs Hypermarked in Tromsø.

Week number	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
18	125	145	140	391	0	330	0
19	222	152	193	223	311	379	0
20	140	174	408	0	417	266	0
21	113	133	141	250	387	385	0
22	0	184	113	210	255	282	0
23	160	152	131	232	290	234	0
24	136	140	147	185	303	375	0
25	183	169	132	171	241	218	0
26	168	149	95	221	243	208	0
27	177	139	129	137	149	226	0
28	142	109	161	84	165	178	0
29	125	113	130	133	192	135	0
30	127	126	114	143	174	175	0
31	197	79	106	113	209	225	0
32	261	131	103	152	211	261	0
33	214	116	179	157	192	190	0
34	140	147	143	125	200	189	0
35	127	172	162	146	273	313	0
36	184	160	148	195	307	326	0
37	179	135	214	152	260	237	0
38	176	130	142	151	260	275	0
39	184	153	165	175	229	346	0
40	99	144	212	183	285	365	0
41	212	166	166	221	295	363	0
42	184	139	128	217	318	328	0
43	164	116	114	180	293	332	0
44	167	175	149	191	308	336	0

45	202	189	169	231	317	320	0
46	118	119	122	244	266	319	0
47	146	149	156	205	284	316	0
48	161	146	180	153	257	341	0
49	227	199	183	183	313	313	50
50	125	153	195	152	255	331	81
51	212	227	204	212	226	316	121
52	380	671	494	82	0	0	0
53	257	309	378	200	0	545	0
1	211	122	142	135	248	289	0
2	164	201	188	194	304	329	0
3	144	145	179	206	268	336	0
4	144	158	155	193	264	307	0
5	140	139	146	211	320	324	0
6	143	159	135	201	328	357	0
7	145	188	201	183	292	331	0
8	153	123	162	158	313	313	0
9	153	124	132	172	296	274	0
10	131	121	113	149	244	334	0
11	152	111	116	209	269	286	0
12	303	287	345	0	0	273	0
13	0	249	94	277	217	259	0
14	126	105	121	129	209	193	0
15	104	163	117	177	269	129	0
16	114	107	147	132	235	276	0
17	110	119					
Sum each day	8571	8631	8739	9126	12761	14888	252
Sum Yearly	62968						