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Optimal Placement of PMUs Considering Logical Topology of Communication Medium and Power System Observability

Bhushan Madan Nikumbh

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Supervisor: Pawan Sharma, Charu Sharma

Principal: UiT – The Arctic University of Norway (Campus Narvik)

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Abstract

A phasor measurement unit (PMU) has a unique ability of providing synchronized phasor measurements of voltage and currents. This ability distinguishes it from all other metering devices. It has been perceived that PMUs hold the capability of revolutionizing the way of power system monitoring and control. However, high per unit cost and challenges related to its communication system has made its judicial placement in an electric grid a significant issue. Therefore, in present work various issues regarding PMU placement are considered. First, Linear programming approach is utilized to find out optimal PMU locations in the given system. Since, PMU installation costs also comprises of PMU communication infrastructure. Therefore, a novel method is employed to find out feasible communication network structure for the given system. The data generated by a PMU needs a reliable and stable communication network. Generally, fiber – optic cables due their high channel capacity, low latency and immunity to electromagnetic interference have become the choice for PMU communication. Therefore, fiber optical communication is considered as communication medium in present work. In this thesis, it is assumed that the fiber – optic network runs parallel to the electric power network. Constant growth and uncertain nature of power system causes problem of congestion. Further, propagation delay associated with optical fibers is also a recent topic of concern. Hence, in order to optimize the congestion and the propagation delay, a logical topology is generally developed for the optical fiber networks. This thesis presents a mathematical formulation based on integer linear programming for logical topology designing. The feasibility of the proposed formulations is checked by applying it on few IEEE systems. Results so obtained, establishes the feasibility of methodology. Present thesis has considered different PMU placement and infrastructure issues independently, which will benefit planning utilities.

Abbreviations

EMS	Energy Management System
GPS	Global Positioning System
IED	Intelligent Electronic Device
PMU	Phasor Measurement Unit
RTU	Remote Terminal Unit
SCADA	Supervisory Control and Data Acquisition
SE	State Estimation
PDC	Phasor Data Concentrator
RMS	Root Mean Square
WAMS	Wide Area Measurement System
IP	Integer Programming
ILP	Integer Linear Programming
BILP	Binary Integer Linear Programming
MILP	Mixed Integer Linear Programming
GA	Genetic Algorithm
OPP	Optimal PMU Placement
SPP	Shortest Path Problem
OPGW	Optical Fiber Power Ground Wire

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1 Introduction

1.1 Background and Motivation

Electric power industry has been transforming rapidly in recent years. Several renewable energy sources are getting integrated into electric power grids along with new loads and storage elements. This has increased the complexity of power systems by raising its dynamics and uncertainties. This in turn has caused concerns regarding the reliability and stability of power systems. In order to address these concerns a concept of smart grid is in the process of constant development for some years now.

State Estimation (SE) is one of the critical application of power systems. It is also a key function in modern energy management systems (EMS). SE creates a complete and accurate database of measurements which can be used as an input for other applications of EMS. To calculate voltage phasors, conventional state estimators utilizes set of measurements consisting of bus voltages, real-reactive power flows and injections. Until recently, supervisory control and data acquisition (SCADA) system was the only means obtain these measurements. SCADA systems can gather these measurements in real-time through remote terminal units (RTUs) which are installed at substations [1].

Advent of global positioning system (GPS) has made possible the inclusion of time synchronized phasor measurements provide by phasor measurement units (PMUs) into the set of measurements. A PMU measures voltage phasor of the bus-bar at which it is installed and current phasors of some or all branches incident to that bus depending on the number of available channels. Use of PMUs at substations can significantly improve monitoring, protection, and control of interconnected power systems [1]. As PMUs have number of advantages over other smart measuring devices such as intelligent electronic devices (IEDs) and smart power meters, its planned installation in near future has grown significantly. However, from economical point of view, high cost of PMU limits the number of installation sites [2].

In order to obtain a sufficient amount of observability of a power system PMU installation sites are dispersed over a wide area. The data generated by these PMUs is then communicated to remotely located phasor data concentrators (PDCs) over a dedicated or a shared communication network [3]. Due to this a robust

communication infrastructure is essential. But the number of limitations while designing a communication system gives rise to a need for an optimal solution that takes into account data loads, propagation delay and congestion in the communication network.

1.2 Objectives

The main objectives of this thesis are:

1. Studying available literature on phasor measurement units.
2. Studying available PMU placement methods.
3. Identification of various problems related with PMU placement
4. Simulating PMU placement problem for complete and incomplete observability.
5. Studying available literature on PMU communication.
6. Identification of various challenges related with PMU communication.
7. Simulating problem of logical topology.
8. Suggesting optimal PMU placement scheme considering observability and communication medium topology.

1.3 Thesis Outline

Chapter 1 of the thesis gives a background that has formed the basis for present work. Main objectives of the thesis have also been presented in this chapter.

Chapter 2 covers the first four objectives of the thesis along with the simulation results of PMU placement problem for various IEEE systems.

In chapter 3 covers the next three objectives related to PMU communication system. The logical topology problem is solved for IEEE 14 – bus system.

Chapter 4 concludes the thesis by summarizing all accomplished tasks and provides an optimal PMU placement scheme thus achieving the last objective of the thesis.

Finally, in chapter 5 a direction for future work is recommended.

2 Phasor Measurement Unit and Placement Problem

2.1 Literature Review

2.1.1 Phasor Measurement Unit

As mentioned in the earlier section, PMU is a device that is able to measure synchronized voltage and current phasors in a power system. One of the most important feature that distinguishes PMU technology from other smart metering techniques is time-stamping of measurements using GPS clock. This assures synchronicity among all the PMUs installed in a power system thus eliminating the parameter of data propagation delay during the use of obtained data.

First PMU was introduced in 1980 at Virginia Tech. As per [4], figure 2.1 below is the generalized configuration of major elements in modern PMU.

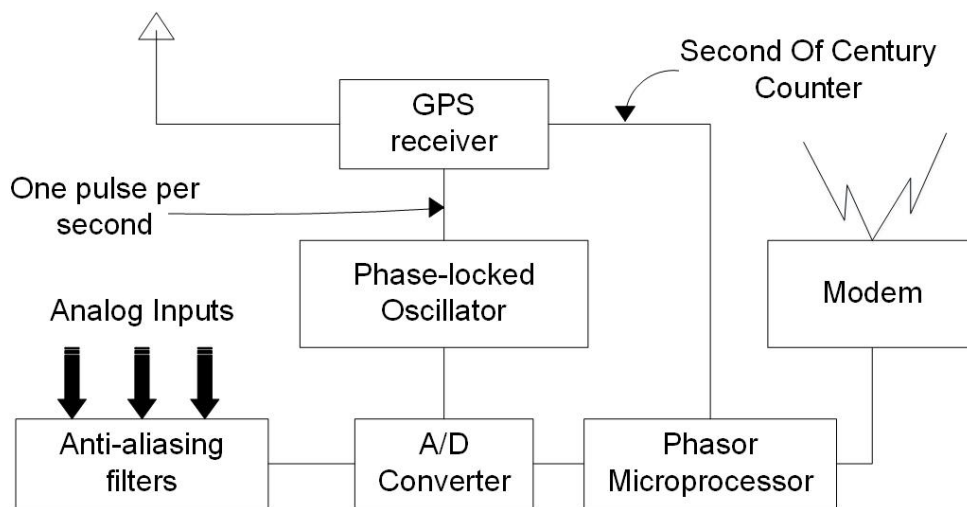


Fig. 2.1 Major elements of modern PMU [4]

Inputs to anti-aliasing filters are currents and voltages of secondary windings of the current and voltage transformers. Input frequencies above the nyquist rate are filtered out by the anti-aliasing filters. GPS clock pulse (one pulse per second) is converted into a high speed timing pulse sequence by the phase locked oscillator. These pulse sequences are then used for waveform sampling. Discrete fourier transformations are executed by phasor microprocessor on the input received through analog-to-

digital(A/D) converter. Thus calculating the positive-sequence estimates of input signal. The phasors are then time-stamped and transmitted to PDC through the modem. Transmission of these phasors is carried out according to IEEE standard format [5].

2.1.2 Synchro-phasor Standard

In [6], phasor is defined as “A complex equivalent of a simple sine wave quantity such that the complex modulus is the sine wave amplitude and the complex angle (in polar form) is the sine wave phase angle.” Classical phasor representation of a sine wave signal given in [7] is as follows. For a sinusoidal signal [7]

$$x(t) = X_m \cos t(\omega t + \phi) \quad (i)$$

The phasor representation of the above signal is [7]

$$X \equiv \frac{X_m}{\sqrt{2}} \varepsilon^{j\phi} = \frac{X_m}{\sqrt{2}} (\cos \phi + j \sin \phi) \quad (ii)$$

Here magnitude of the phasor is $\left(\frac{X_m}{\sqrt{2}}\right)$ that is the root mean square (RMS) value of the sinusoidal signal in (i), while ϕ its phase angle. Illustration of the sinusoidal signal in (i) and its phasor representation in (ii) is as shown in figure 2.2.

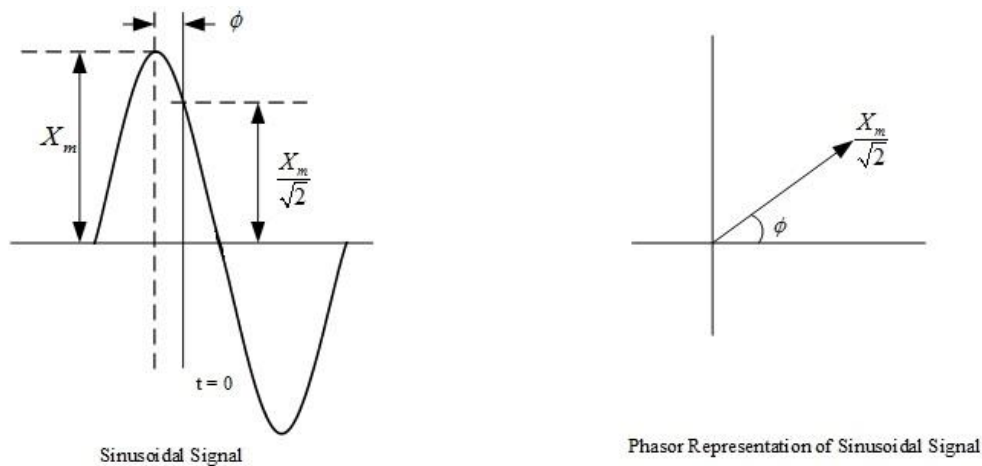


Fig. 2.2 Sinusoidal Signal and its Phasor Representation

A synchro-phasor is “a phasor calculated from data samples using a standard time signal as the reference for the sampling process [6].” It is simply a phasor time-

tagged by a device in synchronism with other similar devices in the power system. For example, in figure 2.2 the marker $t = 0$ is the time-tag for the measurement at that instant. PMU then uses the sampled data of the input signal to provide the phasor measurement given by (ii).

Synchronism is “the state where connected alternating-current systems, machines, or a combination operate at the same frequency and where the phase angle displacement between voltages in them are constant, or vary about a steady and stable average value [6].” To realize this synchronism a sampling clock phase-locked to GPS signal of one-pulse-per-second is used.

2.1.3 Wide Area Measurement System

Wide area measurement system (WAMS) monitors electric power grid and accelerates network calculations using digital measurement devices, control systems and communication infrastructure. It is an intelligent and automatic network [8].

WAMS process is briefly explained in [9]. Data acquisition, data transmitting and data processing are the three main interdependent functions in a WAMS process. Measurement devices like PMUs, RTUs and SCADA perform the function of data acquisition. These devices are dispersed over whole geographic area of the power system. The raw data acquired by the measurement devices is then transmitted over the communication infrastructure set over the entire power system. Finally, the last stage of WAMS process deals with software packages usually referred to as EMS. EMS applications perform data processing operations like control, analysis and optimization of power systems using the obtained data. Some of the EMS applications are state estimation, load flow analysis, optimal power flow design, etc. A conceptual illustration of the WAMS process described above is given in [10] and is as shown in figure 2.3.

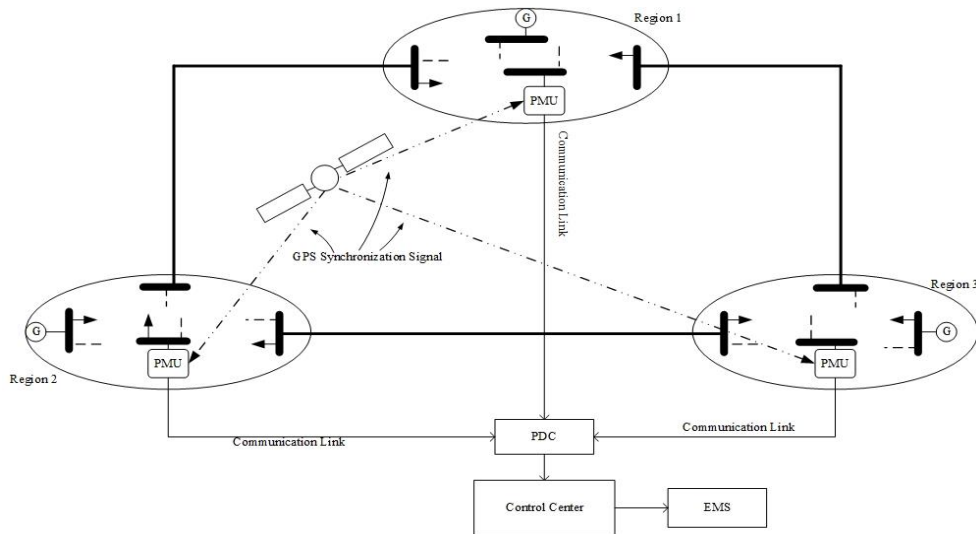


Fig. 2.3 Conceptual Diagram of WAMS using PMUs

2.1.4 PMU Placement Problem

Since PMU is an expensive device, installing it on every bus is highly uneconomical. Hence a PMU placement problem deals with locating the optimal PMU installation sites in a power system considering the desired amount of observability.

2.1.4.1 Concept of Complete and Incomplete Observability

A power system is completely observable if the number and locations of PMUs are sufficient to determine voltages of all the buses in the system [2]. An example of complete observability scenario is as shown in figure 2.4.

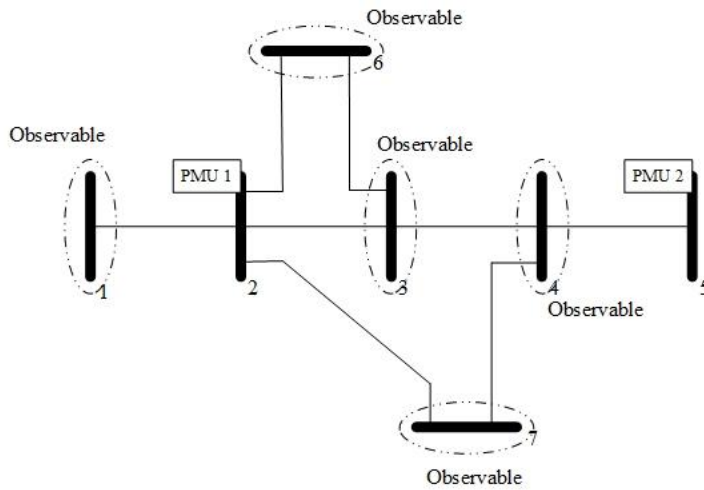


Fig. 2.4 Complete Observability Scenario

A bus is said to be observable if its node voltage can be directly calculated by using known node voltages and branch currents of other buses [11]. As seen from figure 2.4 node voltages and branch currents of bus 2 and bus 5 can be measured by PMU 1 and PMU 2 respectively. Thus bus 2 makes bus 1, 3, 6 and 7 observable, while bus 4 is observable due to its direct connection to bus 5. Hence the 7-bus system shown in above figure can be declared completely observable.

In [12], the concept of incomplete observability and depth of unobservability is described with lucidity. Incomplete observability can be basically referred to as the PMU placement scenario where the number and locations of PMUs are insufficient for determining voltages of all the buses in the system. Further depth-of-one unobservability is a scenario where there is one unobserved bus directly connected to two or more observed buses as shown in figure 2.5. Figure 2.6 depicts depth-of-two unobservability scenario where there are two unobserved buses between two or more observed buses.

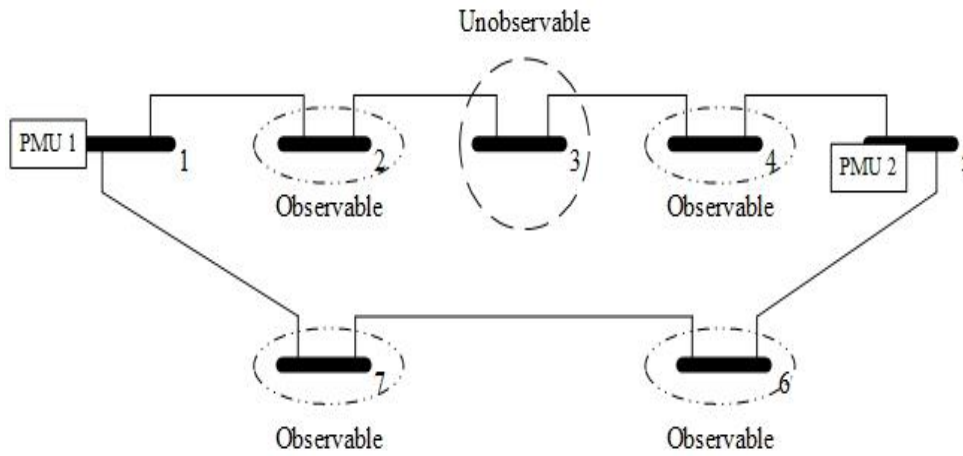


Fig. 2.5 Depth-of-One Unobservability Scenario

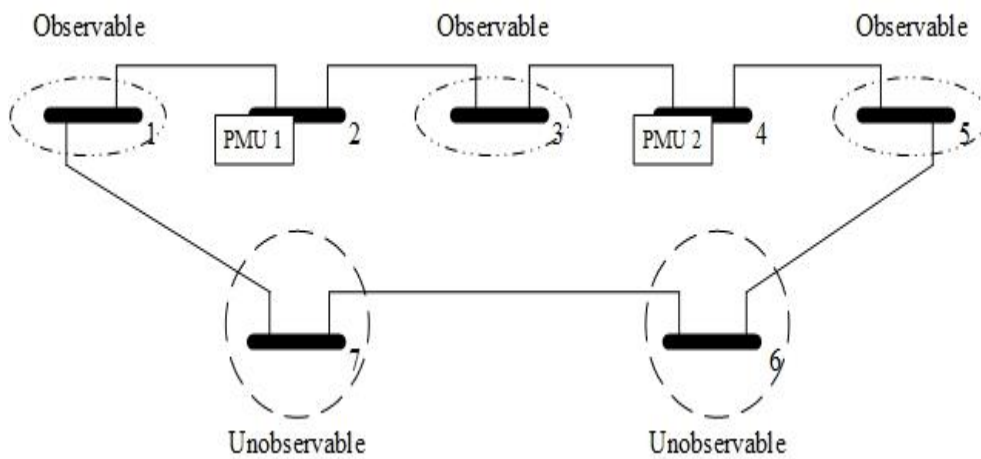


Fig. 2.6 Depth-of-Two Unobservability Scenario

2.1.4.2 Integer Programming

Integer Programming (IP) is mathematical optimization programming for problems having integer variables. IP is referred to as Integer Linear Programming (ILP) when the objective function and constraints are linear in nature. In an ILP, when few variables are integers and others non-integers then ILP is referred to as Mixed Integer Linear Programming (MILP). In case the variables are restricted only to binary terms then it becomes a Binary Integer Linear Programming (BILP) problem.

In [1], an outline of published mathematical and heuristic optimization approaches for optimal PMU placement has been provided. [13] proposes a generalized ILP formulation for redundant PMU placement taking into consideration zero injection measurements. In [14], a similar ILP proposition has been made but with conventional power flow and power injection measurements. A proposition of BILP formulation for optimal PMU placement taking into account PMU channel limits is made in [15]. In [16] an MILP formulation for power system observability taking into consideration specified fixed channel capacity for PMUs is proposed.

Apart from IP several heuristic approaches have been used for optimal PMU placement problem. Simulated Annealing method in [17], Graph Theoretic approach in [2] and Tabu search algorithm in [18] are a few to mention. Authors of [9] use Genetic Algorithm (GA) to present a co-optimization problem in which PMU placement and communication infrastructure designing has been addressed simultaneously.

2.2 Problem Formulation

As mentioned in earlier sections, A PMU is a device that measures voltage phasor of the bus it is installed on and current phasors of all lines directly connected to it. This capability of PMU in turn makes all its direct neighboring buses observable. Since PMUs are expensive its important determine a minimal number of installation locations to attain the desired amount of observability of the system.

In order to achieve the goal of locating minimal number of PMU installation sites a generalized ILP form can be written as in [13],

$$\text{Minimize} \quad \sum_{k=1}^N x_k \quad (\text{iii})$$

$$\text{Subject to:} \quad GT_{PMU} X \geq B_G \quad (\text{iv})$$

Where G is the transformation matrix that varies as per the conventional measurements considered in the PMU placement case. X is the solution stating the installation locations for PMU, where $x_i \in \{0,1\}$ [13].

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ x_N \end{bmatrix} \quad (\text{v})$$

$T_{PMU} = [t_{i,j}]$, is the incidence matrix that describes bus-to-bus connectivity of the given power system where [13],

$$t_{i,j} = \begin{cases} 1, & \text{if } i = j \\ 1, & \text{if } i \text{ and } j \text{ are connected} \\ 0, & \text{otherwise} \end{cases} \quad (\text{vi})$$

B_G is the column vector that indicates the redundancy requirements for the particular case. Matrices G and B_G are dependent on the PMU placement case taken into consideration and hence vary even for the same system. T_{PMU} matrix on the other-hand remains constant regardless of the case the problem is formulated for a system as it simply defines bus connectivity.

Consider IEEE 14 bus system shown in figure 2.7.

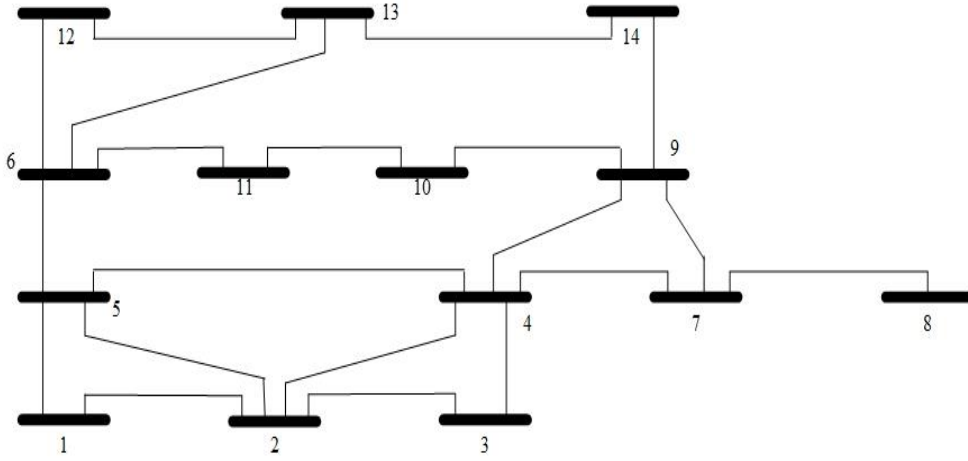


Fig. 2.7 IEEE 14 Bus System

Using conditions given in (vi) the T_{PMU} matrix for the above system is obtained as,

$$T_{PMU} = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \quad (vii)$$

2.2.1 Formulation for Complete Observability

As mentioned earlier A power system is completely observable if the number and locations of PMUs are sufficient to determine voltages of all the buses in the system [2]. In case there is generation or a load on a bus then the parameters like voltage and current are already known and the bus can be considered as observed. These are conventional measurements in power system. Hence an ILP for complete observability can be formulated as follows.

2.2.1.1 Without Conventional Measurements

The optimal PMU placement (OPP) problem without considering conventional measurements is formulated as [14],

$$\begin{aligned} \text{Minimize} \quad & \sum_{k=1}^N x_k \\ \text{Subject to:} \quad & T_{PMU} X \geq b_{PMU} \end{aligned} \quad (\text{viii})$$

Where $b_{PMU} = [b_1 \ b_2 \ \dots \ b_N]^T$ defines the redundancy requirements. Consider IEEE 14 bus system shown in figure 2.7. The bus connectivity matrix T_{PMU} for this system is as given in (vii). From the first row of the T_{PMU} matrix it can be stated that voltage of bus 1 can be measured or calculated if atleast one PMU is placed at bus 1, bus 2 or bus 5. Therefore, the redundancy requirement of the first element of column vector b_{PMU} is $b_1 = 1$. Hence b_{PMU} is obtained as,

$$b_{PMU} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad (ix)$$

On applying the above stated formulation to the given IEEE 14 – bus system in MATLAB version R2016a [19], the solution obtained is,

$$X = [0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 1 \ 0 \ 0 \ 1 \ 0]^T \quad (x)$$

The solution given by (x) states that for IEEE 14 bus system shown in figure 2.7 to be complete observable without considering any conventional measurements, PMUs must be installed on bus 2, bus 8, bus 10, and bus 13.

2.2.1.2 With Conventional Measurements

Let $T_{PMU}X = Y$, where $Y = [y_i] = [y_1 \ y_2 \ \dots \ y_N]^T$. While considering conventional measurements, the element in Y corresponding to any bus in the power system that is associated with the measurement can be zero. This is because the voltage on that bus can be calculated using the measurements associated with it. Following three cases [14] shown in figure 2.8 can elaborate this concept to a certain extent.

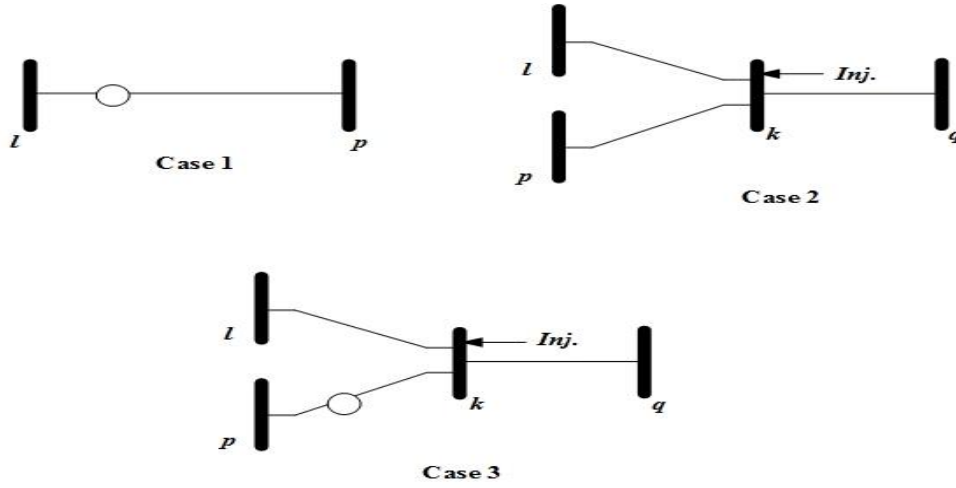


Fig. 2.8 Conventional Measurement Cases

Case 1: If there is power flow measurement on branch $l - p$, then the constraint given below must be considered.

$$y_l + y_p \geq 1 \quad (\text{xi})$$

According to the above constraint, voltage on one of the buses can be calculated using the branch measurement whereas the other remaining bus needs to be covered by a PMU.

Case 2: If there is an injection measurement at bus k , then the constraint given below needs to be held.

$$y_l + y_p + y_k + y_q \geq 3 \quad (\text{xii})$$

Case 3: If there exists a branch measurement on branch $p - k$, then both inequalities given by (xi) and (xii) must be held. But in order to satisfy the inequality in (xi), inequality in (xi) is subtracted from the inequality in (xii) resulting in the inequality $y_k + y_q \geq 1$. Therefore, inequalities for this case are,

$$y_l + y_p \geq 1 \quad \text{and}$$

$$y_k + y_q \geq 1 \quad (\text{xiii})$$

Based on the cases discussed above, ILP formulation for OPP considering conventional measurements can be given as [14],

$$\begin{aligned} \text{Minimize} \quad & \sum_{k=1}^N x_k \\ \text{Subject to:} \quad & T_{con} P T_{PMU} X \geq b_{CON} \end{aligned} \quad (\text{xiv})$$

Where $X = [x_1 \ x_2 \ \dots \ x_N]^T$ and $x_i \in \{0, 1\}$. T_{CON} is transformation matrix that depend on conventional measurements.

$$T_{CON} = \begin{bmatrix} [I_{M \times M}] & [0] \\ [0] & [T_{meas.}] \end{bmatrix} \quad (\text{xv})$$

P is the permutation matrix and b_{CON} is the redundancy matrix which depends on the conventional measurements.

To have better understanding, again consider IEEE 14 – bus system shown in figure 2.7 with injection measurement on bus 7 and branch measurement on line 7 – 8. The buses associated with these measurements bus 4, 7, 8, and 9. Using case 3 the inequalities are,

$$y_7 + y_8 \geq 1 \text{ and} \quad (\text{xvi})$$

$$y_4 + y_9 \geq 1 \quad (\text{xvii})$$

Using the above inequalities T_{meas} can be expressed as,

$$T_{meas} = \begin{array}{c} \text{bus no} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{l} 4 \quad 7 \quad 8 \quad 9 \\ \text{Branch Measurement 7-8} \\ \text{Injection Measurement at 7} \end{array}$$

$I_{M \times M}$ is the identity matrix where,

$$M = (\text{Total Number of buses in system}) - (\text{Number of columns in } T_{meas})$$

In this case $I_{M \times M}$ would be $I_{10 \times 10}$ and hence T_{CON} is as given below,

$$T_{CON} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \quad (\text{xviii})$$

The permutation matrix $P = [p_{i,j}]$. In this case the buses that are not associated with measurements are 1, 2, 3, 5, 6, 10, 11, 12, 13, and 14. Hence the Permutation Matrix P would be given as,

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (\text{xix})$$

On performing the matrix multiplication of T_{CON} , P , and T_{PMU} given by (xviii), (xix) and (vii) respectively the ILP formulation given by (xiv) can be written as below,

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 2 & 2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 2 & 1 & 0 & 2 & 0 & 2 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \\ x_{13} \\ x_{14} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

It is important to note that the last two elements of b_{CON} are the values on the right hand side of inequalities given by (xvi) and (xvii).

On solving the above ILP formulation in MATLAB version R2016a [19] the solution obtained is,

$$X = [0 \ 1 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0]^T$$

According to the above solution, the PMU locations are at bus 2, bus 6 and bus 9 to get a complete observability of the system.

2.2.2 For Depth-of-One Unobservability

As mentioned earlier, depth-of-one unobservability is a scenario where there is one unobserved bus directly connected to two or more observed buses as shown in figure 2.5. Using this description ILP formulation for depth-of-one unobservability can be

modeled as a set of linear inequalities where the sum of y_i corresponding to two connecting buses of a branch must be larger than 1 [13]. Where $T_{PMU} X = Y = [y_i]$.

2.2.2.1 Without Zero Injection Measurement

A bus has zero injection measurement when there is neither generation nor load connected to it. Total flow on all associated branches of this bus equals to zero.

ILP formulation for OPP without any zero injection measurement is as given below [13].

$$\begin{aligned} &\text{Minimize} && \sum_{k=1}^N x_k \\ &\text{Subject to:} && AT_{PMU} X \geq b_1 \end{aligned} \quad (xx)$$

Where $b_1 = [1 \ 1 \ \dots \ 1]_{M_1 \times 1}^T$, $M_1 = \text{number of branches in the system}$.

A is the branch-to-node incident matrix, for IEEE 14 bus system it is illustrated in figure 2.9.

First row of matrix A depicts branch 1 connected to bus 1 and bus 2.

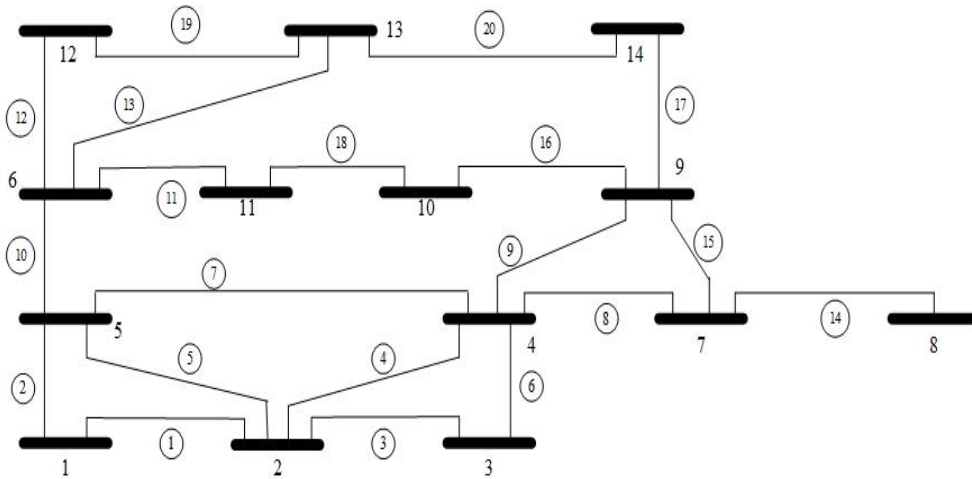


Fig. 2.9 Illustration of Branch-to-Node Matrix

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \quad (\text{xxi})$$

For IEEE 14 – bus system, solving the ILP formulation given by (xx) in MATLAB version R2016a [19] the solution obtained is,

$$X = [0 \ 0 \ 0 \ 1 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]^T$$

The solution above states that PMU locations for depth-of-one unobservability are bus 4 and bus 6.

2.2.2.2 With Zero Injection Measurements

For IEEE 14 bus system shown in figure 2.7 let the zero injection measurement be at bus 7. The only change in the ILP formulation in this case to that of the case without zero injection measurements is that the branches which are not associated with bus having zero injection measurement are taken into consideration.

ILP formulation for this case is as given below [13],

$$\begin{aligned} \text{Minimize} \quad & \sum_{k=1}^N x_k \\ \text{Subject to:} \quad & P_1 A T_{PMU} X \geq P_1 b_1 \end{aligned} \quad (\text{xxii})$$

Where P_1 is a transformation matrix that helps in removing the branches that are associated with zero injection measurements. Hence in case of IEEE 14 – bus system, P_1 is given as,

$$P_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Thus solving the ILP formulation given by (xxii) for IEEE 14 – bus system in MATLAB version R2016a [19] the obtained solution is,

$$X = [0 \ 0 \ 0 \ 1 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]^T$$

The solution above states that PMU locations for depth-of-one unobservability in this case are bus 4 and bus 6.

2.2.3 For Depth-of-Two Unobservability

As mentioned in earlier section, depth-of-two unobservability is a scenario where there are two unobserved buses between two or more observed buses. Using this description ILP formulation for depth-of-two unobservability can be modeled as a set of linear inequalities where the sum of y_i corresponding to three connecting buses must be larger than 1 [13]. Where $T_{PMU} X = Y = [y_i]$.

2.2.3.1 Without Zero Injection Measurement

ILP formulation for OPP without any zero injection measurement for depth-of-two unobservability is as given below [13].

$$\begin{aligned} & \text{Minimize} && \sum_{k=1}^N x_k \\ & \text{Subject to:} && BT_{PMU} X \geq b_2 \end{aligned} \quad (\text{xxiii})$$

Where $b_2 = [1 \ 1 \ \dots \ 1]_{M_2 \times 1}^T$,

$M_2 = \text{Total number of combinations of three connecting buses} .$

Whereas B is the matrix consists of all possible combinations of three connecting buses, which for IEEE 14 bus system in figure 2.7 is as given in (xxiv),

$$B = \begin{bmatrix}
0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1
\end{bmatrix}
\tag{xxiv}$$

Thus solving the ILP formulation given by (xxiii) in MATLAB version R2016a [19] for given IEEE system, the solution obtained is,

$$X = [0 \ 0 \ 0 \ 1 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]^T$$

The above solution states that PMU locations for depth-of-two unobservability without considering any zero injection measurements are bus 4 and bus 6.

2.2.3.2 With Zero Injection Measurements

ILP formulation for OPP considering zero injection measurement for depth-of-two unobservability is as given below [13].

$$\begin{aligned} \text{Minimize} \quad & \sum_{k=1}^N x_k \\ \text{Subject to:} \quad & P_2 B T_{PMU} X \geq P_2 b_2 \end{aligned} \quad (\text{xxv})$$

Consider IEEE 14 bus system shown in figure 2.7 with zero injection measurement at bus 7. Matrix P_2 is a transformation matrix that removes branches not associated with zero injection measurements. In this case P_2 matrix obtained is as given on the next page.

Thus solving the ILP formulation given by (xxv), the solution obtained is,

$$X = [0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0]^T$$

The solution above states that PMU locations for depth-of-two unobservability in this case are bus 5 and bus 9.

2.3 Results

The Above stated formulations are further applied to other standard IEEE systems and Indian bus system. The obtained solutions for all these systems for different observability cases without considering any conventional or zero injection measurements are given below.

Table 1 Optimal PMU Placement for Complete Observability

System	Optimal PMU Locations	Total Number of PMUs
IEEE 14 – bus	2, 8, 10, 13	4
IEEE 30 – bus	1, 5, 8, 10, 11, 12, 19, 23, 26, 29	10
IEEE 57 – bus	1, 4, 9, 20, 23, 27, 29, 30, 32, 36, 38, 41, 45, 46, 50, 54, 57	17
IEEE 118 – bus	4, 5, 7, 9, 12, 17, 19, 23, 26, 34, 37, 42, 46, 48, 51, 55, 60, 62, 65, 67, 71, 75, 79, 84, 87, 91, 95, 99, 103, 106, 112, 114	32
NRPG 246 – bus Indian System	5, 21, 23, 25, 29, 34, 36, 40, 43, 47, 50, 53, 54, 58, 60, 62, 65, 69, 73, 76, 78, 80, 83, 87, 88, 93, 95, 98, 101, 103, 106, 109, 112, 117, 120, 125, 128, 129, 134, 139, 141, 142, 144, 149, 153, 156, 157, 160, 163, 168, 172, 173, 185, 187, 190, 191, 192, 194, 199, 201, 202, 203, 215, 216, 219, 233, 235, 242, 245, 246	70

Table 2 Optimal PMU Placement for Depth-of-One Unobservability

System	Optimal PMU Locations	Total Number of PMUs
IEEE 14 – bus	4, 6	2
IEEE 30 – bus	2, 10, 15, 27	4
IEEE 57 – bus	4, 9, 15, 21, 26, 31, 36, 48, 49, 52, 56	11
IEEE 118 – bus	1, 2, 9, 17, 24, 28, 37, 42, 58, 62, 67, 71, 77, 87, 93, 99, 104, 111	18
NRPG 246 – bus Indian System	8, 11, 22, 32, 53, 54, 56, 65, 70, 71, 83, 88, 89, 91, 101, 121, 126, 130, 139, 140, 141, 147, 158, 160, 166, 170, 190, 191, 194, 199, 203, 204, 205, 219, 226, 229, 233, 245	38

Table 3 Optimal PMU Placement for Depth-of-Two Unobservability

System	Optimal PMU Locations	Total Number of PMUs
IEEE 14 – bus	4, 6	2
IEEE 30 – bus	6, 15, 27	3
IEEE 57 – bus	4, 12, 13, 24, 34, 38, 52, 56	8
IEEE 118 – bus	1, 21, 28, 35, 55, 66, 69, 74, 77, 87, 99, 109	12
NRPG 246 – bus Indian System	13, 21, 32, 49, 56, 65, 69, 83, 86, 89, 101, 113, 130, 137, 139, 147, 157, 160, 170, 185, 191, 194, 200, 203, 207, 211, 229, 233, 245	29

The results show that the ILP formulations presented in the previous sections guarantee a dispersed placement of PMUs around the system and ensures the desired amount of observability for the systems mentioned above.

3 PMU Communication Topology Problem

3.1 Literature Review

3.1.1 PMU's Communication Medium

Communication facilities are essential for transfer of phasor data from PMUs to remotely located PDCs. Channel capacity and Latency are the two significant aspects of data transfer [20]. Channel capacity is the measure of data rate that a data link can sustain. Latency defines data propagation time from source node to destination node. However, data volume created by PMUs is not very large. Hence channel capacity rarely becomes a limiting factor in most applications. Communication channel options for PMU data transfer classified according to physical medium in [21] are leased telephone lines, power line carrier, microwave links, fiber-optic links, etc. Power line carrier communication and microwave links have been more commonly used by electric utilities until recently and are still in use for some applications. But due to unsurpassed channel capacity, low propagation delays and immunity to electromagnetic interference, fiber-optics links have now become a choice of medium for communication facilities of electric utilities.

In [22], fiber optic technology has been discussed and types of fiber, their relative dimensions, modes of data transmission have been described in detail. Figure 3.1 illustrates the construction of a typical fiber optic cable [4]. Such cables are widely

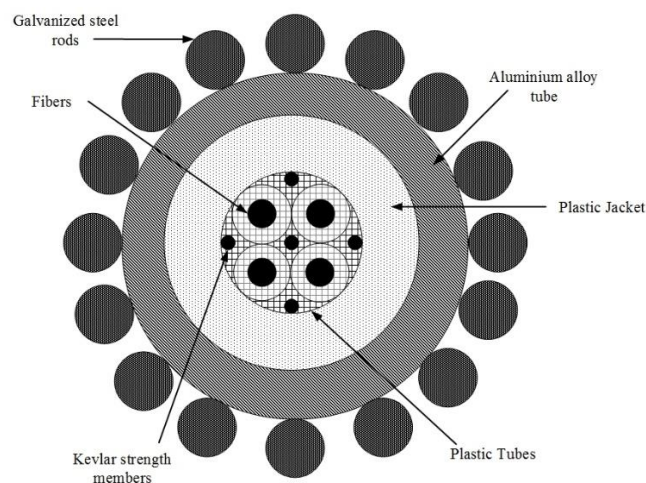


Fig. 3.1 Construction of a Typical Fiber - optic cable [4]

used by electric utilities in their communication infrastructure. In figure 3.2 deployment schemes for optic-fiber cables used by electric utilities is shown. Most popular scheme used is the deployment of fiber-optic cables in ground wire as shown in Figure 3.2(a). Other deployment schemes used by electric utilities involve wrapping of fiber cable around phase conductors, use of separate towers along the transmission lines as in figure 3.2(b) or under-ground deployment of optic-fiber cables as shown in figure 3.2(c) [4].

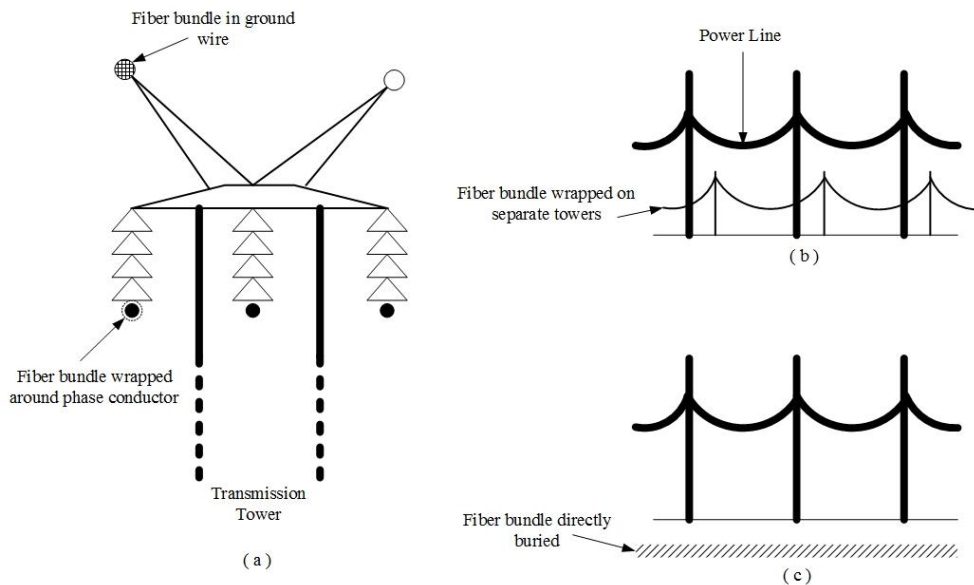


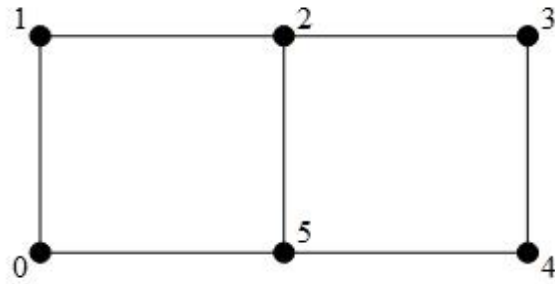
Fig. 3.2 Fiber - optic Cable Deployment Schemes [4]

Optical fiber power ground wire (OPGW) is used for construction of power transmission and distribution lines. An optimal placement of OPGW can not only reduces investment cost but also can improve latency and reliability index within the network [23].

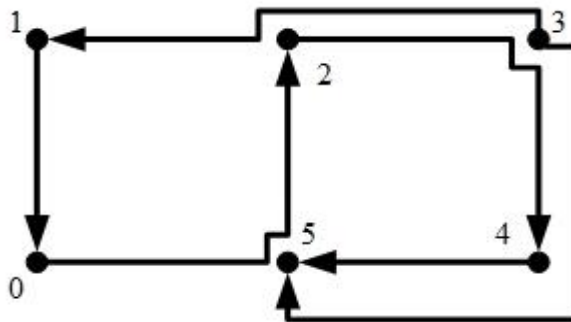
3.1.2 Physical Topology and Logical Topology

High speed wide area networks are mostly used for power system communication now – a – days. These networks use wavelength routed optical networks due to its high bandwidth capability, transparent bit rate, allowance to spatial wavelength, and reliable service provision. To reflect traffic intensities between various nodes it is possible to build logical topology over established physical topology of wavelength routed optical network [24]. As per [25] a physical topology is set routing nodes connected by an optic-fiber cable link whereas logical topology is a set of all possible

lightpaths between routing nodes of a physical topology. A lightpath is a path set up by configuring any two routing nodes in a physical topology. Two lightpaths sharing a physical link on the network need to use different wavelengths [26]. Figure 3.3 (a) shows an example of physical topology of a six node network while figure 3.3 (b) shows a possible logical topology for that physical topology.



(a) Physical Topology of a Six Node Network



(b) Possible Logical Topology of a Six Node Network

Fig. 3.3 Physical and Logical Topology of a Six Node Network [24]

From figure 3.3 (b) it can be seen that data from node 3 to node 1 can be directly sent over the established logical link (3,1). However, to send the data from node 3 to node 2, the data packet has to travel over the logical links (3,1), (1,0) and (0,2) even though there is a direct physical fiber link between node 3 and node 2. This is the basic concept of data routing over a logical topology.

3.1.3 Shortest Path Algorithms

Propagation delay between any two nodes of a network has been considered while designing a logical topology for it. Propagation delay parameter is directly proportional to the physical link distance between two nodes. Hence to calculate the minimum distance between source node to destination node shortest path algorithms need to be used.

A shortest path problem (SPP) in [27] is defined as “a problem to find a path between two vertices in a graph so that the sum of the weight of the constituent channels can reach minimum values.” SPP has widespread practical applications right from logistics, transportation and vehicle routing to robot path planning and communication [28]. Dijkstra, Bellman – Ford, A* search, Floyd – Warshall, Johnson’s, Viterbi, etc. to name a few are the algorithms developed over years in order to solve the SPP. Each algorithm has its own advantages and disadvantages over the other.

In [27] and [29] Dijkstra, A* search and Floyd – Warshall Algorithms have been compared on the basis of computational load, Simulation time and Memory Usage. A Multi – objective Shortest Path (MOSP) algorithm is presented in [8] that exploits advantages of Dijkstra’s algorithm which is a single – source shortest path algorithm to further extend it to multi – source shortest path.

3.2 Communication System Design

Today there are two types of communication control strategies used in a power system, namely, centralized and decentralized [30]. In a centralized strategy communication takes place directly between metering devices and a remotely located control center. A decentralized strategy on the other hand divides the data collection areas and appoints a subordinate control center for each division. These subordinate control centers then act as intermediaries between metering devices and the main control center.

In this thesis, MOSP algorithm is used to find an optimal location for subordinate control center. Reason for using MOSP algorithm are its advantages over other algorithms as discussed in [8]. A subordinate control center is referred to as a central control bus (CCB) further in the thesis.

MOSP approach to be implemented for finding an optimal location of CCB is as follows:

Step 1. Apply Dijkstra’s algorithm to find shortest path from PMU bus to all other buses in the system.

Step 2. Finding the total coverage of each bus.

Step 3. Find the minimum numerical value element in matrix C.

Step 4. Selecting the bus that needs the least maximum number of hops by any PMU bus to reach it. This step is needed if and only if there are two or more buses holding the least distance coverage.

To have better understanding of the concept, IEEE 14 – bus system is again considered here and MOSP approach is applied on it.

The given bus system has total length of transmission line equal to 900 km as shown in figure 3.4. The relative distances between systems buses are obtained from the system admittance matrix [8], [9] and [31].

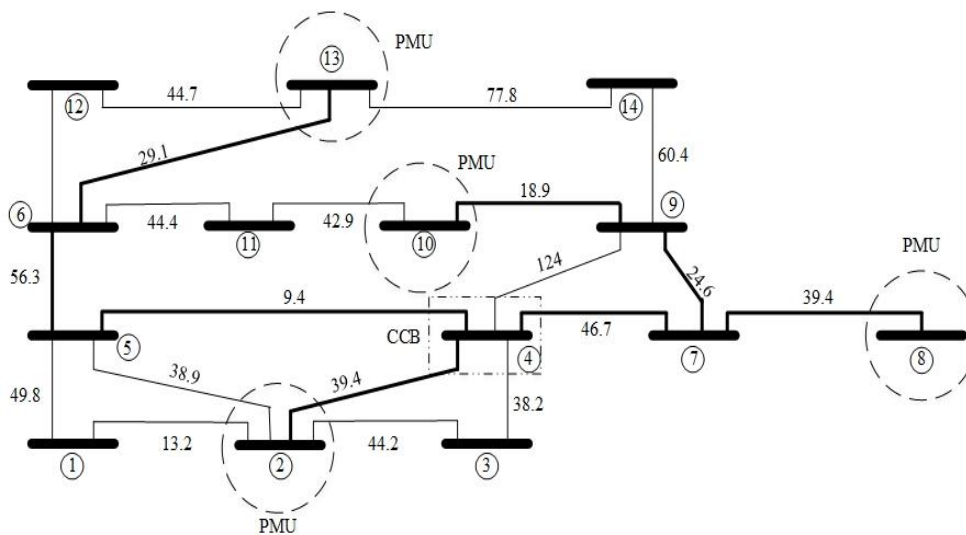


Fig. 3.4 Optimal WAMS design for IEEE 14-Bus System

From table 1 of the previous chapter, it is clear that for complete observability of a IEEE 14 – Bus System, PMU location are selected on bus 2, bus 8, bus 10, and bus 13. The constraint for selection of CCB is the OPGW length. Hence keeping the OPGW link length to the minimum is the main objective.

Dijkstra’s algorithm is applied on bus 2, bus 8, bus 10 and bus 13 to obtain the following distance matrix.

$$D = \begin{matrix} & \begin{matrix} bus\ 2 & bus\ 8 & bus\ 10 & bus\ 13 \end{matrix} \\ \begin{matrix} bus\ 1 \\ bus\ 2 \\ bus\ 3 \\ bus\ 4 \\ bus\ 5 \\ bus\ 6 \\ bus\ 7 \\ bus\ 8 \\ bus\ 9 \\ bus\ 10 \\ bus\ 11 \\ bus\ 12 \\ bus\ 13 \\ bus\ 14 \end{matrix} & \begin{bmatrix} 13.2 & 138.7 & 142.8 & 135.2 \\ 0 & 125.5 & 129.6 & 124.3 \\ 44.2 & 124.3 & 128.4 & 133 \\ 39.4 & 86.1 & 90.2 & 94.8 \\ 38.9 & 95.5 & 99.6 & 85.4 \\ 95.2 & 151.8 & 87.3 & 29.1 \\ 86.1 & 39.4 & 43.5 & 141.5 \\ 125.5 & 0 & 82.9 & 180.9 \\ 110.7 & 64 & 18.9 & 135.3 \\ 129.6 & 82.9 & 0 & 116.4 \\ 139.6 & 125.8 & 42.9 & 73.5 \\ 152.3 & 208.9 & 144.4 & 44.7 \\ 124.3 & 180.9 & 116.4 & 0 \\ 171.1 & 124.4 & 79.3 & 77.8 \end{bmatrix} \end{matrix}$$

Total Coverage of each bus is done by multiplying matrix D by a column vector with number of elements equal to the total number of PMUs in the system. All elements in the column vector must be equal to 1 in order to satisfy its purpose in the algorithm.

$$C = D[1 \ 1 \ 1 \ 1]^T$$

$$C = \begin{bmatrix} 429.9 \\ 379.4 \\ 429.9 \\ 310.5 \\ 319.4 \\ 363.4 \\ 310.5 \\ 389.3 \\ 328.9 \\ 328.9 \\ 381.8 \\ 550.3 \\ 421.6 \\ 452.6 \end{bmatrix} \begin{matrix} bus1 \\ bus2 \\ bus3 \\ bus4 \\ bus5 \\ bus6 \\ bus7 \\ bus8 \\ bus9 \\ bus10 \\ bus11 \\ bus12 \\ bus13 \\ bus14 \end{matrix}$$

According to matrix C bus 4 and bus 7 have the least distance coverage.

From figure 3.4 it can be seen that, for bus 4 maximum number of hops required by any PMU bus in the system are three whereas in case of bus 7 it becomes four. Hence bus 4 is selected as a CCB.

The MOSP algorithm discussed above has been implemented using MATLAB version R2016a [19].

Also an analysis of shortest path algorithms was done during the process of communication system design. Floyd – Warshall and Dijkstra’s algorithm were applied on different IEEE systems in order to check their performance with respect to run – time. Results obtained didn’t show any significant run – time difference for systems having less than 246 nodes. Difference of few milliseconds was noticed for NRPG 246 – bus Indian system.

3.3 Logical Topology Design

After computing an optimal location for the subordinate control center, an approach to design logical topology is also discussed in this thesis. In order to solve logical topology problem, the MILP formulation developed in [26] is utilized in present work.

The MILP formulated for logical topology in [26] is as follows,

$$\text{Minimize} \quad \lambda_{\max}$$

Subject to:

Flow conservation at each node:

$$\sum_j \lambda_{ij}^{sd} - \sum_j \lambda_{ji}^{sd} = \begin{cases} \lambda^{sd} & \text{if } s = i \\ -\lambda^{sd} & \text{if } d = i \\ 0 & \text{otherwise} \end{cases} \quad \text{for all } s, d, i$$

Total flow on a logical link:

$$\begin{aligned} \lambda_{ij} &= \sum_{s,d} \lambda_{ij}^{sd} && \text{for all } i, j \\ \lambda_{ij} &\leq \lambda_{\max} && \text{for all } i, j \\ \lambda_{ij}^{sd} &\leq b_{ij} \lambda^{sd} && \text{for all } i, j, s, d \end{aligned}$$

Average delay constraint for each s – d pair:

$$\sum_{i,j} \lambda_{ij}^{sd} \delta_{ij} \leq \lambda^{sd} \alpha \delta_{sd}$$

Degree Constraints:

$$\begin{aligned} \sum_i b_{ij} &= \Delta_l && \text{for all } j \\ \sum_j b_{ij} &= \Delta_l && \text{for all } i \\ \lambda_{ij}^{sd}, \lambda_{ij}, \lambda_{\max} &\geq 0 && \text{for all } i, j, s, d \\ b_{ij} &\in \{0, 1\} && \text{for all } i, j \end{aligned}$$

Where, s is the source node of the data packet

d is the destination node of the data packet

$b_{ij} = 1$ if there is a logical link from node i to node j

$b_{ij} = 0$ if there is no logical link from node i to node j

Δ_i denotes the number of transceivers available at each node.

$(\alpha \delta_{sd})$ is the maximum permissible average propagation delay on the physical topology between any (s, d) pair

δ_{ij} is the propagation delay on the logical link (i, j)

λ^{sd} is the arrival rate of packets at node s that are destined for node d

λ_{ij}^{sd} is the arrival rate of packets from (s, d) pair on logical link (i, j)

λ_{ij} is the arrival rate of packets on link (i, j) from all (s, d) pairs

λ_{\max} is maximum data load on any link, also called congestion

The given formulation is applied to the network shown in figure 3.5 which is obtained by using MOSP algorithm on IEEE 14 – bus system.

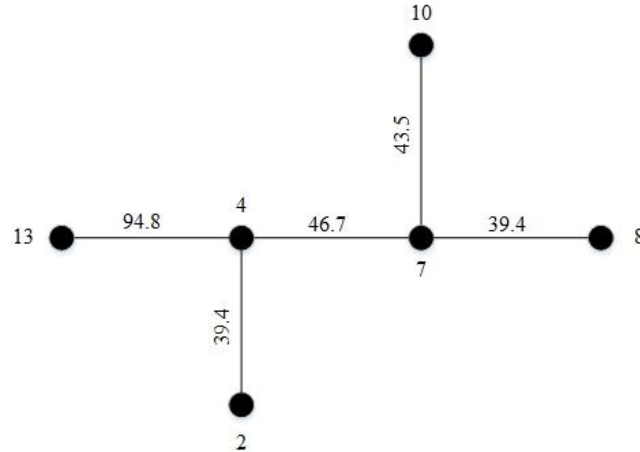


Fig. 3.5 Physical Topology of WAMS of IEEE 14-bus system

In [3], the traffic generated by a PMU is considered to be 19.2 kilo – bits per second.

Hence the traffic matrix $T = [\lambda^{sd}]$ for this case is created as,

$$\begin{array}{r}
\text{node } d \quad 2 \quad 4 \quad 7 \quad 8 \quad 10 \quad 13 \\
T = \begin{bmatrix} 0 & 19.2 & 0 & 0 & 0 & 0 \\ 0 & 0 & & 0 & 0 & 0 \\ 0 & 0 & & 0 & 0 & 0 \\ 0 & 19.2 & 0 & 0 & 0 & 0 \\ 0 & 19.2 & 0 & 0 & 0 & 0 \\ 0 & 19.2 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{array}{l} 2 \\ 4 \\ 7 \\ 8 \\ 10 \\ 13 \end{array} \\
\text{node } s
\end{array}$$

δ is the distance matrix for the network shown in figure 3.5. It is obtained using Floyd – Warshall algorithm as discussed in [27].

$$\begin{array}{r}
\text{node} \quad 2 \quad 4 \quad 7 \quad 8 \quad 10 \quad 13 \\
\delta = \begin{bmatrix} 0 & 39.4 & 86.1 & 125.5 & 129.6 & 134.2 \\ 39.4 & 0 & 46.7 & 86.1 & 90.2 & 94.8 \\ 86.1 & 46.7 & 0 & 39.4 & 43.5 & 141.5 \\ 125.5 & 86.1 & 39.4 & 0 & 82.9 & 180.9 \\ 129.6 & 90.2 & 43.5 & 82.9 & 0 & 185 \\ 134.2 & 94.8 & 141.5 & 180.9 & 185 & 0 \end{bmatrix} \begin{array}{l} 2 \\ 4 \\ 7 \\ 8 \\ 10 \\ 13 \end{array} \\
\text{node}
\end{array}$$

Solving the MILP formulation mentioned above in IBM LOG CPLEX OPTIMIZATION STUDIO [32]. Assuming $\Delta_l = 1$ as there is only one PMU at each node and $\alpha \geq 1$ the following logical topology is obtained with $\lambda^{sd} = 76.8$ kilo – bits per second.

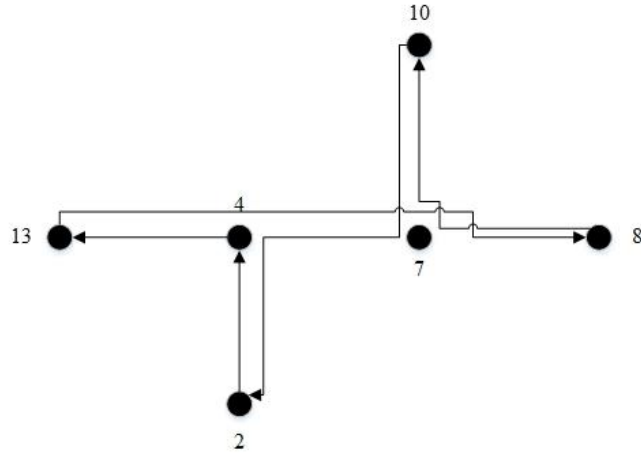


Fig. 3.6 Logical topology for $\Delta_l = 1$

Bus 4 is the CCB and hence the destination for data generated by PMUs at bus 2, 8, 10 and 13. Physical topology of the network in figure 3.5 shows a direct fiber link between bus 13 and bus 4. However, data generated by PMU on bus 13 has to travel over the logical links (13, 8), (8,10), (10, 2) and (2, 4) in order to reach CCB at bus 4 as shown in figure 3.6. This prevents congestion and stacking of data packets at the destination. Similarly, data from bus 8 travels over 3 logical links whereas that from bus 10 travels over 2. Data from bus 2 travels over only one logical link that is (2, 4).

4 Conclusion

This thesis addresses three different issues regarding planning of PMU installation in a power system.

First the PMU placement problem is considered in detail. Here ILP algorithm is used due to its computational efficiency over other available optimization methodologies. The obtained results show that the given ILP formulations applied on different IEEE system guarantee a dispersed placement of PMUs around the system and hence ensure the desired amount of observability.

Later, MOSP algorithm provided in the thesis is used for optimal designing of communication infrastructure for a given system. The result obtained for IEEE 14 – bus system is a communication network in terms of CCB and PMU locations.

Finally, logical topology is designed for the communication network obtained through MOSP algorithm. For designing of logical topology, the MILP algorithm proposed in [26] is utilized. The given algorithm is successfully applied on IEEE 14 – bus system. The obtained result is a routing pattern for data generated by the PMUs in order to prevent congestion at the CCB.

5 Future work

In this thesis, the problem of optimal PMU placement and that of logical topology of PMU's communication medium has been addressed independently.

However, Co – optimization of PMU locations and communication system using GA has already been attempted in [9]. GA being a heuristic approach has its own benefits and drawbacks. On the other hand, ILP, a deterministic approach requires less computational time and provides a mathematical solution that is hard to challenge. This makes it more reliable compared to several heuristic and meta – heuristic approaches.

Unification of optimization process using a deterministic approach may be difficult but definitely not impossible, thus making it the basis for future work.

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Appendix

A.1 MATLAB program for PMU placement problem

```
% UiT - The Arctic University of Norway, Narvik
% Department of Technology
% Master Thesis
% Bhushan Madan Nikumbh

%% main script 'main'

n = input('enter the system bus number:');
p = input('enter the injection bus number (enter 0 if no
injection):');
type = input('Comp_Obs = 0, dep_1_inobsv = 1, dep_2_inobsv= 2
:');
switch type
    case 0
        BM = input('enter 0 if no Brn mrsmt, 1 if Brn mrsmt
present:');
        if BM ~= 0
            prompt = 'enter brn mrsmt matrix :';
            BM_Mat = input(prompt);
        else BM_Mat = 0;
        end
        [X, grf] = com_obsv( n, p, BM, BM_Mat );
    case 1
        if p == 0;
            [X, grf] = dep_1( n );
        else [X, grf] = dep_1_inj( n, p );
        end
    case 2
        if p == 0;
            [X, grf] = dep_2( n, p );
        else [X, grf] = dep_2_inj( n, p );
        end
end
[rx, cnx]=size(X);
PMU_pos = [];
cnt = 1;
for c=1:cnx
    if X(1,c) > 0
        PMU_pos(1,cnt)= c;
        cnt = cnt + 1;
    end
end
end
```

```

%% com_obs function
% function for complete observability
% with and without conventional measurements
function [ X, grf ] = com_obs( n, p, BM, BM_Mat )
Info_Mat = Info_mat( n );
grf = plot_G( Info_Mat );
func = ones(n,1);
Tmult_Mat = Tmult( p, BM, BM_Mat, Info_Mat, n );
bcon_Mat = bcon( p, BM, BM_Mat, Info_Mat, n );
    intcon = 1:n;
lb = zeros(n,1);
ub = ones(n,1);
x = intlinprog(func,intcon,Tmult_Mat,bcon_Mat,[],[],lb,ub);
X = x';
end

%% dep_1 function
% function for depth-of-one observability
% without considering zero injection measurement
function [ X, grf ] = dep_1( n )

Info_Mat = Info_mat( n );
grf = plot_G( Info_Mat );
Tpmu_Mat = Tpmu ( Info_Mat, n );
Brn2node_Mat = Brn2node( Info_Mat );
b1_Mat = b1( Brn2node_Mat );
Rmult = Brn2node_Mat*Tpmu_Mat;
Lmult = b1_Mat;
RHS = -Rmult;
LHS = -Lmult;
func = ones(n,1);
intcon = 1:n;
lb = zeros(n,1);
ub = ones(n,1);
x = intlinprog(func,intcon,RHS,LHS,[],[],lb,ub);
X = x';
end

%% dep_1_inj function
% function for depth-of-one unobservability
% with zero injection measurements
function [ X, grf ] = dep_1_inj( n, p )
    Info_Mat = Info_mat( n );

```

```

grf = plot_G( Info_Mat );
Tpmu_Mat = Tpmu ( Info_Mat, n );
Brn2node_Mat = Brn2node( Info_Mat );
P1_Mat = P1( Info_Mat, p );
b1_Mat = b1( Brn2node_Mat );
Rmult = P1_Mat*Brn2node_Mat*Tpmu_Mat;
Lmult = P1_Mat*b1_Mat;
RHS = -Rmult;
LHS = -Lmult;
func = ones(n,1);
intcon = 1:n;
lb = zeros(n,1);
ub = ones(n,1);
x = intlinprog(func,intcon,RHS,LHS,[],[],lb,ub);
X = x';
end

```

```

%% dep_2 function
% function for depth-of-two observability
% without considering zero injection measurement
function [ X, grf ] = dep_2( n, p )
Info_Mat = Info_mat( n );
grf = plot_G( Info_Mat );
Tpmu_Mat = Tpmu ( Info_Mat, n );
P_new=unique(Info_Mat,'rows');
node3brn_Mat = node3brn( Info_Mat, p, n );
New_B_mat = node3brn_Mat;
[z,~]=size(New_B_mat);
b2=ones(z,1);
Rmult = New_B_mat*Tpmu_Mat;
Lmult = b2;
RHS = -Rmult;
LHS = -Lmult;
func = ones(n,1);
intcon = 1:n;
lb = zeros(n,1);
ub = ones(n,1);
x = intlinprog(func,intcon,RHS,LHS,[],[],lb,ub);
X = x';
end

```

```

%% dep_2_inj function
% function for depth-of-two unobservability
% with zero injection measurements
function [ X, grf ] = dep_2_inj( n, p )
Info_Mat = Info_mat( n );

```

```

grf = plot_G( Info_Mat );
Tpmu_Mat = Tpmu ( Info_Mat, n );
node3brn_Mat = node3brn( Info_Mat, p, n );
P2_Mat = P2( Info_Mat, p, n );
b2_Mat = b2( Info_Mat, p, n );
Rmult = P2_Mat*node3brn_Mat*Tpmu_Mat;
Lmult = P2_Mat*b2_Mat;
RHS = -Rmult;
LHS = -Lmult;
func = ones(n,1);
intcon = 1:n;
lb = zeros(n,1);
ub = ones(n,1);
x = intlinprog(func,intcon,RHS,LHS,[],[],lb,ub);
X = x';
end

%% b1 function
% function for b1 matrix
function [ b1_Mat ] = b1( Brn2node_Mat )
[b,~]=size(Brn2node_Mat);
b1_Mat=ones(b,1);
end

%% b2 function
% function for b2 matrix
function [ b2_Mat ] = b2( Info_Mat, p, n )
P2_Mat = P2( Info_Mat, p, n );
[rw2,~]=size(P2_Mat);
b2_Mat=ones(rw2,1);
end

%% bcon function
% function for bcon matrix
function [ bcon_Mat ] = bcon( p, BM, BM_Mat, Info_Mat, n )
if p~=0
    Tmeans_Mat = Tmeans( p, BM, BM_Mat, Info_Mat, n );
    [a,b]=size(Tmeans_Mat);
    cnt = 0;
    for m = 1:b
        if Tmeans_Mat(a,m)==1
            cnt = cnt + 1;
        end
    end
    bcon_Mat = Tmult( p, BM, BM_Mat, Info_Mat, n );
end

```



```

        [x,~]=size(Tmult_Mat);
        LHS = [ones(1,x-1),cnt-1];
        LHS_trn = LHS';
        bcon_Mat = -LHS_trn;
    else
        LHS = ones(n,1);
        bcon_Mat = -LHS;
    end
end

%% bM_Mat function
% function for branch measurement matrix
function [ New_BM_Mat ] = bM_Mat(BM, BM_Mat )
if BM ~= 0
    BM_trn=BM_Mat';
%convert BM_Trn into single row matrix
    Mer = [BM_trn(1,:),BM_trn(2,:)] ;
    Sort_Mer = sort (Mer);
    New_BM_Mat = Remv_dup (Sort_Mer);
else
    New_BM_Mat = [];
end
end

%% Brn2node function
% function for Branch-to-Node Matrix
function [ Brn2node_Mat ] = Brn2node( Info_Mat )
[a,~]=size(Info_Mat);
Brn2node_Mat = [];
for n=1:a
    Brn2node_Mat(n,Info_Mat(n,1))=1;
    Brn2node_Mat(n,Info_Mat(n,2))=1;
end
end

%% Info_mat function
% function for collecting connectivity information
function [ Info_Mat ] = Info_mat( n )
num=n;
linedt = linedatas(num);
[a,~]=size(linedt);
Info=linedt(1:a,1:2);
Info_Mat=unique(Info,'rows');
end

```

```

%% inj_Mat function
% function for determining buses connected to injection bus
function [ Inj_Mat ] = inj_Mat( p, Info_Mat )
Inj = [];
Inj(1,1) = p;
[i,~] = size(Info_Mat);
cnt = 2;
for m = 1:i
    X = Info_Mat(m,1);
    Y = Info_Mat(m,2);
    if p == X
        Inj(1,cnt) = Y;
        cnt = cnt+1;
    elseif p == Y
        Inj(1,cnt) = X;
        cnt = cnt+1;
    end
end
Inj_Mat = sort (Inj);
end

%% node3brn function
% function from generating B matrix
% matrix depicting 3 connecting buses
function [ node3brn_Mat ] = node3brn( Info_Mat, p, n )
[a,~]=size(Info_Mat);
Trial_Mat_1 = [];
q = 1;
for m = 1:a
    if Info_Mat(m,1)~=p && Info_Mat(m,2)~=p
        Trial_Mat_1(q,1)=Info_Mat(m,1);
        Trial_Mat_1(q,2)=Info_Mat(m,2);
        q=q+1;
    end
end
New_node_Mat = New_node( p, Info_Mat, n );
[~,cn7]=size(New_node_Mat);
[b,~]=size(Trial_Mat_1);
Q = [];
y = 1;
for cnt0=1:cn7
    for m = 1:b
        if Trial_Mat_1(m,1)==New_node_Mat(1,cnt0)
            Q(y,1)=Trial_Mat_1(m,1);
            Q(y,2)=Trial_Mat_1(m,2);
            y=y+1;
        elseif Trial_Mat_1(m,2)==New_node_Mat(1,cnt0)

```

```

                Q(y,1)=Trial_Mat_1(m,1);
                Q(y,2)=Trial_Mat_1(m,2);
                y=y+1;
            end
        end
    end
P_new=unique(Q, 'rows');
[x,~]=size(P_new);
Initial_3brn=[];
d=1;
s=1;
for c=1:x-1
for cntx=s:x-1
    if P_new(c,2)==P_new(cntx+1,1)
        Initial_3brn(d,1)=P_new(c,1);
        Initial_3brn(d,2)=P_new(c,2);
        Initial_3brn(d,3)=P_new(cntx+1,2);
        d=d+1;
    end
end
for cntx=s:x-1
    if P_new(c,1)==P_new(cntx+1,1)
        Initial_3brn(d,1)=P_new(c,1);
        Initial_3brn(d,2)=P_new(c,2);
        Initial_3brn(d,3)=P_new(cntx+1,2);
        d=d+1;
    end
end
s=s+1;
end
[rwi,cni]=size(Initial_3brn);
B_mat=[];
for cnt_1=1:rwi
    for cnt_2=1:cni
        Ele=Initial_3brn(cnt_1,cnt_2);
        B_mat(cnt_1,Ele)=1;
    end
end
node3brn_Mat=B_mat;
end

%% New_node function
function [ New_node_Mat ] = New_node( p, Info_Mat, n )
Inj_Mat = inj_Mat( p, Info_Mat );
New_node=[];
cnt7=1;
[~,cn4]=size(Inj_Mat);

```

```

node_Mat=1:n;
[~,cn3]=size(node_Mat);
cnt6=0;
cnt5=1;
while cnt6<cn3
    cnt6=cnt6+1;
    if Inj_Mat(1,cnt5)~=node_Mat(1,cnt6)
        New_node(1,cnt7)=node_Mat(1,cnt6);
        cnt7=cnt7+1;
        cnt5=cnt5-1;
    end
    if cnt5<cn4
        cnt5=cnt5+1;
    end
end
New_node_Mat=New_node;
end

%% Per_new function
% function creates permutation matrix P
function [ Per_Mat ] = Per_new( n, p, BM_Mat, Info_Mat, BM )
if p~=0
    Inj_Mat = inj_Mat(p, Info_Mat);
    if BM~=0
        New_BM_Mat = bM_Mat(BM, BM_Mat);
        BM_Inj = [New_BM_Mat, Inj_Mat];
        BM_Inj_1 = sort (BM_Inj);
        U = Remv_dup(BM_Inj_1);
    else
        U = Inj_Mat;
    end
    [a,b] = size(U);
    PER = zeros(n,n);
    c = 1;
    d = 1;
    cnt = 0;
    while cnt<b;
        cnt=cnt+1;
        X = U(a,cnt);
        if d==X
            PER(c,d) = 0 ;
        else
            PER(c,d) = 1;
            c=c+1;
            cnt=cnt-1;
        end
        d=d+1;
    end
end

```

```

end
while d<=n;
    PER(c,d)=1;
    c=c+1;
    d=d+1;
end
d = 1;
cnt1 = 0;
while cnt1<b;
    cnt1=cnt1+1;
    X=U(a,cnt1);
    if d==X
        PER(c,d)=1;
        c=c+1;
    else PER(c,d)=0;
        cnt1=cnt1-1;
    end
    d=d+1;
end
else
    PER = eye(n);
end
Per_Mat = PER;
end

%% P1 function
% function creates P1 matrix
function [ P1_Mat ] = P1( Info_Mat,p )
P1_Mat=[];
cnt=1;
[a,~]=size(Info_Mat);

for m = 1:a
    if Info_Mat(m,1)~=p && Info_Mat(m,2)~=p
        P1_Mat(cnt,m)=1;
        cnt=cnt+1;
    end
end
end

%% P2 function
% function creates P2 matrix
function [ P2_Mat ] = P2( Info_Mat, p, n )
node3brn_Mat = node3brn( Info_Mat, p, n );
[rw,~]=size(node3brn_Mat);

```

```

P_2=eye(rw);
P2_Mat = P_2;
end

%% Rmv_dup function
% function removes duplicate elements from a matrix
function [ Rmv_Dup ] = Rmv_dup( in_Mat )
inp_Mat = in_Mat;
Rmv_Dup = [];
cnt = 1;
cntx = 1;
[~,b]=size(inp_Mat);
c = b-1;
cnty = 0;
while cnty < c
    cnty = cnty+1;
    if inp_Mat(1,cnt)==inp_Mat(1,cnt + 1)
        cnt = cnt+1;
    else
        Rmv_Dup(1,cntx) = inp_Mat(1,cnt);
        cntx = cntx + 1;
        cnt = cnt + 1;
    end
end
if inp_Mat(1,c) ~= inp_Mat(1,b)
    Rmv_Dup(1,cntx) = inp_Mat(1,b);
end
end

%% Tcon function
% function creates Tcon matrix
function [ Tcon_Mat ] = Tcon( p, BM, BM_Mat, Info_Mat, n )
Tmeans_Mat = Tmeans( p, BM, BM_Mat, Info_Mat, n );
[a,b]=size(Tmeans_Mat);
if p~=0
    A = eye(n-b);
    B = zeros(n-b,b);
    C = zeros(a,n-b);
    D = Tmeans_Mat;
    Tcon_Mat = [A B
                C D];
else
    Tcon_Mat = Tmeans_Mat;
end
end
end

```

```

%% Tmeans function
% function creates Tmeans matrix
function [ Tmeans_Mat ] = Tmeans( p, BM, BM_Mat, Info_Mat, n
)
Mat = [];
if p~=0
    Inj_Mat = inj_Mat(p, Info_Mat);
    [~,e] = size(Inj_Mat);
    if BM~=0
        New_BM_Mat = bM_Mat(BM, BM_Mat);
        BM_Inj = [New_BM_Mat, Inj_Mat];
        BM_Inj_1 = sort (BM_Inj);
        New_BM_Inj = Remv_dup(BM_Inj_1);
        [~,b] = size(New_BM_Mat);
        [~,a] = size(New_BM_Inj);
        m = 0;
        n = a;
        while m < b
            m = m+1;
            if New_BM_Mat(1,m)<=n
                Mat(1,New_BM_Mat(1,m)) = 1;
            else
                Mat(1,n+1) = 1;
                n = n+1;
            end
        end
        [~,c]=size(Mat);
        if c<a
            for f = c+1:a
                Mat(1,f)=0;
            end
        end
        [~,g]=size(Mat);
        for d=1:g
            if Mat(1,d)==0
                Mat(2,d) = 1;
            else
                Mat(2,d) = 0;
            end
        end
    else
        Mat = ones(1,e);
    end
else
    Mat = eye(n);
end
Tmeans_Mat = Mat;
end

```

```

%% Tmult function
% function multiplies Tcon, P and Tpmu matrix
function [ Tmult_Mat ] = Tmult( p, BM, BM_Mat, Info_Mat, n )
Tcon_Mat = Tcon( p, BM, BM_Mat, Info_Mat, n );
Per_Mat = Per_new( n, p, BM_Mat, Info_Mat, BM );
Tpmu_Mat = Tpmu ( Info_Mat, n );
RHS = Tcon_Mat*Per_Mat*Tpmu_Mat;
Tmult_Mat = -RHS;
end

%% Tmult function
% function multiplies Tcon, P and Tpmu matrix
function [ Tmult_Mat ] = Tmult( p, BM, BM_Mat, Info_Mat, n )
Tcon_Mat = Tcon( p, BM, BM_Mat, Info_Mat, n );
Per_Mat = Per_new( n, p, BM_Mat, Info_Mat, BM );
Tpmu_Mat = Tpmu ( Info_Mat, n );
RHS = Tcon_Mat*Per_Mat*Tpmu_Mat;
Tmult_Mat = -RHS;
end

%% plot_G function
% function creates a graphical representation of IEEE systems
function [ grf ] = plot_G( Info_Mat )
trn = Info_Mat';
s = trn(1,:);
t = trn(2,:);
G = graph(s,t);
grf = plot(G);
end

%% linedatas function
% function stores line-data of IEEE system
function [linedt] = linedatas(num)

% line data of IEEE systems should be entered here
% for example linedat7 = [1 2
%                          2 3
%                          2 6
%                          2 7
%                          3 4
%                          3 6
%                          4 5
%                          4 7];

switch num

```



```

case 3
    linedt = linedat3;
case 4
    linedt = linedat4;
case 5
    linedt = linedat5;
case 6
    linedt = linedat6;
case 7
    linedt = linedat7;
case 8
    linedt = linedat8;
case 14
    linedt = linedat14;
case 30
    linedt = linedat30;
case 57
    linedt = linedat57;
case 118
    linedt = linedat118;
case 246
    linedt=linedat246;
end
end

```

A.2 MATLAB program for MOSP algorithm

```

% UiT - The Arctic University of Norway, Narvik
% Department of Technology
% Master Thesis
% Bhushan Madan Nikumbh
% MOSP algorithm for shortest path
% IEEE 14 - bus system

s = [1 1 2 2 2 3 4 4 4 5 6 6 6 7 7 9 9 10 12 13];
t = [2 5 3 4 5 4 5 7 9 6 11 12 13 8 9 10 14 11 13 14];
weights = [13.2 49.8 44.2 39.4 38.9 38.2 9.4 46.7 124 56.3
44.4 57.1 29.1 39.4 24.6 18.9 60.4 42.9 44.7 77.8];
G = graph(s,t,weights);
plot(G, 'EdgeLabel',G.Edges.Weight);
PMU = [2 8 10 13];
[~,pmu_c]=size(PMU);
dist = [];
path=[];
n = 14;
for cnt = 1:pmu_c

```

```

    pmu_num = PMU(1, cnt);
    for c = 1:n
        [~,d] = shortestpath(G, pmu_num, c);
        dist(c, cnt) = d;
    end
end
[~, col] = size(dist);
uni = ones(col, 1);
tot_dist = dist*uni;
[min_dist, CCB] = min(tot_dist);
for cnt = 1:pmu_c
    pmu_num = PMU(1, cnt);
    [p, ~] = shortestpath(G, pmu_num, CCB);
    [~, cnp] = size(p);
    for cntx = 1:cnp
        path(cnt, cntx) = p(1, cntx);
    end
end
end

```

A.3 CPLEX program for Logical Topology

```

/*****
* OPL 12.6.3.0 Model
* Author: Bhushan Madan Nikumbh
* Creation Date: 11. juni 2016 at 20:38:27
*****/
// parameters
int nodes = 6;
int deg = 1;

range sorc = 1..nodes;
range dest = 1..nodes;
range in_node = 1..nodes;
range out_node = 1..nodes;

float Traffic_Mat[sorc][dest] = ...;
float Dist_Mat[sorc][dest] = ...;

// variables
dvar float+ L_sdi[j][sorc][dest][in_node][out_node];
dvar float+ L_ij[in_node][out_node];
dvar float+ L_max;
dvar boolean b[in_node][out_node];
dvar float+ alpha;

minimize L_max;

```

```

subject to
{
    alpha_const:
    alpha >= 1;

    forall (s in sorc, d in dest, i in in_node)

        if (s == i)
            flow_consv_1:
            (sum(j in out_node) (L_sdijs[s][d][i][j])) - (sum(j in
out_node) (L_sdijs[s][d][j][i])) == Traffic_Mat[s][d];

            else if (d == i)
                flow_consv_2:
                (sum(j in out_node) (L_sdijs[s][d][i][j])) - (sum(j in
out_node) (L_sdijs[s][d][j][i])) == -Traffic_Mat[s][d];

            else
                flow_consv_3:
                (sum(j in out_node) (L_sdijs[s][d][i][j])) - (sum(j in
out_node) (L_sdijs[s][d][j][i])) == 0;

            forall (i in in_node, j in out_node)
                total_flow_1:
                L_ij[i][j] == sum(s in sorc, d in
dest) ((L_sdijs[s][d][i][j]));

            forall (i in in_node, j in out_node)
                total_flow_2:
                L_ij[i][j] <= L_max;

            forall (s in sorc, d in dest, i in in_node, j in out_node)
                total_flow_3:
                ((L_sdijs[s][d][i][j])) <= (b[i][j])*(Traffic_Mat[s][d]);

            forall (s in sorc, d in dest)
                Avg_delay_constraint:
                sum(i in in_node, j in
out_node) ((L_sdijs[s][d][i][j])*(Dist_Mat[i][j])) <=
(((Traffic_Mat[s][d]))*alpha*(Dist_Mat[s][d]));

            forall (j in out_node)
                Deg_constraint1:
                sum(i in in_node) b[i][j] == deg;

            forall (i in in_node)
                Deg_constraint2:

```

```

    sum(j in out_node)b[i][j] == deg;

}

/*****
* OPL 12.6.3.0 Data
* Author: Bhushan
* Creation Date: 11. juni 2016 at 20:38:27
*****/

Traffic_Mat = [[0 19.2 0 0 0 0]
               [0 0 0 0 0 0]
               [0 0 0 0 0 0]
               [0 19.2 0 0 0 0]
               [0 19.2 0 0 0 0]
               [0 19.2 0 0 0 0]];

Dist_Mat = [[0 39.4 86.1 125.5 129.6 134.2]
            [39.4 0 46.7 86.1 90.2 94.8]
            [86.1 46.7 0 39.4 43.5 141.5]
            [125.5 86.1 39.4 0 82.9 180.9]
            [129.6 90.2 43.5 82.9 0 185]
            [134.2 94.8 141.5 180.9 185 0]];

```