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Working Paper Series in Economics and Management
No. 02/07, February 2007

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March 22, 2007

Abstract

We consider sequential bargaining between three firms that are all essential in creating a surplus. One of the firms is dominant in the sense that it ultimately decides whether the surplus will be created. The other firms have an incentive to get a large share of the pie for themselves, but leaving enough for the dominant firm that it finds it profitable to create the surplus. Hence, the smaller firms have preferences over who they take their share from. Of all of the bargaining protocols that we consider, we identify the set of Pareto optimal protocols, and show which of them will be uniquely preferred by each firm.

JEL Classification: C78

Keywords: bargaining, surplus division, asymmetry, protocols.

*The work on this paper was started while Clark was visiting Université de Marne-la-Vallée. He would like to thank the hosts for their hospitality. We should like to thank participants at seminars at the University of Tromsø, Norwegian School of Economics and Business Administration and Ludvig Maximillian University, Munich for helpful comments. Comments from Christian Riis have been especially insightful. Clark would like to acknowledge funding through project 172603/V10 at the Research Council of Norway. Remaining errors are our own.

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1 Introduction

Bargaining models attempt to capture situations involving two or many parties, who can cooperate in the creation of a commonly desirable surplus, but over whose distribution all parties are in conflict. In such a situation the more one party can get, the less that remains for the other parties. Each party inflicts a one-to-one externality on each other opponent. However many social, political and economic problems of relevance do not exhibit this underlying feature. In this paper we consider a three-player bargaining situation in which the players have preferences over whom they take a share of the surplus from. Coase's (1960) famous example of the negotiations between a railroad company and a group of farmers is a case in point.[1] In order to create the surplus, the railroad is dependent on securing an agreement with each farmer; however, in pursuing the largest possible share of the surplus, each farmer is mindful of the fact that the project must be profitable enough for the railroad to want to instigate the project that creates the surplus. Hence each farmer wants to increase his share at the expense of the other farmers, not the railroad.

In this paper we consider bargaining situations between three firms. To fix ideas, consider the following example. Suppose that two small firms are dependent upon a larger one to create the surplus. The dominant firm depends upon the inputs of the smaller ones to create the surplus, but will only take the step of creating if it is sufficiently profitable. Suppose that the cost of creating the surplus is not known *ex ante* when the firms bargain over the shares they will get. The smaller firms then realize that the creation of the surplus will only occur if the share that the dominant firm is offered covers the cost once it is revealed. Hence, we have a situation in which the smaller firms want to secure a large share at the expense of each other, and not at the expense of the dominant firm. In this situation we investigate the effect that the bargaining protocols between the three players have on the shares obtained, and on the probability that the surplus is created. Hence the model exhibits an asymmetric externality, and also endogenizes the expected size of the surplus created.

The non-cooperative model of Rubinstein (1982) offers a useful tool to tackle the strategic dimension of such bargaining situations. However it is well-known that the Rubinstein result on the uniqueness of the subgame perfect equilibrium outcome cannot be extended to three or more players. As shown by Shaked (see Osborne and Rubinstein, 1990, section 3.13), there

[1] Cai (2000) recasts Coase's example in a bargaining model. The relationship of the current paper to Cai (2000) is discussed later.

exist a multiplicity of equilibria when unanimity is required. In such a game, a proposal made by one player has to be accepted by all the other players in order to be implemented. But if only one of them rejects it, negotiations continue until the next period to a new offer made by another player. The result is that every feasible agreement can be supported as a subgame perfect equilibrium. The uniqueness of the outcome can be restored in the multilateral extension of the Rubinstein model by introducing an exit rule as in Krishna and Serrano (1996). This rule asserts that after a proposal has been made to all the players, any player can accept the offered share, leave the negotiation table with the awarded share and let the remaining players continue to bargain over the rest of the pie. A new division of the surplus is then offered until agreement is reached. In this paper, we concentrate on a sequential negotiation involving bilateral bargaining protocols as in the multi-issue model of Fershtman (2000) or in the model of Suh and Wen (2006).[2] Since the sequence is finite, there always exists a unique subgame perfect equilibrium for any bargaining procedure.

We then examine different sequential bilateral bargaining protocols and calculate the equilibrium agreements and expected payoffs that they yield. Our results emphasize the role of the bargaining agenda and show how players can manipulate it. In a situation involving three parties, there exist 24 combinations of pairs of the players where each is represented in at least one round of bargaining. However, we show that only few of them are not Pareto dominated. As in the model of Rubinstein, we show that there is always an advantage for a player to be proposer in the first round of bargaining but it may be not enough to ensure a high payoff. In some protocols, we show that it might be better for one player to be present in the first round but also in the second round or to be in the first round but not in the second. One of the main results in the paper is that out of the 24 possible protocols, we can uniquely determine the one that each player will want to see implemented given only information on the discount factor.

The bargaining situations that we consider in this paper have some common features with other works in this field. As mentioned, the bargaining protocols that we examine exhibit exit by one player as in Krishna and Serrano (1996). Cai (2000, 2003) also considers bargaining situations involving one central player who has a profitable project and needs cooperation from each of the other players to undertake it. In these works, only the central player is active in the bargaining process since he has to bargain in a bilateral

[2]These papers concentrate on the limiting case where the discount factor approaches unity. Our model is solved for all values of the discount factor. Indeed, the aim of Suh and Wen (2006) is to find bargaining protocols that give the Nash solution in this limit.

manner with all the other passive players according the following rule. In each round, the central player bargains with one passive player. If an agreement is reached, the passive player leaves the game with a binding share while in case of disagreement, he/she is moved to the end of the queue. Then the bargaining process moves to the next round with a new passive player until the emergence of a global agreement with all the passive players ensuring that the project will be implemented.

Our model differs in several ways. In Cai's models, there is no link between the implementation of the project and the bargaining process. Even if the value of the project is common knowledge (and normalized to one), the problem remains a version of the Rubinstein model of splitting a pie among many players with symmetric externalities. Moreover, we allow bilateral bargaining under protocols in which all of the players may get to actively make offers and counteroffers.

In Fershtman (2000), a "buyer" negotiates with two firms over the prices of two goods; each firm benefits from generally high prices, but each would prefer to see its price highest. An alternative interpretation is that each firm places more weight on an "issue" that is bargained over. Several protocols are considered, but the buyer is present in all negotiations; some of these protocols involve simultaneous resolution of issues and some are placed in a sequential framework. Our perspective differs to that of Fershtman (2000) since we consider only one issue, and there is pure conflict among the smaller firms over this; furthermore, we do not constrain protocols to always involve the "strong" player, giving our framework more of a multilateral perspective.

Suh and Wen (2006) have a somewhat different focus to the current paper and consider a multilateral Rubinstein game with n players and a fixed pie. To establish the link to our model, consider the case of three players. These authors show that the backward-induction equilibrium of the procedure in which players 1 and 2 bargains in a first round followed by a bargain between 2 and 3 in a second round always gives an advantage to the player (here 1) who can exit first from the game. In order to eliminate this advantage of one player over the others, Suh and Wen (2006) develop two particular procedures under which multilateral bargaining (through bilateral rounds) converges to the Nash solution. The two procedures specify at each bilateral round who exits the game and who moves to the next round. To establish equivalence with the Nash solution, they consider the case when the discount factor goes to one.

Our work also shares some common features with the model of Calvo-Armengol (1999) who considers a particular version of the "three-player/one pie" game where players hold asymmetric positions. The bargaining process involves a central player who is the only one that can bargain in a bilateral manner for a fixed length of time with the two different partners. However, to reach a agreement, only one partner is needed. Hence, the author examines which position favors the central player. It is shown that the outcome for the central player will be better when he bargains with the relatively more impatient partner because the latter is more eager to accept tougher proposals.

The paper is organized as follows: In Section 2 the model is presented, and the different bargaining protocols are discussed and solved in Section 3. Section 4 looks at which of the protocols each player would prefer and Section 5 concludes.

2 The model

We consider a two stage model. Stage 1 defines a bargaining model of some kind, the exact details of which we return to in the next section. The bargains made at this stage reflect shares of a surplus that will be obtained contingent upon the surplus actually being created at stage 2. Stage 2 analyses the creation of the surplus. Inputs from all of the firms are essential in creating the surplus, but suppose firm X takes on the job of coordinating its creation. Given that X has obtained binding agreements on surplus division with firms 1 and 2 at stage 1, it must then decide whether it is profitable to create the surplus and share it accordingly. Fix the size of the surplus at $B > 0$. At the start of stage 2, the cost of creating the surplus is made known as the result of a draw from a uniform distribution on $[0, T]$ where T is a known positive parameter. Let x be the realized cost. Firm X then creates the surplus as long as the share of the surplus it receives at least covers the cost of its creation.

Given shares s_1 and s_2 from the first stage, X will create the surplus as long as

$$(1 - s_1 - s_2)B \geq x$$

Seen from stage 1, the probability that the surplus gets created is then

$$P = \Pr((1 - s_1 - s_2)B \geq x) = \frac{(1 - s_1 - s_2)B}{T}$$

At stage 1, none of the firms know whether the surplus will be created or not. Then the expected profit of firms 1, 2 and X seen from stage 1 are

$$\begin{aligned}
\pi_1^* &= Ps_1B = \frac{(1-s_1-s_2)s_1B^2}{T} \\
\pi_2^* &= Ps_2B = \frac{(1-s_1-s_2)s_2B^2}{T} \\
\pi_X^* &= P(1-s_1-s_2)B = \frac{(1-s_1-s_2)^2B^2}{T}
\end{aligned}$$

Writing $\psi = B^2/T$ and $s_X = 1 - s_1 - s_2$, and defining $1 \geq \delta > 0$ to be the common discount factor, the expected profit of X obtained at time t can be rewritten as

$$\pi_X^*(s_1, s_2, t) = \delta^t (s_X)^2 \psi \tag{1}$$

and for firms 1 and 2

$$\pi_1^*(s_1, s_2, t) = \delta^t s_1 s_X \psi \tag{2}$$

$$\pi_2^*(s_1, s_2, t) = \delta^t s_2 s_X \psi \tag{3}$$

As explained in the introduction, the payoff of firm 1 does not only depend on the share that he/she is able to get at the expense of the other players. His/her payoff is defined as a share of what the firm X will be able to keep in the bargaining process with both firms. Firm 1 faces a trade off between its own share and the share of X. If a higher share demanded by firm 1 reduces the share of X then this will reduce the probability that there is a surplus to share. Hence firm 1 prefers to get the highest share possible but by preserving the share of X, and at the expense of firm 2. The logic here is supported by the symmetric Nash bargaining solution for which we would maximize

$$\max_{s_1, s_2} \Omega = (1 - s_1 - s_2)^4 s_1 s_2$$

with first order condition for s_1:

$$\frac{\partial \Omega}{\partial s_1} = s_2 (5s_1 + s_2 - 1)(s_1 + s_2 - 1)^3 = 0$$

Evaluating at a symmetric situation gives $s_1 = s_2 = \frac{1}{6}$ and $s_X = \frac{2}{3}$ (with corresponding expected payoffs $(\frac{\psi}{9}, \frac{\psi}{9}, \frac{4\psi}{9})$). Hence an equal division of the surplus would not be the outcome of static Nash bargaining in this model. We note below that none of the protocols that we consider converge to the Nash solution in the limit as $\delta \to 1$.

3 Possible bargaining protocols at stage 1

3.1 The different cases

At the bargaining stage of the game we assume that all three players must participate. This gives 24 combinations of pairs of the players where each is represented in at least one round of bargaining. Of these, 12 involve X and firm 1 bargaining in the first round and these are represented in the table below.[3]

Round 1\Round 2	$X \to 2$	$1 \to 2$	$2 \to 1$	$2 \to X$	$1 \to X$	$X \to 1$
$X \to 1$	A	B	C	D	-	-
$1 \to X$	E	F	G	H	-	-
$1 \to 2$	E	-	-	H	I	J

The rows represent the first round of bargaining and the columns are the second. The letters in the table represent distinct cases for the bargaining protocols. Notice that the protocol where 1 offers to X in the first round and X offers to 2 in the second is called E, as is the case in which 1 offers to 2 in the first round and X offers to 2 in the second. This is because the equilibria for these cases turns out to be identical, and we have chosen at this early stage to economize on notation for the number of cases.[4] The other twelve cases are obtained by exchanging 1 and 2 in the first round. The shares and payoffs obtained by 1 and 2 are then also interchanged for these cases. Hence we focus on the ten distinct cases that are represented in the table.

3.2 Method of solution

Let $s_j^{(i)}$ be the offer or the counteroffer made by player i with $i = X, 1, 2$ concerning the share s_j received by player j with $j = 1, 2$. Consider case A where X makes the first offer in round 1 to firm 1 and then in round 2 to firm 2. At each round, we solve a Rubinstein bargaining situation. We thus have in mind the following situation: in round 1, X makes an offer of a share $s_1^{(X)}$ to firm 1. If 1 accepts then he leaves the bargaining table and round 2 begins. If 1 declines the offer it then makes a counteroffer $s_1^{(1)}$. The offer-counteroffer procedure continues until agreement is reached. Upon agreement, 1 leaves the bargaining table, and X and firm 2 then bargain in the same manner over the share that 2 will be given (with X making the first

[3] To avoid confusion with the two stages in the game, we shall refer to different periods in the bargaining stage as "rounds" in what follows.

[4] Notice that protocols E and H are the only ones in which firm 1 proposes at the first stage, and then leaves the negotiation table.

offer in case A). Since there are two rounds of negotiations, we solve for the unique subgame perfect Nash equilibrium by backwards induction. In round 2, X offers $s_2^{(X)}$ and accepts $s_2^{(2)}$, and 2 offers $s_2^{(2)}$ and accepts $s_2^{(X)}$ where the equilibrium offers $s_2^{(X)}$ and $s_2^{(2)}$ have to satisfy the following two conditions

$$\pi_2^* \left(s_1, s_2^{(X)}, 0 \right) = \pi_2^* \left(s_1, s_2^{(2)}, 1 \right) \tag{4}$$

$$\pi_X^* \left(s_1, s_2^{(2)}, 0 \right) = \pi_X^* \left(s_1, s_2^{(X)}, 1 \right) \tag{5}$$

which leads to

$$\left(1 - s_1 - s_2^{(X)} \right) s_2^{(X)} = \delta s_2^{(2)} \left(1 - s_1 - s_2^{(2)} \right) \tag{6}$$

$$\left(1 - s_1 - s_2^{(2)} \right)^2 = \delta \left(1 - s_1 - s_2^{(X)} \right)^2 \tag{7}$$

Equation (4) asserts that 2 is indifferent in terms of expected payoffs between accepting X's offer $s_2^{(X)}$ in the current period or rejecting it, and making the counteroffer $s_2^{(2)}$ in the following period that will be accepted by X. (5) reflects the same indifference for X given the share demanded by 2 $s_2^{(2)}$, and the counteroffer by X ($s_2^{(X)}$). Note that s_1 is the share that firm 1 secures in the first round of bargaining. Solving (6) and (7) simultaneously for the offers yields a unique positive solution

$$s_2^{(X)} = \frac{\delta^{\frac{3}{2}} (1 - s_1)}{\left(\sqrt{\delta} + 1 \right) (\delta + 1)}, \; s_2^{(2)} = \frac{1 - s_1}{\left(\sqrt{\delta} + 1 \right) (\delta + 1)} \tag{8}$$

Since we are assuming here that X makes the first offer, $s_2^{(X)}$ is the relevant offer. Turning to round 1 and using $s_2^{(X)}$ from (8), subgame perfect offers must satisfy

$$\left(1 - s_1^{(X)} - \frac{\delta^{\frac{3}{2}} \left(1 - s_1^{(X)} \right)}{\left(\sqrt{\delta} + 1 \right) (\delta + 1)} \right) s_1^{(X)} = \delta s_1^{(1)} \left(1 - s_1^{(1)} - \frac{\delta^{\frac{3}{2}} \left(1 - s_1^{(1)} \right)}{\left(\sqrt{\delta} + 1 \right) (\delta + 1)} \right) \tag{9}$$

$$\left(1 - s_1^{(1)} - \frac{\delta^{\frac{3}{2}} \left(1 - s_1^{(1)} \right)}{\left(\sqrt{\delta} + 1 \right) (\delta + 1)} \right)^2 = \delta \left(1 - s_1^{(X)} - \frac{\delta^{\frac{3}{2}} \left(1 - s_1^{(X)} \right)}{\left(\sqrt{\delta} + 1 \right) (\delta + 1)} \right)^2 \tag{10}$$

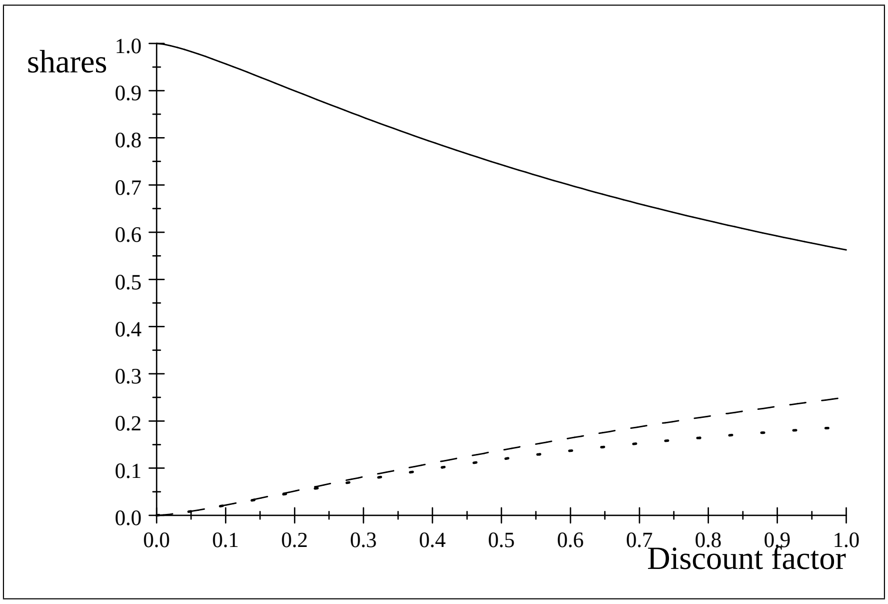
Note that the offers made by the players in the first round of the bargaining process affect the share given to firm 2 in the next round, and the optimal offers take account of this. Solving (9) and (10) simultaneously gives

$$s_1^{(X)} = \frac{\delta^{\frac{3}{2}}}{\left(\sqrt{\delta}+1\right)(\delta+1)},\ s_1^{(1)} = \frac{1}{\left(\sqrt{\delta}+1\right)(\delta+1)} \tag{11}$$

as the unique positive solutions. Since X makes the first offer, $s_1^{(X)}$ is the relevant share here. Inserting this into $s_2^{(X)}$ in (8), and noting $s_X = 1 - s_1 - s_2$ gives the subgame perfect equilibrium shares of the entire game for protocol A as

$$\begin{aligned} s_1^A(\delta) &= \frac{\delta^{\frac{3}{2}}}{\left(\sqrt{\delta}+1\right)(\delta+1)}, \\ s_2^A(\delta) &= \frac{\delta^2+\delta^{\frac{3}{2}}+\delta^{\frac{5}{2}}}{\left(\sqrt{\delta}+1\right)^2(\delta+1)^2}, \\ s_X^A(\delta) &= \frac{\left(\delta+\sqrt{\delta}+1\right)^2}{\left(\sqrt{\delta}+1\right)^2(\delta+1)^2} \end{aligned}$$

which are depicted in the figure. The solid line is s_X, the dashed one is s_1, and the dotted line is s_2.



It is easily verified that $s_1^A(\delta) > s_2^A(\delta)$ so that firm 1 reaps an advantage from being involved in the first round of negotiations. To understand this recall that each of firms 1 and 2 want X to get as large a share as possible. In the first round, firm 1 knows that making higher bids will reduce the claims made by firm 2 in the next round (see (8)) since the firms that bargain in round 2 are essentially bargaining over the surplus that is left after firm 1 has committed to a share and left the negotiations. Firm 2 knows that attempting to secure a large share will reduce the probability that the surplus is created and hence it accepts offers that are lower than the share that firm 1 has obtained. Since X makes all offers in case A, we note that firms 1 and 2 are held to a low share by X when they are impatient, and that the share rises as the players become more patient.

The limiting shares for $\delta = 1$ are $s_X^A(1) = 9/16, s_1^A(1) = 1/4, s_2^A(1) = 3/16$. Inserting the shares into (1), (2) and (3), the expected payoffs in equilibrium using protocol A can be determined as

$$\begin{aligned}
\pi_X^A(\delta) &= \frac{\left(\delta + \sqrt{\delta} + 1\right)^2}{\left(\sqrt{\delta} + 1\right)^2 (\delta + 1)^2}\psi \\
\pi_1^A(\delta) &= \frac{\delta^{\frac{3}{2}}\left(\delta + \sqrt{\delta} + 1\right)^2}{\left(\sqrt{\delta} + 1\right)^3 (\delta + 1)^3}\psi \\
\pi_2^A(\delta) &= \frac{\left(\delta^2 + \delta^{\frac{3}{2}} + \delta^{\frac{5}{2}}\right)\left(\delta + \sqrt{\delta} + 1\right)^2}{\left(\sqrt{\delta} + 1\right)^4 (\delta + 1)^4}\psi
\end{aligned}$$

where it is straightforward to verify that $\pi_X^A(\delta) > \pi_1^A(\delta) > \pi_2^A(\delta)$. In the limiting case we have that $\pi_X^A(1) = (81/256)\,\psi$, $\pi_1^A(1) = (9/64)\,\psi$, $\pi_2^A(1) = (27/256)\,\psi$.

3.3 General solutions

The problem that we are considering at stage 1 of the game has some general properties that we now discuss. Consider the bargaining that occurs in round two. In the cases considered, the participants will either be firm 1 and 2, X and firm 2 or X and firm 1. When 1 and 2 bargain in the second round, the program to be solved for optimal offers is

$$\begin{aligned}
\left(1 - s_1 - s_2^{(2)}\right) s_1 &= \delta s_1 \left(1 - s_1 - s_2^{(1)}\right) \\
\left(1 - s_1 - s_2^{(1)}\right) s_2^{(1)} &= \delta \left(1 - s_1 - s_2^{(2)}\right) s_2^{(2)}
\end{aligned} \tag{12}$$

and this has unique positive solution[5]

$$s_2^{(2)} = \frac{(1-s_1)}{\delta+\delta^2+1} \equiv \eta(1-s_1),\ s_2^{(1)} = \frac{\delta^2(1-s_1)}{\delta+\delta^2+1} \equiv \beta(1-s_1) \qquad (13)$$

On the other hand if X and 2 negotiate in the second round the optimal offers must satisfy

$$\begin{aligned} \left(1-s_1-s_2^{(X)}\right)s_2^{(X)} &= \delta s_2^{(2)}\left(1-s_1-s_2^{(2)}\right) \\ \left(1-s_1-s_2^{(2)}\right)^2 &= \delta\left(1-s_1-s_2^{(X)}\right)^2 \end{aligned} \qquad (14)$$

with unique positive solutions

$$s_2^{(X)} = \frac{\left(\sqrt{\delta}\right)^3(1-s_1)}{\left(\sqrt{\delta}+1\right)(\delta+1)} \equiv \alpha\left(1-s_1\right),\ s_2^{(2)} = \frac{1-s_1}{\left(\sqrt{\delta}+1\right)(\delta+1)} \equiv \gamma\left(1-s_1\right) \qquad (15)$$

By analogy, when X and 1 negotiate in the second round, the offers made by the players are for 1's share, and take the form

$$s_1^{(X)} = \alpha\left(1-s_2\right),\ s_1^{(1)} = \gamma\left(1-s_2\right) \qquad (16)$$

Hence, in protocols A-J, the optimal offer in the second stage will belong to the set $\{\alpha\left(1-s_1\right), \beta(1-s_1), \gamma\left(1-s_1\right), \eta(1-s_1), \alpha\left(1-s_2\right), \gamma\left(1-s_2\right)\}$. Note that the second stage offer is a proportion of the pie that is left after the round 1 share is deducted.

The possibilities for the first round of bargaining are that 1 and X or 1 and 2 participate. In the former case firm 2 must be present in the second round. This means that the offer in the second round will take the form $\alpha\left(1-s_1\right)$ or $\beta(1-s_1)$ or $\gamma\left(1-s_1\right)$ or $\eta(1-s_1)$ depending upon who is involved in this round and which of the participants makes the first offer. Suppose that $\tau\left(1-s_1\right)$ is the offer where $1 > \tau \geq 0$.[6] Then when 1 and X bargain at the first stage optimal offers satisfy

$$\begin{aligned} \left(1-s_1^{(X)}-\tau\left(1-s_1^{(X)}\right)\right)s_1^{(X)} &= \delta s_1^{(1)}\left(1-s_1^{(1)}-\tau\left(1-s_1^{(1)}\right)\right) \\ \left(1-s_1^{(1)}-\tau\left(1-s_1^{(1)}\right)\right)^2 &= \delta\left(1-s_1^{(X)}-\tau\left(1-s_1^{(X)}\right)\right)^2 \end{aligned}$$

[5]The dependence of α, β, γ and η on δ is supressed in the following.

[6]Strictly speaking, one should remember the fact that in representing α, β, γ and η, τ is a function of δ. However, we wish to demonstrate the generic nature of the solutions here, and in particular their independence of the term that τ represents.

with unique solution

$$s_1^{(X)} = \frac{\left(\sqrt{\delta}\right)^3}{\left(\sqrt{\delta}+1\right)(\delta+1)} = \alpha,\ s_1^{(1)} = \frac{1}{\left(\sqrt{\delta}+1\right)(\delta+1)} = \gamma$$

The point to note is that the solution is independent of τ whatever value this should take in the second round. When 1 and 2 participate in the first round of negotiations, the program solved if firm 2 is present in the second round is

$$\begin{aligned}
\left(1 - s_1^{(2)} - \tau\left(1 - s_1^{(2)}\right)\right) s_1^{(2)} &= \delta s_1^{(1)}\left(1 - s_1^{(1)} - \tau\left(1 - s_1^{(1)}\right)\right) \\
\left(1 - s_1^{(1)} - \tau\left(1 - s_1^{(1)}\right)\right)\tau\left(1 - s_1^{(1)}\right) &= \delta\tau\left(1 - s_1^{(2)}\right)\left(1 - s_1^{(2)} - \tau\left(1 - s_1^{(2)}\right)\right)
\end{aligned}$$

with solution $s_1^{(2)} = \alpha, s_1^{(1)} = \gamma$ which are again independent of τ. When firms 1 and 2 open the negotiations and 2 is not present in the second round (cases I and J), the second round offer takes the form $\alpha(1 - s_2), \gamma(1 - s_2)$. Again representing this generically as $\tau(1 - s_2)$, the first round program is

$$\begin{aligned}
\left(1 - \tau\left(1 - s_2^{(2)}\right) - s_2^{(2)}\right)\tau\left(1 - s_2^{(2)}\right) &= \delta\tau\left(1 - s_2^{(1)}\right)\left(1 - \tau\left(1 - s_2^{(1)}\right) - s_2^{(1)}\right) \\
\left(1 - \tau\left(1 - s_2^{(1)}\right) - s_2^{(1)}\right) s_2^{(1)} &= \delta s_2^{(2)}\left(1 - \tau\left(1 - s_2^{(2)}\right) - s_2^{(2)}\right)
\end{aligned}$$

It can readily be verified that the following values solve these equations uniquely: $s_2^{(1)} = \alpha, s_2^{(2)} = \gamma$.

Hence we can conclude the following

Proposition 1 *i) When firm* 1 *opens the negotiations in the first round with firm* 2 *and firm* 2 *is not present in the second round of bargaining (cases I and J) then the subgame perfect equilibrium share obtained by firm* 2 *is* $s_2 = \frac{\left(\sqrt{\delta}\right)^3}{\left(\sqrt{\delta}+1\right)(\delta+1)} \equiv \alpha$.

ii) When firm 1 *opens the negotiations in the first round with* X *or firm* 2 *and* 2 *is present in the second round (cases E, F, G, H), then the subgame perfect equilibrium share obtained by firm* 1 *is* $s_1 = \frac{1}{\left(\sqrt{\delta}+1\right)(\delta+1)} \equiv \gamma$.

iii) When X *opens the negotiations in round* 1 *(cases A, B, C, D), the subgame perfect equilibrium share obtained by firm* 1 *is* $s_1 = \frac{\left(\sqrt{\delta}\right)^3}{\left(\sqrt{\delta}+1\right)(\delta+1)} \equiv \alpha$.

In each case A-I, the share of the other firm is determined by using the appropriate expression from (13), (15) or (16). The share of X *is then* $s_X = 1 - s_1 - s_2$.

The limit case $\delta \to 1$ for the shares and expected payoffs that result in the subgame perfect equilibrium of the bargaining protocols in A-I are summed up in Proposition 2.

Proposition 2 *Let $\delta \to 1$. Then in cases B, C, F and G, the shares and expected payoffs for firms 1, 2 and X are $\left(\frac{1}{4}, \frac{1}{4}, \frac{1}{2}\right), \left(\frac{1}{8}\psi, \frac{1}{8}\psi, \frac{1}{4}\psi\right)$. In cases A, D, E, and H they are $\left(\frac{1}{4}, \frac{3}{16}, \frac{9}{16}\right), \left(\frac{9}{64}\psi, \frac{27}{256}\psi, \frac{81}{256}\psi\right)$, and for cases I and J $\left(\frac{3}{16}, \frac{1}{4}, \frac{9}{16}\right), \left(\frac{27}{256}\psi, \frac{9}{64}\psi, \frac{81}{256}\psi\right)$.*

Note that in cases B, C, F and G, the total surplus is $\frac{\psi}{2}$ whilst it is higher at $\frac{9\psi}{16}$ in all of the other cases in the limit. In the former cases, the share obtained by X is lower and this affects the likelihood that the surplus is created. Common for the cases B, C, F and G is that X does not participate in the second round of negotiations. In all of the other cases, X is present in the second round and in the limit case it manages to "steal" an extra $\frac{1}{16}$ of the pie from its partner in this round. Hence, in the limit case it is an advantage for X to be present in the second round, and firms 1 and 2 would wish to avoid meeting this player there.

4 The choice of protocol

The following table gives a complete picture of the expected payoffs that can be achieved by each player in the different protocols.

	$\pi_1(\delta)$	$\pi_2(\delta)$	$\pi_X(\delta)$
A	$\frac{(\sqrt{\delta})^3(\delta+\sqrt{\delta}+1)^2}{(\sqrt{\delta}+1)^3(\delta+1)^3}\psi$	$\frac{(\sqrt{\delta})^3(\delta+\sqrt{\delta}+1)^3}{(\sqrt{\delta}+1)^4(\delta+1)^4}\psi$	$\frac{(\delta+\sqrt{\delta}+1)^4}{(\sqrt{\delta}+1)^4(\delta+1)^4}\psi$
B	$\frac{(\sqrt{\delta})^3}{(\delta+1)(\delta-\sqrt{\delta}+1)(\sqrt{\delta}+1)^2}\psi$	$\frac{\delta^2}{(\delta+1)(\delta^{\frac{3}{2}}+1)^2}\psi$	$\frac{1}{(\delta^{\frac{3}{2}}+1)^2}\psi$
C	$\frac{(\sqrt{\delta})^5}{(\sqrt{\delta}+1)^2(\delta-\sqrt{\delta}+1)(\delta+1)}\psi$	$\frac{\delta}{(\sqrt{\delta}+1)^2(\delta-\sqrt{\delta}+1)^2(\delta+1)}\psi$	$\frac{\delta^2}{(\delta^{\frac{3}{2}}+1)^2}\psi$
D	$\frac{\delta^2(\delta+\sqrt{\delta}+1)^2}{(\delta+1)^3(\sqrt{\delta}+1)^3}\psi$	$\frac{\sqrt{\delta}(\delta+\sqrt{\delta}+1)^3}{(\delta+1)^4(\sqrt{\delta}+1)^4}\psi$	$\frac{\delta(\delta+\sqrt{\delta}+1)^4}{(\delta+1)^4(\sqrt{\delta}+1)^4}\psi$
E	$\frac{\sqrt{\delta}(\delta+\sqrt{\delta}+1)^2}{(\delta+1)^3(\sqrt{\delta}+1)^3}\psi$	$\frac{(\sqrt{\delta})^5(\delta+\sqrt{\delta}+1)^3}{(\delta+1)^4(\sqrt{\delta}+1)^4}\psi$	$\frac{\delta(\delta+\sqrt{\delta}+1)^4}{(\delta+1)^4(\sqrt{\delta}+1)^4}\psi$
F	$\frac{\sqrt{\delta}}{(\delta+1)(\delta-\sqrt{\delta}+1)(\sqrt{\delta}+1)^2}\psi$	$\frac{\delta^3}{(\sqrt{\delta}+1)^2(\delta-\sqrt{\delta}+1)^2(\delta+1)}\psi$	$\frac{\delta}{(\delta^{\frac{3}{2}}+1)^2}\psi$
G	$\frac{(\sqrt{\delta})^3}{(\sqrt{\delta}+1)^2(\delta-\sqrt{\delta}+1)(\delta+1)}\psi$	$\frac{\delta^2}{(\sqrt{\delta}+1)^2(\delta-\sqrt{\delta}+1)^2(\delta+1)}\psi$	$\frac{\delta^3}{(\sqrt{\delta}+1)^2(\delta-\sqrt{\delta}+1)^2}\psi$
H	$\frac{\delta(\delta+\sqrt{\delta}+1)^2}{(\delta+1)^3(\sqrt{\delta}+1)^3}\psi$	$\frac{(\sqrt{\delta})^3(\delta+\sqrt{\delta}+1)^3}{(\delta+1)^4(\sqrt{\delta}+1)^4}\psi$	$\frac{\delta^2(\delta+\sqrt{\delta}+1)^4}{(\delta+1)^4(\sqrt{\delta}+1)^4}\psi$
I	$\frac{\sqrt{\delta}(\delta+\sqrt{\delta}+1)^3}{(\delta+1)^4(\sqrt{\delta}+1)^4}\psi$	$\frac{\delta^2(\delta+\sqrt{\delta}+1)^2}{(\delta+1)^3(\sqrt{\delta}+1)^3}\psi$	$\frac{\delta(\delta+\sqrt{\delta}+1)^4}{(\delta+1)^4(\sqrt{\delta}+1)^4}\psi$
J	$\frac{\delta^3(\delta+\sqrt{\delta}+1)^2}{(\delta+1)^4(\sqrt{\delta}+1)^4}\psi$	$\frac{\delta^3(\delta+\sqrt{\delta}+1)}{(\delta+1)^3(\sqrt{\delta}+1)^3}\psi$	$\frac{\delta^3(\delta+\sqrt{\delta}+1)^2}{(\delta+1)^4(\sqrt{\delta}+1)^4}\psi$

The following comparisons are immediate by inspection.

Between cases B,C,F,G (converging in the limit $\delta \to 1$ to $\frac{1}{8}\psi$ for each of 1 and 2, and $\frac{1}{4}\psi$ for X) we can verify

$$\begin{aligned}\pi_1^F(\delta) &\geq \pi_1^B(\delta) = \pi_1^G(\delta) \geq \pi_1^C(\delta)\\ \pi_2^F(\delta) &\geq \pi_2^G(\delta) = \pi_2^C(\delta) \geq \pi_2^B(\delta)\\ \pi_X^B(\delta) &\geq \pi_X^F(\delta) \geq \pi_X^C(\delta) \geq \pi_X^G(\delta)\end{aligned}$$

For cases A,D,E,H $\left(\left(\frac{9}{64}\psi, \frac{27}{256}\psi, \frac{81}{256}\psi\right)\right.$ in the limit) we get

$$\begin{aligned}\pi_1^E(\delta) &\geq \pi_1^H(\delta) = \pi_1^A(\delta) \geq \pi_1^D(\delta)\\ \pi_2^D(\delta) &\geq \pi_2^H(\delta) = \pi_2^A(\delta) \geq \pi_2^E(\delta)\\ \pi_X^A(\delta) &\geq \pi_X^D(\delta) = \pi_X^E(\delta) = \pi_X^I(\delta) \geq \pi_X^H(\delta) \geq \pi_X^J(\delta)\end{aligned}$$

where the comparison in the last line includes cases I and J for X as it converges to the same limit as the other cases.

Cases I,J give $\left(\frac{27}{256}\psi, \frac{9}{64}\psi, \frac{81}{256}\psi\right)$ as payoffs in the limit, and the comparison is as follows:

$$\begin{array}{rcl} \pi_1^I(\delta) & \geq & \pi_1^J(\delta) \\ \pi_2^I(\delta) & \geq & \pi_2^J(\delta) \\ \pi_X^I(\delta) & \geq & \pi_X^J(\delta) \end{array}$$

From these comparisons we can see that I weakly Pareto dominates J, A dominates H and F is at least as good for all players as C and G. Hence we have only six protocols that are not weakly Pareto dominated: A, B, D, E, F, I. When considering which protocols are likely candidates for implementation, we can concentrate on the expected payoffs of X and firm 1, since there exist complement protocols in which 2 can achieve the same payoffs as firm 1. For example protocol A has complement A' in which X offers to 2 in the first round and then X offers to 1 in the second; hence $\pi_1^A(\delta) = \pi_2^{A'}(\delta)$. In the above, the largest payoff that 2 can achieve in equilibrium is either from D or F.[7] However 2 would prefer to swap places with 1 if it could gain more by being a proposer in the first round. Consider the complement protocol to E; in this case the expected equilibrium payoff to 2 would be $\pi_2^{E'}(\delta) = \pi_1^E(\delta)$ and it is straightforward to verify that this expression is larger than $\pi_2^D(\delta)$ and $\pi_2^F(\delta)$. Hence we can conclude that if firm 2 could choose protocol then it would be one of the complements to the cases A-J.

Thus we consider the protocols that are not Pareto dominated and ask which of these X and firm 1 would prefer if they could choose. The results of the comparison that is based upon the expected equilibrium payoffs are summarized in the following proposition.

Proposition 3 *Let* $\widehat{\delta} = 0.46557$*.*

i) If X can choose the protocol then it selects A for $\delta > \widehat{\delta}$ and B otherwise.

ii) If firm 1 can choose the protocol then it selects E for $\delta > \widehat{\delta}$ and F otherwise.

Concerning the preferred choice of X, both protocols A and B involve X making the offer to firm 1 in the first round. We know that this elicits an equilibrium offer of $s_1 = \alpha$. In protocol A, the second round offer of X to 2 is $s_2 = \alpha(1-s_1) = \alpha(1-\alpha)$, while in protocol B it is $s_2 = \beta(1-s_1) = \beta(1-\alpha)$. When $\delta > \widehat{\delta}$ we have that $\alpha < \beta$ and hence X gives a lower share to 2 in protocol A, compared to B. A similar argument can be made for part ii) of the proposition. In protocols E and F $s_1 = \gamma$ in the first round, and

[7] F is preferred by firm 2 for $\delta > 0.9197$.

$s_2^E = \alpha(1-\gamma)$ whilst $s_2^F = \beta(1-\gamma)$. Hence 1 has a constant share in both protocols but makes sure that less is given to 2 in E when $\delta > \widehat{\delta}$ since this makes $s_2^E < s_2^F$. Given that it has a constant share, firm 1 is interested in making the share of X as large as possible since the expected profit of 1 is a proportion of $s_1 s_X$ from (2).

When δ is large, then X prefers to be proposer in both rounds and prefers to make the first offer in round 1 and then leave when δ is small. For firm 1, the picture is reversed. It prefers to make both offers when δ is low, and to make the first offer in round 1 and then let X bargain with 2 in round 2 when δ is high.

To explain the intuition here, consider part (i) of the proposition. The first round share is the same in each of cases A and B. Given the bargain struck over s_1 in the first round, when 1 and 2 bargain at the second round, the rule used by 1 to solve (12) is

$$s_2^{(1)} = \frac{s_2^{(2)} - (1-s_1)(1-\delta)}{\delta}$$

which is decreasing in the size of the pie left after round 1 $(1-s_1)$. When X and 2 bargain in the second round, the offer used by X satisfies (14):

$$s_2^{(X)} = \frac{s_2^{(2)} + (1-s_1)(\sqrt{\delta}-1)}{\sqrt{\delta}}$$

which is increasing in $(1-s_1)$. Hence, when $(1-s_1)$ is large, X prefers to send 1 to the second round to negotiate with 2 (case B) since the fact that $(1-s_1)$ is large elicits tough offers from 1. Firm 1 has only received a small share in round one, and wants most of the remaining pie to go to X to secure provision of the surplus. When $(1-s_1)$ is small, 1 becomes less tough at round 2 and X prefers to negotiate himself (case A). When δ is large, $(1-s_1)$ is low, and X prefers case A, and vice versa for low δ. The same logic applies when firm 1 chooses the protocol.

5 Conclusion

In this paper we have considered bargaining between three firms, one of whom is central in the creation of the surplus to be shared. In order to ensure that the surplus gets created, a firm wants to obtain a large share for itself but not at the expense of the central player. Of the 24 possible bargaining protocols that can be envisaged in bilateral bargaining over two rounds with each player active in at least one round, we have managed to identify the one that each firm would prefer to see implemented. The model has many applications such as to the railroad example of Coase (1960). Alternatively, firms 1 and 2 could own components or patents that are essential in the creation of a new product for example, and these must be combined with a third component/patent that the central firm attempts to discover by investing in R&D. The willingness of this firm to expend resources, and hence the probability of innovation, will depend upon the bargains that are struck. The smaller firms have to take account of this when they enter negotiations. Hence, the bargaining procedure will be an important factor in determining how much R&D that is carried out in the attempt to innovate. Given sequential multilateral bargaining between all players, our results can identify the protocol that will be expected to be implemented according to the value of the discount factor and which of the players that can make the choice.

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